Computer Vision

Spring 2006 15-385,-685

Instructor: S. Narasimhan

Wean 5403

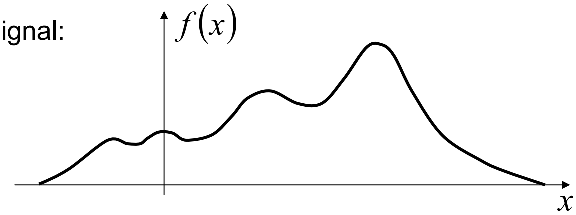
T-R 3:00pm – 4:20pm

Image Resampling and Pyramids

Lecture #8

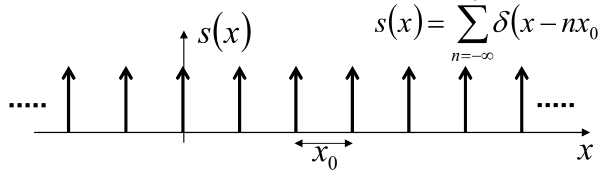
RECAP - Sampling Theorem

Continuous signal:



Shah function (Impulse train):

$$s(x)=\sum_{n=-\infty}^{\infty}\delta(x-nx_0)$$



Sampled function:

$$f_s(x)=f(x)s(x)=f(x)\sum_{n=-\infty}^{\infty}\delta(x-nx_0)$$

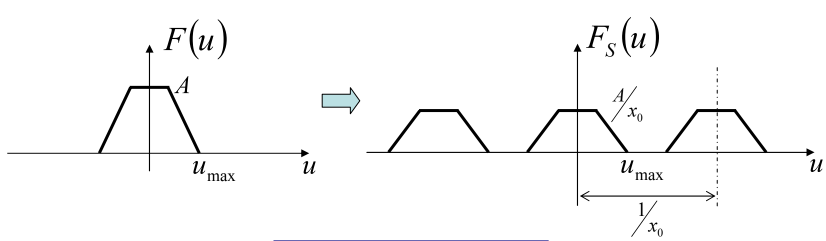
RECAP - Sampling Theorem

Sampled function:

$$f_s(x)=f(x)s(x)=f(x)\sum_{n=-\infty}^{\infty}\delta(x-nx_0)$$

Sampling frequency $\frac{1}{x_0}$

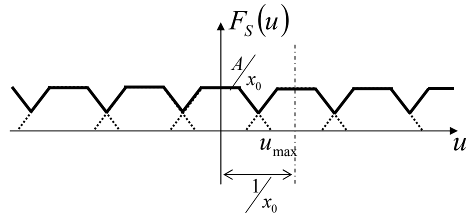
$$F_S(u)=F(u)*S(u)=F(u)*\frac{1}{x_0}\sum_{n=-\infty}^{\infty}\delta\left(u-\frac{n}{x_0}\right)$$



Only if $u_{\max}\le\frac{1}{2x_0}$

RECAP - Nyquist Theorem

If $u_{\max} > \dfrac{1}{2x_0}$



Aliasing

When can we recover $F(u)$ from $F_S(u)$?

$\boxed{\text{Only if } u_{\max} \le \dfrac{1}{2x_0} \text{ (Nyquist Frequency)}}$

We can use

$$C(u) = \begin{cases} x_0 & |u| < 1/2x_0 \\ 0 & \text{otherwise} \end{cases}$$

Then $F(u) = F_S(u)C(u)$ and $f(x) = \mathrm{IFT}[F(u)]$

$\boxed{\text{Sampling frequency must be greater than } 2u_{\max}}$

Aliasing

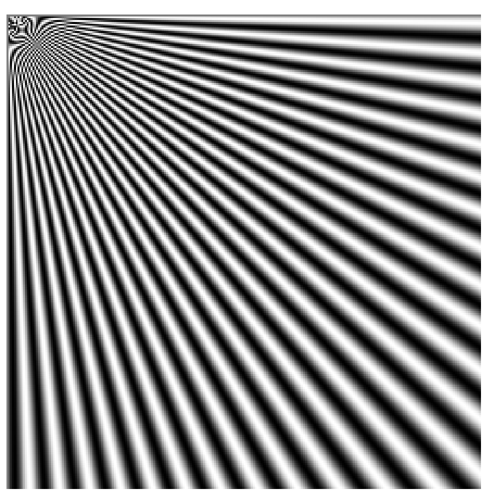


Image Scaling

This image is too big to fit on the screen. How can we reduce it?

How to generate a half-sized version?

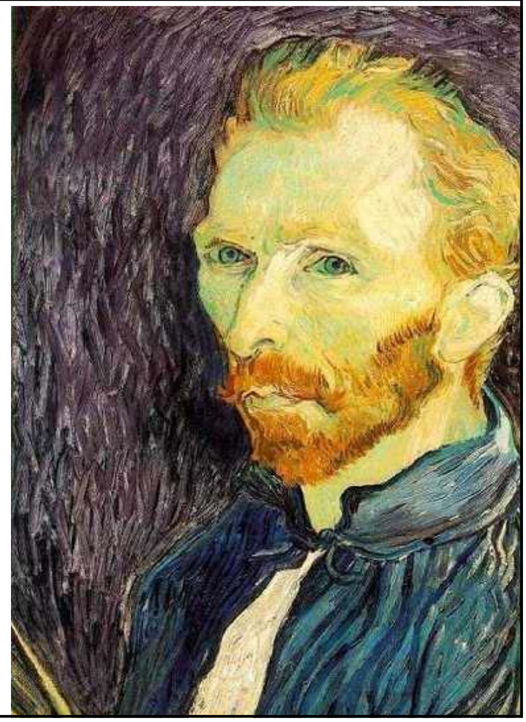
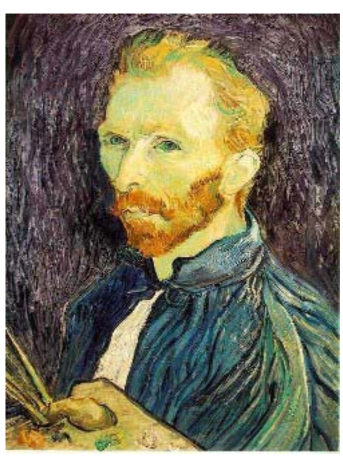


Image Sub-Sampling

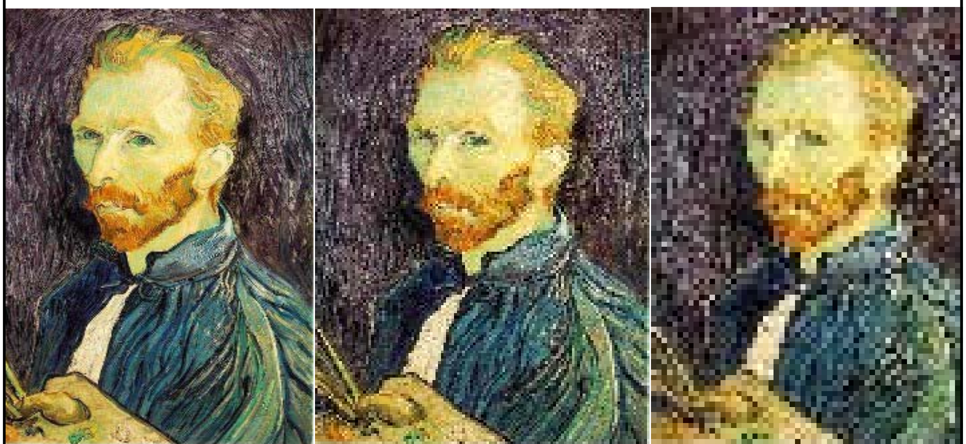


1/4

1/8

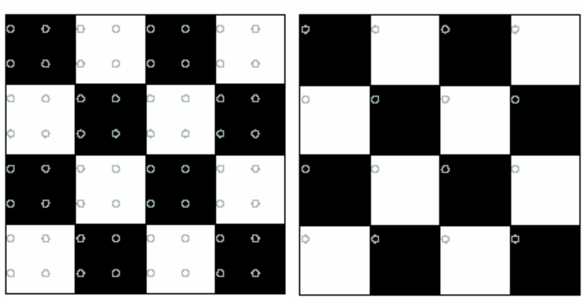
Throw away every other row and column to create a 1/2 size image - called *image sub-sampling*

Image Sub-Sampling

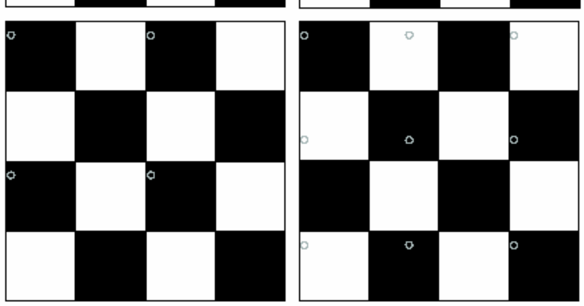


1/2 1/4 (2x zoom) 1/8 (4x zoom)

Good and Bad Sampling

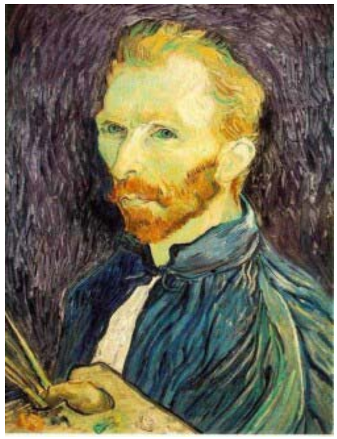


Good sampling:
•Sample often or,
•Sample wisely



Bad sampling:
•see aliasing in action!

Sub-Sampling with Gaussian Pre-Filtering



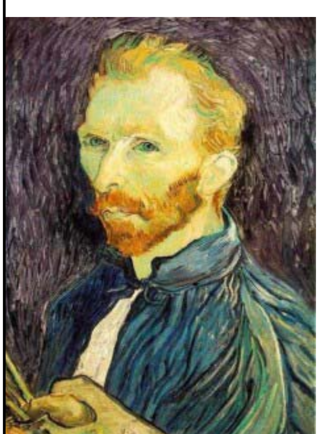
Gaussian 1/2

G 1/4

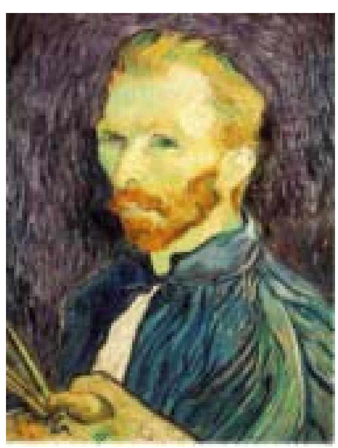
G 1/8

- Solution: filter the image, *then* subsample
  - Filter size should double for each ½ size reduction. Why?

Sub-Sampling with Gaussian Pre-Filtering



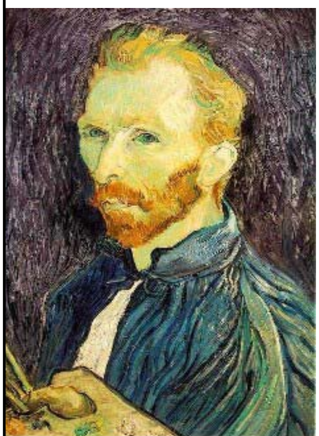
Gaussian 1/2



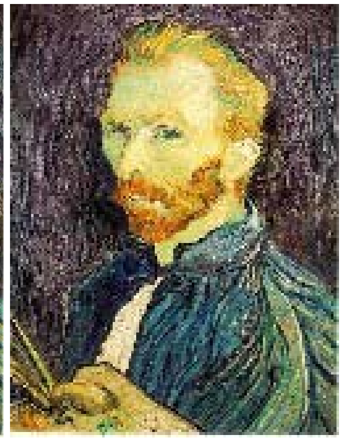
G 1/4

G 1/8

Compare with...



1/2

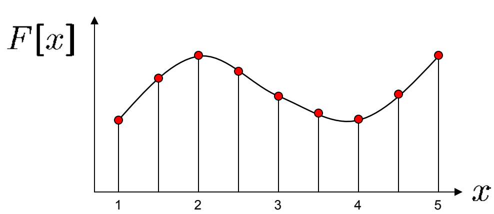


1/4 (2x zoom)

1/8 (4x zoom)

Image Resampling

- What about arbitrary scale reduction?
- How can we increase the size of the image?



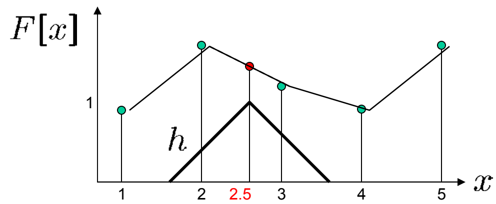
- Recall how a digital image is formed

$$F[x, y] = \text{quantize}\{f(xd, yd)\}$$

  - It is a discrete point-sampling of a continuous function
  - If we could somehow reconstruct the original function, any new image could be generated, at any resolution and scale

Image Resampling

- So what to do if we don't know f
  - Answer: guess an approximation $\tilde{f}$
  - Can be done in a principled way: filtering



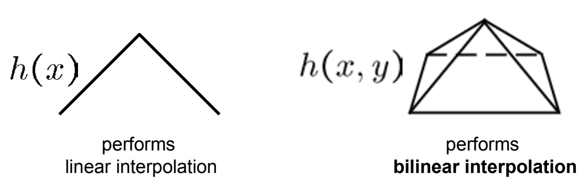
- Image reconstruction
  - Convert F to a continuous function

    $f_F(x) = F(\frac{x}{d})$ when $\frac{x}{d}$ is an integer, 0 otherwise
  - Reconstruct by cross-correlation:

$$\tilde{f} = h \otimes f_F$$

Resampling filters

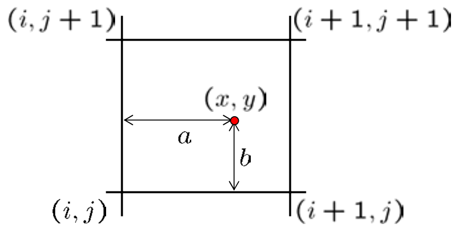
- What does the 2D version of this hat function look like?



- Better filters give better resampled images
  - Bicubic is common choice

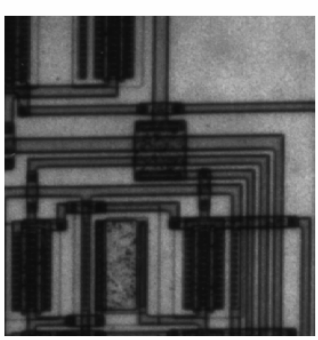
Bilinear interpolation

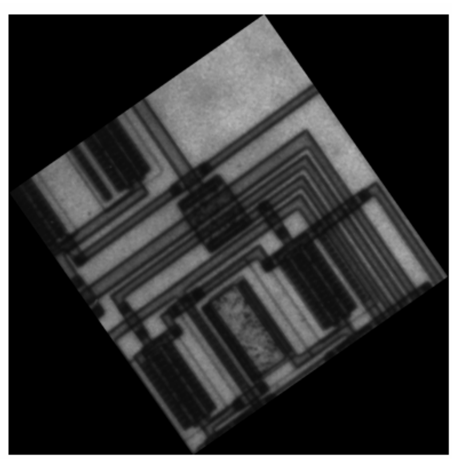
- A common method for resampling images



$$
\begin{aligned}
F(x,y) = \;& (1-a)(1-b) \quad F(i,j) \\
& +a(1-b) \quad F(i+1,j) \\
& +ab \quad F(i+1,j+1) \\
& +(1-a)b \quad F(i,j+1)
\end{aligned}
$$

Image Rotation



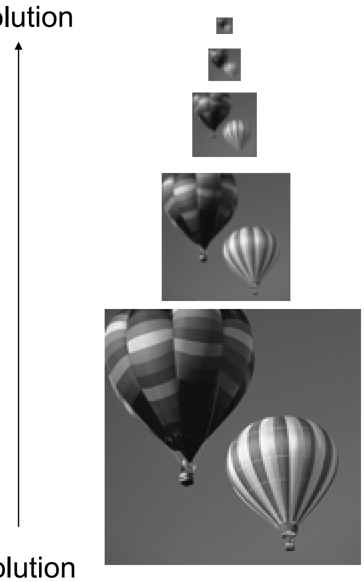


Multi-Resolution Image Representation

- Fourier domain tells us “what” (frequencies, sharpness, texture properties), but not “where”.
- Spatial domain tells us “where” (pixel location) but not “what”.
- We want a image representation that gives a local description of image “events” – what is happening where.
- Naturally, think about representing images across varying scales.

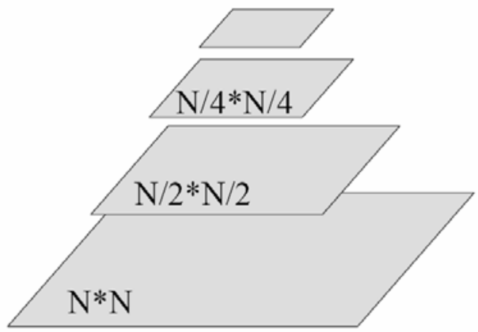
Multi-resolution Image Pyramids

Low resolution



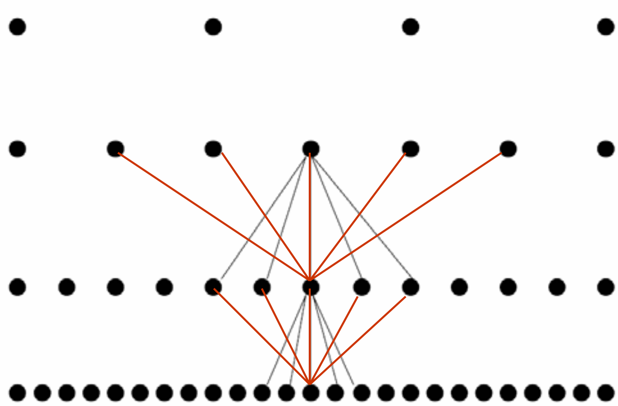
High resolution

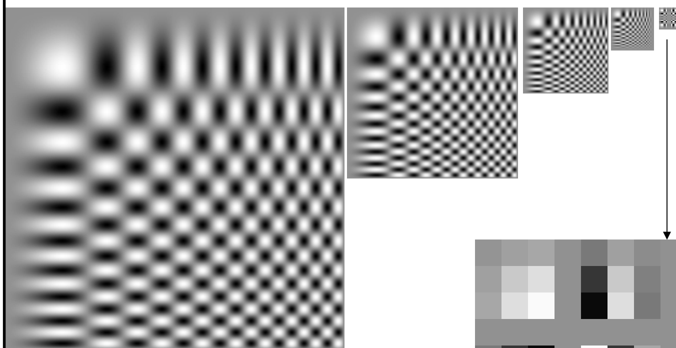
Space Required for Pyramids

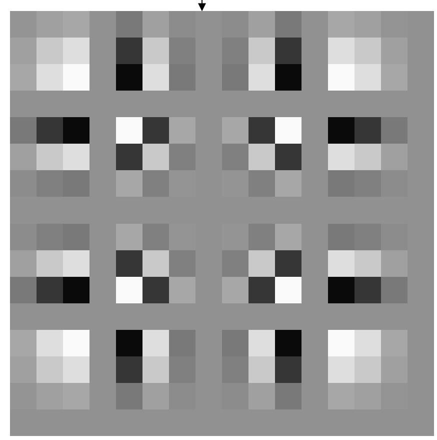


$$N^2 + \frac{1}{4}N^2 + \frac{1}{16} + N^2 + \cdots = 1\frac{1}{3}N^2$$

Pyramid construction

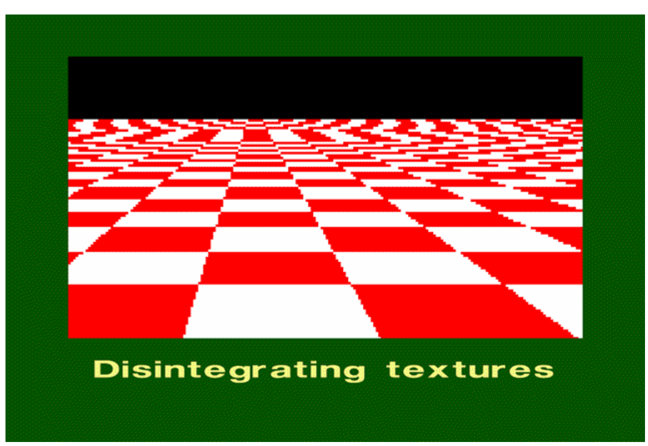




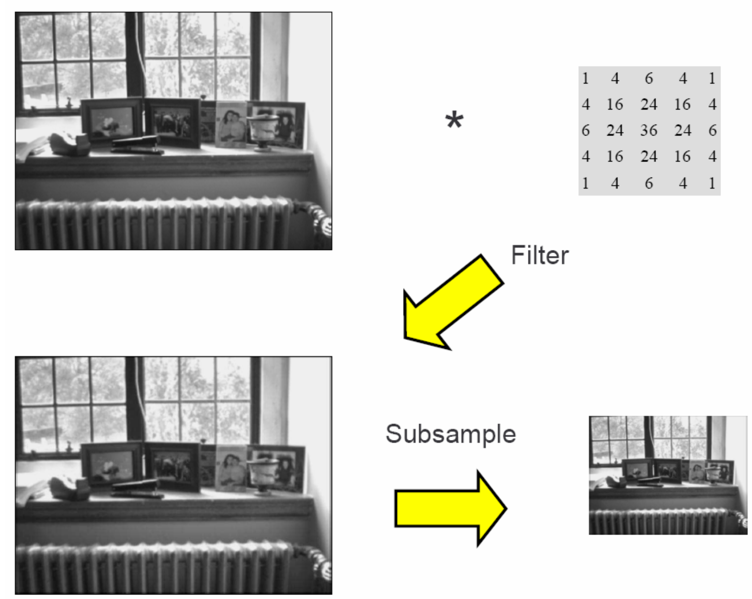


Constructing a pyramid by taking every second pixel leads to layers that badly misrepresent the top layer

Even worse for synthetic images

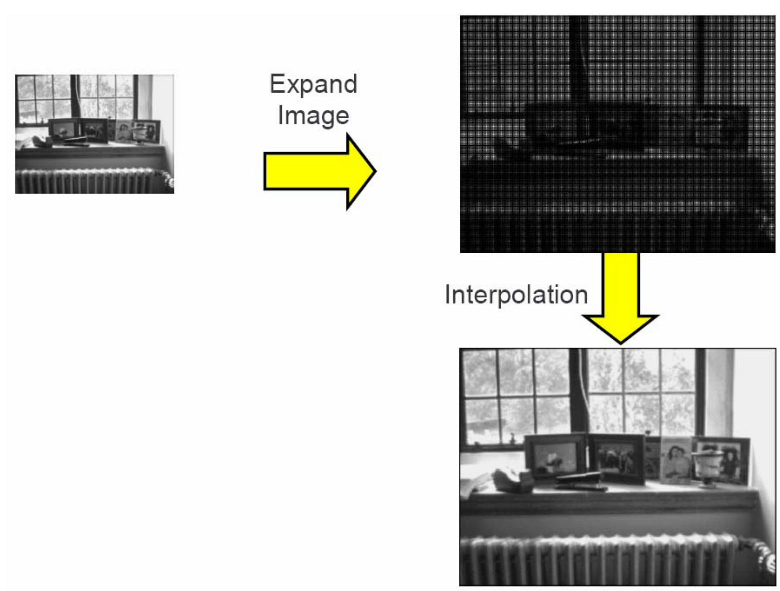


Decimation

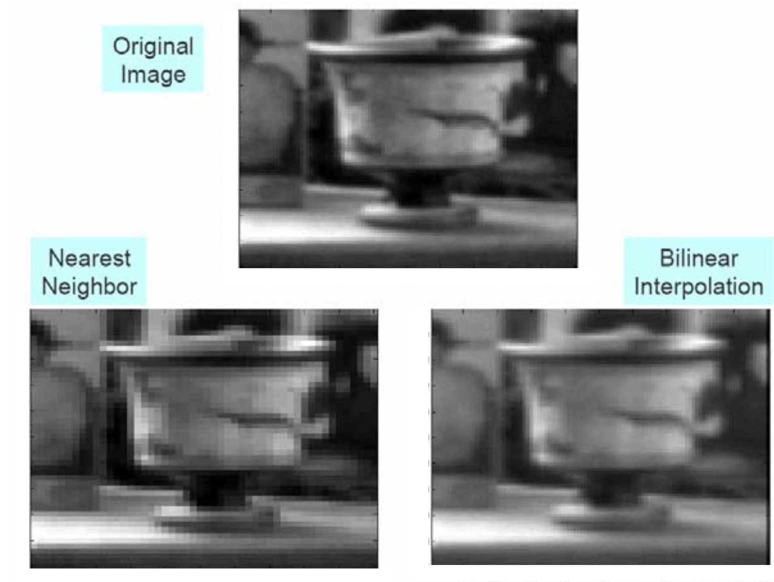


1	4	6	4	1
4	16	24	16	4
6	24	36	24	6
4	16	24	16	4
1	4	6	4	1

Expansion

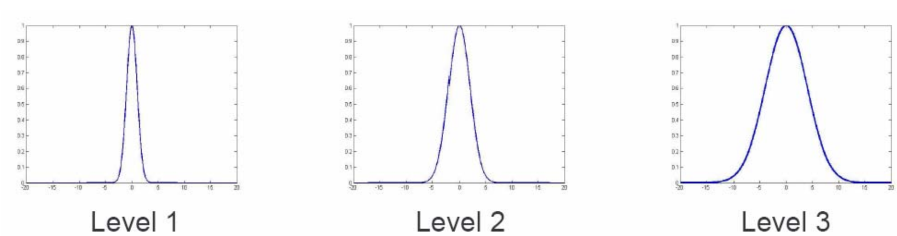


Interpolation Results



The Gaussian Pyramid

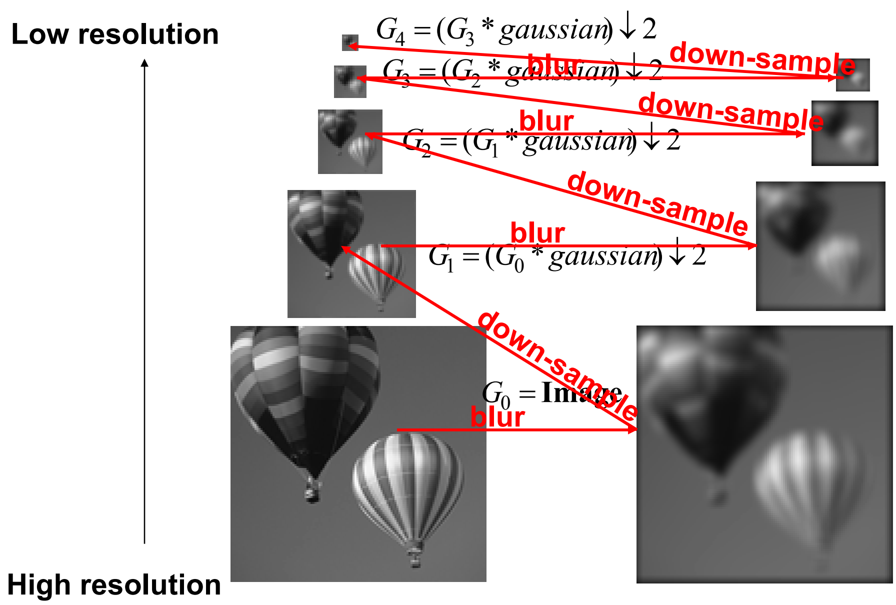
- Smooth with Gaussians because
  - a Gaussian*Gaussian=another Gaussian
- Synthesis
  - smooth and downsample
- Gaussians are low pass filters, so repetition is redundant
- Kernel width doubles with each level

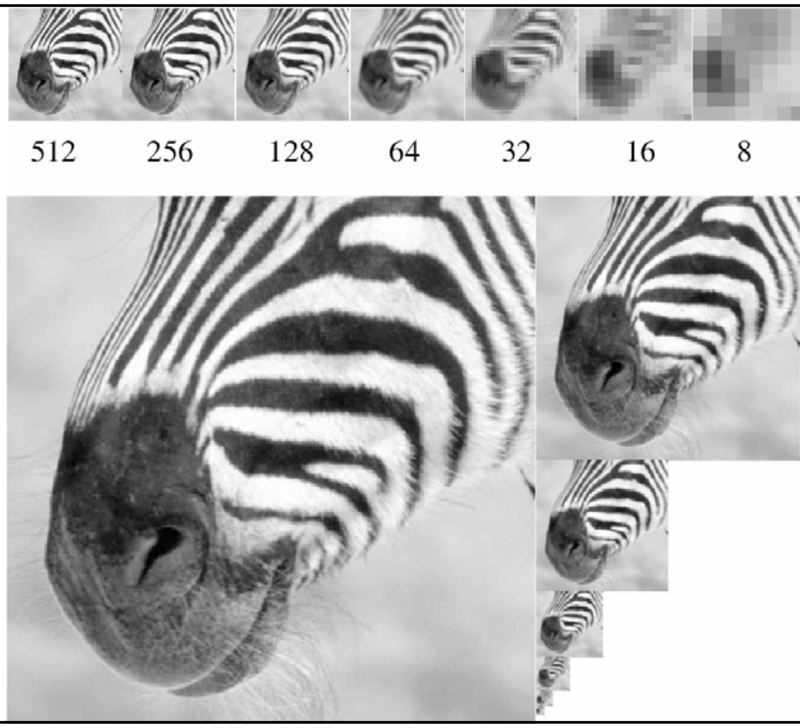


Smoothing as low-pass filtering

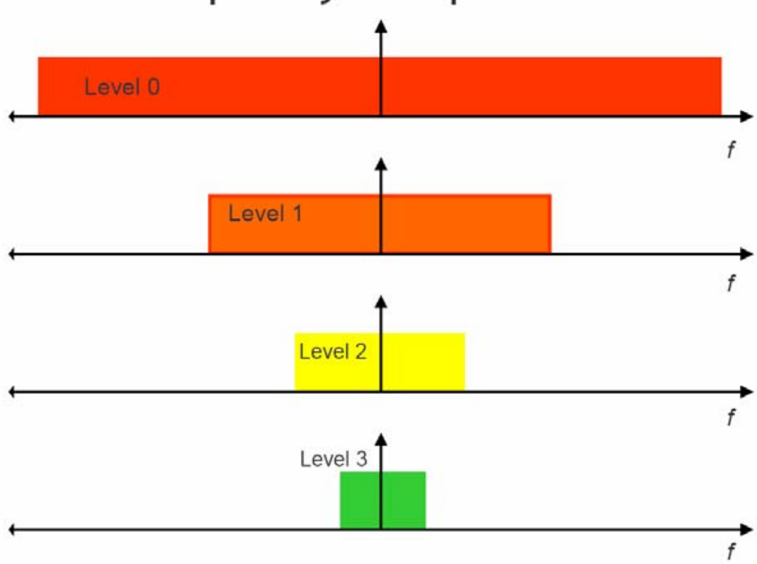
- High frequencies lead to trouble with sampling.
- Suppress high frequencies before sampling !
  - truncate high frequencies in FT
  - or convolve with a low-pass filter
- Common solution: use a Gaussian
  - multiplying FT by Gaussian is equivalent to convolving image with Gaussian.

The Gaussian Pyramid

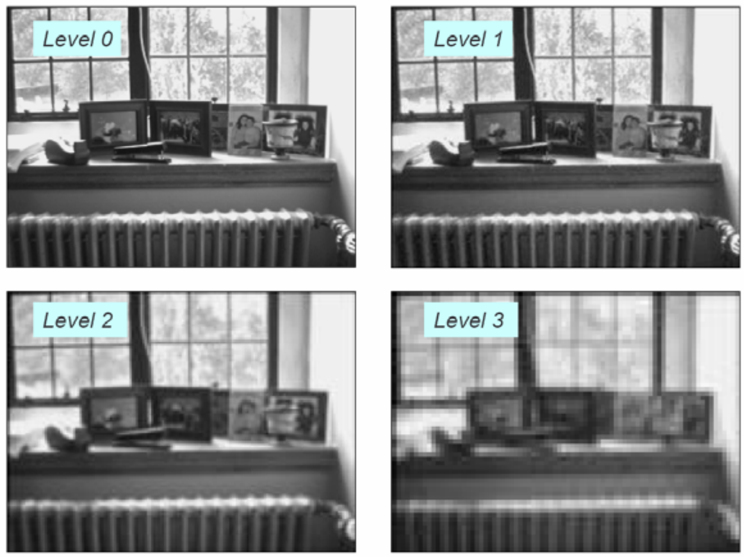




Gaussian Pyramid Frequency Composition



Pyramids at Same Resolution



Difference of Gaussians (DoG)

- Laplacian of Gaussian can be approximated by the difference between two different Gaussians

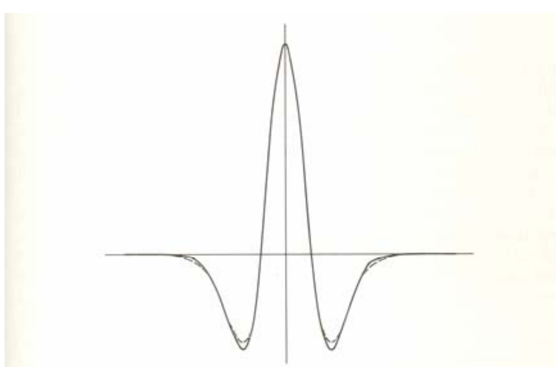
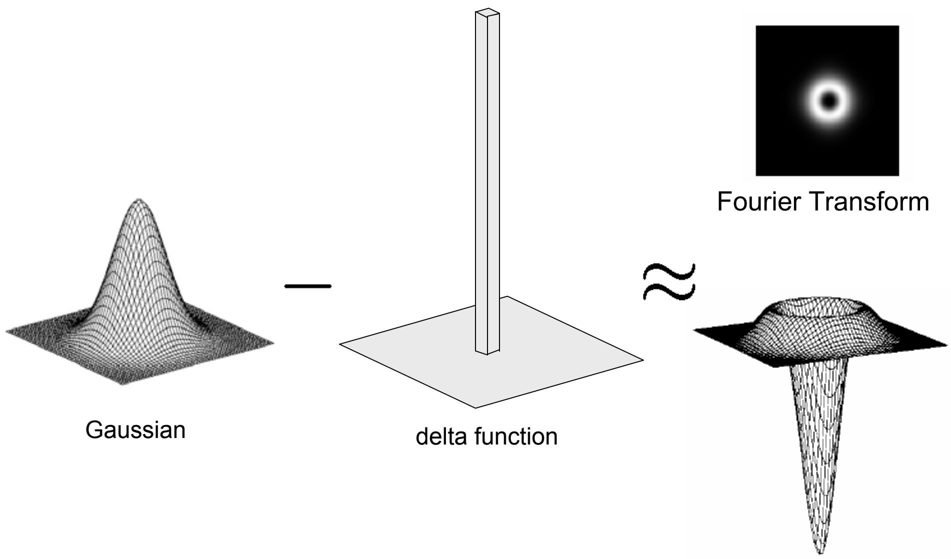


Figure 2–16. The best engineering approximation to $\nabla^2 G$ (shown by the continuous line), obtained by using the difference of two Gaussians (DOG), occurs when the ratio of the inhibitory to excitatory space constraints is about 1:1.6. The DOG is shown here dotted. The two profiles are very similar. (Reprinted by permission from D. Marr and E. Hildreth, "Theory of edge detection," *Proc. R. Soc. Lond. B 204*, pp. 301–328.)

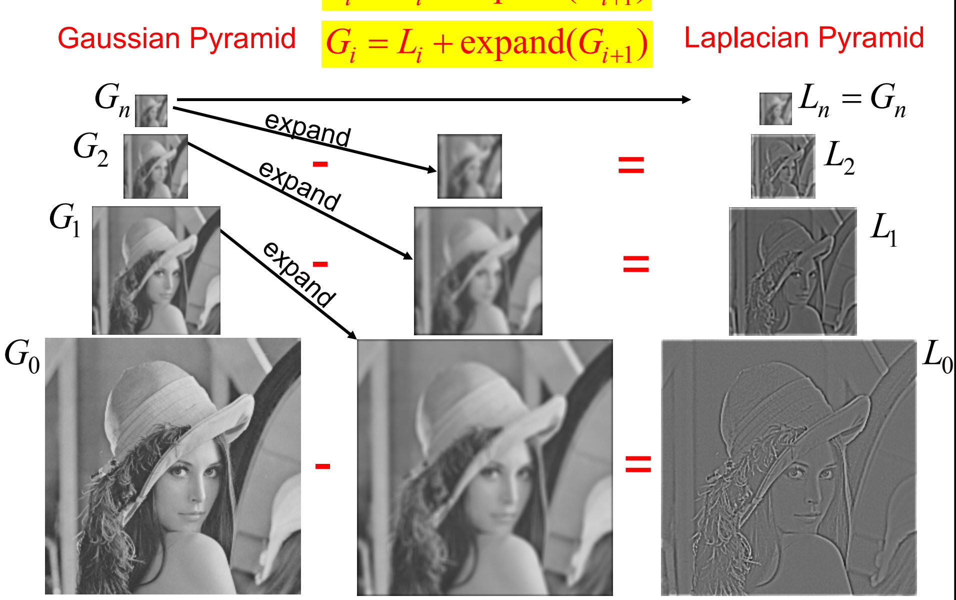
Gaussian – Image filter

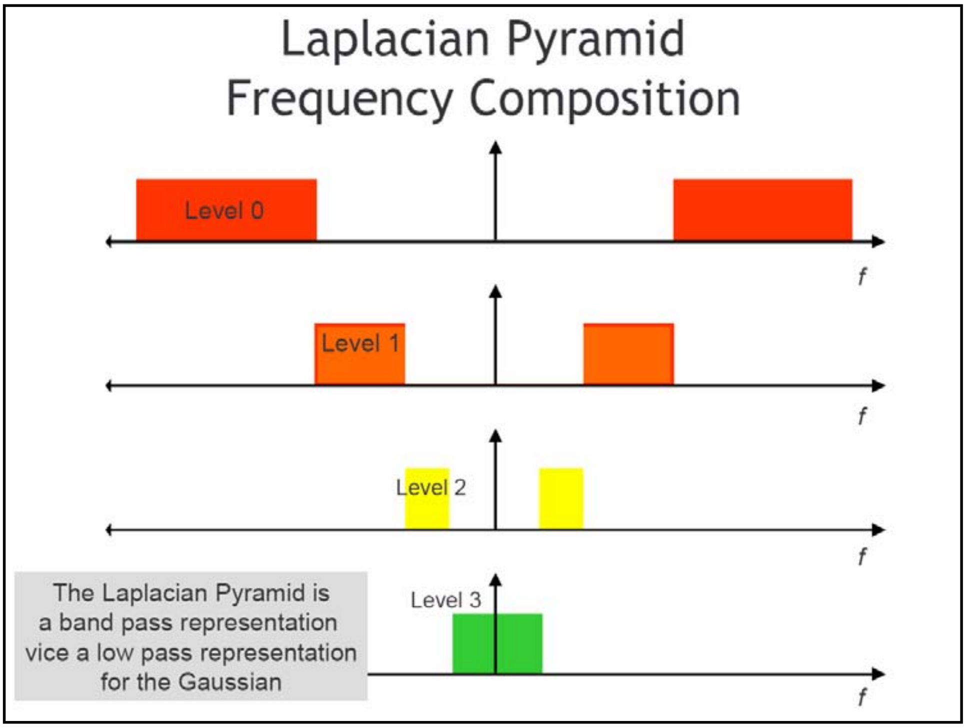


The Laplacian Pyramid

$$L_i = G_i - \text{expand}(G_{i+1})$$

$$G_i = L_i + \text{expand}(G_{i+1})$$





Applications of Image Pyramids

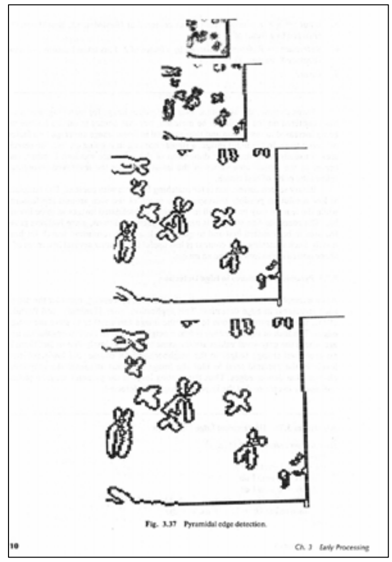
- Coarse-to-Fine strategies for computational efficiency.
- Search for correspondence
  - look at coarse scales, then refine with finer scales
- Edge tracking
  - a “good” edge at a fine scale has parents at a coarser scale
- Control of detail and computational cost in matching
  - e.g. finding stripes
  - very important in texture representation
- Image Blending and Mosaicing
- Data compression (laplacian pyramid)

Edge Detection using Pyramids

Coarse-to-fine strategy:

• Do edge detection at higher level.

• Consider edges of finer scales only near the edges of higher scales.



Fast Template Matching

Template

Search Region

- For an m x n image…
- For a p x q template…
- The complexity of the 2D pattern recognition task is O(mnpq) ☹
- This gets even worse for a family of templates (*e.g.*, to address scale and/or rotational effects)

Fast Template Matching

Template

Search Region

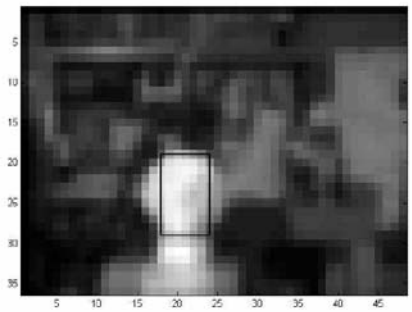
Original Image

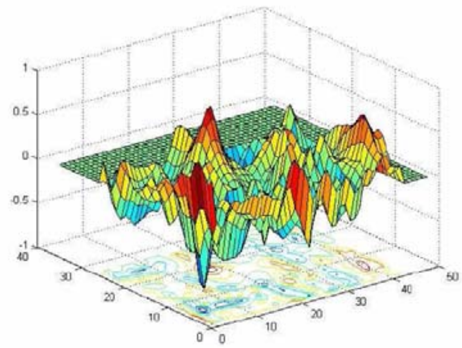
Multi-resolution correlation

- Multi-resolution template matching
  - reduce resolution of both template and image by creating an **image pyramid**
  - match small template against small image
  - identify locations of strong matches
  - expand the image and template, and match higher resolution template selectively to higher resolution image
  - iterate on higher and higher resolution images
- Issue:
  - how to choose detection thresholds at each level
    - too low will lead to too much cost
    - too high will miss match

Level 3 Search

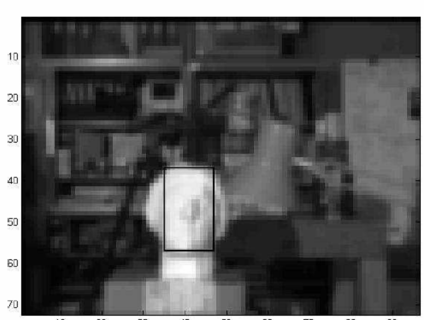
- At the lowest pyramid level, we search the entire image with the correlation template

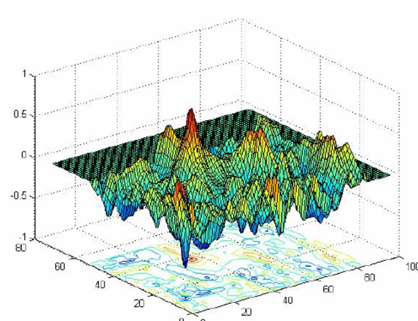




Level 2 Search

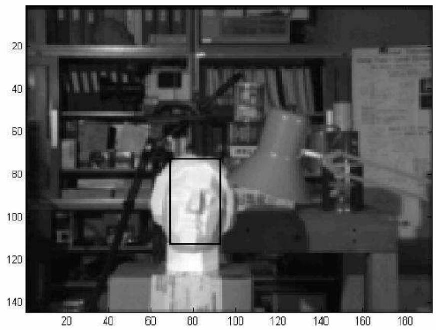
- Subsequent searches are constrained to a neighborhood of only several pixels in the x and y directions

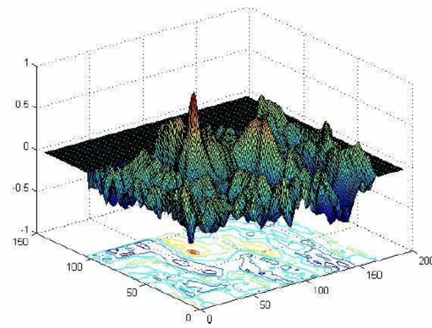




Level 1 Search

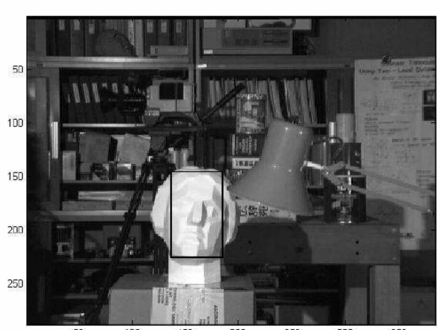
- Subsequent searches are constrained to a neighborhood of only several pixels in the x and y directions





Level 0 Search

- In the end, the total time (in Matlab) was reduced from ≈ 31 seconds to ≈ 0.5 seconds while obtaining the same template match



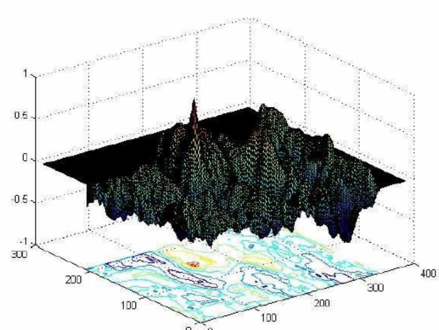


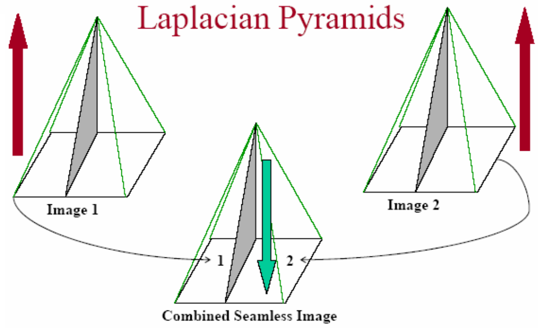
Image Blending and Mosaicing

- Given two images A and B
- Construct Laplacian Pyramid L_a and L_b
- Create a third Laplacian Pyramid L_c where for each level l

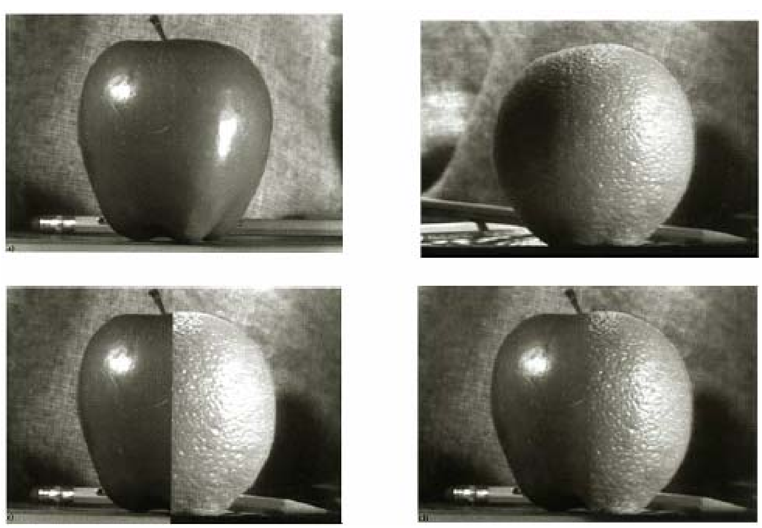
$$L_c(i,j) = \begin{cases} L_a(i,j) & \text{if } i < width/2 \\ (L_a(i,j) + L_b(i,j))/2 & \text{if } i = width/2 \\ L_b(i,j) & \text{if } i > width/2 \end{cases}$$

- Sum all levels L_c in to get the blended image

Image Merging with Laplacian Pyramids



Blending Apples and Oranges



Pyramid blending of Regions

- Given two images A and B, and a mask M
- Construct Laplacian Pyramids L_a and L_b
- Construct a Gaussian Pyramid G_m
- Create a third Laplacian Pyramid L_c where for each level l

$$L_c(i,j) = G_m(i,j)L_a(i,j) + (1 - G_m(i,j))L_b(i,j)$$

- Sum all levels L_c in to get the blended image

Horror Photo

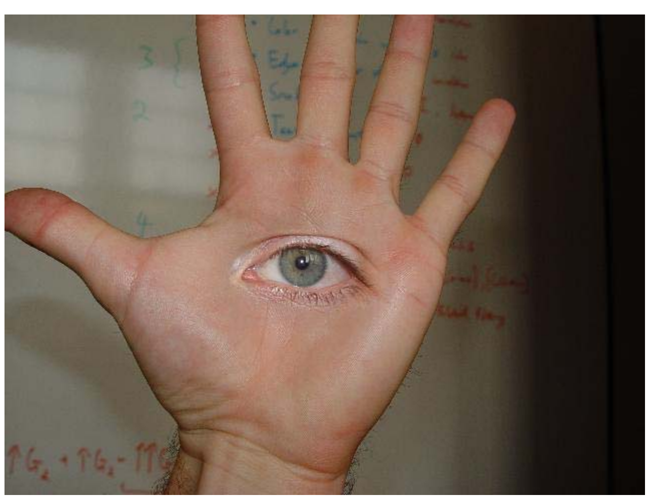
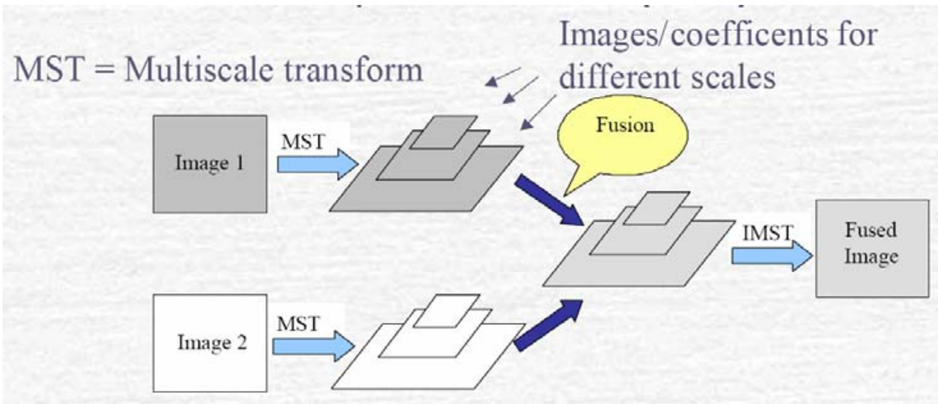


Image Fusion



Multi-scale Transform (MST) = Obtain Pyramid from Image
Inverse Multi-scale Transform (IMST) = Obtain Image from Pyramid

Multi-Sensor Fusion

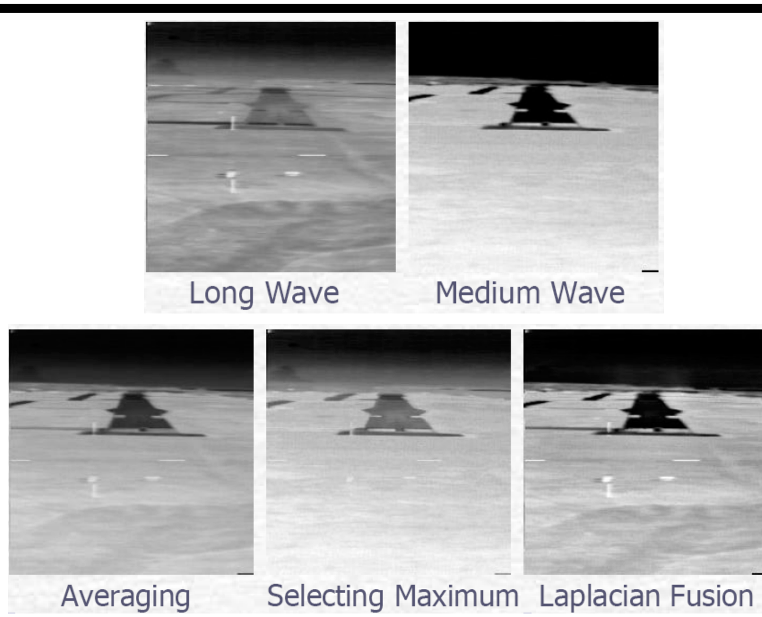


Image Compression

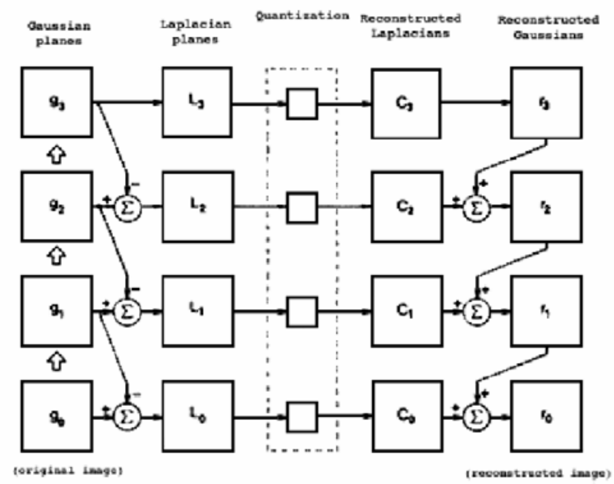


Fig. 10. A summary of the steps in Laplacian pyramid coding and decoding. First, the original image g_0 (lower left) is used to generate Gaussian pyramid levels g_1, g_2, … through repeated local averaging. Levels of the Laplacian pyramid L_0, L_1, … are then computed as the differences between adjacent Gaussian levels. Laplacian pyramid elements are quantized to yield the Laplacian pyramid code C_0, C_1, C_2, …. Finally, a reconstructed image r_0 is generated by summing levels of the code pyramid.

Next Class

- Hough Transform