In [85]:

```
import numpy as np
import pandas as pd
import scipy
import matplotlib.pyplot as plt
from scipy.integrate import odeint
from scipy.interpolate import UnivariateSpline
```

In [180]:

```
# 1986-2004年的化石燃料排放CO2
year = np.linspace(1987,2004,18)
gamma1987_2004 = [5755,5968,6088,6151,6239,6178,6172,6284,6422,6550,6663,6638,6584,6750,6916,6981,7397,7782]
gamma1 = []

# 将化石燃料转化为PgC单位
for i in gamma1987_2004:
    j = i/1000
    gamma1.append(j)
# print(gamma1)

# 运用样条插值，确保gamma能够连续取值
gamma2 = UnivariateSpline(year, gamma1)
gamma2.set_smoothing_factor(0.5)
# print(gamma2(2000))

# 读取观测值数据集，跳过注释部分，取1986-2004内的数据
observation = pd.read_csv("co2_annmean_mlo.csv" , skiprows = 55)
observation1986_2004 = observation.loc[ (observation['year'] >1985) & (observation['year'] <2005) ]
observation1986_2004.head()
```

Out[180]:

	year	mean	unc
27	1986	347.61	0.12
28	1987	349.31	0.12
29	1988	351.69	0.12
30	1989	353.20	0.12
31	1990	354.45	0.12

1.1 Caculation without buffer efffect

In [181]:

```
def fun1(y,t):
    y1 = y[0]
    y2 = y[1]
    k12=105/740
    k21=102/900
    gamma=gamma2(t)
    dydt = [-k12*y1+k21*y2+gamma, k12*y1-k21*y2]
    return dydt

# 初始条件
y0 = [740,900]
t = np.linspace(1987,2004,18)
# print(t)

# 求解
Solution1 = odeint(fun1,y0,t)[:, 0]/2.13
print(Solution1)
```

```
[347.41784038 348.74561854 350.10879164 351.49253483 352.88637601
 354.28350841 355.68019984 357.07539017 358.47033775 359.86836229
 361.27464319 362.69606337 364.14108621 365.61966444 367.14316646
 368.72431922 370.3771653  372.11702912]
```

1.2 Caculation with buffer efffect

In [182]:

```
def fun2(y,t):
    y1 = y[0]
    y2 = y[1]
    k12=105/(740+79)
    k21=102/(900-79)
    N20=821
    Xi=9.92#根据公式求得
    gamma=gamma2(t)
    dydt = [-k12*y1+k21*(N20+Xi*(y2-N20))+gamma, k12*y1-k21*(N20+Xi*(y2-N20))]
    return dydt

# 初始条件
yy = [740+79,900-79]
tt = np.linspace(1987,2004,18)

# 求解
Solution2 = odeint(fun2,yy,tt)[:, 0]/2.13
print(Solution2)
```

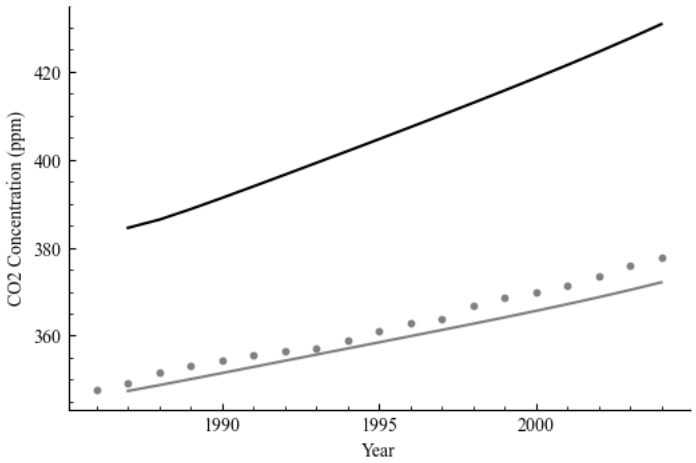
```
[384.50704225 386.36989916 388.76039838 391.31561695 393.93541573
 396.58882399 399.2643802  401.95739333 404.66668137 407.39372071
 410.14243515 412.9191631  415.73260831 418.59386013 421.51638127
 424.51601516 427.61097888 430.82186945]
```

1.3 reproduce Figure 2

In [183]:

```
fig, ax = plt.subplots(figsize=(6,4),dpi=100)
ax.spines['right'].set_visible(False)                #去除上右边框
ax.spines['top'].set_visible(False)
plt.rc('font',family='Times New Roman')              #设置字体
plt.rcParams['xtick.direction'] = 'in'               #将坐标轴的刻度线方向设置向内
plt.rcParams['ytick.direction'] = 'in'
ax.xaxis.set_major_locator(plt.MultipleLocator(5))   #设置坐标轴主次刻度
ax.xaxis.set_minor_locator(plt.MultipleLocator(1))
ax.yaxis.set_major_locator(plt.MultipleLocator(20))
ax.yaxis.set_minor_locator(plt.MultipleLocator(5))

plt.plot(t,Solution1, c='gray', label='caculation without buffer effect')
plt.plot(t,Solution2, c='k')
plt.scatter(observation1986_2004['year'],observation1986_2004['mean'],s=10,c='gray',marker='o')
plt.xlabel("Year")
plt.ylabel("CO2 Concentration (ppm)")
# plt.legend()
plt.show()
```



1.4 reproduce Figure 4

In [202]:

```
    def fun3(y,t):
        y1 = y[0], y2 = y[1], y3 = y[2], y4 = y[3], y5 = y[4], y6 = y[5], y7 = y[6]
        k12=60/615
        k21=60/842
        k23=9/842
        k24=43/842
        k32=52/9744
        k34=162/9744
        k43=205/26280
        k45=0.2/26280
        k51=0.2/90000000
        k67=62/731
        k71=62/1328
        N20=821
        Xi=8.91#根据大气二氧化碳浓度公式求得
        gamma=gamma2(t)
        f=31.1
        Delte=0.08
        dydt = [ -k12*y1+k21*(N20+Xi*(y2-N20))+gamma-f+Delte+k51*y5+k71*y7,
                            k12*y1-k21*(N20+Xi*(y2-N20))-k23*y2+k32*y3-k24*y2,
                            k23*y2-k32*y3-k34*y3+k43*y4,
                            k34*y3-k43*y4+k24*y2-k45*y4,
                            k45*y4-k51*y5,
                            f-k67*y6-2*Delte,
                            k67*y6-k71*y7+Delte]
        return dydt

    bata=0.38
    f0=62/2.13

    # 初始条件
    yyy = [615,842,9744,26280,90000000,731,1238]
    ttt = np.linspace(1750,2000,251)

    # 求解
    Solution3 = odeint(fun3,yyy,ttt)[:, 0]/2.13
    print(Solution3)
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
Input In [202], in <cell line: 36>()
     33 ttt = np.linspace(1750,2000,251)
     35 # 求解
---> 36 Solution3 = odeint(fun3,yyy,ttt)[:, 0]/2.13
     37 print(Solution3)

File F:\Anaconda3\lib\site-packages\scipy\integrate\odepack.py:241, in odeint(func, y0, t, args, Dfun, col_deriv, full_o
utput, ml, mu, rtol, atol, tcrit, h0, hmax, hmin, ixpr, mxstep, mxhnil, mxordn, mxords, printmessg, tfirst)
    239 t = copy(t)
    240 y0 = copy(y0)
--> 241 output = _odepack.odeint(func, y0, t, args, Dfun, col_deriv, ml, mu,
    242                           full_output, rtol, atol, tcrit, h0, hmax, hmin,
    243                           ixpr, mxstep, mxhnil, mxordn, mxords,
    244                           int(bool(tfirst)))
    245 if output[-1] < 0:
    246     warning_msg = _msgs[output[-1]] + " Run with full_output = 1 to get quantitative information."

Input In [202], in fun3(y, t)
      1 def fun3(y,t):
----> 2     y1 = y[0], y2 = y[1], y3 = y[2], y4 = y[3], y5 = y[4], y6 = y[5], y7 = y[6]
      3     k12=60/615
      4     k21=60/842

TypeError: cannot unpack non-iterable numpy.float64 object
```

In [197]:

```
def fun4(y,t):
    y1 = y[0], y2 = y[1], y3 = y[2], y4 = y[3], y5 = y[4], y6 = y[5], y7 = y[6]
    k12=60/615
    k21=60/842
    k23=9/842
    k24=43/842
    k32=52/9744
    k34=162/9744
    k43=205/26280
    k45=0.2/26280
    k51=0.2/90000000
    k67=62/731
    k71=62/1328
    N20=821
    Xi=8.91#根据大气二氧化碳浓度公式求得
    gamma=gamma2(t)
    f=31.7
    Delte=0.08
    dydt = [ -k12*y1+k21*(N20+Xi*(y2-N20))+gamma-f+Delte+k51*y5+k71*y7,
                        k12*y1-k21*(N20+Xi*(y2-N20))-k23*y2+k32*y3-k24*y2,
                        k23*y2-k32*y3-k34*y3+k43*y4,
                        k34*y3-k43*y4+k24*y2-k45*y4,
                        k45*y4-k51*y5,
                        f-k67*y6-2*Delte,
                        k67*y6-k71*y7+Delte]
    return dydt

bata=0.50

# 求解
Solution4 = odeint(fun4,yyy,ttt)[:, 0]/2.13
print(Solution4)
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
Input In [197], in <cell line: 31>()
     28 bata=0.50
     30 # 求解
---> 31 Solution4 = odeint(fun4,yyy,ttt)[:, 0]/2.13
     32 print(Solution4)

File F:\Anaconda3\lib\site-packages\scipy\integrate\odepack.py:241, in odeint(func, y0, t, args, Dfun, col_deriv, full_o
utput, ml, mu, rtol, atol, tcrit, h0, hmax, hmin, ixpr, mxstep, mxhnil, mxordn, mxords, printmessg, tfirst)
    239 t = copy(t)
    240 y0 = copy(y0)
--> 241 output = _odepack.odeint(func, y0, t, args, Dfun, col_deriv, ml, mu,
    242                           full_output, rtol, atol, tcrit, h0, hmax, hmin,
    243                           ixpr, mxstep, mxhnil, mxordn, mxords,
    244                           int(bool(tfirst)))
    245 if output[-1] < 0:
    246     warning_msg = _msgs[output[-1]] + " Run with full_output = 1 to get quantitative information."

Input In [197], in fun4(y, t)
      1 def fun4(y,t):
----> 2     y1 = y[0], y2 = y[1], y3 = y[2], y4 = y[3], y5 = y[4], y6 = y[5], y7 = y[6]
      3     k12=60/615
      4     k21=60/842

TypeError: cannot unpack non-iterable numpy.float64 object
```

看不懂报错，下载燃料燃烧速率的csv文件出错，无法得到gamma的函数

In [ ]: