Computer Vision

Spring 2006 15-385,-685

Instructor: S. Narasimhan

Wean 5403

T-R 3:00pm – 4:20pm

Lecture #13

Announcements

Homework 4 will be out today. Due 4/4/06.
Please start early.

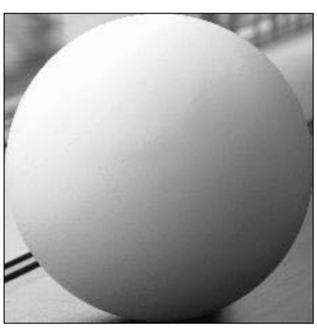
Midterm stats: A range → 40+, B range → 30+

40+ → 13 students
30+ → 9 students
Below 30 → 6 students

Shape from Shading

Lecture #13

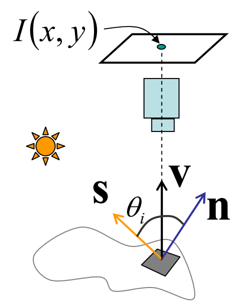
Image Intensity and 3D Geometry



- Shading as a cue for shape reconstruction
- What is the relation between intensity and shape?
  - Reflectance Map

Reflectance Map - RECAP

- Relates image irradiance *I(x,y)* to surface orientation *(p,q)* for given source direction and surface reflectance
- Lambertian case:



k : source brightness

ρ : surface albedo (reflectance)

c : constant (optical system)

Image irradiance:

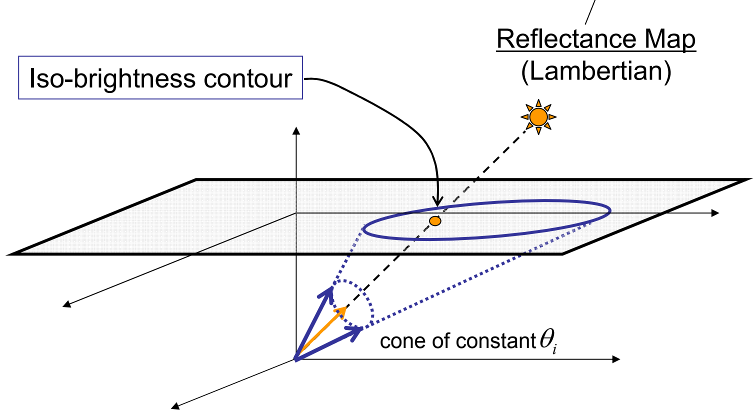
$$I = \frac{\rho}{\pi} kc \cos\theta_i = \frac{\rho}{\pi} kc \mathbf{n}\cdot\mathbf{s}$$

Let $\frac{\rho}{\pi} kc = 1$ then $\boxed{I = \cos\theta_i = \mathbf{n}\cdot\mathbf{s}}$

Reflectance Map - RECAP

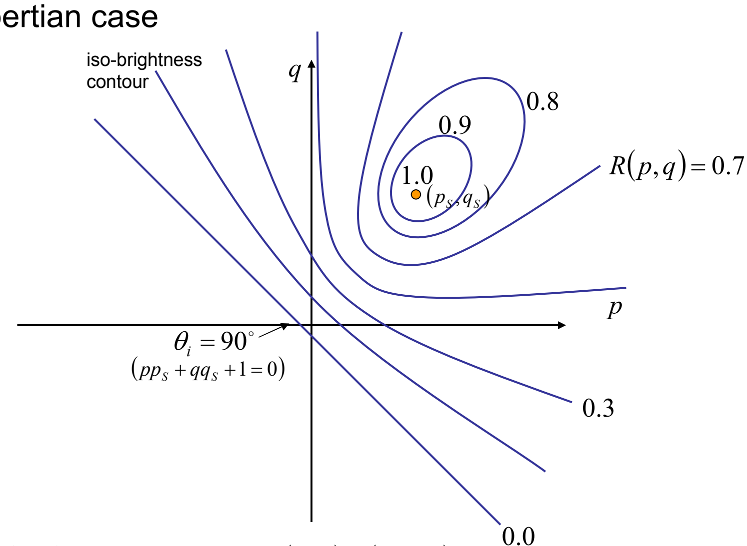
- Lambertian case

$$I = \cos\theta_i = \mathbf{n}\cdot\mathbf{s} = \frac{(pp_s + qq_s + 1)}{\sqrt{p^2+q^2+1}\sqrt{p_S^2+q_S^2+1}} = \boxed{R(p,q)}$$



Reflectance Map - RECAP

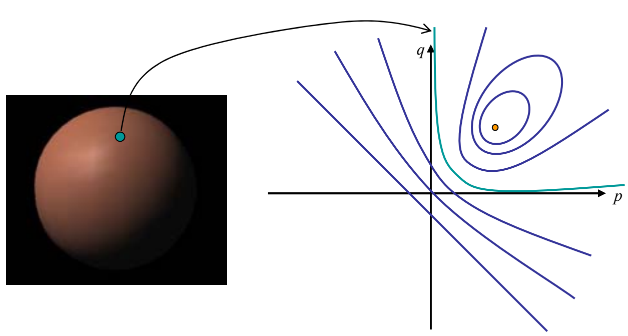
- Lambertian case



Note: $R(p,q)$ is maximum when $(p,q) = (p_S, q_S)$

Shape from a Single Image?

- Given a single image of an object with known surface reflectance taken under a known light source, can we recover the shape of the object?
- Given *R(p,q)* (*(p_S,q_S)* and surface reflectance) can we determine *(p,q)* uniquely for each image point?

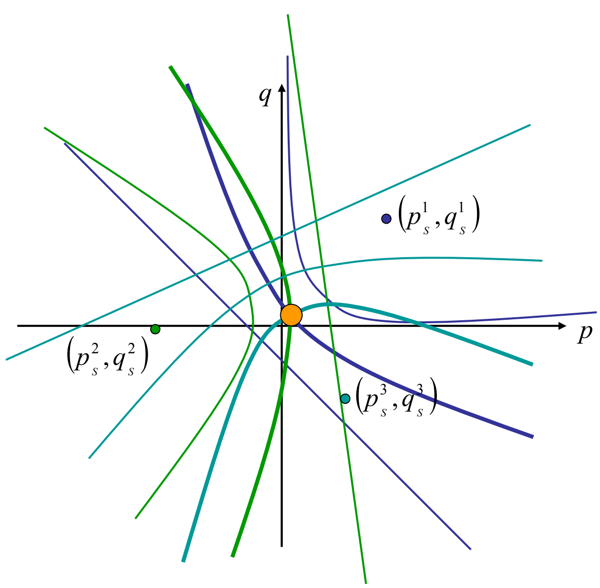


NO

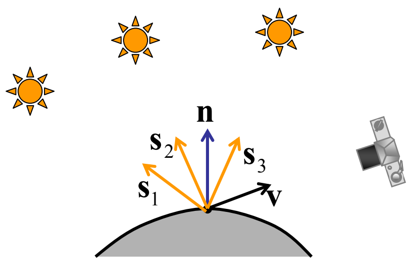
Solution

- Take more images
  - Photometric stereo (previous class)
- Add more constraints
  - Shape-from-shading (this class)

Photometric Stereo



Photometric Stereo



Lambertian case:

$$I = \frac{\rho}{\pi} kc \cos\theta_i = \rho \mathbf{n} \cdot \mathbf{s} \quad \left(\frac{kc}{\pi} = 1\right)$$

Image irradiance:

$$I_1 = \rho \mathbf{n} \cdot \mathbf{s}_1$$
$$I_2 = \rho \mathbf{n} \cdot \mathbf{s}_2$$
$$I_3 = \rho \mathbf{n} \cdot \mathbf{s}_3$$

- We can write this in matrix form:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_2 \end{bmatrix} = \rho \begin{bmatrix} \mathbf{s}_1^T \\ \mathbf{s}_2^T \\ \mathbf{s}_3^T \end{bmatrix} \mathbf{n}$$

Solution

- Take more images
  - Photometric stereo (previous class)

- Add more constraints
  - Shape-from-shading (this class)

Human Perception

- Our brain often perceives shape from shading.
- Mostly, it makes many assumptions to do so.
- For example:

    Light is coming from above (sun).

    Biased by occluding contours.

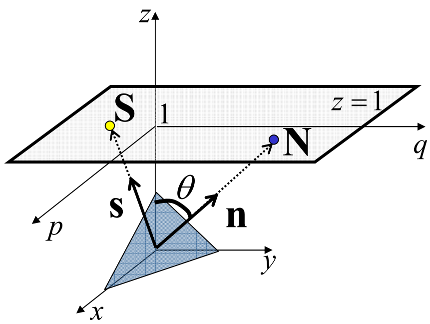
by V. Ramachandran

See Ramachandran's work on Shape from Shading by Humans

http://psy.ucsd.edu/chip/ramabio.html

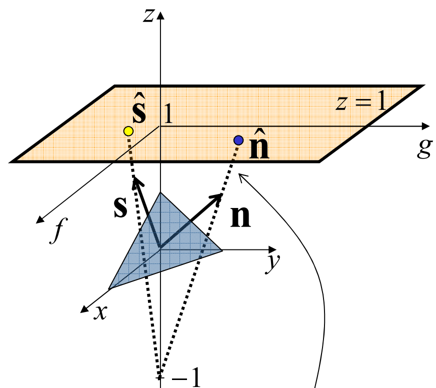
Stereographic Projection

(p,q)-space (gradient space)



Problem
(p,q) can be infinite when $\theta = 90^\circ$

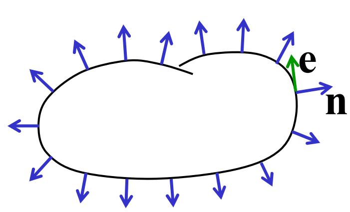
(f,g)-space

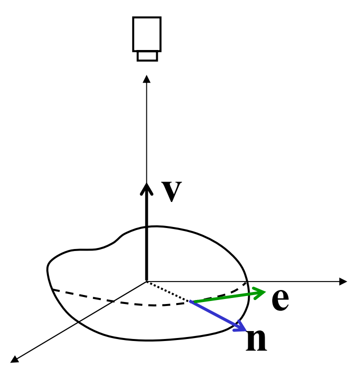


$$f = \frac{2p}{1+\sqrt{1+p^2+q^2}} \qquad g = \frac{2q}{1+\sqrt{1+p^2+q^2}}$$

Redefine reflectance map as $R(f,g)$

Occluding Boundaries





$\mathbf{n} \perp \mathbf{e}, \quad \mathbf{n} \perp \mathbf{v} \quad \therefore \quad \mathbf{n} = \mathbf{e} \times \mathbf{v}$ $\quad$ $\mathbf{e}$ and $\mathbf{v}$ are known

The $\mathbf{n}$ values on the occluding boundary can be used as the boundary condition for shape-from-shading

Image Irradiance Constraint

- Image irradiance should match the reflectance map

Minimize

$$\boxed{e_i = \iint_{\text{image}} \left(I(x,y) - R(f,g)\right)^2 dxdy}$$

(minimize errors in image irradiance in the image)

Smoothness Constraint

- Used to constrain shape-from-shading
- Relates orientations *(f,g)* of neighboring surface points

Minimize

$$\boxed{e_s = \iint_{\text{image}} \left(f_x^2 + f_y^2\right) + \left(g_x^2 + g_y^2\right) dxdy}$$

(f,g) : surface orientation under stereographic projection

$$f_x = \frac{\partial f}{\partial x}, f_y = \frac{\partial f}{\partial y}, g_x = \frac{\partial g}{\partial x}, g_y = \frac{\partial g}{\partial y}$$

(penalize rapid changes in surface orientation *f* and *g* over the image)

Shape-from-Shading

- Find surface orientations *(f,g)* at all image points that minimize

$$e = e_s + \lambda e_i$$

weight

smoothness constraint

image irradiance error

Minimize

$$e = \iint_{\text{image}} \left(f_x^2 + f_y^2\right) + \left(g_x^2 + g_y^2\right) + \lambda\left(I(x,y) - R(f,g)\right)^2 dxdy$$

Numerical Shape-from-Shading

- **Smoothness error** at image point *(i,j)*

$$s_{i,j} = \frac{1}{4}\left(\left(f_{i+1,j} - f_{i,j}\right)^2 + \left(f_{i,j+1} - f_{i,j}\right)^2 + \left(g_{i+1,j} - g_{i,j}\right)^2 + \left(g_{i,j+1} - g_{i,j}\right)^2\right)$$

Of course you can consider more neighbors (smoother results)

- **Image irradiance error** at image point *(i,j)*

$$r_{i,j} = \left(I_{i,j} - R\left(f_{i,j}, g_{i,j}\right)\right)^2$$

Find $\{f_{i,j}\}$ and $\{g_{i,j}\}$ that minimize

$$e = \sum_i \sum_j \left(s_{i,j} + \lambda r_{i,j}\right)$$

(Ikeuchi & Horn 89)

Numerical Shape-from-Shading

Find $\{f_{i,j}\}$ and $\{g_{i,j}\}$ that minimize $e = \sum_i \sum_j (s_{i,j} + \lambda r_{i,j})$

If $f_{k,l}$ and $g_{k,l}$ minimize e, then $\frac{\partial e}{\partial f_{k,l}} = 0, \frac{\partial e}{\partial g_{k,l}} = 0$

$$\frac{\partial e}{\partial f_{k,l}} = 2(f_{k,l} - \bar{f}_{k,l}) - 2\lambda(I_{k,l} - R(f_{k,l}, g_{k,l}))\left.\frac{\partial R}{\partial f}\right|_{f_{k,l}} = 0$$

$$\frac{\partial e}{\partial g_{k,l}} = 2(g_{k,l} - \bar{g}_{k,l}) - 2\lambda(I_{k,l} - R(f_{k,l}, g_{k,l}))\left.\frac{\partial R}{\partial g}\right|_{g_{k,l}} = 0$$

where $\bar{f}_{k,l}$ and $\bar{g}_{k,l}$ are 4-neighbors average around image point *(k,l)*

$$\bar{f}_{k,l} = \frac{1}{8}(f_{i+1,j} + f_{i,j+1} + f_{i-1,j} + f_{i,j-1})$$

$$\bar{g}_{k,l} = \frac{1}{8}(g_{i+1,j} + g_{i,j+1} + g_{i-1,j} + g_{i,j-1})$$

(Ikeuchi & Horn 89)

Numerical Shape-from-Shading

$$\frac{\partial e}{\partial f_{k,l}} = 2(f_{k,l} - \bar{f}_{k,l}) - 2\lambda(I_{k,l} - R(f_{k,l}, g_{k,l}))\left.\frac{\partial R}{\partial f}\right|_{f_{k,l}} = 0 \qquad \frac{\partial e}{\partial g_{k,l}} = 2(g_{k,l} - \bar{g}_{k,l}) - 2\lambda(I_{k,l} - R(f_{k,l}, g_{k,l}))\left.\frac{\partial R}{\partial g}\right|_{g_{k,l}} = 0$$

Update rule

$$f^{n+1}_{k,l} = \bar{f}^{n}_{k,l} + \lambda(I_{k,l} - R(f_{k,l}, g_{k,l}))\left.\frac{\partial R}{\partial f}\right|_{f_{k,l}}$$

$$g^{n+1}_{k,l} = \bar{g}^{n}_{k,l} + \lambda(I_{k,l} - R(f_{k,l}, g_{k,l}))\left.\frac{\partial R}{\partial g}\right|_{g_{k,l}}$$

- Use known (f, g) values on the occluding boundary to constrain the solution (boundary conditions)
- Compare $(f^{n+1}_{k,l}, g^{n+1}_{k,l})$ with $(f^{n}_{k,l}, g^{n}_{k,l})$ for convergence test
- As the solution converges, increase λ to remove the smoothness constraint

(Ikeuchi & Horn 89)

Calculus of Variations

Minimize

$$e = \iint_{\text{image}} F\left(f, g, f_x, f_y, g_x, g_y\right) dx\,dy$$

$$F = \left(f_x^2 + f_y^2\right) + \left(g_x^2 + g_y^2\right) + \lambda\left(I(x,y) - R(f,g)\right)^2$$

Euler equations for *F* (read Horn A.6)

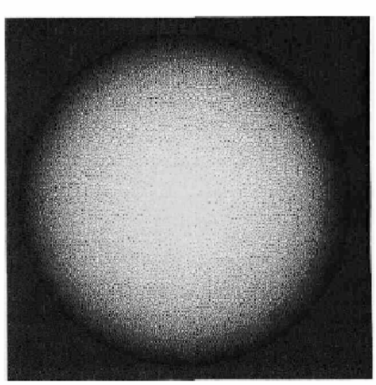
$$F_f - \frac{\partial}{\partial x} F_{f_x} - \frac{\partial}{\partial y} F_{f_y} = 0, F_g - \frac{\partial}{\partial x} F_{g_x} - \frac{\partial}{\partial y} F_{g_y} = 0$$

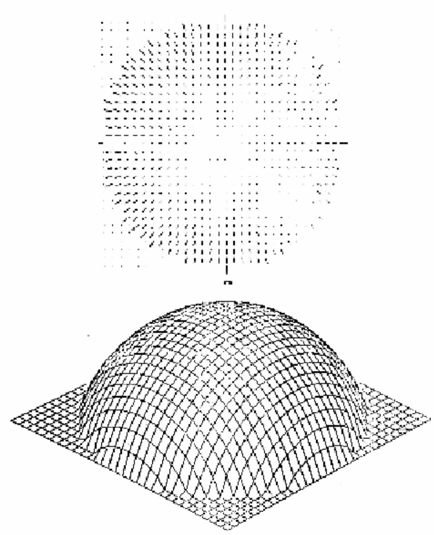
Euler equations for shape-from-shading

$$\nabla^2 f = -\lambda\left(I(x,y) - R(f,g)\right)\frac{\partial R}{\partial f}, \nabla^2 g = -\lambda\left(I(x,y) - R(f,g)\right)\frac{\partial R}{\partial g}$$

Solve this coupled pair of second-order partial differential equations **with the occluding boundary conditions**!

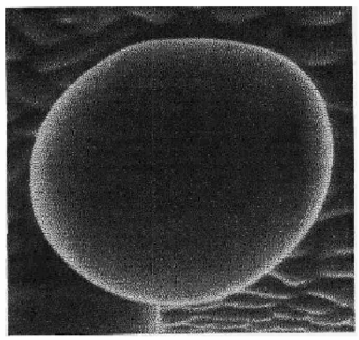
Results



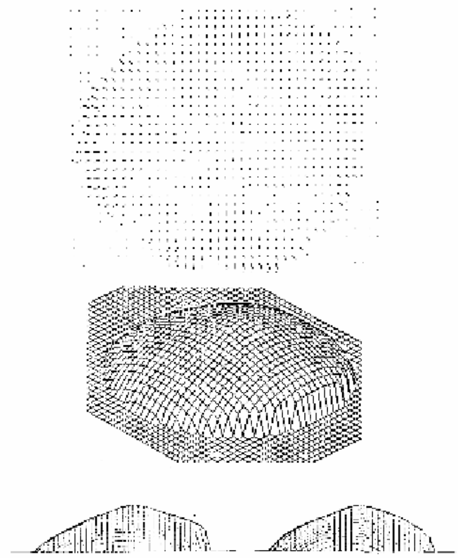


by Ikeuchi and Horn

Results



Scanning Electron Microscope image
(inverse intensity)



by Ikeuchi and Horn

Next Two Classes

- Binocular Stereo
- Reading: Horn, Chapter 13.