A Theoretical Study of the Input Impedance of a Circular Microstrip Disk Antenna

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***Abstract*—A theoretical study is presented of the input impedance of a circular microstrip disk antenna excited by a coaxial line. The theory is based on Green's function technique applied to the disk cavity with the boundary admittance at the edge. Both the feed pin size and the boundary admittance are shown to be important in deriving the analytical expression for the input impedance. The boundary admittance is obtained by considering the radiated power and the electric and magnetic stored energies in the fringe capacitance. The analytical expression for the input impedance includes the feed location, the feed pin size, the disk size and thickness, and the dielectric constant of the material, and is useful for optimizing various parameters. The calculations are compared with experimental data showing a good agreement.**

I. INTRODUCTION

IN RECENT YEARS, interest in microstrip antennas has been increasing, primarily because of their low profile, small size, and light weight [1], [2]. Even though many experimental and theoretical studies have been reported [1]-[11], there is a need for more analytical investigations in order to make systematic studies of the characteristics of microstrip antennas.

In this paper we present a theoretical analysis of the input impedance of a circular disk antenna. This paper extends the earlier preliminary study [11] to include the effects of the feed pin size and the fringe capacitance. The formula for the input impedance is presented in a form convenient for optimizing various design parameters. The calculated input impedances and percent bandwidths are compared with experimental results showing excellent agreement. It is shown that it is important to take into account the feed pin size, the radiation from the disk edge, and the fringe capacitance in the derivation of the analytical expression of the input impedance.

II. GENERAL FORMULATION OF INPUT IMPEDANCE

Consider a circular disk antenna with a radius a and a thickness d, as shown in Fig. 1, excited by a line current $I_0(z')$ on the feed pin. The feed pin has radius r_f and is located at (ρ_0, ϕ_0); and the current density on the pin is given by $\bar{J}(\bar{r})$.

The input impedance Z_{in} of the antenna is given by

$$Z_{in} = \frac{2}{I_0(0)I_0^*(0)}\left\{\frac{-1}{2}\int_{S_0} \bar{E}\cdot\bar{J}^*\,dS + \text{copper loss}\right\}, \tag{1}$$

$$\text{copper loss} = R_s \int_{S_d} |\bar{J}_s|^2\,ds, \tag{2}$$

Manuscript received February 10, 1980; revised July 22, 1980. This work was supported in part by the National Science Foundation.

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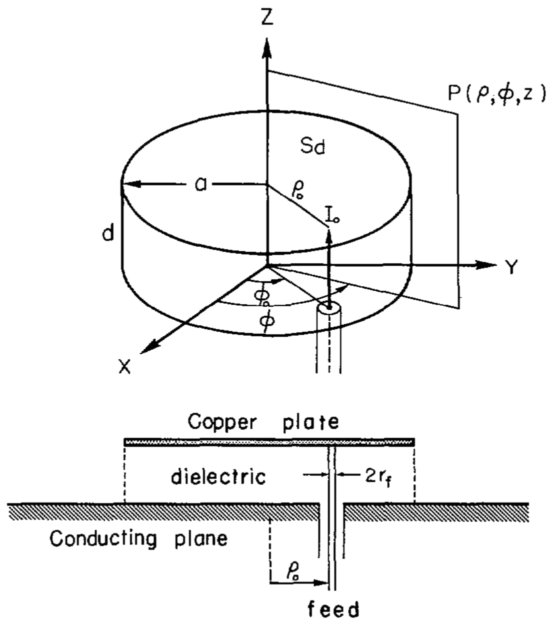

Fig. 1. Geometry of circular microstrip disk antenna.

where $\bar{E}$ and $\bar{H}$ are the electric and the magnetic fields for the perfectly conducting plates; S_0 and S_d are the surfaces on the feed pin and on the disk plate, respectively, R_s denotes the surface resistance of the disk plates, and $\bar{J}_s$ is the surface current on the disk plates. Here we used the classical induced electromotive force (EMF) method based on the conservation of complex power [12].

In evaluating (1) and (2) the field quantities are calculated taking into account the dielectric loss, the radiation, and the electric and magnetic stored energy in the fringe of the disk, but assuming the perfectly conducting disk plate. The copper loss on the the top and the bottom surfaces of the disk is then calculated using (2). Note that the EMF method is more convenient to include these approximations than the variational method.

Next we assume that the current on the feed pin and the field in the disk are uniform in the z direction, so that (1) is reduced to the following:

$$Z_{in} \cong \frac{2}{I_0 I_0^*}\left\{-\frac{dI_0^*}{4\pi}\int_0^{2\pi} E_z\,d\alpha + \text{copper loss}\right\}, \tag{3}$$

where the integration is taken on the surface S_0 of the feed pin (see Fig. 2). In the next section the field E_z on the feed pin surface is obtained using Green's function technique.

III. DERIVATION OF THE FIELD INSIDE THE DISK

The electric field E_z at (ρ, ϕ) due to a line current I_0 located at (ρ_0, ϕ_0) is given by the following:

$$E_z = j\omega\mu_0 I_0 G(\rho, \phi; \rho_0, \phi_0) \tag{4}$$

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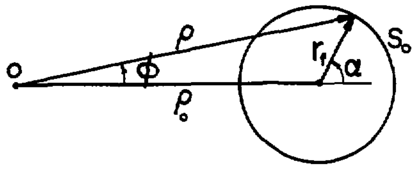

Fig. 2. Integration around feed pin.

where G is Green's function given by

$$G = \sum_{n=1}^{\infty} \frac{\cos n\phi \cdot \cos n\phi_0}{\pi} \frac{y_1(\rho_<)y_2(\rho_>)}{D_n} \tag{5}$$

where $\rho_<$ and $\rho_>$ denote the lesser or the greater of ρ and ρ_0. Thus $\rho_< = \rho$ and $\rho_> = \rho_0$ if $\rho < \rho_0$ and $\rho_< = \rho_0$ and $\rho_> = \rho$ if $\rho > \rho_0$. The denominator D_n is constant and is given by

$$D_n = \rho_0 \Delta\{y_1(\rho_0), y_2(\rho_0)\}$$

where $\Delta\{y_1, y_2\}$ is the Wronskian. In these expressions, y_1 and y_2 are given by

$$y_1(\rho) = J_n(k\rho)$$
$$y_2(\rho) = J_n(k\rho) + A_n N_n(k\rho) \tag{6}$$

where J_n and N_n are the Bessel and Neumann functions and $y_1(\rho)$ does not include $N_n(k\rho)$ as $y_1(\rho)$ should remain finite at $\rho = 0$. The constants A_n are chosen to satisfy the boundary conditions:

$$Y_n \equiv -\frac{H_\phi}{E_z} = \frac{j}{\omega\mu_0} \frac{\partial E_z/\partial\rho}{E_z} \quad \text{at } \rho = a. \tag{7}$$

Using (4), (5), and (6), together with (7), we obtain the unknown constants A_n in terms of the boundary admittance Y_n:

$$A_n = -\frac{Y_n Z_0 J_n(ka) - j\sqrt{\epsilon_r} J_n'(ka)}{Y_n Z_0 N_n(ka) - j\sqrt{\epsilon_r} N_n'(ka)} \tag{8}$$

where $Z_0 = 120\pi\ \Omega$ is the intrinsic impedance and J_n' and N_n' mean the derivatives with respect to the argument $k\rho$, and k is the complex wavenumber given by

$$k = 2\pi f\sqrt{\epsilon_r}/C. \tag{9}$$

$\Delta(y_1, y_2)$ is the Wronskian determinant given by

$$\Delta(y_1, y_2) = \frac{2A_n}{\pi\rho_0}. \tag{10}$$

The electric field E_z at (ρ, ϕ) due to the line current at $(\rho_0, 0)$ is then given by

$$E_z = \frac{j}{2}\omega\mu_0 I_0 \sum_{n=1}^{\infty} \cos n\phi \cdot \left\{\frac{J_n(k\rho_<)J_n(k\rho_>)}{A_n} + J_n(k\rho_<)N_n(k\rho_>)\right\}. \tag{11}$$

The magnetic field is obtained by the following:

$$H_\rho = \frac{j}{\omega\mu_0} \frac{1}{\rho} \frac{\partial E_z}{\partial\phi} \qquad H_\phi = \frac{-j}{\omega\mu_0} \frac{\partial E_z}{\partial\rho}. \tag{12}$$

IV. EVALUATION OF THE INPUT IMPEDANCE

The input impedance is given by (3) as the integral of the electric field (11) over the feed pin surface. In order to obtain an analytical expression of the input impedance, it is necessary to perform the integration taking into account the feed pin size. This will be done in this section. The electric field (11) depends on the boundary admittance Y_n, and this will be discussed in the next section.

Let us calculate the integral in (3) using E_z in (11). First we rewrite (3)

$$Z_{in} = -j60\pi k_0 d\left[\frac{1}{2\pi}\int_0^{2\pi}(S_1 + S_2)\,d\alpha\right] + \frac{2}{I_0 I_0^*}\ (\text{copper loss}), \tag{13}$$

where

$$S_1 = \sum_{n=1}^{\infty} \cos n\phi \frac{J_n(k\rho)J_n(k\rho_0)}{A_n} \tag{14}$$

$$S_2 = \sum_{n=1}^{\infty} \cos n\phi J_n(k\rho)N_n(k\rho_0), \qquad \text{for } \rho < \rho_0$$
$$= \sum_{n=1}^{\infty} \cos n\phi J_n(k\rho_0)N_n(k\rho), \qquad \text{for } \rho > \rho_0. \tag{15}$$

We note that $\rho \sin\phi = r_f \sin\alpha$, and for $r_f \ll \rho_0$ we use the following approximations:

$$\phi \cong \frac{r_f}{\rho_0}\sin\alpha \qquad \rho \cong \rho_0 + r_f\cos\alpha. \tag{16}$$

We then expand $J_n(k\rho)$ in (14):

$$J_n(k\rho) \cong J_n(k\rho_0) + kr_f\cos\alpha J_n'(k\rho_0). \tag{17}$$

Performing the integration with respect to α, we get

$$\frac{1}{2\pi}\int_0^{2\pi} S_1\,d\alpha = \sum_{n=1}^{\infty} \frac{J_0\left(n\frac{r_f}{\rho_0}\right)J_n^2(k\rho_0)}{A_n}. \tag{18}$$

Similarly, we expand $J_n(k\rho)$ and $N_n(k\rho)$ in (15), and integrating with respect to α, we get

$$\frac{1}{2\pi}\int_0^{2\pi} S_2\,d\alpha = \sum_{n=1}^{\infty} J_0\left(n\frac{r_f}{\rho_0}\right)J_n(k\rho_0)N_n(k\rho_0) + \frac{1}{\pi^2}\left(\pi - \frac{r_f}{\rho_0}\right). \tag{19}$$

To improve convergence in (19), we write

$$\sum_{n=1}^{\infty} J_0\left(n\frac{r_f}{\rho_0}\right) J_n(k\rho_0)N_n(k_{\rho_0})$$
$$= \sum_{n=1}^{\infty} J_0\left(n\frac{r_f}{\rho_0}\right)\left[J_n(k\rho_0)N_n(k\rho_0)+\frac{1}{\pi n}\right]$$
$$-\sum_{n=1}^{\infty}\frac{1}{\pi n}J_0\left(n\frac{r_f}{\rho_0}\right). \tag{20}$$

Using the integral representation of the Bessel function, the last term in (20) is given by [13]:

$$-\sum_{n=1}^{\infty}\frac{1}{\pi n}J_0\left(n\frac{r_f}{\rho_0}\right)$$
$$=-\sum_{n=1}^{\infty}\frac{2}{\pi^2 n}\int_0^{\pi/2}\cos\left(n\frac{r_f}{\rho_0}\sin\beta\right)d\beta$$
$$=\frac{2}{\pi^2}\int_0^{\pi/2}\ln\left[2\sin\left(\frac{r_f}{2\rho_0}\sin\beta\right)\right]d\beta. \tag{21}$$

The convergence of the infinite series in (20) is good, and the series can be truncated at $n = M$ when the Mth term is 10^{-6} of the sum up to the $M-1$ term. The values of M are between 10 and 20 for both the V band and X band.

Using (18)-(21), we finally obtain the input impedance

$$Z_{\text{in}} = -j60\pi k_0 d\left\{\sum_{n=1}^{M} J_0\left(n\frac{r_f}{\rho_0}\right)\right.$$
$$\cdot\left[\frac{J_n^2(k\rho_0)}{A_n}+J_n(k\rho_0)N_n(k\rho_0)+\frac{1.}{\pi n}\right]$$
$$\left.+\frac{2}{\pi^2}\int_0^{\pi/2}\ln\left[2\sin\left(\frac{r_f}{2\rho_0}\sin\beta\right)\right]d\beta+\frac{1}{\pi^2}\left[\pi-\frac{r_f}{\rho_0}\right]\right\}$$
$$+\frac{2}{I_0 I_0{}^*}\text{(copper loss)}. \tag{22}$$

This expression includes the feed pin size r_f, the feed location ρ_0, the frequency, and the dielectric constant. The constant A_n depends on the equivalent boundary admittance Y_n at the edge $\rho = a$ as shown in (8). The derivation of this admittance Y_n will be discussed in the next section.

The input impedance of a circular disk antenna was previously obtained by Shen [18] who neglected the finite size of the feed pin but used the single term $n = 1$ to calculate the reactance. We summed all terms ($n = 1 \sim \infty$) and therefore the impedance (22) agrees well with experimental data. It is expected that the input resistance is insensitive to the pin size, but the input reactance is greatly affected by the pin size. Also, note that the impedance with the infinite series in (13) is divergent if the pin size is reduced to zero. In [18] this did not happen since only the first term is taken.

V. EVALUATION OF THE BOUNDARY ADMITTANCE Y_n

The characteristics of microstrip antennas are often approximated by those of the corresponding cavity with a magnetic wall at the edge of the disk. However, in order to determine the input impedance of the antenna, it is necessary to take into account the reactive power due to the electric and magnetic stored energy in the fringe of the disk and the real power due to the radiation. These reactive and active powers are represented by the equivalent boundary admittance at the disk edge.

In order to obtain the boundary admittance, we consider the power flow through the side surface S_a of the disk at $\rho = a$. For each mode, we write

$$\frac{1}{2}\int_{S_a} Y_n{}^*|E_z|^2\,dS = P_r\text{ (radiated power)}$$
$$+jP_i\text{ (reactive power)}. \tag{23}$$

The boundary admittance Y_n normalized to free space characteristic admittance Y_0 is then given by

$$y_n = \frac{Y_n}{Y_0} = g_n + jb_n$$
$$g_n = \frac{Z_0 P_r}{\frac{1}{2}\int_{S_a}|E_z|^2\,dS}$$
$$b_n = \frac{[-Z_0 P_i]}{\frac{1}{2}\int_{S_a}|E_z|^2\,dS}. \tag{24}$$

The radiated power P_r for mode $n = 1$ can be calculated by

$$P_r = \text{radiation power}$$
$$=\frac{1}{2Z_0}\int_0^{2\pi}\int_0^{\pi/2}(|E_\theta|^2+|E_\phi|^2)r^2\sin\theta\,d\theta\,d\phi, \tag{25}$$

where

$$E_\theta = -jE_0\frac{e^{-jk_0 r}}{r}a\frac{\sin(k_0 d\cos\theta)}{\cos\theta}\cos\phi J_1{}'(k_0 a\sin\theta)$$
$$E_\phi = jE_0\frac{e^{-jk_0 r}}{k_0 r}\frac{\sin(k_0 d\cos\theta)}{\sin\theta}\sin\phi J_1(k_0 a\sin\theta)$$
$$E_0 = j\omega\mu_0 I_0\left\{\frac{J_1(k\rho_0)J_1(ka)}{A_1}+J_1(k\rho_0)N_1(ka)\right\}$$
$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \cong 120\pi. \tag{26}$$

The contribution of the higher mode $n \neq 1$ is found to be negligible compared with the mode $n = 1$, since the antenna is excited at frequencies close to the resonant frequency of the

$n = 1$ mode. Thus we obtain

$$g_n = \frac{1}{k_0 a k_0 d} \int_0^{\pi/2} (I_1^2 + I_2^2) \sin\theta \, d\theta, \quad \text{for } n = 1$$

$$= 0, \quad \text{for } n \neq 1$$

$$I_1 = \frac{k_0 a \sin(k_0 d \sin\theta)}{\cos\theta} J_1'(k_0 a \sin\theta)$$

$$I_2 = \frac{\sin(k_0 d \cos\theta)}{\sin\theta} J_1(k_0 a \sin\theta). \tag{27}$$

Estimation of the reactive power can be made by assuming that the electric and magnetic fields are constant throughout the equivalent volume ΔV and are equal to the fields at the edge of the disk (see Fig. 3). We then get

$$P_i = \text{reactive power} = -2\omega[W_e - W_h]$$

$$W_e = \int_{\Delta V} \frac{\epsilon |E_z|^2}{4} dV \cong \frac{\epsilon |E_z|^2}{4} \Delta V$$

$$W_h = \int_{\Delta V} \frac{\mu_0}{4} [|H_\rho|^2 + |H_\phi|^2] \, dV$$

$$\cong \frac{\mu_0}{4} [|H_\rho|^2 + |H_\phi|^2] \Delta V, \tag{28}$$

where E_z, H_ρ, and H_ϕ are evaluated at the disk edge. In (28), $\epsilon = \epsilon_0$ if ΔV is in free space, $\epsilon = \epsilon_0 \epsilon_r'$ if the dielectric substrate is extended beyond the disk, and ϵ_r' is the real part of the relative complex dielectric constant ϵ_r. The equivalent volume ΔV is taken to be equal to the volume of the static fringe capacitance ΔC between the radius a and the equivalent radius a_d of the fringe edge:

$$\Delta C = \frac{\epsilon \Delta S}{d} = \frac{\epsilon \Delta V}{d^2}$$

$$\Delta S = \pi(a_d^2 - a^2) = \frac{\Delta V}{d} \tag{29}$$

where the static fringe capacitance ΔC is given by the Kirchhoff equation

$$\Delta C = \epsilon_0 2a \left(\ln \frac{\pi a}{2d} + 1.7726 \right)$$

$$a_d = a \left\{ 1 + \frac{d}{\epsilon \pi a^2} \Delta C \right\}^{1/2}. \tag{30}$$

When the dielectric substrate is extended beyond the disk, we may use the expressions given by Coen and Gladwell originally proposed by Itoh and Mittra to calculate the fringe capacitance ΔC [15], [16]. They show that when $d/a < 0.2$, the gate function for the charge distribution gives a good

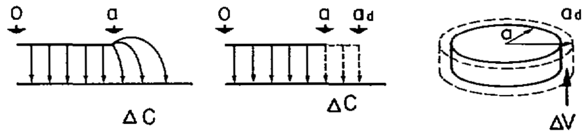

Fig. 3. Fringe capacitance.

approximation for ΔC while when $d/a > 0.2$, Maxwell's function gives a better approximation.

$$\Delta C = \frac{\pi \alpha a}{2} \frac{1}{\frac{4}{3\pi} - (1-\beta) \sum_{m=1}^{\infty} \beta^{m-1} J_m} - \frac{\epsilon_0 \epsilon_r' \pi a^2}{d},$$

$$\text{for } d \leqslant \frac{a}{5} \tag{31}$$

where

$$J_m = \frac{4}{3\pi \xi^3} \{(2\xi^2 - 1)E(\xi) + (1 - \xi^2)K(\xi)\} - m\gamma,$$

$$\xi^2 = \frac{1}{1 + m^2\gamma^2},$$

and $K(\xi)$ and $E(\xi)$ are the complete elliptic integral of the first and second kinds, respectively:

$$\Delta c = \frac{2\pi \alpha a}{\frac{\pi}{2} - (1-\beta) \sum_{m=1}^{\infty} \beta^{m-1} I_m} - \frac{\epsilon_0 \epsilon_r' \pi a^2}{d},$$

$$\text{for } d \geqslant \frac{a}{5} \tag{32}$$

where

$$I_m = \frac{1}{2} \left\{ \pi - 2 \tan^{-1}(m\gamma) + m\gamma \ln\left(\frac{m^2\gamma^2}{1 + m^2\gamma^2} \right) \right\}.$$

In (31) and (32), α, β, and γ are given by

$$\alpha = \epsilon_0(1 + \epsilon_r'), \qquad \beta = \frac{1 - \epsilon_r'}{1 + \epsilon_r'}, \qquad \gamma = \frac{d}{a}$$

Using (28) and (29) with the fringe capacitance, we get

$$b_n = k_0 \epsilon_r' \Delta l \left[1 - \frac{W_h}{W_e} \right] \tag{33}$$

where

$$\Delta l = \frac{\int_{\Delta V} \frac{|E_z|^2}{2} dV}{\int_{S_a} \frac{|E_z|^2}{2} dS} \cong \frac{\Delta V}{S_a} \tag{34}$$

Note that in (34), the same ϕ dependence of the field E_z appears both in the numerator and the denominator, and therefore the dynamic capacitance [14] need not be considered. The quantity Δl in (34) represents the equivalent length of the fringe capacitance and using (29) and noting $S_a = 2\pi ad$, it is given by

$$\Delta C = \frac{\epsilon(2\pi a)\Delta l}{d}. \tag{35}$$

For the circular disk under consideration, we get

$$b_n = \frac{k_0 a \epsilon_r'}{2} \left\{ 1 - \frac{n^2}{\epsilon_r'(k_0 a)^2} - |Y_n Z_0|^2 \right\} \frac{a_d^2 - a^2}{a^2}. \tag{36}$$

Y_n in (36) is obtained from (24) by the iteration method. Thus the first iteration, by letting $Y_n = 0$ in (36) yields

$$Y_n Z_0 = g_n + j \frac{k_0 a \epsilon_r'}{2} \left\{ 1 - \frac{n^2}{\epsilon_r'(k_0 a)^2} \right\} \frac{a_d^2 - a^2}{a^2}. \tag{37}$$

These expressions (24), (27), (33), and (36) are used in (8) to obtain the constant A_n, which is then substituted in (22) to arrive at the final expression for the input impedance Z_{in}.

VI. THEORETICAL CALCULATION OF INPUT IMPEDANCE AND COMPARISON WITH EXPERIMENTAL DATA

The input impedance (22) is calculated using the following constants:

$$f = 1200 \text{ MHz} \sim 1400 \text{ MHz}, \qquad \text{increment} = 5 \text{ MHz}$$

$$a = 1.65 \text{ in}, \qquad \rho_0 = a/5 \sim a/2, \qquad d = 0.019\, a \sim 0.3a$$

$$r_f = 0.026 \text{ in},$$

$$\text{real } \epsilon_r' = 1.0 \sim 3.0, \qquad \tan\delta = 0 \sim 5/1000.$$

Figs. 4 and 5 show the calculated impedance loci obtained by varying the frequency f and the feed location ρ_0. One of those loci is compared with the measured value which appears as a dashed line. For the thinner substrate case shown in Fig. 4, there is some discrepancy between the calculated and the measured loci and frequency locations. However, Fig. 5, which shows the results for the thicker substrate, indicates very good agreement between the calculated and the measured values.

As was expected, the radius of the locus depends greatly upon the real part of the boundary admittance which is based on the radiation power; whereas the imaginary part which is based on the fringe capacitance, has a greater effect on the frequency dependence. The measurement was done for the disk with a Teflon–glass substrate so that we used the value of 2/1000 for the tan δ for the frequency of 3 GHz given by Von Hippel *et al.* [17].

Figs. 6 and 7 show the comparison between the calculated and measured bandwidths for voltage standing-wave ratios (VSWR) less than two as the substrate thickness in wavelength d/λ_ϵ is varied. The small circles show the measured value obtained by Bailey and Parks [3] for $\rho_0/a = 1/3$ and for the substrate with a dielectric constant of 2.5. They show good

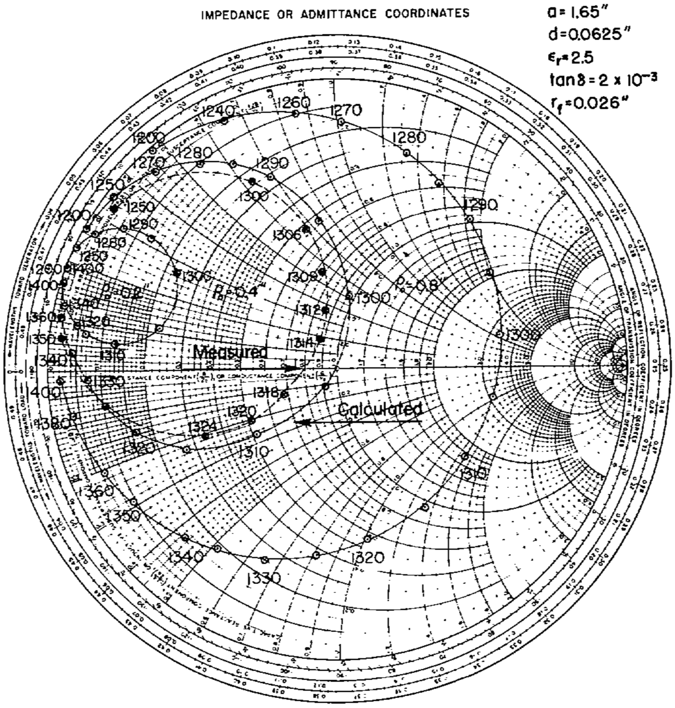

Fig. 4. Impedance loci of thin disk as functions of frequency (MHz).

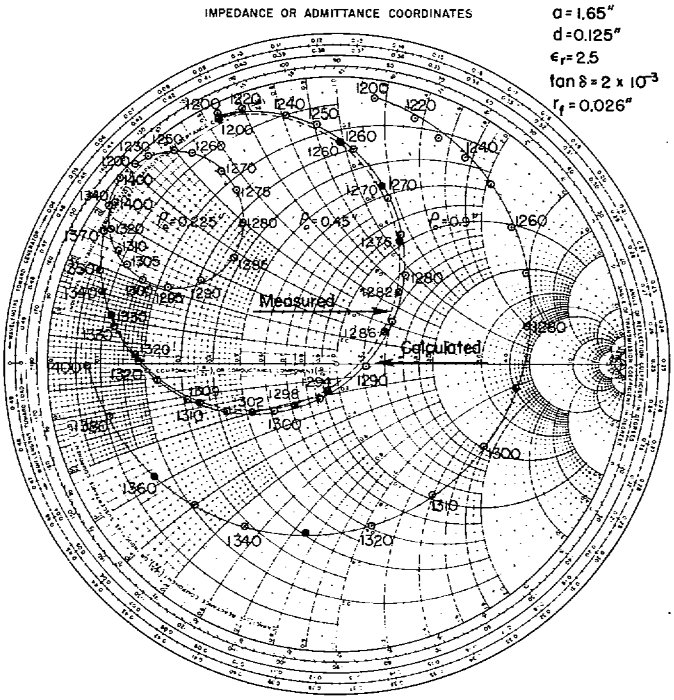

Fig. 5. Impedance loci of thick disk as functions of frequency (MHz).

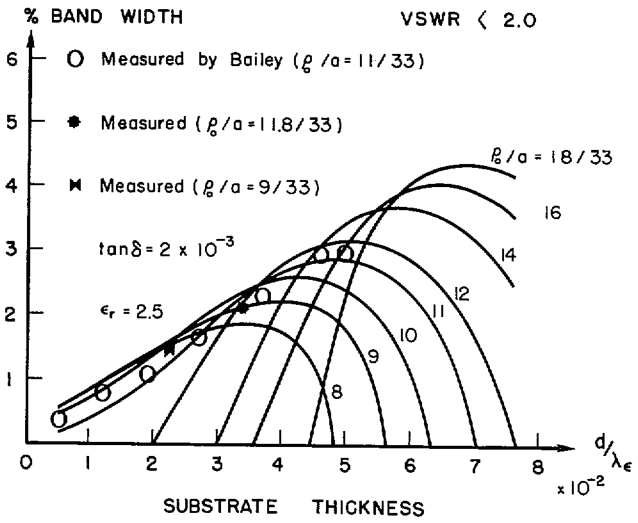

Fig. 6. Percent bandwidth as functions of substrate thickness for various feed location ρ_0/a when the dielectric constant is $\epsilon_r = 2.5$. The feed pin size for Figs. 6–8 is $r_f = 0.026$ in.

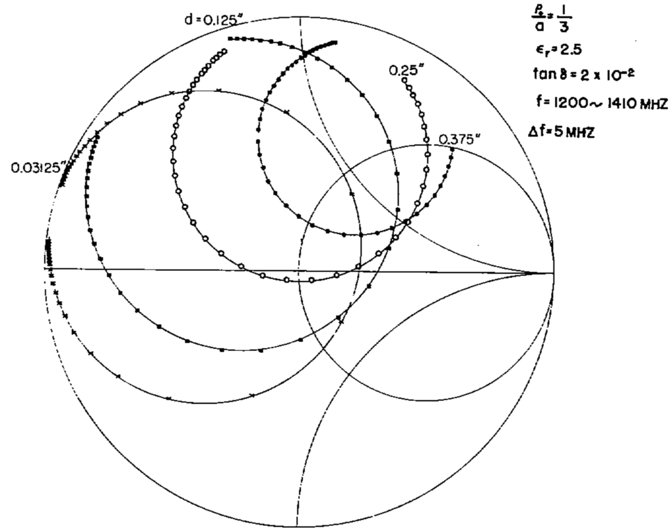

Fig. 8. Change of impedance loci for different substrate thicknesses. For each thickness, the impedance moves in a clockwise direction with the increase of frequency.

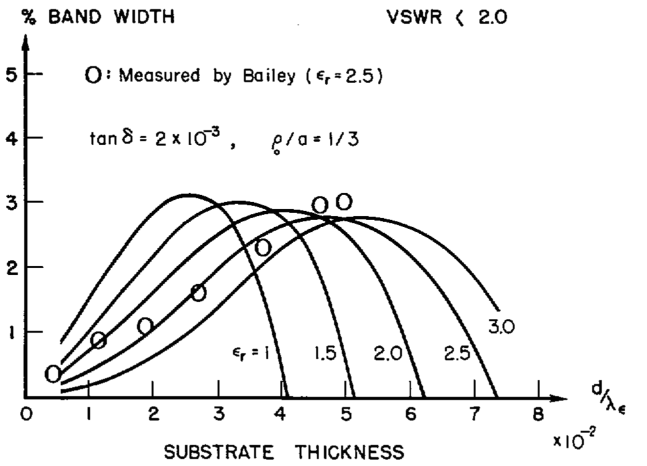

Fig. 7. Percent bandwidth as functions of substrate thickness for different dielectric constant ϵ_r when the feed location is $\rho_0/a = 1/3$.

agreement with a curve marked $\rho_0/a = 11/33$. In Fig. 6, the stars indicate the measured values of $\rho_0/a = 11.8/33$ and 9/33.

As can be seen, the bandwidth tends to increase with ρ_0/a, and there is an optimum d to obtain a wider bandwidth for a given VSWR and ρ_0/a. These characteristics may be very important in a disk antenna design. Although the higher ρ_0/a can yield the wider bandwidth, optimization of d requires a thicker substrate and consideration of the z-dependence of the electric field in the disk, which is ignored in our calculation. It is found that the bandwidth becomes wider as the tan δ diminishes. This is due to the shift of the impedance locus and the decrease of its radius.

Fig. 7 shows the bandwidth for a constant value of $\rho_0/a = 1/3$ as the dielectric constant is varied. This figure shows that there is an optimum substrate thickness for a given dielectric constant. Fig. 8 shows why there is an optimum d for a given VSWR. There is a clockwise movement of impedance loci and a decrease in diameter of loci with the increase of the substrate thickness.

VII. CONCLUSION

The following conclusions can be drawn from this work.

1) Expression (22) of the input impedance of a circular disk antenna is obtained using Green's function and the equivalent impedance boundary condition. It includes the effects of the feed pin size, the radiated power from the disk edge, and the fringe capacitance. The results show good agreement with the measured values.

2) The radius of the impedance locus is affected primarily by the real part of the boundary admittance obtained from the radiation power, whereas the frequency on the locus is affected primarily by the imaginary part of the admittance obtained from the reactive power.

3) At constant ρ_0/a, as the dielectric loss increases, the bandwidth decreases, and the diameter of the impedance locus also becomes smaller.

4) There is an optimum d for any given VSWR when the feed location and the disk radius are kept constant.

5) The smaller the dielectric constant, the thinner the substrate must be for maximum bandwidth.

ACKNOWLEDGMENT

The authors wish to thank Dr. James Yee of the Boeing Company for introducing them to this problem and supplying experimental data.

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