Finally, the scalar products of current expansion functions with Floquet modes are

$$\langle \hat{j}_m, \hat{e}_n \rangle = (m\pi/a)d^{-1/2} \exp(-jk_{xn}d) \cdot \frac{1-(-1)^m \exp(jk_{xn}a)}{(m\pi/a)^2 - k_{xn}^2}. \tag{A7}$$

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The Synthesis of Shaped Patterns with Series-Fed Microstrip Patch Arrays

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Abstract—**A method of designing series-fed microstrip patch arrays to produce a shaped pattern is described. The technique is based on a circuit representation of the patches in the array environment with experimentally determined parameters. The positions and widths of the patches are derived from the amplitudes and phases of the elements of a uniform array, which produces the desired pattern and which has the same extent as the patch array. For the array to have high efficiency, the amplitude distribution must not be strongly peaked. An algorithm for obtaining an approximation to the desired far-field amplitude distribution while retaining control over the amplitude of excitation is presented. Very close agreement has been obtained between calculated and measured performance of such arrays.**

I. INTRODUCTION

THE MECHANISM of radiation from rectangular microstrip patches has been described by several authors [1]-[3], and in particular the use of series fed arrays of microstrip patches to achieve pencil beams has been described by Metzler [4]. A method of accurately synthesizing shaped beams with series fed microstrip patch array is described in this paper.

The form of the arrays considered is shown in Fig. 1. The radiating patches themselves are resonant so that the input line to a patch is matched if the output line is terminated. In the simple transmission line model of a patch this corresponds to a length of $\lambda g/2$. Excitation at one end of the array with a termination at the other results in a traveling wave on the microstrip line, from which power is radiated at the edges of each patch which behave as a pair of in-phase slots [5]. The phases of the radiation sources can be controlled by selection of the positions of the patches on the line and their amplitudes by choice of the widths of the patches. Because the amplitude and phase of the radiation at each patch is determined by the cumulative transmission characteristics of the preceding patches on the line, the transmission characteristics of the patches must be determined with some accuracy in order to achieve a desired amplitude and phase distribution of radiating sources along the array. A circuit representation of the patches based on the transmission line model of Derneryd [6], but with empirically determined parameters, is used to represent the patch array.

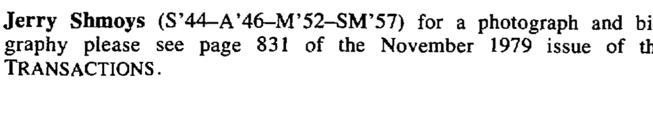

Fig. 1. Series fed microstrip patch array.

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The properties of series fed microstrip patch arrays are similar to those of edge slotted waveguide antennas as far as bandwidth, polarization, and efficiency are concerned, however, microstrip arrays appear to be capable of cheaper fabrication.

II. CIRCUIT REPRESENTATION OF PATCHES AND ARRAYS

The circuit representation used here for a single patch is shown in Fig. 2. The patch itself is represented by a length of microstrip line of width equal to that of the patch. The characteristic impedance and complex propagation constant are taken from the formulas for microstrip line properties given by Bahl and Trivedi [7]. The effects of fringing fields and of the junction between the wide and narrow lines are represented by the extensions Δ and δ in the wide and narrow lines, respectively, as shown. This representation of the junctions was chosen because Δ and δ are not strongly dependent on frequency. Radiation at the discontinuities is accounted for by shunt conductances g, which are referred to the impedance of the narrow line. Experimental characterization of the patches requires a determination of the parameters g, Δ, and δ as a function of patch width W and, since the parameters are intended to characterize the patch in the presence of mutual effects from other patches, it is expected that g and Δ should also be functions of the patch spacing S. δ which principally represents local junction effects is assumed to be independent of S. It has, however, been found experimentally that the dependence of Δ on S is negligible.

The assumption is made that the parameters for a patch in the array environment depend only on the patch dimensions and the local spacing of patches.

When the dependence of Δ, δ, and g on patch width and of Δ and g on patch spacing has been ascertained, analysis of the performance of a patch array can be carried out using the circuit representation of the elements. This is conveniently done using a T-matrix representation of line sections, shunt conductances, and extensions. Assuming unit forward and zero backward propagating wave at the terminating load, the forward and backward wave components at any other point in the array can be found by multiplication of the T-matrices. Insertion loss, return loss, and radiation efficiency (ratio of power dissipation in the radiation conductances to the input power), can be readily found. The radiation pattern in the principal plane (E-plane) is assumed to be that of an array of point sources located at the patch ends with amplitude $v\sqrt{g}$, where v is the complex voltage at the radiation conductance of the patch in the circuit representation.

III. EXPERIMENTAL CHARACTERIZATION OF PATCHES

In order that an approximate design can be carried out to establish the range of patch widths and spacings required, the parameters g, Δ, and δ can be determined for single patches, ignoring the dependence of g and Δ on S. When the approximate range of widths and spacings has been established, several uniform arrays spanning the range of spacings and widths can be made and the dependence on S of these quantities can be determined.

The parameters g, Δ, and δ can be determined from measurements on several patches of different widths and of length near $\lambda g/2$ where λg is the wavelength of the quasi-transverse electromagnetic (TEM) mode in microstrip of the width of the patch given by the formulas of [7].

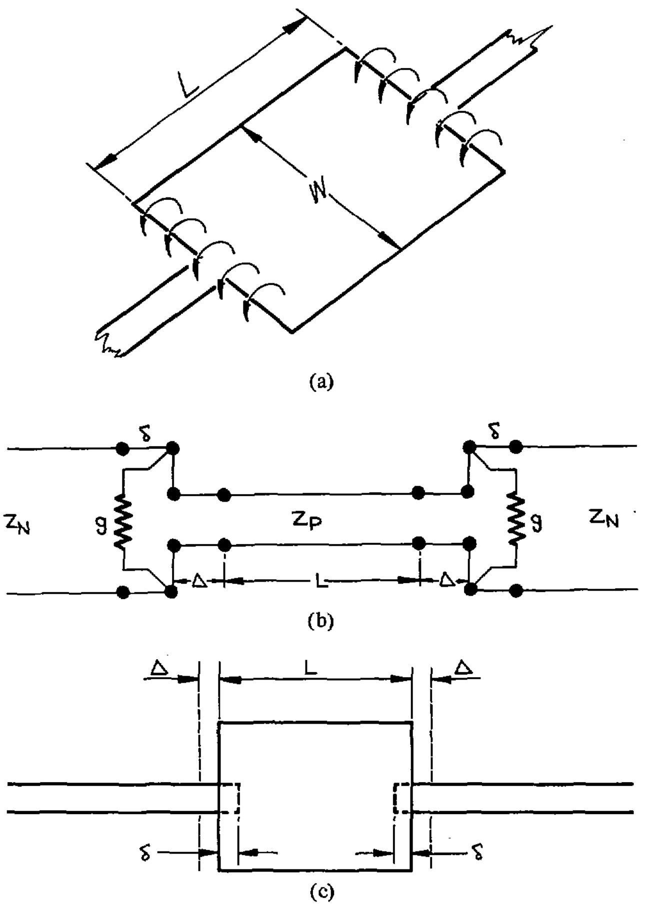


Fig. 2. Single patch and its equivalent circuit respresentation.

Δ may be found by terminating the output line from the patch and finding the resonant frequency defined by a minimum standing-wave ratio (SWR) on the input line. Δ is then defined by

$$L + 2\Delta = \lambda g/2 \tag{1}$$

at the resonant frequency, where g is evaluated from the formulas of [7].

δ is found from a measurement of the transmission phase

$$2\delta = \frac{\lambda_N}{2\pi}\,\phi_{\text{exc}} \tag{2}$$

where ϕ_{exc} is the phase angle in excess of π between the patch ends at resonance and λ_N is the wavelength in narrow microstrip line at resonance.

The conductance g can be measured by connecting an adjustable short circuit to the output line, finding the ratio or the input reflection coefficients when the short is positioned so that radiation from the patch is minimized, ρ_1 and at a quarter wavelength from this position ρ_2. The conductance is given by

$$2g = \frac{\rho_1 + \rho_2}{\rho_1 - \rho_2}. \tag{3}$$

This technique largely separates the distributed line loss from the localized radiation loss. However, localized radiation loss from connectors still causes some inaccuracy. Graphs of the three parameters are shown in Fig. 3. Expressions for g for isolated patches without the feedline are given by Derneryd [6]. These are shown in the diagram for comparison.

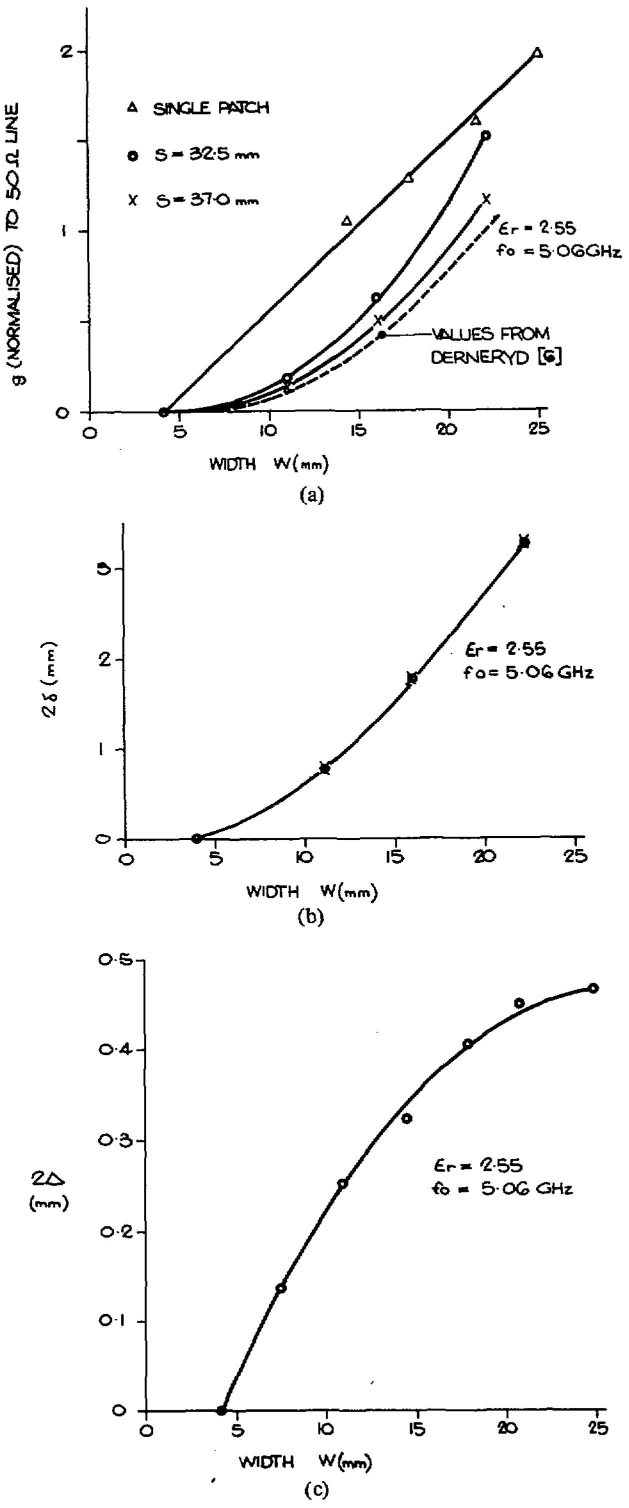


Fig. 3. Measured radiation conductance g and extensions Δ and δ as a function of patch width, W, and spacing S.

An initial design of the patch array can be carried out based on the circuit parameters determined from measurements on single patches using the techniques described in the following sections. From this initial design, the approximate ranges of patch widths and spacings which will occur in the final design are found. A more accurate determination of the patch characteristics that apply to patches in the array environment over the appropriate ranges of widths and spacings is made from measurements on several uniformly spaced arrays of constant width patches.

The insertion loss and insertion phase of the uniform array is first calculated using the T-matrix circuit analysis and the values of Δ, δ, and g found from the single patch measurements. The analysis is repeated using slightly perturbed values of Δ and g until the measured values of insertion loss and phase are obtained. It is found that the value of Δ so obtained is not significantly different from that obtained from the single patch method, although a higher accuracy is obtainable in this way. The values of g, however, differ significantly from those for a single patch. These results are also shown in Fig. 3.

IV. PATCH ARRAY GEOMETRY

The design of an array consists of finding the location of the center of each patch and its width. Its length follows from the width of the patch since it is chosen to make the patch resonant, i.e.,

$$l(w) = \frac{\lambda(w)}{2} - 2\Delta. \tag{4}$$

The approach which we have adopted to the design is firstly to design a uniform array of point source elements which produces a good approximation to the required pattern and which extends over approximately the same length as the required microstrip patch array. The spacing of the elements should be similar to the average element spacing in the final array. A suitable spacing may be found by equating the phase gradient of the sources formed by a uniform patch array to that appropriate for the direction of maximum radiation in the desired pattern. This leads to

$$S(1/\lambda_N - 1/\lambda_0 \cos \zeta) = 1/2 + \frac{1/2\lambda g - 2\Delta - 2\delta}{\lambda_N} \tag{5}$$

where ζ is the angle between the traveling wave and the direction of maximum radiation. From the amplitudes and phases of the elements of this uniform array, a set of patch widths and locations which will produce approximately the same pattern as the uniform array may be found by a process of interpolation.

Following is a technique that has been found to give satisfactory results.

1) A smooth curve of phase as a function of position $\Phi(x)$ is fitted to the phases φ_n of the elements of the uniform array, i.e., $\Phi(x_n) = \varphi_n$ where x_n is the location of the nth element.
2) A smooth curve is fitted to the cumulative amplitude distribution of the uniform array, as a function of position. The "cumulative amplitude" function $C(x_n)$ at element n located at x_n refers to

$$C(x_n) = \sum_{1}^{n} |a_i| \tag{6}$$

where a_i is the amplitude of excitation of the ith element.
3) The patches of the nonuniform patch array are located so that the phases of the sources which they form lie on the curve of condition 1). The widths of the patches are selected so that the cumulative amplitudes at each of the sources

$$C(z_n) = \sum_{1}^{n} |a_i| \tag{7}$$

lie on the curve of condition 2).

For this purpose, a patch is represented by a single source located at z_n, the center of the patch, with amplitude.

$$\alpha_n = (v_1 + v_2) \cdot \sqrt{g} \tag{8}$$

where v_1 and v_2 are the voltages at the ends of the patch and g is the radiation conductance at each end of the nth patch. The index of the patches is taken as increasing from the termination to the feed.

To do this in practice it is assumed that unit power is incident on the termination and a value for the total radiated power γ is assumed. The amplitudes of the elements of the uniform array are therefore normalized so that

$$\sum_1^N |a_i|^2 = \gamma \tag{9}$$

where N is the number of elements. The first patch (nearest the termination) is colocated with the first element of the uniform array. By condition (3) we have therefore $C(x_1) = \alpha_1$. The voltage is known and hence the conductance. The experimental data give the patch width and length necessary to achieve this conductance. If it is assumed that the second patch is identical to the first, using the T-matrix circuit analysis, the phase $\varphi(z_2)$ of the source representing this patch can be found as a function of z_2, its location. The patch is located so that $\varphi(z_2) = \Phi(z_2)$. The solution to this equation nearest to $z_2 = z_1$ is selected.

The amplitude of this patch is then given by $\alpha_2 = C(z_2) - C(z_1)$. The conductance of the patch is calculated and appropriate values of the patch width and length are found (leaving the phase slightly in error). This procedure is continued until all the patches have been located and assigned a width and length.

The entire procedure is iterated several times using the lengths, widths, and element spacing of each preceding iteration until the conditions in (3) are realized. If at the end of this process it is found that the widths of some patches are unacceptably high, the entire process is repeated with a reduced value of γ.

The radiation pattern of the final configuration can be calculated by the method outlined in Section II. In practice the patterns obtained agree closely with those of the uniform arrays from which they were derived. The radiation efficiency, insertion loss, and return loss are also given by the analysis.

V. OPTIMIZATION OF EXCITATION

It is desirable that the distribution of amplitude along the length of the array should not be highly peaked. A peak amplitude distribution requires high values of radiation conductance if good efficiency is to be achieved. A limit to the achievable conductance is set by the increasing excitation of the first transverse mode as the width of the patch approaches one wavelength (in dielectric).

If a limit (of, e.g., 0.6 λg) is set to the width of the patches then a peaked amplitude distribution will lead to a design with low total radiating power and consequently low efficiency. Maximum efficiency is achieved if all the patches have the maximum allowable width. The amplitude distribution would then drop exponentially away from the feed point.

A method of synthesis of the desired far-field amplitude pattern with an array is therefore required in which some control is maintained over the amplitudes of excitation of the array. Good approximations to the desired far-field amplitude pattern are possible with a variety of amplitude distributions on the array because only the far-field amplitude pattern is prescribed while the phase may be chosen arbitrarily.

If the phase of the far field is assigned, the amplitudes and phases of excitation which give the best approximation to the prescribed pattern in the least squares sense can be found directly. It suffices to find suitable amplitudes and phases of excitation for a uniform array since the interpolation procedure described in Section IV can be used to derive the geometry of the actual patch array from this. The complex excitations $\{a_n\}$ of the uniform array are found by seeking to match the pattern to the prescribed one $F(\theta_\nu)$, at a large number M (more than the number of elements) of angles θ_ν.

$$F(\theta_\nu) = \sum_{n=1}^N a_n b_{\nu n} \tag{11}$$

where

$$b_{\nu n} = \exp\{-jk_n \sin\theta_\nu\} \tag{12}$$

$$z_n = \left\{n - \frac{(N+1)}{2}\right\} d$$

where d is the element spacing.

In matrix notation:

$$F = Ba \tag{13}$$

where

$$F = \begin{bmatrix} F(\theta_1) \\ F(\theta_2) \\ \vdots \\ F(\theta_m) \end{bmatrix} \quad B = [b_{\nu n}] \quad a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}$$

The least squares solution to this overdetermined set of equations gives the complex excitation of the elements.

$$a = (B^*B)^{-1} B^*F \tag{14}$$

where the asterisk denotes transpose conjugate.

The application of this method to the ideal far-field amplitude pattern shown in Fig. 4(a) with the phase arbitrarily set constant leads to the array amplitude distribution and far-field amplitude distribution given in Fig. 4. This peaked distribution is not well suited to implementation of a patch array. An efficiency of 32 percent is obtained with a maximum patch width of 0.4 λg.

To obtain a more favorable amplitude distribution, we must remove the constraint on the far-field phase distribution and impose a constraint on the excitation amplitude distribution. An iterative algorithm which has been found to yield excellent results is the following. An initial coarse estimate is made of the far-field phase which is compatible with the desired far-field amplitude pattern and the desired distribution of excitation amplitude. A ray-optics method based on Chu's synthesis technique [8] has been found to be satisfactory.

In this method it is assumed that the power radiated by the nth element of the array is confined to the range of angle θ_{n-1} to θ_n. Referring to Fig. 5, we have

$$\sum_1^n |a_i|^2 = \int_{\theta_0}^{\theta_n} p(\theta) \cos\theta \, d\theta \tag{15}$$

where $p(\theta)$ is the prescribed power pattern, normalized so that

$$\sum_1^N |a_i|^2 = \int_{\theta_0}^{\theta_N} p(\theta) \cos\theta \, d\theta. \tag{16}$$

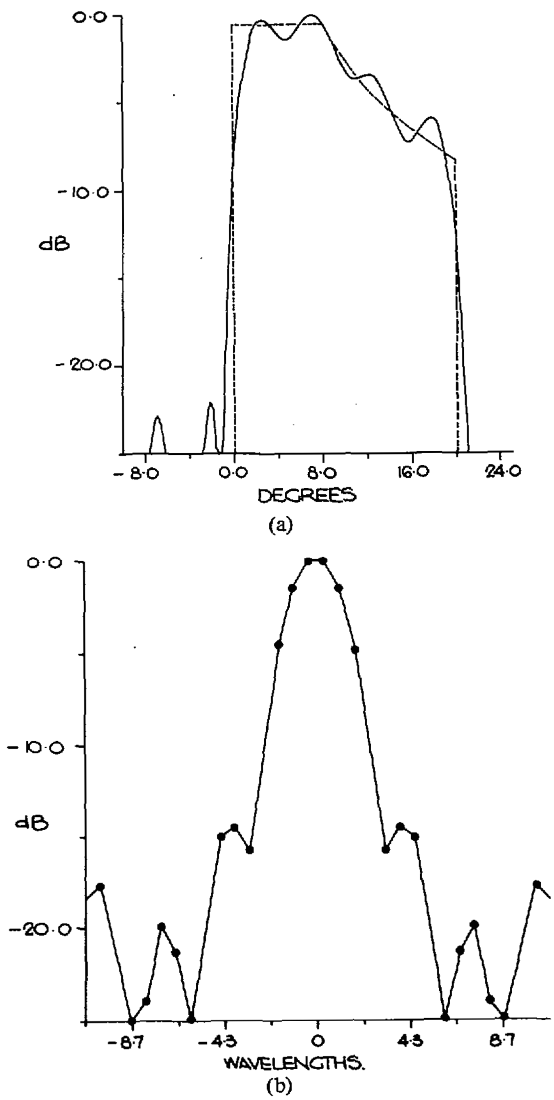


Fig. 4. (a) Ideal amplitude pattern and pattern obtained by least squares fitting assuming phase of far field to be zero. (b) Amplitudes of excitation of patch array obtained by this method.

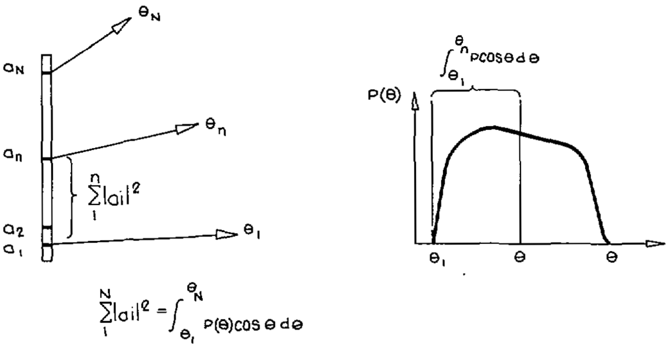


Fig. 5. Approximation used in Chu's optical method of pattern synthesis.

Using the equation, given the required amplitudes of excitation $|a_i|$, and the prescribed power distribution $p(\theta)$, the angles θ_n can be found.

Knowing the angles at which the radiation from each element is centered, a smooth phase distribution can be set up so that the phase gradient at each element is that appropriate to radiation in the required direction.

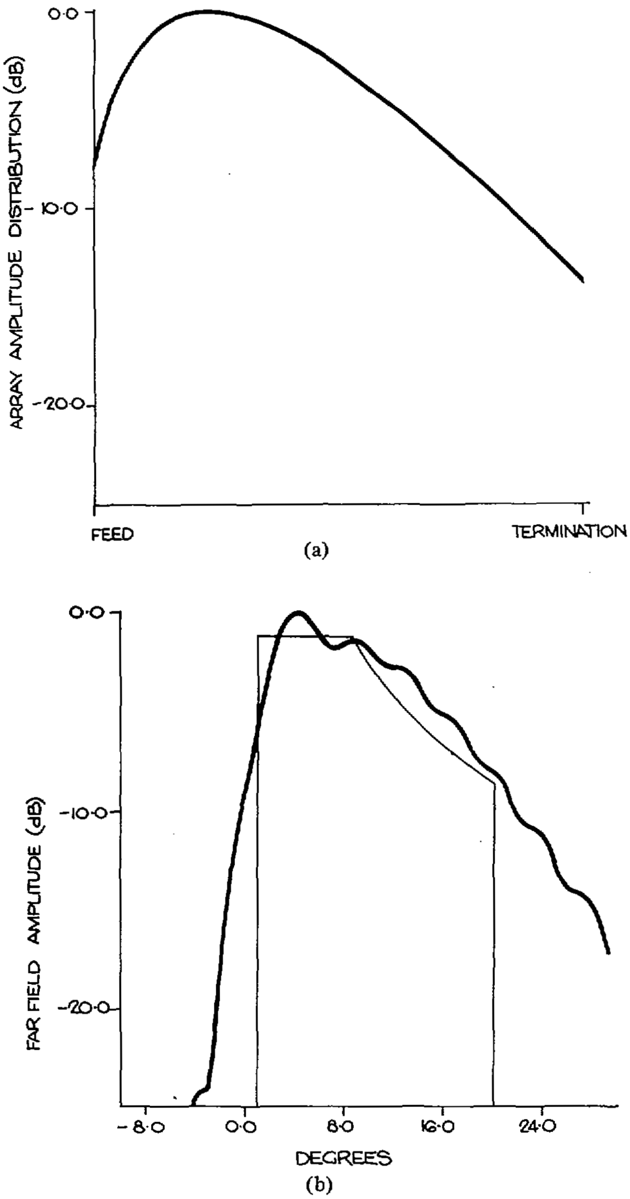


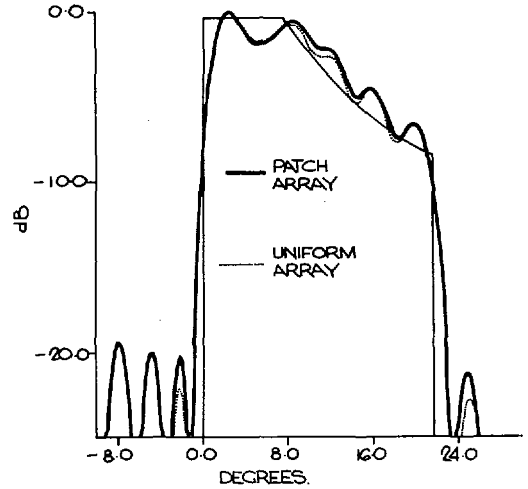
Fig. 6. (a) Assigned amplitude of excitation. (b) Amplitude pattern obtained by Chu's optical method of pattern synthesis.

The method becomes accurate if the θ_n are closely spaced so that adequate length of the array is available to confine the radiation to the required range of directions. A desired amplitude distribution and the far-field amplitude and phase pattern obtained by this method is shown in Fig. 6.

This coarse estimate of the far-field phase and the ideal far-field amplitude distribution are now used to calculate the required amplitude and phase of excitation of the array elements using the least squares method described above. The amplitude of excitation is discarded and replaced by the desired amplitude of excitation before calculating the far-field pattern with an improved estimate of the required far-field phase distribution.

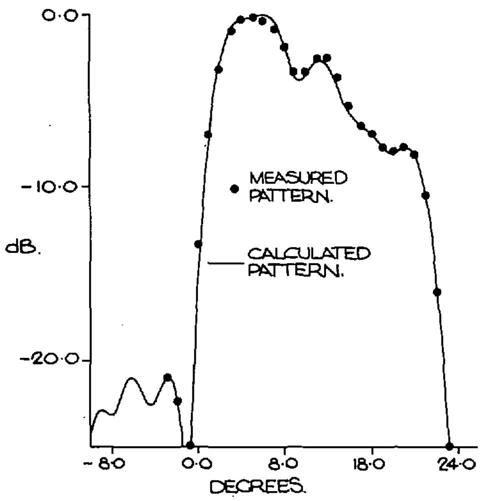
The process is continued until convergence has occurred. The result is illustrated in Fig. 7. The amplitude of excitation obtained from the final least squares synthesis resembles closely the desired one. For comparison the amplitudes and pattern of the patch array derived from this by the method of Section VI are also given. It is seen that application of the method results in little degradation of the pattern.

(a)

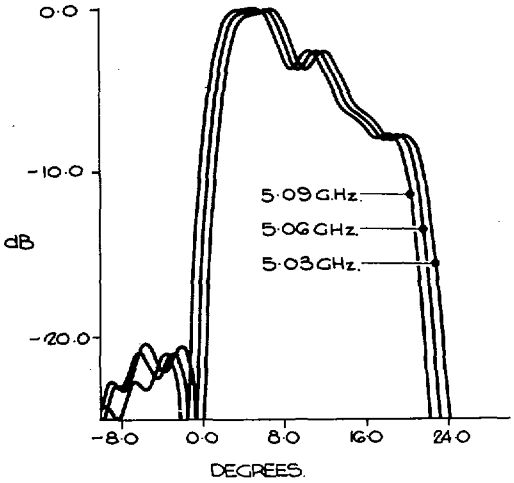


(b)

Fig. 7. (a) Amplitudes of excitation for uniform and patch array obtained by algorithm of text. (b) Far-field amplitude patterns of uniform and patch arrays.



(a)



(b)

Fig. 8. (a) Comparison of measured and calculated amplitude patterns of a patch array. (b) Effect on pattern of a 1 percent change in frequency.

TABLE I
PARAMETERS OF PATCH ARRAY

Substrate	PTFE Fiberglass, thickness 1.5 mm $\epsilon_r = 2.55$.
Frequency	5 GHz
Number of patches	32
Length of array	1100 mm
Characteristic impedance of connecting line	50 Ω
Radiation efficiency	71 percent
Maximum patch width	21 mm
Power dissipated in microstrip	22 percent
Power dissipated in termination	7 percent
Return loss	−25 dB

The pattern obtained with this distribution is also shown. The radiation efficiency obtained with this distribution and the parameters of Table I is 71 percent which may be compared with the figure of 32 percent obtained for the array of Fig. 4.

This iterative technique is similar to one described by Mautz and Harrington [9] for optimizing the excitation of an array for synthesizing a far-field amplitude pattern when the far-field phase is unconstrained.

VI. RESULTS

An array was designed using the techniques described to give the pattern of Fig. 4(a) with the characteristics given in Table I. The measured pattern at the design frequency is given in Fig. 8(a), together with the pattern calculated from the circuit analysis. It is seen that the agreement is excellent. Difficulty was initially experienced with disturbance to the pattern caused by radiation from the coaxial to microstrip transitions. Small metal shields lined with absorber were later placed over these, and the pattern of Fig. 8 was obtained. The effect on the pattern of a 1 percent change in frequency is also shown. The effect is almost entirely a translation of the pattern resulting from a net change of phase gradient along the array. This dispersion is accurately predicted by the circuit model and is common to all antennas, such as slotted waveguide antennas, consisting of radiating elements loosely coupled to a traveling wave. The return loss of the antenna was better than −20 dB over a 3 percent frequency range.

VII. CONCLUSION

Using the design techniques described here it is possible to synthesize accurately shaped patterns with low sidelobe levels with series fed microstrip patch arrays and with high efficiencies. The arrays themselves are an attractive alternative to edge-slotted waveguide antennas.

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