Water Resources Research

Supporting Information for

Diagnostic Analysis of Bank Storage Effects on Sloping Floodplain

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S1. Solution of 1-D advection-dispersion equation

Solution of the boundary value problem for function $\Theta(\xi,t)$ based on equation with a constant U,

$$\frac{\partial\Theta}{\partial t}=D\frac{\partial^2\Theta}{\partial\xi^2}+U\frac{\partial\Theta}{\partial\xi}, \quad 0<\xi<\infty, \quad t>0 \tag{S1-1}$$

with a boundary condition

$$\Theta(0,t)=\Theta_b(t) \tag{S1-2}$$

and initial condition:

$$\Theta(\xi,0)=\Theta_i(\xi) \tag{S1-3}$$

can be written using many standard methods (Menziani et al., 2007; Roubinet et al., 2012). The inhomogeneous Dirichlet's boundary condition (S1-2), however, will cause what the solution is invalid at the location of this boundary (e.g., Menziani et al., 2007). To overcomes this issue, the boundary condition (S1-2) should be homogenized. We defined a new variable to homogenize Equation (S1-2) as follows,

$$\Psi(\xi,t)=\Theta(\xi,t)-\Theta_b(t) \tag{S1-4}$$

Substituting Equation (S1-4) into Equations (S1-1)- (S1-3) yields

$$\frac{\partial\Psi}{\partial t}=D\frac{\partial^2\Psi}{\partial\xi^2}+U\frac{\partial\Psi}{\partial\xi}-W(t), \quad 0\le\xi<\infty, \quad t>0 \tag{S1-5}$$

$$\Psi(0,t)=0 \tag{S1-6}$$

$$\Psi(\xi,0)=\Psi_i(\xi) \tag{S1-7}$$

Where

$$W(t)=\frac{\mathrm{d}\Theta_b(t)}{\mathrm{d}t} \tag{S1-8}$$

$$\Psi_i(\xi)=\Theta_i(\xi)-\Theta_b(0) \tag{S1-9}$$

The auxiliary Green's function of Equation (S1-5) can be written as (Roubinet et al., 2012)

$$G(\xi,\xi',t-t')=\frac{1}{\sqrt{4\pi Dt'}}\left\{\exp\left[-\frac{\left(\xi-\xi'+Ut'\right)^2}{4Dt'}\right]-\exp\left[-\frac{\left(\xi+\xi'-Ut'\right)^2+4\xi Ut'}{4Dt'}\right]\right\} \tag{S1-10}$$

Solutions of the above boundary value problem, expressed in terms of the corresponding Green's functions, can be written as

$$\Psi(\xi,t)=\int_0^{\infty}\Psi_i(\xi')G(\xi,\xi',t)\mathrm{d}\xi'-\int_0^{t}\int_0^{\infty}W(t')G(\xi,\xi',t-t')\mathrm{d}\xi'\mathrm{d}t'=\Theta_{ic}(\xi,t)-\Theta_{bc}(\xi,t) \tag{S1-11}$$

where

$$\Theta_{ic}(\xi,t)=\frac{e^{\frac{-2U\xi-U^2t}{4D}}}{\sqrt{4\pi Dt}}\int_0^{\infty}\left[\Theta_i(u)-\Theta_b(0)\right]\left\{\exp\left[\frac{Uu}{2D}-\left(\frac{\xi-u}{\sqrt{4Dt}}\right)^2\right]-\exp\left[\frac{Uu}{2D}-\left(\frac{\xi+u}{\sqrt{4Dt}}\right)^2\right]\right\}\mathrm{d}u \tag{S1-12}$$

$$\Theta_{bc}(\xi,t)=\frac{1}{2}\int_0^{t}\left(-\frac{\mathrm{d}\Theta_b(t-u)}{\mathrm{d}u}\right)\left[\mathrm{erfc}\left(-\frac{\xi+Uu}{\sqrt{4Du}}\right)-\exp\left(-\frac{\xi U}{D}\right)\mathrm{erfc}\left(\frac{\xi-Uu}{\sqrt{4Du}}\right)\right]\mathrm{d}u \tag{S1-13}$$

Substituting Equation (S1-11) into Equation (S1-4) yields

$$\Theta(\xi,t)=\Theta_b(t)+\Theta_{ic}(\xi,t)-\Theta_{bc}(\xi,t) \tag{S1-14}$$

S2. Water fluxes to the stream bank

According to mass conservation, inflow rate per unit stream length $Q(t)$ is related to the mass $M(t)$ injected into the aquifer per unit stream length:

$$Q(t)=\frac{dM(t)}{dt} \tag{S2-1}$$

where $M(t)$ can be calculated as follows:

$$\begin{aligned} M(t)&=S_y\int_0^{\infty}[h(x,t)-H_0]dx=S_y\left[\int_0^{X(t)}[h(x,t)-H_0]dx+\int_{X(t)}^{\infty}[h(x,t)-H_0]dx\right]= \\ &=S_y\left[\int_0^{X(t)}x\tan\theta dx+\int_{X(t)}^{\infty}[h(x,t)-H_0]dx\right]=S_y\left[\frac{\tan\theta}{2}[X(t)]^2+\int_{X(t)}^{\infty}[h(x,t)-H_0]dx\right]\end{aligned} \tag{S2-2}$$

Use of expression (S2-1) includes differentiation of the integral

$$I(t)=\int_{X(t)}^{\infty}[h(x,t)-H_0]dx \tag{S2-3}$$

where the integrand and integration limits depend on this parameter. In general, the following relation applies

$$\frac{d}{dt}\left[\int_{\alpha(t)}^{\beta(t)}f(x,t)dx\right]=\int_{\alpha(t)}^{\beta(t)}\frac{\partial f(x,t)}{\partial t}dx+\frac{d\beta(t)}{dt}f(\beta(t),t)-\frac{d\alpha(t)}{dt}f(\alpha(t),t) \tag{S2-4}$$

where $f(x,t)=h(x,t)-H_0,\ \alpha(t)=X(t),\ \beta(t)=\infty$. Therefore,

$$Q(t)=\frac{dM(t)}{dt}==T\left.\frac{\partial h(x,t)}{\partial x}\right|_{x=X(t)}=-T\left.\frac{\partial h(\xi,t)}{\partial \xi}\right|_{\xi=0} \tag{S2-5}$$

Now, water flux rate across stream-aquifer interface per unit stream length can be obtained by substitution of Equation (20) into (S2-5):

$$Q(t)=\Theta^*_{ic}(t-t_{i-1})+\Theta^*_{bc}(t-t_{i-1}) \tag{S2-6}$$

where

$$\Theta^*{}_{ic}(t)=-\frac{T}{\sqrt{4\pi D^3t^3}}\int_0^{\infty}\left[h(u,t_{i-1})-H(t_{i-1})\right]u\exp\left[-\frac{(V_it-u)^2}{4Dt}\right]du \tag{S2-7}$$

$$\Theta^*{}_{bc}(t)=T\alpha_i\int_0^t\frac{1}{\sqrt{\pi Du}}\exp\left(-\frac{V_i^2u}{4D}\right)+\frac{V_i}{2D}\mathrm{erfc}\left(-\sqrt{\frac{V_i^2u}{4D}}\right)\mathrm{d}u \tag{S2-8}$$

Figure S3 displays the changes of the water fluxes of the stream bank with time using the semi-analytical solution (S2-6) (solid curves) and COMSOL (circle symbols) for the different bank slope angles, where the other parameters are same in Figure 3a. The semi-analytical solution of the water fluxes agrees with the numerical solution very well. In addition, Figure S3 shows that the bank slope has the significant impacts on the water fluxes between the stream-aquifer. More specifically, the smaller angle leads to the larger amount of stream water infiltrate into the aquifer during the period of stream stage rise and the larger discharge from the aquifer during the period of stream stage decline. Therefore, for the smaller slopes, the stream-aquifer interactions will be more significant and the fluxes will be underestimated if the impact of the sloping stream bank is neglected.

S3. MATLAB scripts

```
% Solution evaluation
function [ val ] = h( xi,t,T,D,V,Ht,alpha)

% h is solution of equation 20 in manuscript
% xi is new spatial variable, xi=x-X(t)
% T is array with time intervals: T=[t_0, t_1, …,t_i]
% D is diffusion coefficient (equation 14 in manuscript)
% V is defined by equation 14 in manuscript
% Ht is stream stage.
% alpha is defined by equation 1 in manuscript

ii=sum(t>T);                    % number of the time interval before time t.
du=0.02;
maxu=100;
u=0:du:maxu;

Vi=V(ii);
Ti=T(ii);
Hi=H(ii);
alphai=alpha(ii);
Ht=Hfunc(t,H,alpha,T);

hic=HIC(T,ii,D,V,H,alpha,u);

tic
bc=BC( xi,t,Ti,D,Vi,Hi,alphai);
toc
ic=IC( xi,t,D,Vi,hic,u,Ti);
val=Ht+ic-bc;
end

% Evaluation of initial heads for each time intervals.
function [ val ] = HIC(T,ii,D,V,H,alpha,u)
xi=u;
if ii==1
    val=0;
else
    hic=0;
     for i=1:ii-1
        t=T(i+1);

        bcW=BCW( xi,t,T(i),D,V(i),alpha(i));
```

```
            ic=IC( xi,t,D,V(i),hic,u,T(i));

            hic=ic-bcW;
        end
        val=hic;
end
end

% Evaluation of equation (22).
function val=BC( xi,t,Ti,D,Vi,Hi,alphai)
t=t-Ti;
temp1=@(u) erfc(-(xi+Vi*u)./sqrt(4*D*u));
temp2=@(u) erfc((xi-Vi*u)./sqrt(4*D*u));
temp3=exp(-xi*Vi/D);
fun=@(u) temp1(u)-temp3.*temp2(u);
val=alphai/2.*integral(fun,0,t,'ArrayValued',true);
end

% Evaluation of equation (21).
function val=IC( xi,t,D,Vi,hic,u,Ti)
t=t-Ti;
[X,U]=meshgrid(xi,u);
if length(hic)==1
      HI=hic;
else
      [X,HI]=meshgrid(xi,hic);
end
temp1=exp(-(2*Vi*xi+Vi^2*t)/(4*D))/sqrt(4*pi*D*t);
temp2=exp(Vi*U/(2*D)-((X-U)/sqrt(4*D*t)).^2)-exp(Vi*U/(2*D)-
((X+U)/sqrt(4*D*t)).^2);
val=temp1.*trapz(u,temp2.*HI);
end

% Evaluation of equation (18).
function [ val ] = Hfunc( t,H,alpha,T)
Ht=0;
for i=1:length(H)-1
Ht=Ht+(H(i)+alpha(i)*(t-T(i)))*(t<=T(i+1)&&t>T(i));
end
val=Ht;
end
```

MATLAB script for evaluation of *X*(*t*)

```
function [ val ] = X ( t,H,alpha,T,theta)
```

```
% X(t) is defined by equation 3a in manuscript.

Ht=0;
for i=1:length(H)-1
Ht=Ht+(H(i)+alpha(i)*(t-T(i)))*(t<=T(i+1)&&t>T(i));
end
val=(Ht-H(1))/tan(theta);
end
```

Figure S1

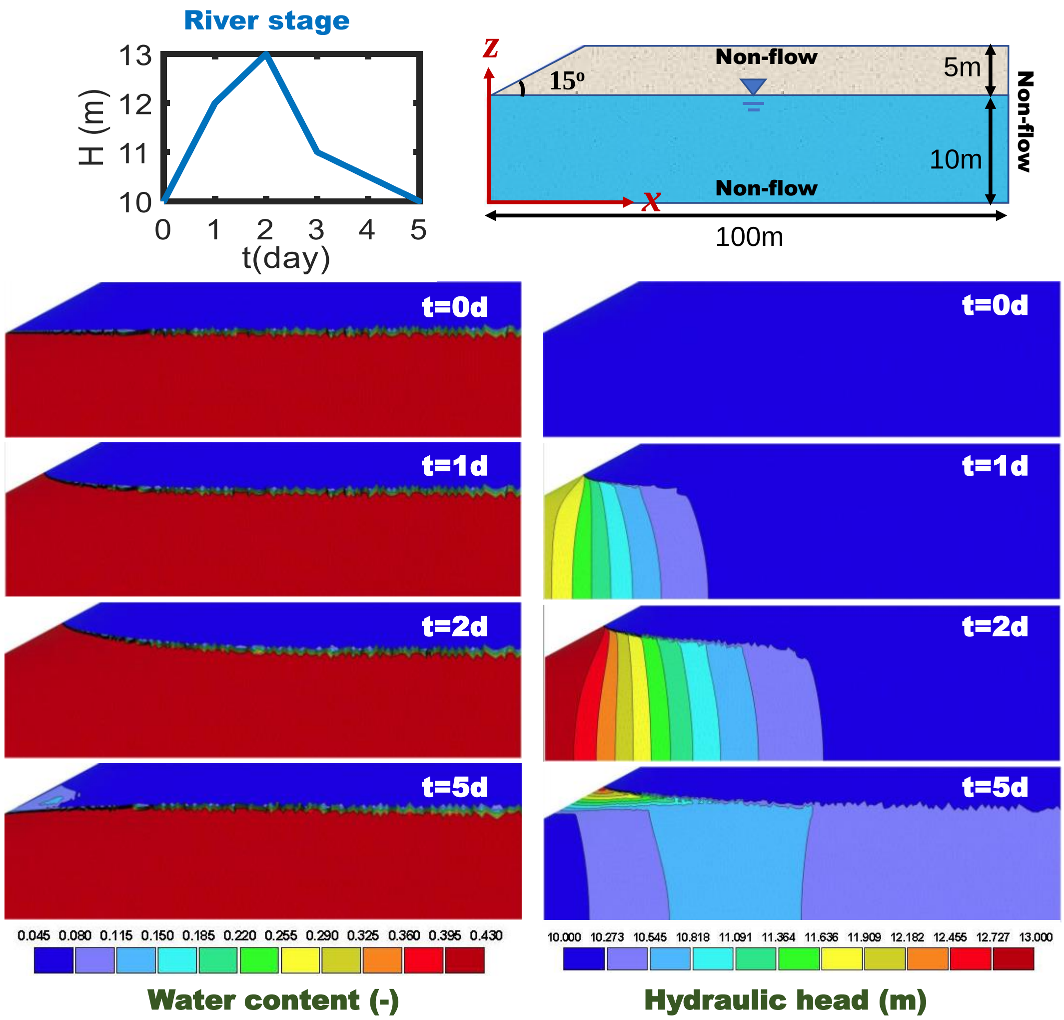

Figure S1 Contours of water contents and the hydraulic head predicted by the 2-D variably saturated flow model HYDRUS 2D/3D at different times for the synthetic case.

Figure S2

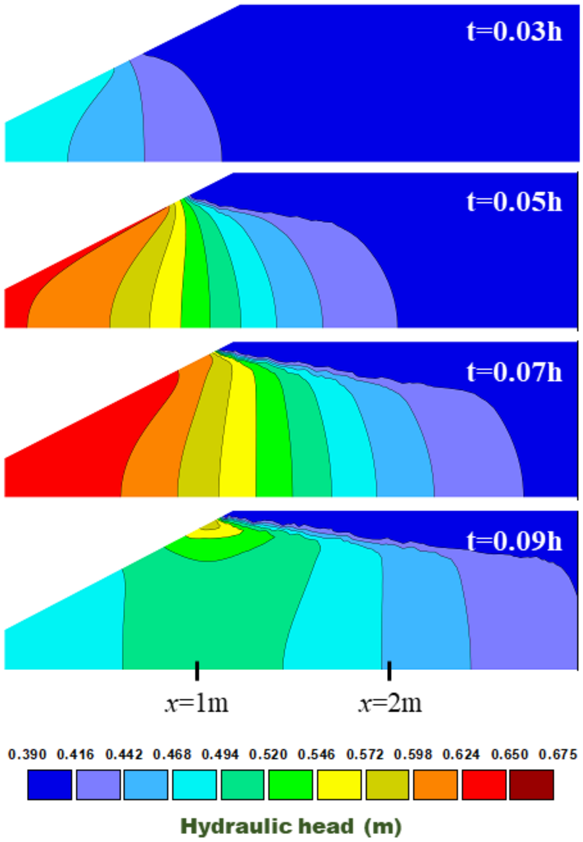

Figure S2 Contours of the hydraulic head predicted by the 2-D variably saturated flow model HYDRUS 2D/3D at different times for the laboratory experiment.

Figure S3

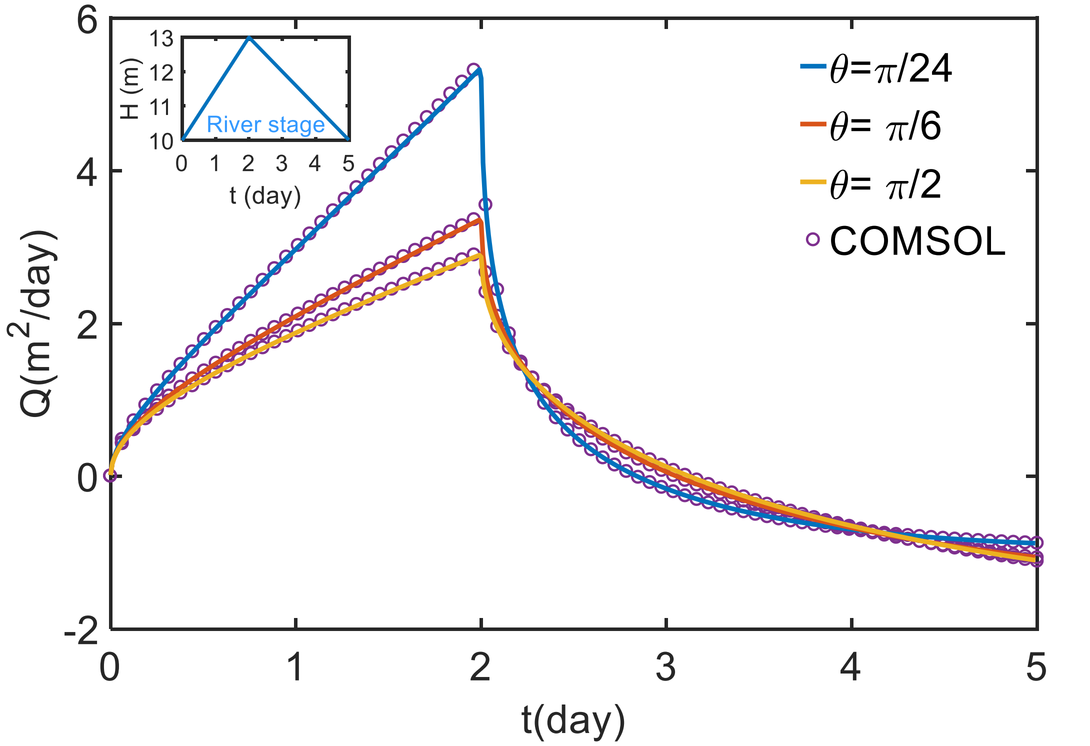

Figure S3 Dynamics of water fluxes at stream bank using solution (S2-6) and COMSOL for the different bank slope angle, where the other parameters are same in Figure 3a.