

# Computer Vision

Spring 2006 15-385,-685

Instructor: S. Narasimhan

Wean 5403

T-R 3:00pm – 4:20pm

## Image Formation and Optics

### Lecture #2

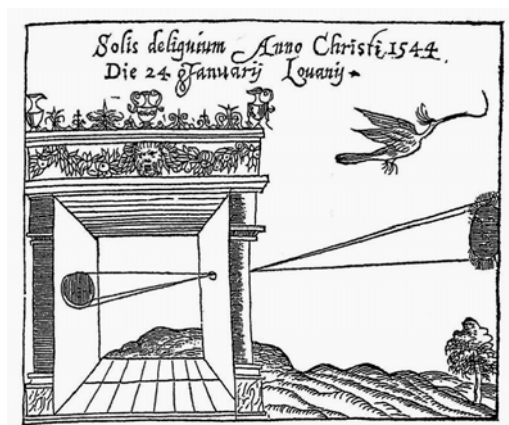
## Topics to be Covered

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- Brief History of Images
- Pinhole and Perspective Projection
- Approximations to Perspective Projection
- Image Formation using Lenses
- Lens related issues

## A Brief History of Images

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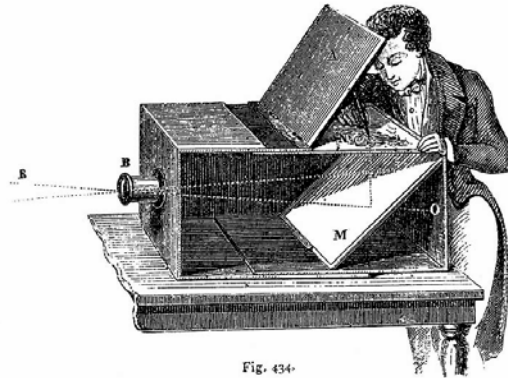
*Camera Obscura*, Gemma Frisius, 1544

1544

## A Brief History of Images

1558

1568



Lens Based Camera Obscura, 1568

## A Brief History of Images

1558

1568

1837



*Still Life*, Louis Jaques Mande Daguerre, 1837

## A Brief History of Images



Silicon Image Detector, 1970



1558

1568

1837

1970

## A Brief History of Images



Digital Cameras

1558

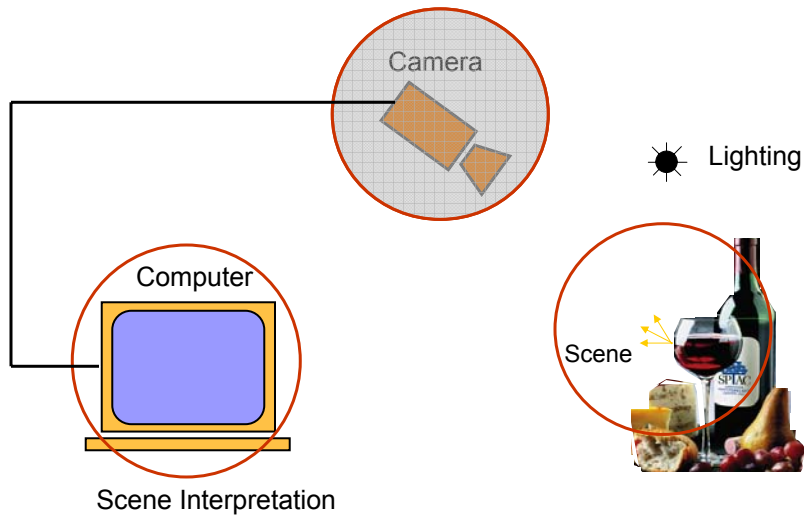
1568

1837

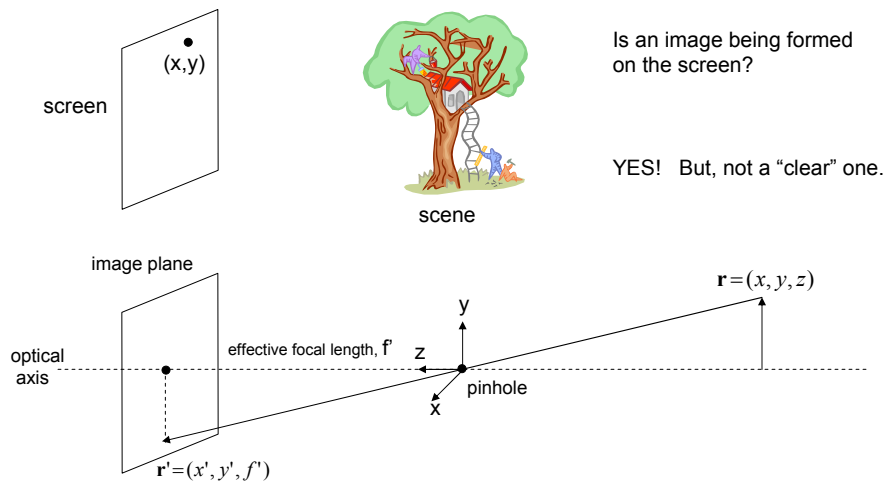
1970

1995

## Components of a Computer Vision System



## Pinhole and the Perspective Projection



Is an image being formed on the screen?

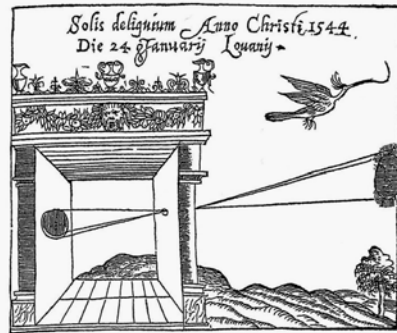
YES! But, not a "clear" one.

$$\frac{\mathbf{r}'}{f'} = \frac{\mathbf{r}}{z} \quad \Rightarrow \quad \frac{x'}{f'} = \frac{x}{z} \quad \frac{y'}{f'} = \frac{y}{z}$$

# Pinhole Camera

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- Basically a pinhole camera is a box, with a tiny hole at one end and film or photographic paper at the other.
- Mathematically: out of all the light rays in the world, choose the set of light rays passing through a point and projecting onto a plane.



# Pinhole Photography

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©Charlotte Murray Untitled, 4" x 5" pinhole photograph, 1992



Image Size inversely proportional to Distance

Reading: <http://www.pinholeresource.com/>

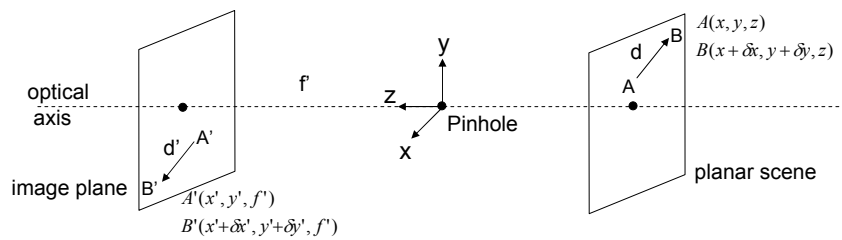
# Pinhole Photography



Wide Field of View and Sharp Image

©Clarissa Carnell, *Stonehenge*, 5" x 7" Gold Toned Printing-Out Paper Pinhole Photograph, 1986

## Magnification



From perspective projection:

$$\frac{x'}{f'} = \frac{x}{z} \quad \frac{y'}{f'} = \frac{y}{z}$$

$$\frac{x' + \Delta x'}{f'} = \frac{x + \Delta x}{z} \quad \frac{y' + \Delta y'}{f'} = \frac{y + \Delta y}{z}$$

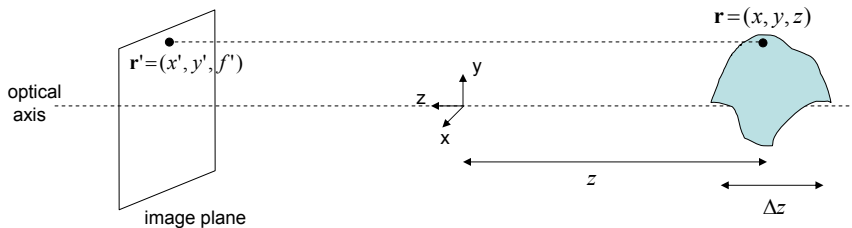
Magnification:

$$m = \frac{d'}{d} = \frac{\sqrt{(\Delta x')^2 + (\Delta y')^2}}{\sqrt{(\Delta x)^2 + (\Delta y)^2}} = \frac{f'}{z}$$

$$\frac{Area_{image}}{Area_{scene}} = m^2$$

# Orthographic Projection

- Magnification:  $x' = m x$   $y' = m y$
- When  $m = 1$ , we have orthographic projection

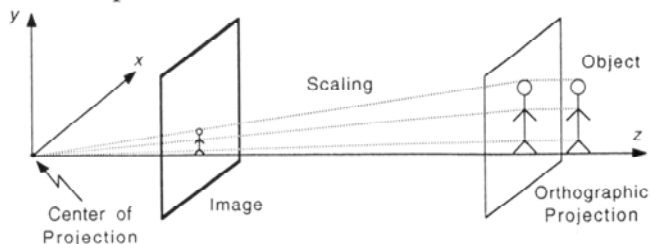


- This is possible only when  $z \gg \Delta z$
- In other words, the range of scene depths is assumed to be much smaller than the average scene depth.

But, how do we produce non-inverted images?

## Better Approximations to Perspective Projection

- Weak-Perspective

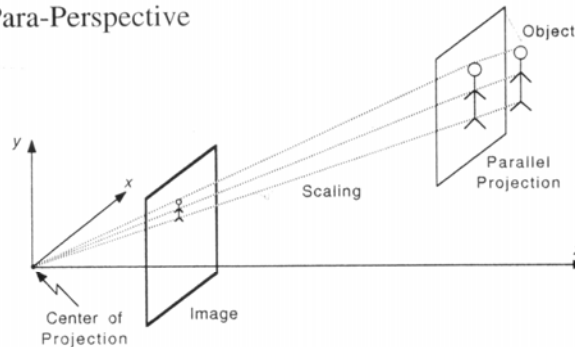


**Figure 2.8** Approximation of perspective projection by orthographic projection. Perspective projection onto a plane can be approximated by orthographic projection, followed by scaling, when (1) the object dimensions are small compared to the distance of the object from the center of projection, and (2) compared to this distance, the object is close to the straight line that passes through the center of projection and is orthogonal to the image plane (this line is the  $z$ -axis here).



## Better Approximations to Perspective Projection

### • Para-Perspective



**Figure 2.9** Approximation of perspective projection by parallel projection. *Parallel projection* onto a plane is a generalization of orthographic projection in which all the object points are projected along a set of parallel straight lines that may or may not be orthogonal to the projection plane. Perspective projection onto a plane can be approximated by parallel projection, followed by scaling, whenever the object dimensions are small compared to the distance of the object from the center of projection. The direction of parallel projection in such an approximation is along the "average direction" of perspective projection.

## Problems with Pinholes

- Pinhole size (aperture) must be "very small" to obtain a clear image.
- However, as pinhole size is made smaller, less light is received by image plane.
- If pinhole is comparable to wavelength  $\lambda$  of incoming light, DIFFRACTION blurs the image!
- Sharpest image is obtained when:

$$\text{pinhole diameter } d = 2 \sqrt{f' \lambda}$$

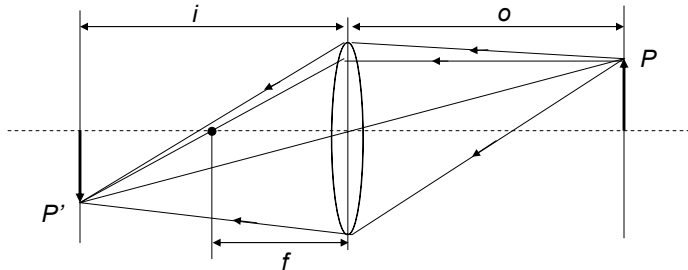
Example: If  $f' = 50\text{mm}$ ,  
 $= 600\text{nm}$  (red),  
 $d = 0.36\text{mm}$



**Fig. 5.96** The pinhole camera. Note the variation in image clarity as the hole diameter decreases. [Photos courtesy Dr. N. Joel, UNESCO.]

## Image Formation using Lenses

- Lenses are used to avoid problems with pinholes.
- Ideal Lens: Same projection as pinhole but gathers more light!



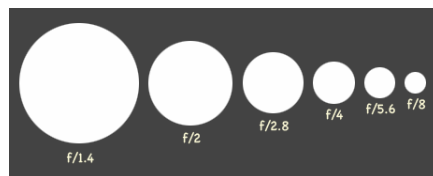
- Gaussian Thin Lens Formula:  $\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$
- $f$  is the focal length of the lens – determines the lens's ability to refract light
- $f$  different from the effective focal length  $f'$  discussed before!

## Aperture, F-Number

- Aperture : Diameter  $D$  of the lens that is exposed to light.

- F-Number ( $f/\#$ ):

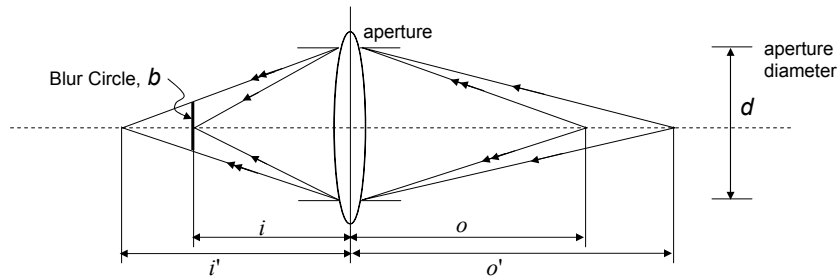
$$f/\# = \frac{f}{D},$$



Copyright: © Jared C. Benedict.

- For example, if  $f$  is 16 times the pupil diameter, then  $f/\# = f/16$ .
- The greater the  $f/\#$ , the less light per unit area reaches the image plane.
- f-stops represent a convenient sequence of  $f/\#$  in a geometric progression.

## Focus and Defocus



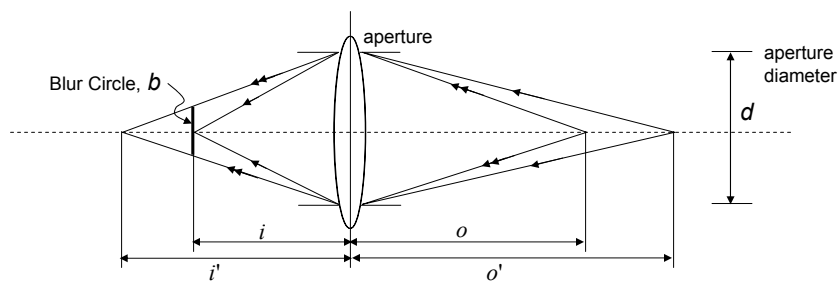
- Gaussian Law:

$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \quad \Rightarrow \quad (i' - i) = \frac{f}{(o' - f)} \frac{f}{(o - f)} (o - o')$$

$$\frac{1}{i'} + \frac{1}{o'} = \frac{1}{f}$$

- In theory, only one scene plane is in focus.

## Circle of Confusion



- Blur Circle Diameter **b** : Derive using similar triangles

$$b = \frac{d}{i'} (i' - i)$$

## Depth of Field

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- Range of object distances over which image is sufficiently well focused.
- Range for which *blur circle* is less than the resolution of the sensor.



[http://images.dpchallenge.com/images\\_portfolio/27920/print\\_preview/116336.jpg](http://images.dpchallenge.com/images_portfolio/27920/print_preview/116336.jpg)

## Depth of Field

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Both near and farther scene areas are blurred

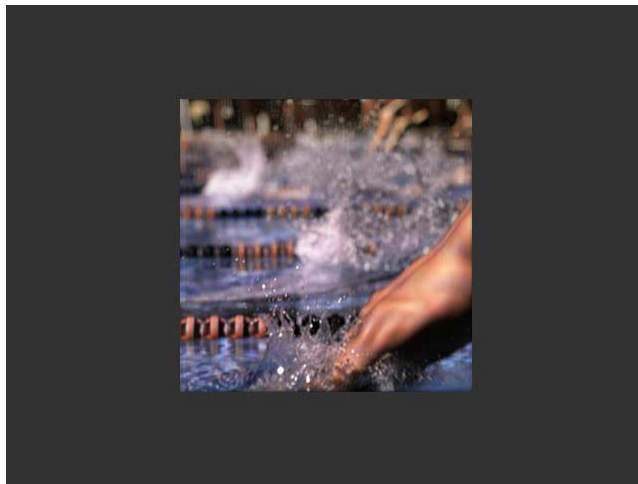
## Controlling Depth of Field



→  
Increase Aperture, decrease Depth of Field

[www.cambridgeincolour.com/.../depth-of-field.htm](http://www.cambridgeincolour.com/.../depth-of-field.htm)

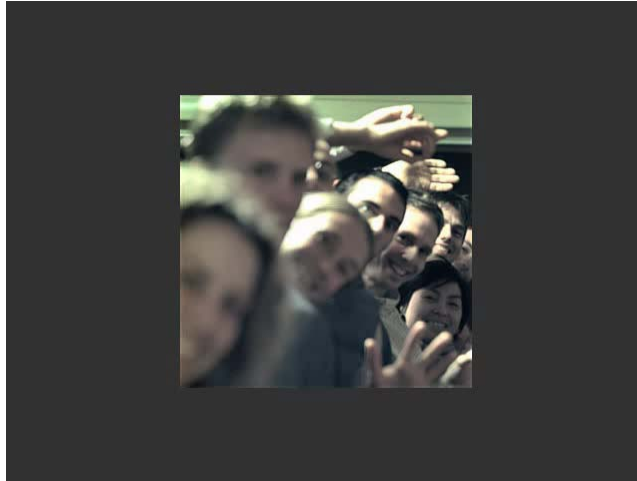
## Light Field Camera – Digital Refocusing



Use a microlens array in front of the CCD/Film

Ted Adelson, Wang, MIT; Ren Ng, Marc Levoy, Pat Hanrahan, Stanford

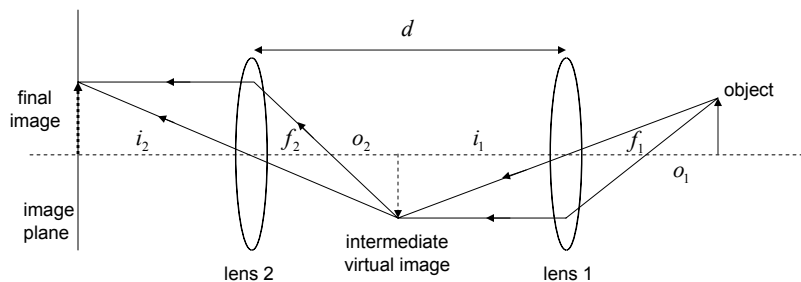
## Light Field Camera – Digital Refocusing



Use a microlens array in front of the CCD/Film

Ted Adelson, Wang, MIT; Ren Ng, Marc Levoy, Pat Hanrahan, Stanford

## Optics of a Two Lens System



- Rule : Image formed by first lens is the object for the second lens.
- Main Rays : Ray passing through focus emerges parallel to optical axis.  
Ray through optical center passes un-deviated.

- Magnification: 
$$m = \frac{i_2}{o_2} \frac{i_1}{o_1}$$

**Exercises:** What is the combined focal length of the system?  
What is the combined focal length if  $d = 0$ ?

## Lens Vignetting

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- Usually brighter at the center and darker at the periphery.

Reading: <http://www.dpreview.com>

## Chromatic Abberations

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Reading: <http://www.dpreview.com>

## Lens Glare

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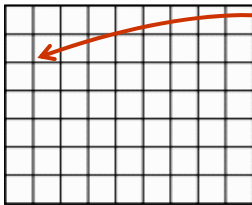


- Stray interreflections of light within the optical lens system.
- Happens when very bright sources are present in the scene.

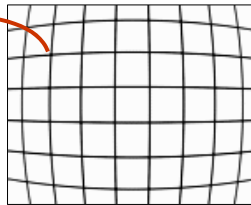
Reading: <http://www.dpreview.com>

## Radial Lens Distortions

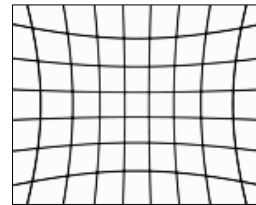
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No Distortion



Barrel Distortion



Pincushion Distortion

- Radial distance from Image Center:

$$r_u = r_d + k_1 r_d^3$$



## Correcting Radial Lens Distortions



Before

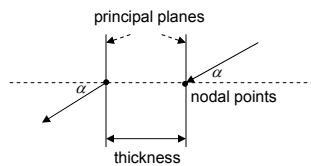


After

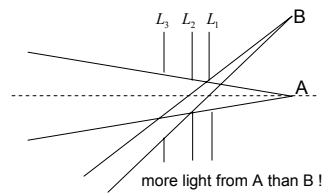
[http://www.grasshopperonline.com/barrel\\_distortion\\_correction\\_software.html](http://www.grasshopperonline.com/barrel_distortion_correction_software.html)

## Common Lens Related Issues - Summary

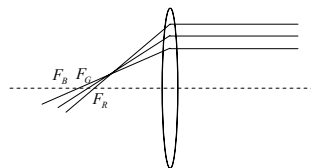
Compound (Thick) Lens



Vignetting

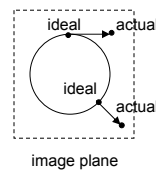


Chromatic Abberation



Lens has different refractive indices for different wavelengths.

Radial and Tangential Distortion



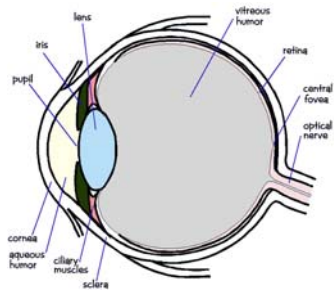
## Next Class

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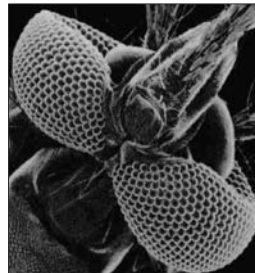
- Image Sensing
- Horn, Chapter 2

## Biological Cameras

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Human Eye



Mosquito Eye