Computer Vision

Spring 2006 15-385,-685

Instructor: S. Narasimhan

Wean 5403

T-R 3:00pm – 4:20pm

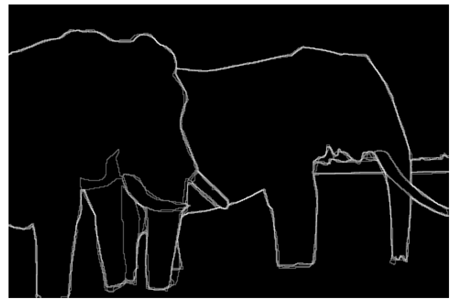
Boundary Detection: Hough Transform

Lecture #9

Reading: Computer Vision (Ballard and Brown): Chapter 4

"Use of the Hough Transform to detect lines and curves in pictures", Comm. ACM 15, 1, January 1972 (pgs 112-115)

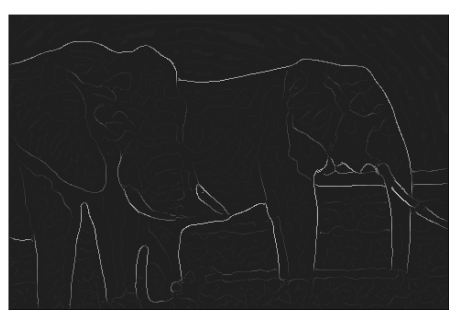
Boundaries of Objects



Marked by many users

http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/bench/html/images.html

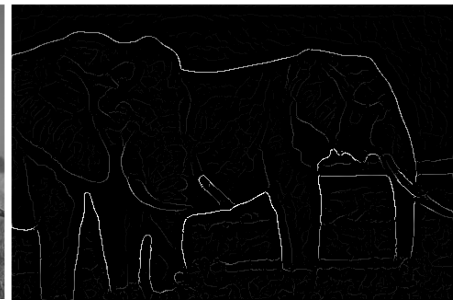
Boundaries of Objects from Edges



Brightness Gradient (Edge detection)

- Missing edge continuity, many spurious edges

Boundaries of Objects from Edges

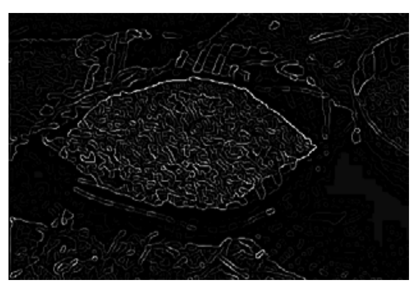


Multi-scale Brightness Gradient

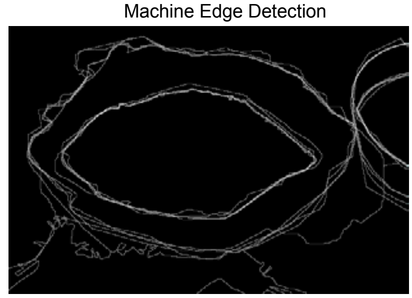
- But, low strength edges may be very important

Boundaries of Objects from Edges

Image



Machine Edge Detection



Human Boundary Marking

Boundaries in Medical Imaging

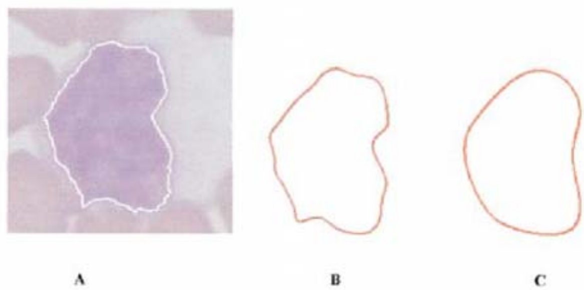
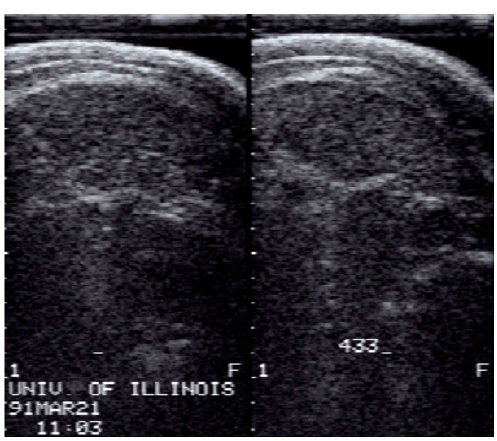


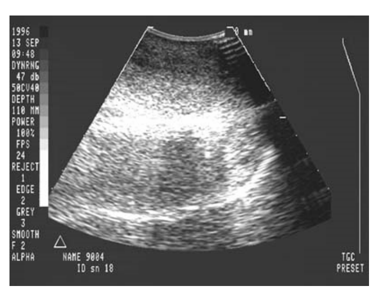
Fig. 2. Representation of a closed contour by elliptic Fourier descriptors. (a) Input. (b) Series truncated at 16 harmonics. (c) Series truncated to four harmonics.

Detection of cancerous regions.

[Foran, Comaniciu, Meer, Goodell, 00]

Boundaries in Ultrasound Images





Hard to detect in the presence of large amount of speckle noise

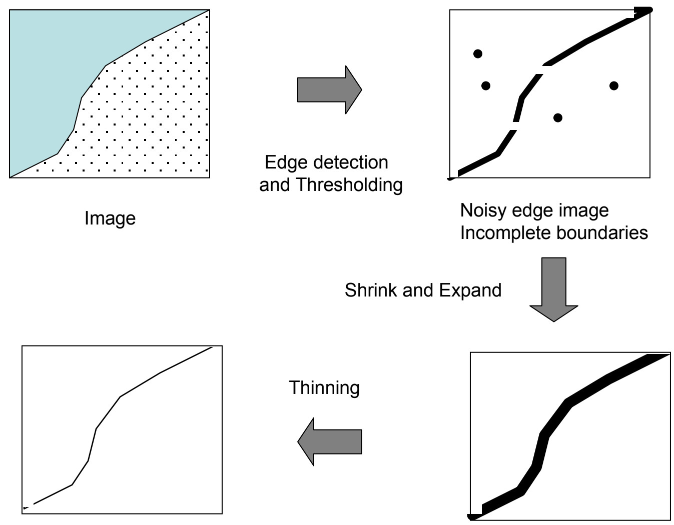
Boundaries of Objects

Sometimes hard even for humans!

Topics

- Preprocessing Edge Images
- Edge Tracking Methods
- Fitting Lines and Curves to Edges
- The Hough Transform

Preprocessing Edge Images



Edge Tracking Methods

Adjusting a priori Boundaries:

Given: Approximate Location of Boundary

Task: Find Accurate Location of Boundary

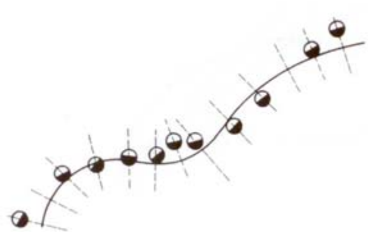


Fig. 4.2 Search orientations from an approximate boundary location.

• Search for STRONG EDGES along normals to approximate boundary.

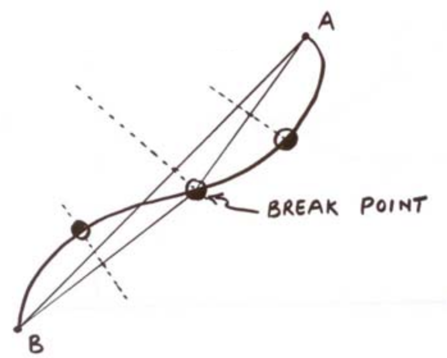
• Fit curve (eg., polynomials) to strong edges.

Edge Tracking Methods

Divide and Conquer:

Given: Boundary lies between points A and B

Task: Find Boundary

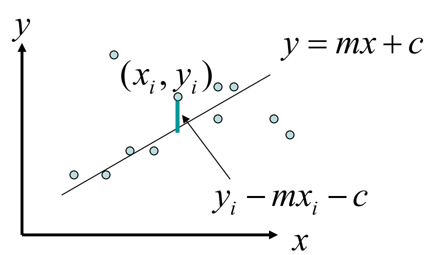


- Connect A and B with Line
- Find strongest edge along line bise
- Use edge point as break point
- Repeat

Fitting Lines to Edges (Least Squares)

Given: Many (x_i, y_i) pairs

Find: Parameters (m, c)



Minimize: Average square distance:

$$E = \sum_i \frac{(y_i - mx_i - c)^2}{N}$$

Using:

$$\frac{\partial E}{\partial m} = 0 \quad \& \quad \frac{\partial E}{\partial c} = 0$$

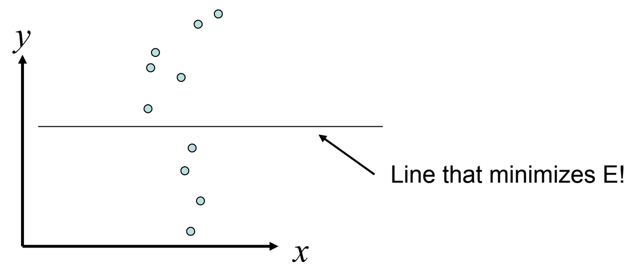
Note:

$$\bar{y} = \frac{\sum_i y_i}{N} \qquad \bar{x} = \frac{\sum_i x_i}{N}$$

$$c = \bar{y} - m\,\bar{x}$$

$$m = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}$$

Problem with Parameterization

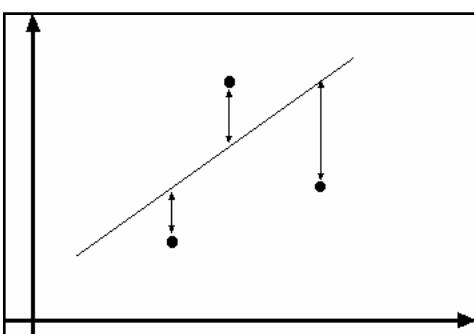


Solution: Use a different parameterization

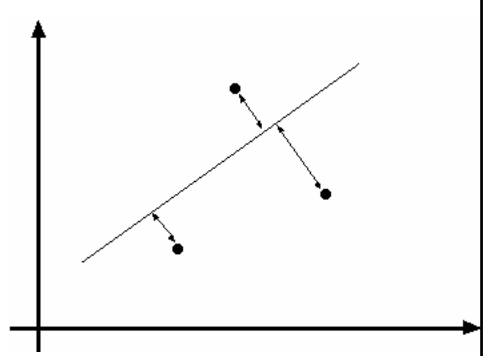
(same as the one we used in computing Minimum Moment of Inertia)

$$E = \frac{1}{N}\sum_i (\rho - x_i \cos\theta + y_i \sin\theta)^2$$

Note: Error E must be formulated carefully!



Line fitting can be max. likelihood - but choice of model is important

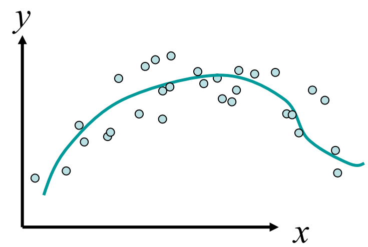


Curve Fitting

Find Polynomial:

$$y = f(x) = ax^3 + bx^2 + cx + d$$

that best fits the given points (x_i, y_i)



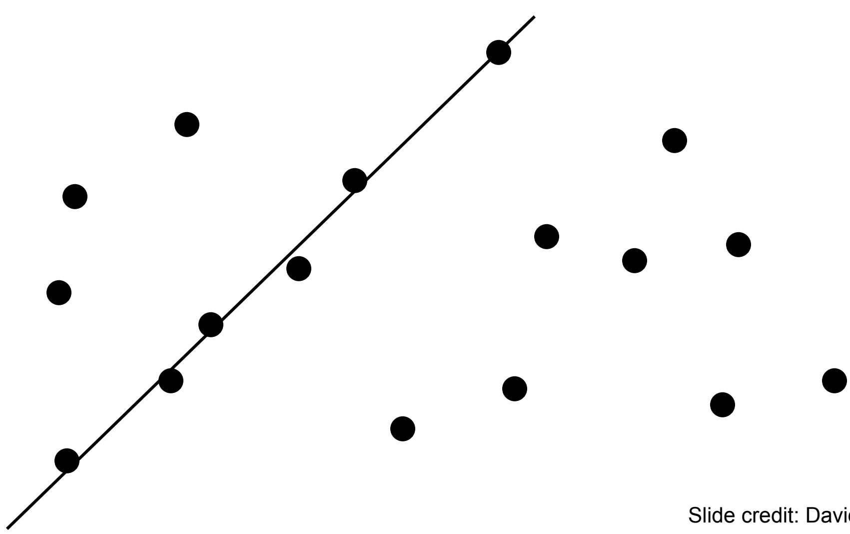
Minimize:

$$\frac{1}{N}\sum_i [y_i - (ax_i^3 + bx_i^2 + cx_i + d)]^2$$

Using: $\frac{\partial E}{\partial a} = 0$, $\frac{\partial E}{\partial b} = 0$, $\frac{\partial E}{\partial c} = 0$, $\frac{\partial E}{\partial d} = 0$

Note: $f(x)$ is LINEAR in the parameters (a, b, c, d)

Line Grouping Problem



Slide credit: David Jacobs

This is difficult because of:

- Extraneous data: clutter or multiple models
  - We do not know what is part of the model?
  - Can we pull out models with a few parts from much larger amounts of background clutter?
- Missing data: only some parts of model are present
- Noise

- **Cost:**
  - It is not feasible to check all combinations of features by fitting a model to each possible subset

Hough Transform

- Elegant method for direct object recognition
  - Edges need not be connected
  - Complete object need not be visible
  - Key Idea: Edges VOTE for the possible model

Image and Parameter Spaces

Equation of Line: $y = mx + c$

Find: (m, c)

Consider point: (x_i, y_i)

$$y_i = mx_i + c \quad or \quad c = -x_i m + y_i$$

Parameter space also called Hough Space

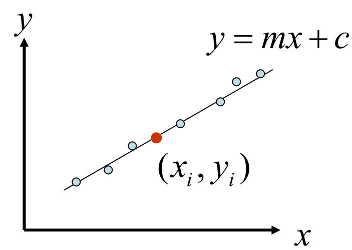
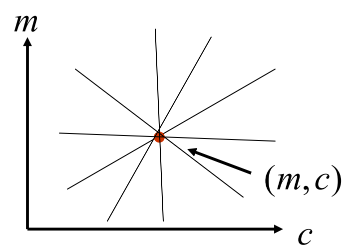


Image Space



Parameter Space

Line Detection by Hough Transform

Algorithm:

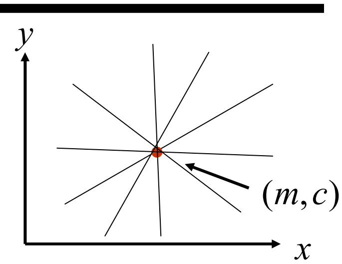
- Quantize Parameter Space (m, c)
- Create Accumulator Array $A(m, c)$
- Set $A(m, c) = 0 \quad \forall m, c$
- For each image edge (x_i, y_i) increment:

$$A(m, c) = A(m, c) + 1$$

- If (m, c) lies on the line:

$$c = -x_i m + y_i$$

- Find local maxima in $A(m, c)$



Parameter Space

$A(m, c)$

	1						1		
		1				1			
			1		1				
				2					
			1		1				
		1				1			
	1						1		

Better Parameterization

NOTE: $-\infty \leq m \leq \infty$

Large Accumulator

More memory and computations

Improvement: (Finite Accumulator Array Size)

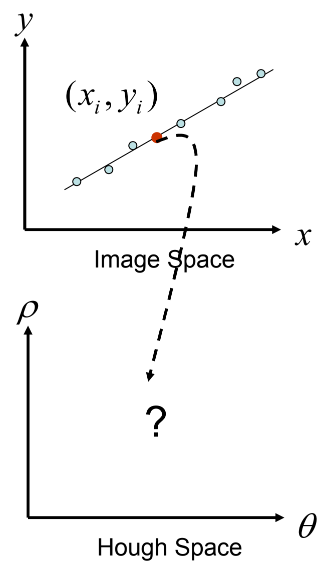
Line equation: $\rho = -x\cos\theta + y\sin\theta$

Here $0 \leq \theta \leq 2\pi$

$0 \leq \rho \leq \rho_{\max}$

Given points (x_i, y_i) find (ρ, θ)

Hough Space Sinusoid



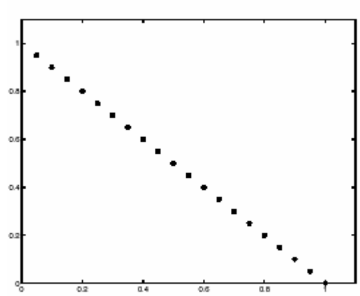
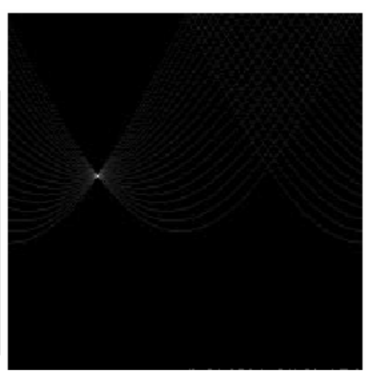
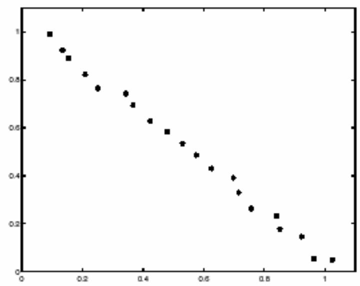


Image space



Votes

Horizontal axis is θ, vertical is rho.



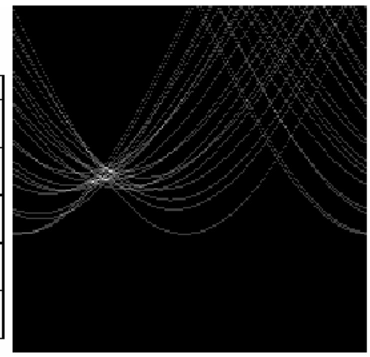
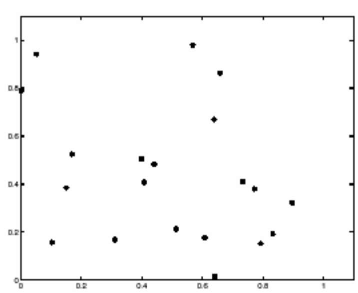
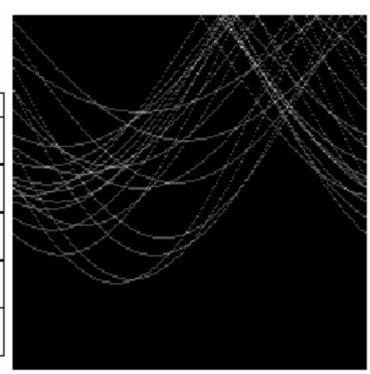


Image space

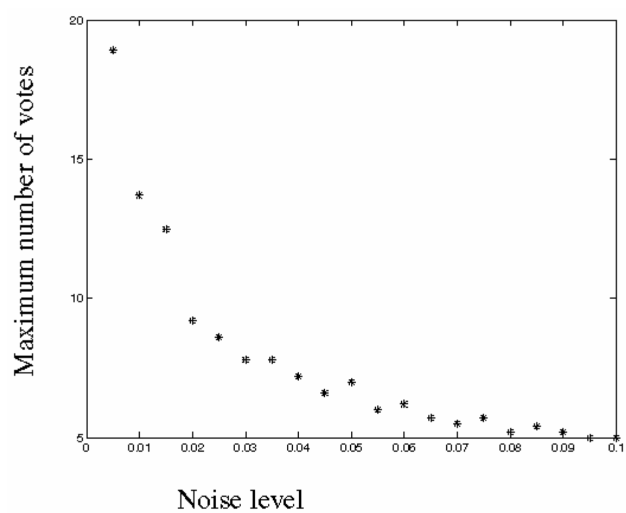
votes



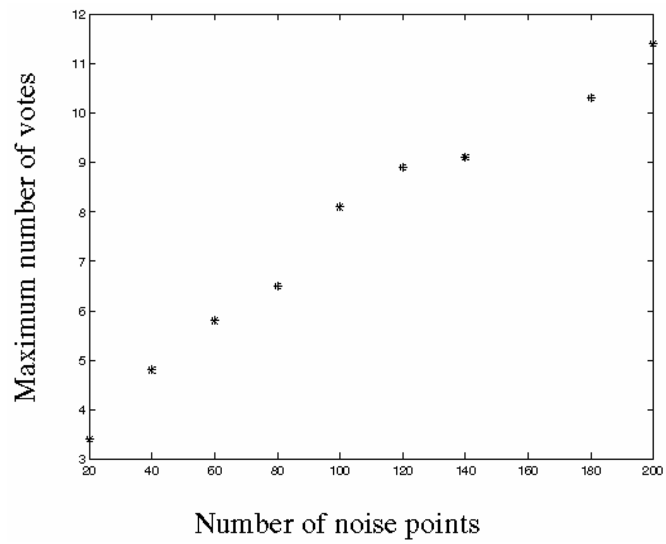


Mechanics of the Hough transform

- Difficulties
  - how big should the cells be? (too big, and we merge quite different lines; too small, and noise causes lines to be missed)
- How many lines?
  - Count the peaks in the Hough array
  - Treat adjacent peaks as a single peak
- Which points belong to each line?
  - Search for points close to the line
  - Solve again for line and iterate



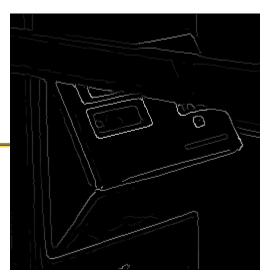
Fewer votes land in a single bin when noise increases.



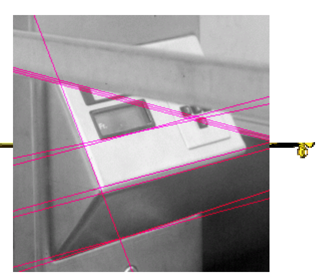
Adding more clutter increases number of bins with false peaks.

Real World Example

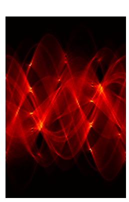
Original



Edge Detection



Found Lines

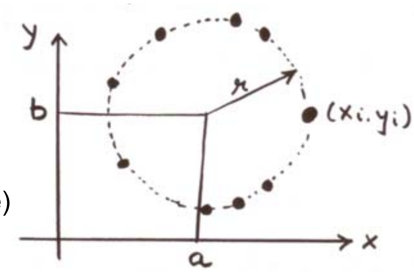


Parameter Space

Finding Circles by Hough Transform

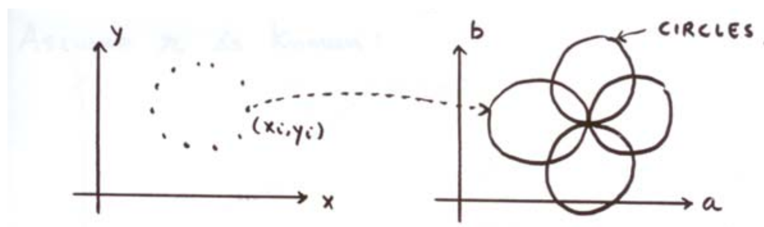
Equation of Circle:

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$



If radius is known: (2D Hough Space)

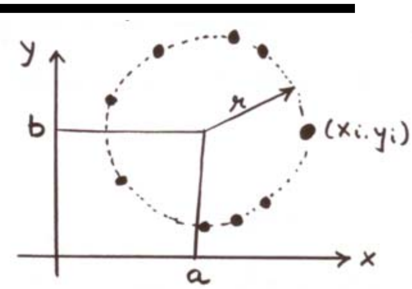
Accumulator Array $A(a,b)$



Finding Circles by Hough Transform

Equation of Circle:

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$



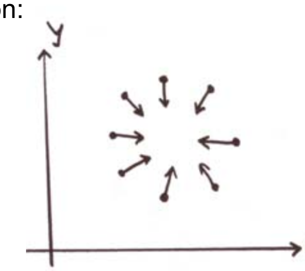
If radius is not known: 3D Hough Space!

Use Accumulator array $A(a,b,r)$

What is the surface in the hough space?

Using Gradient Information

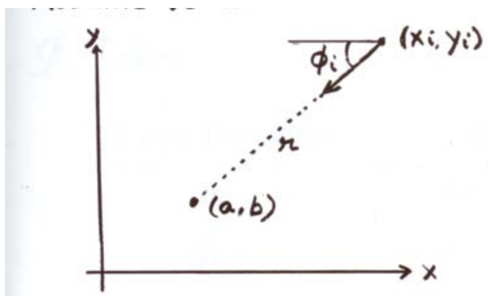
- Gradient information can save lot of computation:



Edge Location (x_i, y_i)

Edge Direction ϕ_i

Assume radius is known:



$$a = x - r\cos\phi$$

$$b = y - r\sin\phi$$

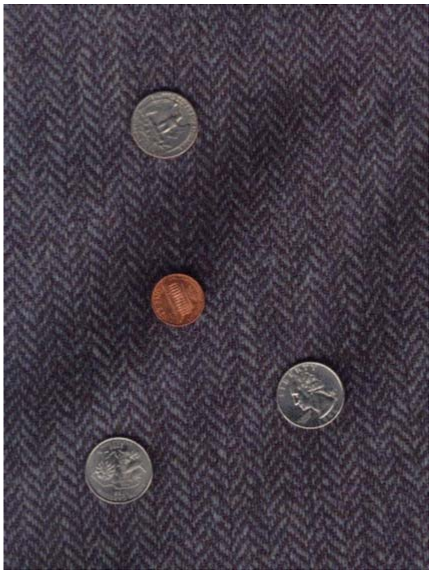
Need to increment only one point in Accumulator!!

Real World Circle Examples

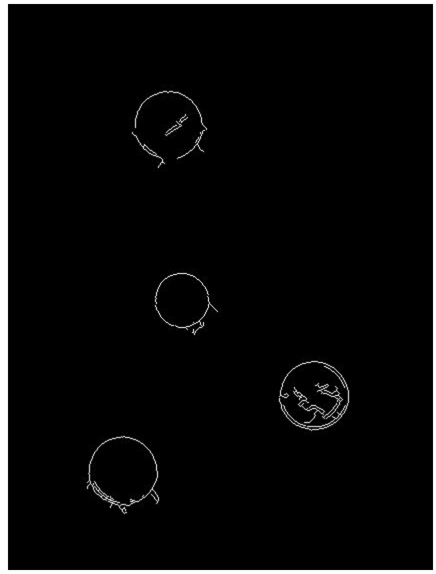
Crosshair indicates results of Hough transform, bounding box found via motion differencing.

Finding Coins

Original

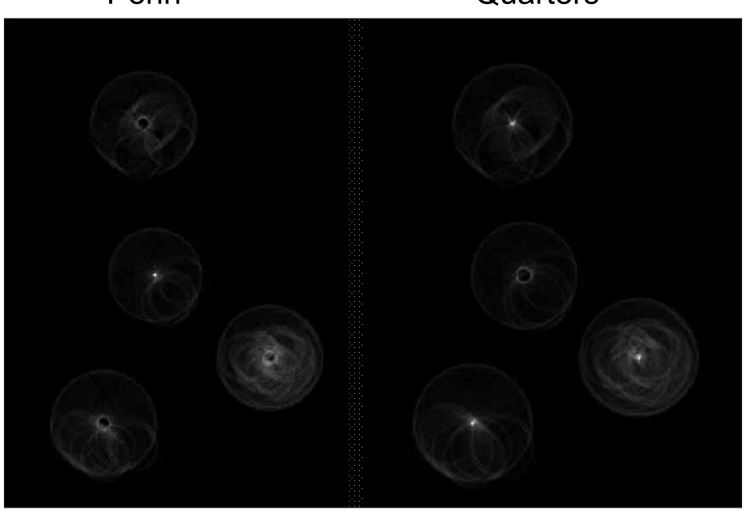


Edges (note noise)

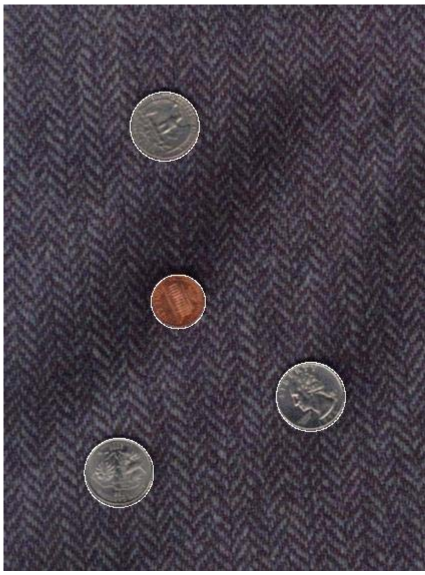


Finding Coins (Continued)

Penn Quarters



Finding Coins (Continued)

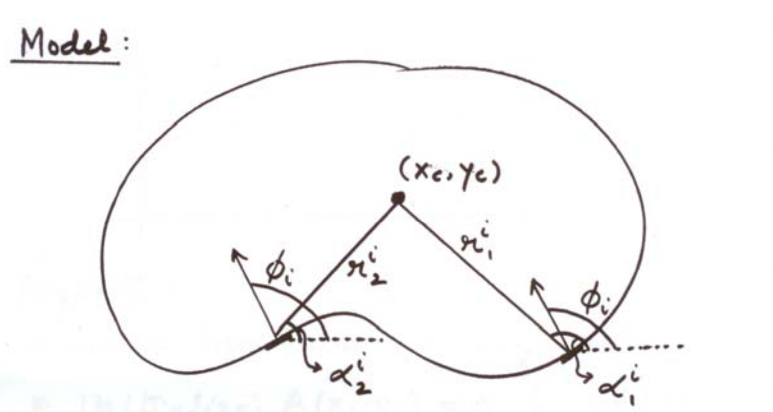


Note that because the quarters and penny are different sizes, a different Hough transform (with separate accumulators) was used for each circle size.

Coin finding sample images from: Vivek Kwatra

Generalized Hough Transform

- Model Shape NOT described by equation



Generalized Hough Transform

- Model Shape NOT described by equation

ϕ - Table

Edge Direction	$\bar{r} = (r, \alpha)$
ϕ_1	$\bar{r}^1_1, \bar{r}^1_2, \bar{r}^1_3$
ϕ_2	$\bar{r}^2_1, \bar{r}^2_2$
ϕ_i	$\bar{r}^i_1 \vdots \bar{r}^i_2$
ϕ_n	$\bar{r}^n_1, \bar{r}^n_2$

Generalized Hough Transform

Find Object Center (x_c, y_c) given edges (x_i, y_i, ϕ_i)

Create Accumulator Array $A(x_c, y_c)$

Initialize: $A(x_c, y_c) = 0 \quad \forall (x_c, y_c)$

For each edge point (x_i, y_i, ϕ_i)

For each entry $\overline{r}_k^i$ in table, compute:

$$x_c = x_i + r_k^i \cos \alpha_k^i$$

$$y_c = y_i + r_k^i \sin \alpha_k^i$$

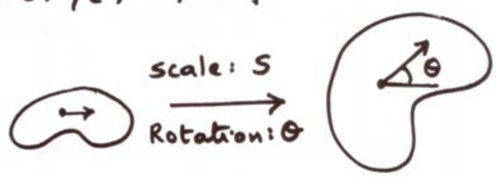
Increment Accumulator: $A(x_c, y_c) = A(x_c, y_c) + 1$

Find Local Maxima in $A(x_c, y_c)$

Scale & Rotation:

Use Accumulator Array:

$$A[x_c, y_c, S, \theta]$$



Use:

$$x_c = x_i + r^i_k S \cos(\alpha^i_k + \theta)$$

$$y_c = y_i + r^i_k S \sin(\alpha^i_k + \theta)$$

$$A(x_c, y_c, S, \theta) = A(x_c, y_c, S, \theta) + 1.$$

Hough Transform: Comments

- Works on Disconnected Edges
- Relatively insensitive to occlusion
- Effective for simple shapes (lines, circles, etc)
- Trade-off between work in Image Space and Parameter Space
- Handling inaccurate edge locations:
    - Increment Patch in Accumulator rather than a single point

Next Class

- Lightness and Retinex.
- Reading: Horn, Chapter 9.
- Research webpages of Edward Adelson (MIT).
- Google for illusions.