Computer Vision

Spring 2006 15-385,-685

Instructor: S. Narasimhan

Wean 5403

T-R 3:00pm – 4:20pm

Lecture #11

Principles of Radiometry and Surface Reflectance

Lecture #11

Announcements

Homework 3 due on Thursday before class.

Submit programming part on blackboard
and hand in written part.

Midterm – March 9

Syllabus – until and including Lightness and Retinex

Closed book, closed notes exam in class.

Time: 3:00pm – 4:20pm

Midterm review class next Tuesday (March 7)
(Email me by March 6 specific questions)

If you have read the notes and readings, attended all classes, done assignments well, it should be a walk in the park☺

Course Schedule

1/17/2006: Introduction and Course Fundamentals

PART 1 : Cameras and Imaging

1/19/2006:	Image Formation and Projection	
1/24/2006:	Matlab Review	
1/26/2006:	Image Sensing	[Homework 1 OUT]

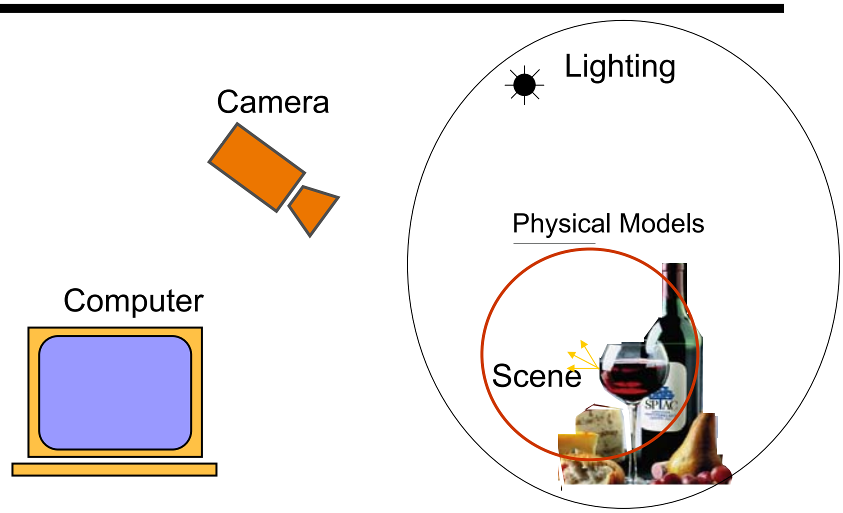
PART 2 : Signal and Image Processing

1/31/2006:	Binary Image Processing	
2/2/2006:	1D Signal Processing	[Homework 1 DUE; Homework 2 OUT]
2/7/2006:	2D Image Processing	
2/9/2006:	Edge Detection	
2/14/2006:	Image Pyramids	
2/16/2006:	Hough Transform	[Homework 2 DUE; Homework 3 OUT]

PART 3: Physics of the World

2/21/2006:	Basic Principles of Radiometry	
2/23/2006:	Retinex Theory	
2/28/2006:	Surface Reflectance and BRDF	
3/2/2006:	Photometric Stereo	[Homework 3 DUE]
3/7/2006:	**Midterm Review**	
3/9/2006:	**Midterm Exam**	
3/13/2006:	**Midterm Grades Due**	
3/21/2006:	Shape from Shading	[Homework 4 OUT]

Physics-based Methods in Vision

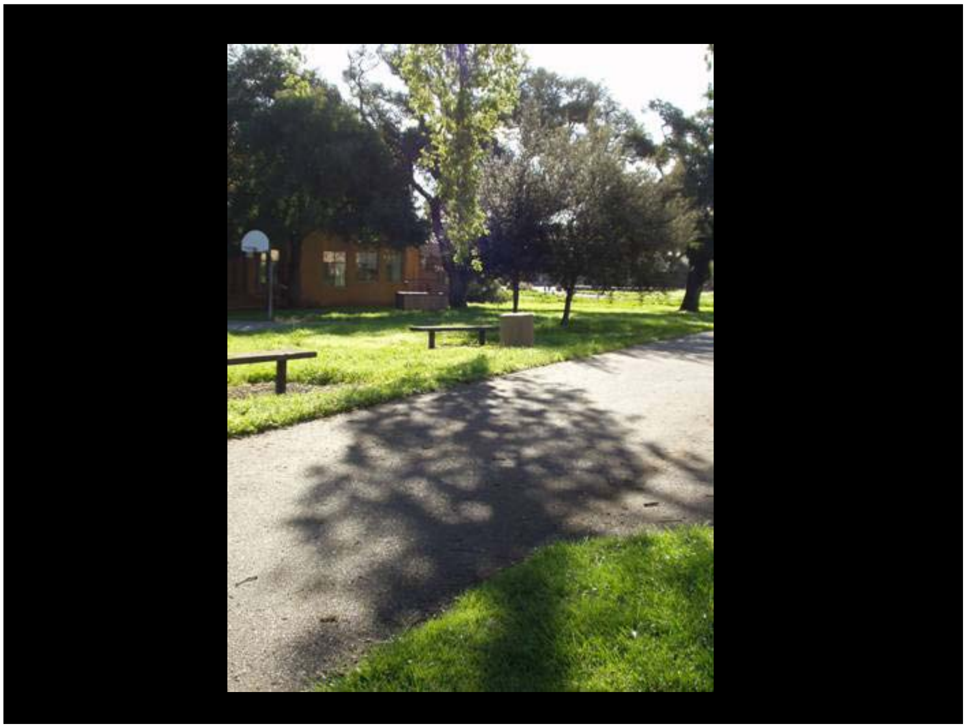


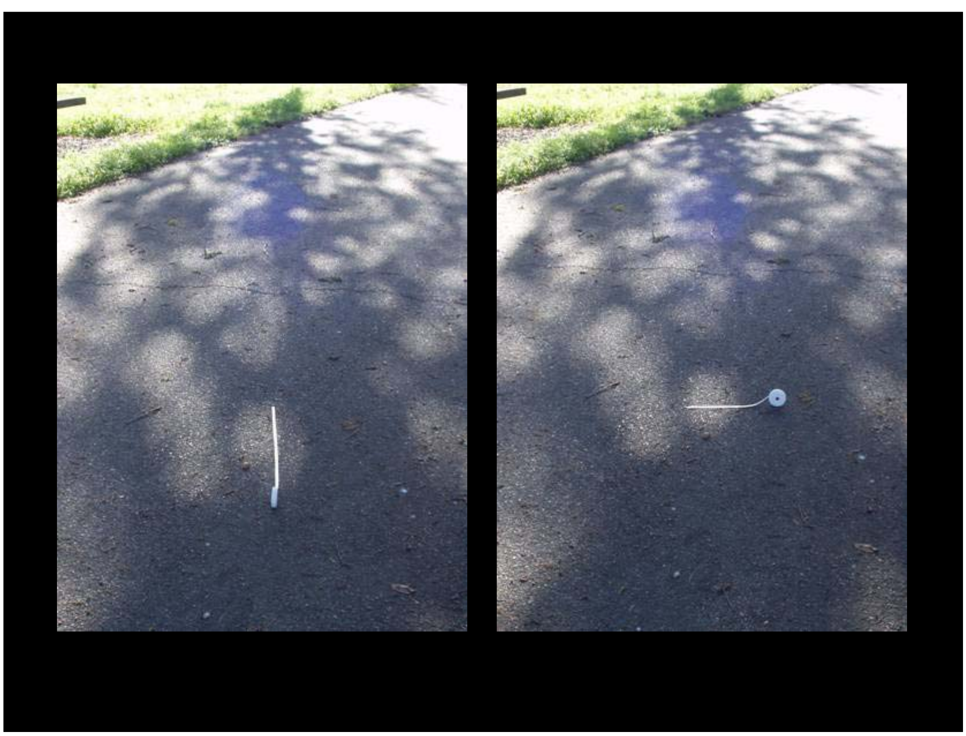
We need to understand the relation between the lighting, surface reflectance and medium and the image of the scene.

Why study the physics (optics) of the world?

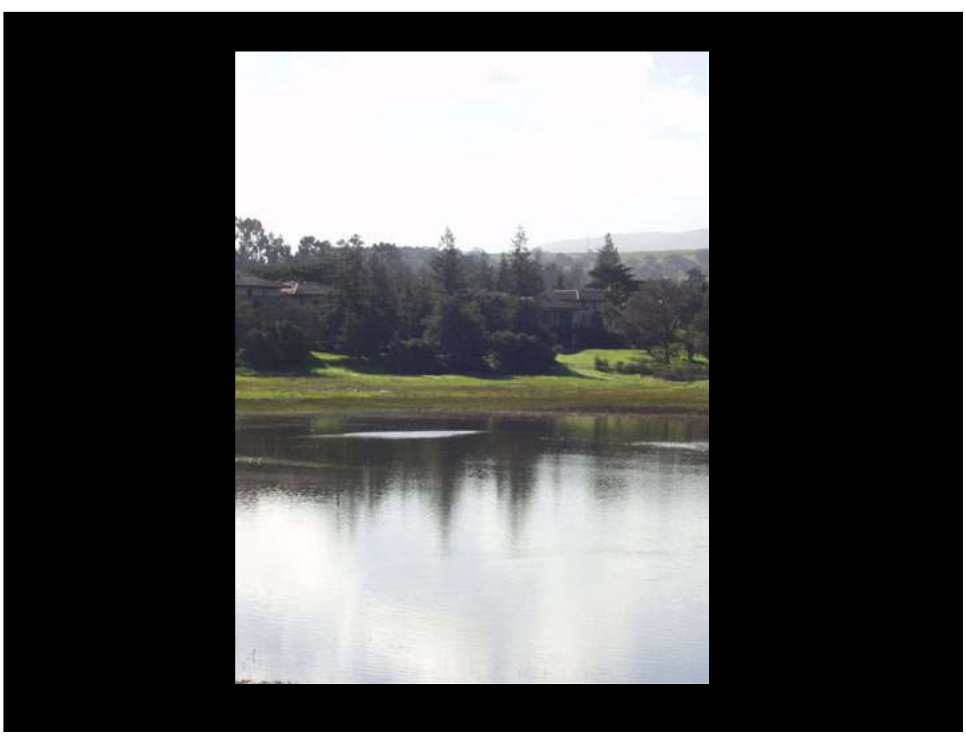
Lets see some pictures!

Light and Shadows

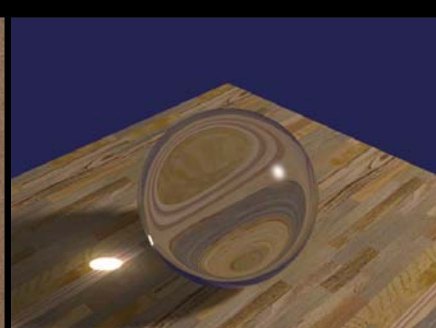


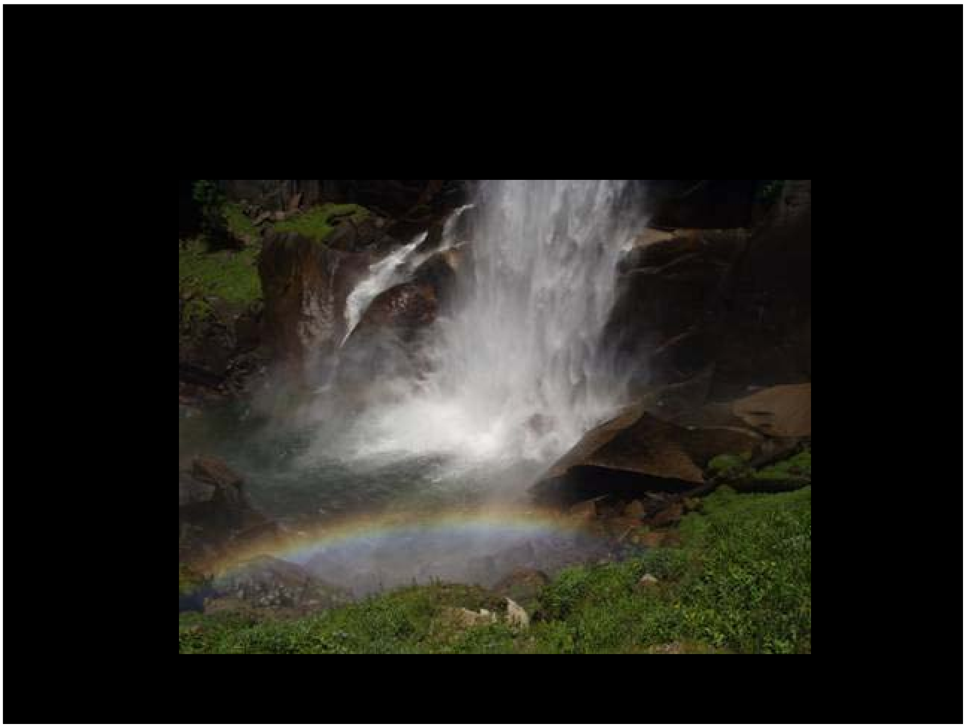


Reflections



Refractions



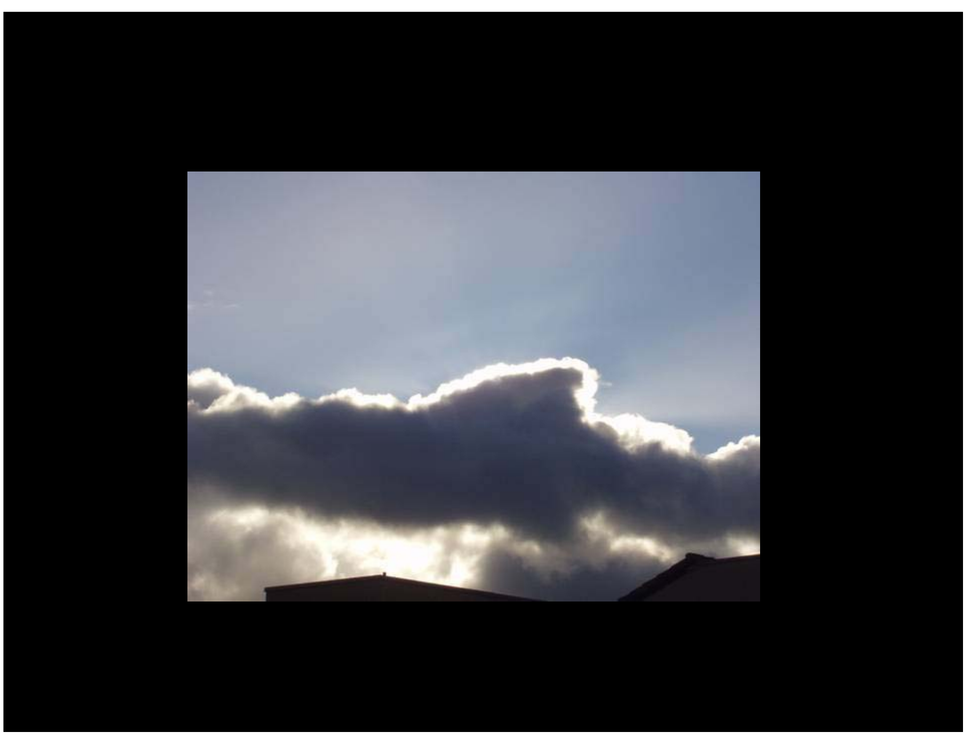


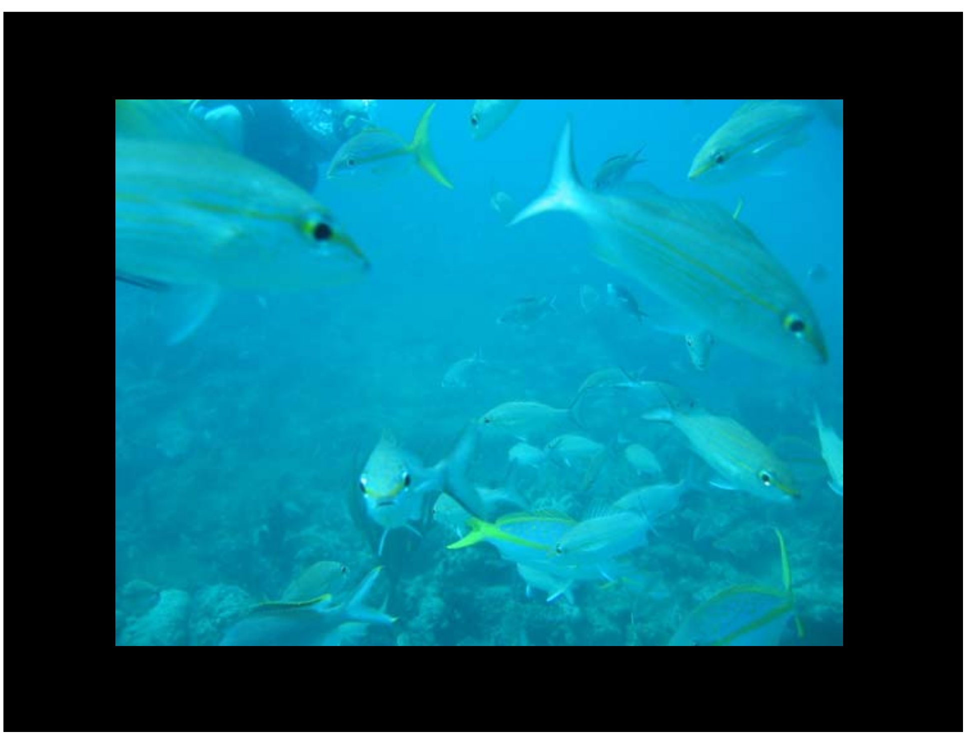
Interreflections

Scattering

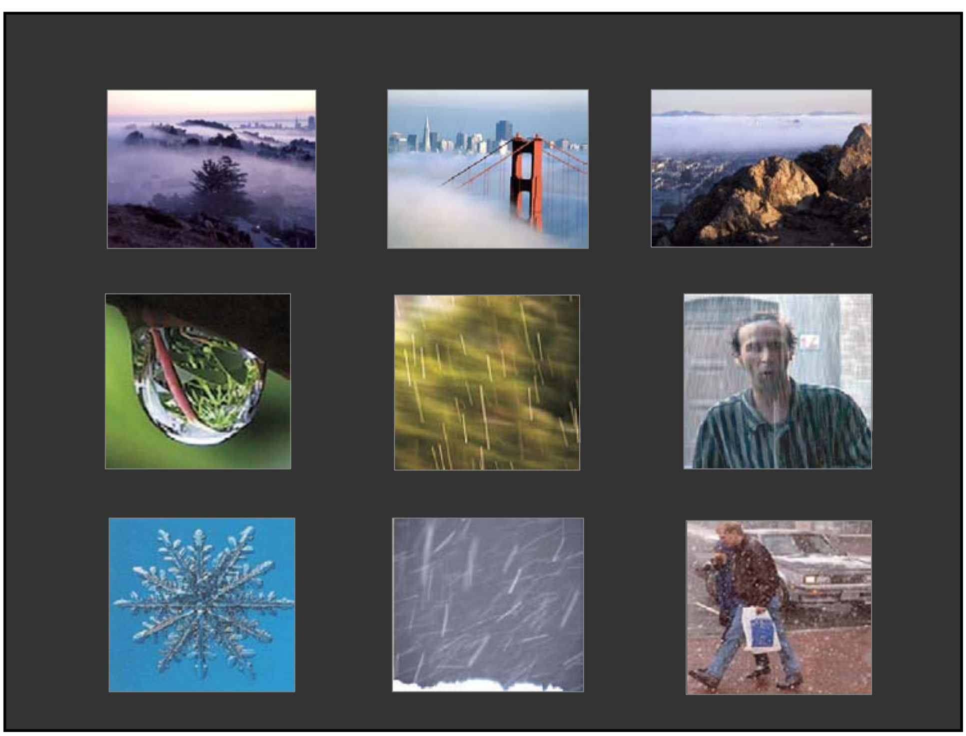

Haze

De-hazed





More Complex Appearances



For in-depth study of Appearance, take fall Graduate class "Physics-based methods in Vision" (previously "Appearance Modeling")

Radiometry and Image Formation

- To interpret image intensities, we need to understand Radiometric Concepts and Reflectance Properties.
- Topics to be Covered:

1) Image Intensities: Overview

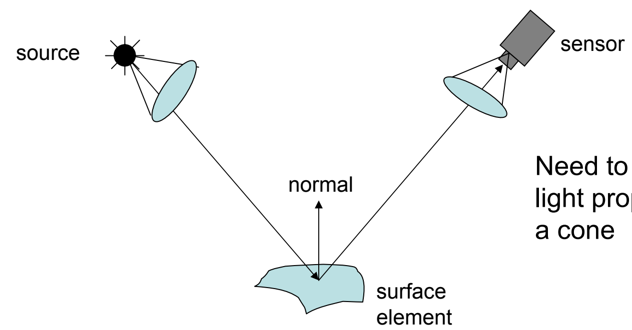
2) Radiometric Concepts:

    Radiant Intensity
    Irradiance
    Radiance
    BRDF

3) Image Formation using a Lens

4) Diffuse and Specular Reflectance

Image Intensities

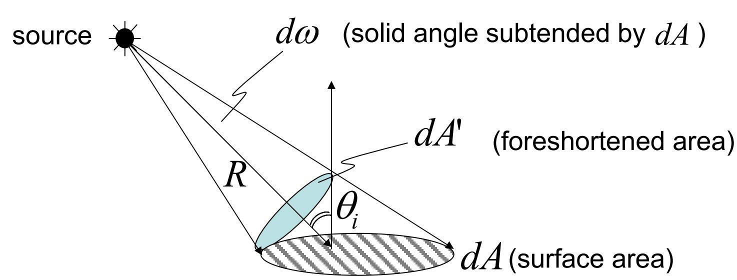


Need to consider light propagation in a cone

Image intensities = *f* (normal, surface reflectance, illumination)

Note: Image intensity understanding is an under-constrained problem!

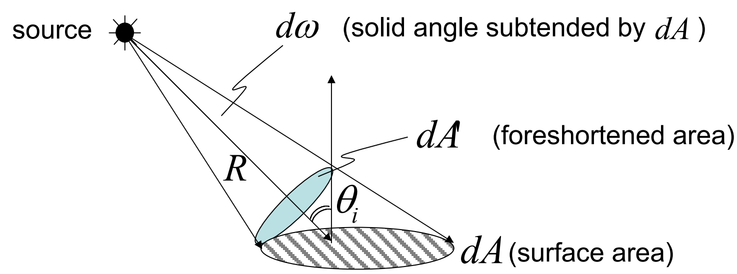
Solid Angle



Solid Angle : $$d\omega = \frac{dA'}{R^2} = \frac{dA\cos\theta_i}{R^2}$$ (steradian)

What is the solid angle subtended by a hemisphere?

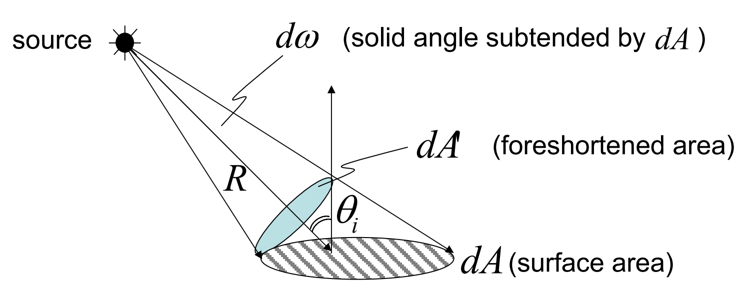
Radiant Intensity of Source



Radiant Intensity of Source : $J = \dfrac{d\Phi}{d\omega}$ (watts / steradian)

Light Flux (power) emitted per unit solid angle

Surface Irradiance

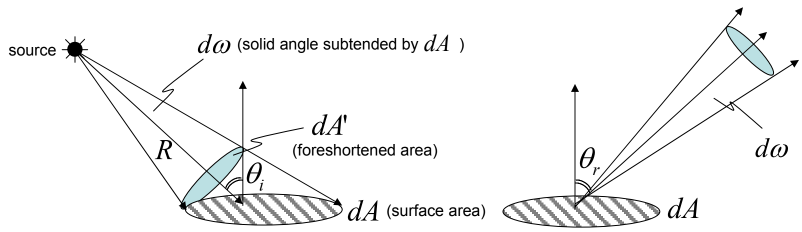


Surface Irradiance : $E = \dfrac{d\Phi}{dA}$ (watts / m^2)

Light Flux (power) incident per unit surface area.

Does not depend on where the light is coming from!

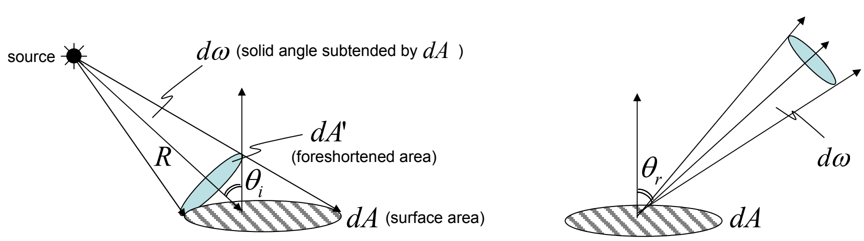
Surface Radiance (tricky!)



$$L = \frac{d^2\Phi}{(dA\cos\theta_r)\ d\omega}$$ (watts / m^2 steradian)

- Flux emitted per unit foreshortened area per unit solid angle.
- *L* depends on direction θ_r
- Surface can radiate into whole hemisphere.
- *L* depends on reflectance properties of surface.

Radiometric concepts – boring…but, important!



(1) Solid Angle : $d\omega = \frac{dA'}{R^2} = \frac{dA\cos\theta_i}{R^2}$ (steradian)

What is the solid angle subtended by a hemisphere?

(2) Radiant Intensity of Source : $J = \frac{d\Phi}{d\omega}$ (watts / steradian)

Light Flux (power) emitted per unit solid angle

(3) Surface Irradiance : $E = \frac{d\Phi}{dA}$ (watts / m)

Light Flux (power) incident per unit surface area.

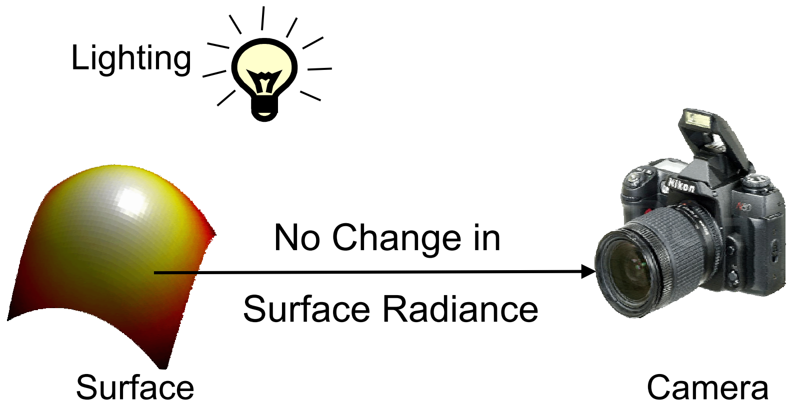
Does not depend on where the light is coming from!

(4) Surface Radiance (tricky) :

$$L = \frac{d^2\Phi}{(dA\cos\theta_r)\ d\omega}$$ (watts / m^2 steradian)

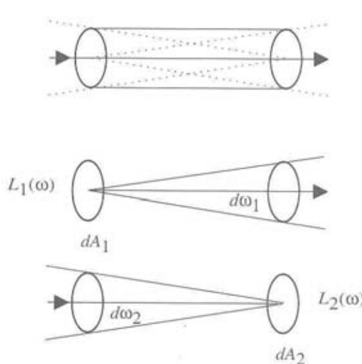
- Flux emitted per unit foreshortened area per unit solid angle.
- *L* depends on direction θ_r
- Surface can radiate into whole hemisphere.
- *L* depends on reflectance properties of surface.

The Fundamental Assumption in Vision



Radiance property

- Radiance is constant as it propagates along ray
  - Derived from conservation of flux
  - Fundamental in Light Transport.



$$d\Phi_1 = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2 = d\Phi_2$$

$$d\omega_1 = dA_2 / r^2 \qquad d\omega_2 = dA_1 / r^2$$

$$d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2$$

$$\therefore L_1 = L_2$$

Relationship between Scene and Image Brightness

- Before light hits the image plane:

Scene → Scene Radiance L → Lens → Image Irradiance E

Linear Mapping!

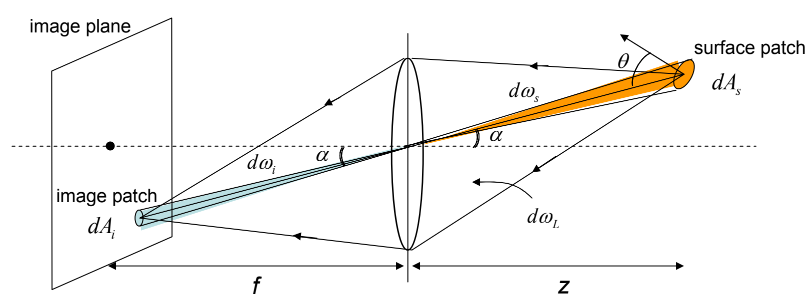
- After light hits the image plane:

Image Irradiance E → Camera Electronics → Measured Pixel Values, I

Non-linear Mapping!

Can we go from measured pixel value, I, to scene radiance, L?

Relation between Image Irradiance E and Scene Radiance L



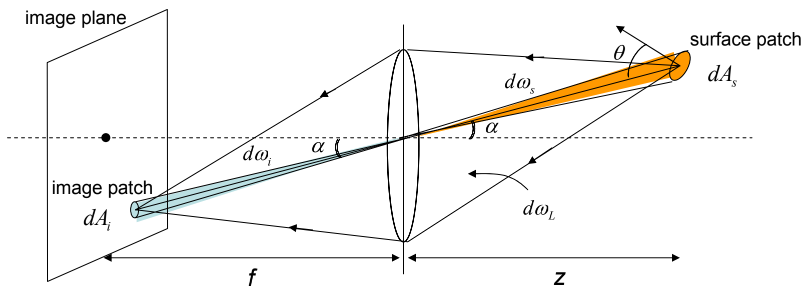
- Solid angles of the double cone (orange and green):

$$d\omega_i = d\omega_s \qquad \frac{dA_i \cos\alpha}{(f/\cos\alpha)^2} = \frac{dA_s \cos\theta}{(z/\cos\alpha)^2} \qquad \boxed{\frac{dA_s}{dA_i} = \frac{\cos\alpha}{\cos\theta}\left(\frac{z}{f}\right)^2} \rightarrow (1)$$

- Solid angle subtended by lens:

$$\boxed{d\omega_L = \frac{\pi d^2}{4} \frac{\cos\alpha}{(z/\cos\alpha)^2}} \rightarrow (2)$$

Relation between Image Irradiance E and Scene Radiance L



- Flux received by lens from dA_s = Flux projected onto image dA_i

$$\boxed{L\,(dA_s \cos\theta)\,d\omega_L = E\,dA_i} \longrightarrow (3)$$

- From (1), (2), and (3): $\boxed{E = L\,\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos\alpha^4}$

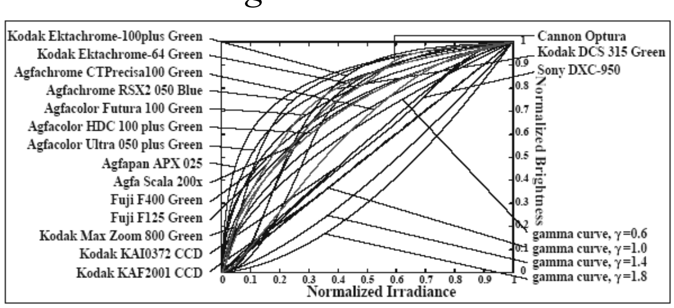
  - Image irradiance is proportional to Scene Radiance!
  - Small field of view → Effects of 4th power of cosine are small.

Relation between Pixel Values I and Image Irradiance E

Image Irradiance E →	Camera Electronics	→ Measured Pixel Values, I

- The camera response function relates image irradiance at the image plane to the measured pixel intensity values.

$$g : E \rightarrow I$$



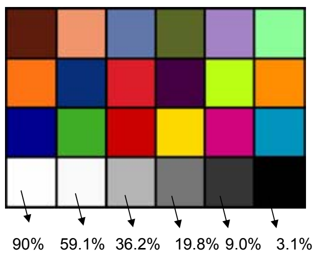
(Grossberg and Nayar)

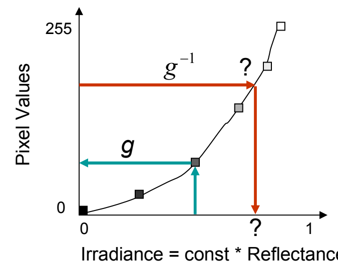
Radiometric Calibration - RECAP

•Important preprocessing step for many vision and graphics algorithms such as photometric stereo, invariants, de-weathering, inverse rendering, image based rendering, etc.

$$g^{-1} : I \to E$$

•Use a color chart with precisely known reflectances.





- Use more camera exposures to fill up the curve.
- Method assumes constant lighting on all patches and works best when source is far away (example sunlight).
- Unique inverse exists because g is monotonic and smooth for all cameras.

Surface Appearance

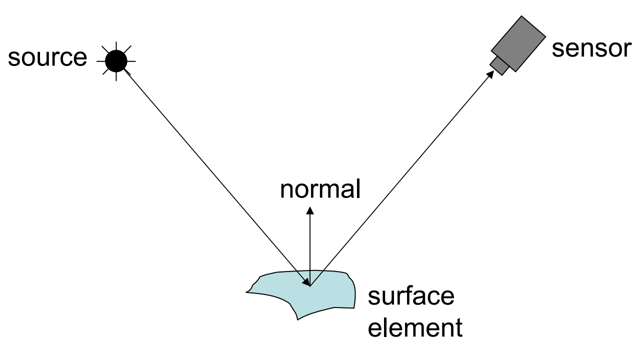
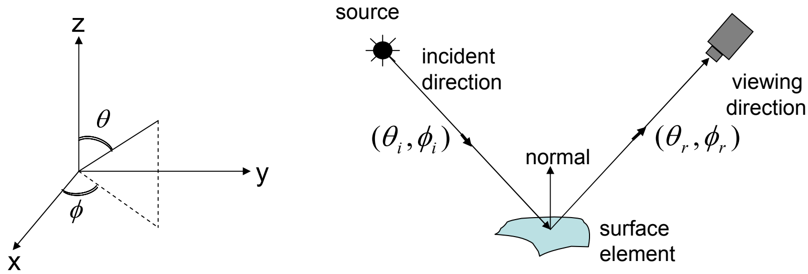


Image intensities = f (normal, surface reflectance, illumination)

Surface reflection depends on both the viewing and illumination directions.

BRDF: Bidirectional Reflectance Distribution Function

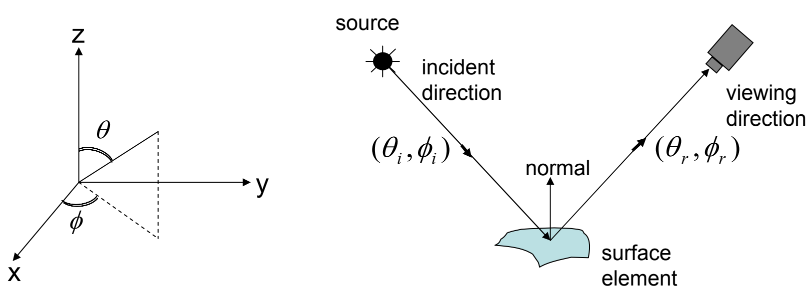


$E^{surface}(\theta_i, \phi_i)$ Irradiance at Surface in direction (θ_i, ϕ_i)

$L^{surface}(\theta_r, \phi_r)$ Radiance of Surface in direction (θ_r, ϕ_r)

BRDF : $f(\theta_i, \phi_i; \theta_r, \phi_r) \quad = \quad \dfrac{L^{surface}(\theta_r, \phi_r)}{E^{surface}(\theta_i, \phi_i)}$

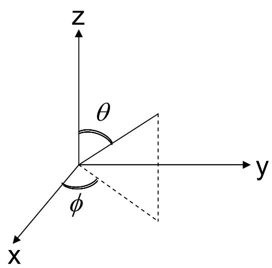
Important Properties of BRDFs

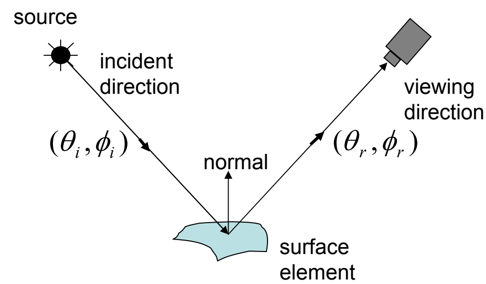


- Conservation of Energy:

$$\int_{hemisphere} f(\theta_i, \phi_i; \theta_r, \phi_r)\, d\omega_i \le 1$$

Important Properties of BRDFs



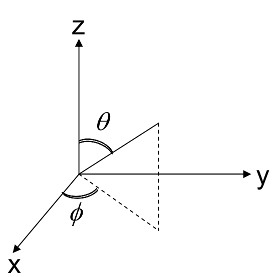


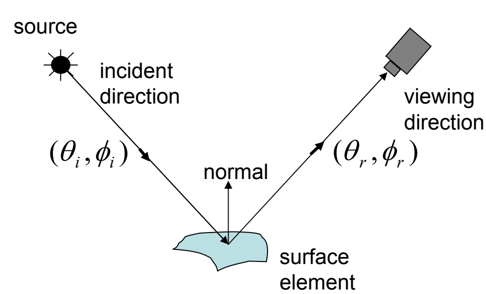
- Helmholtz Reciprocity: (follows from 2nd Law of Thermodynamics)

BRDF does not change when source and viewing directions are swapped.

$$f(\theta_i, \phi_i; \theta_r, \phi_r) \quad = \quad f(\theta_r, \phi_r; \theta_i, \phi_i)$$

Important Properties of BRDFs



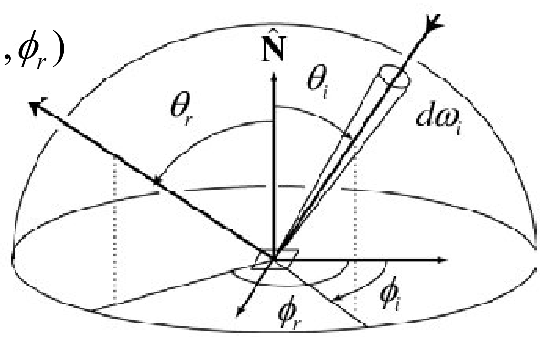


- Rotational Symmetry (Isotropy):

BRDF does not change when surface is rotated about the normal.

Can be written as a function of 3 variables : $f(\theta_i, \theta_r, \phi_i - \phi_r)$

Derivation of the Scene Radiance Equation



From the definition of BRDF:

$$L^{surface}(\theta_r,\phi_r) \quad = \quad E^{surface}(\theta_i,\phi_i)\, f(\theta_i,\phi_i;\theta_r,\phi_r)$$

Derivation of the Scene Radiance Equation

From the definition of BRDF:

$$L^{surface}(\theta_r,\phi_r) \quad = \quad \underline{E^{surface}(\theta_i,\phi_i)}\, f(\theta_i,\phi_i;\theta_r,\phi_r)$$

Write Surface Irradiance in terms of Source Radiance:

$$L^{surface}(\theta_r,\phi_r) \quad = \quad \underline{L^{src}(\theta_i,\phi_i)}\, f(\theta_i,\phi_i;\theta_r,\phi_r)\, \underline{\cos\theta_i d\omega_i}$$

Integrate over entire hemisphere of possible source directions:

$$L^{surface}(\theta_r,\phi_r) \quad = \quad \int_{2\pi} L^{src}(\theta_i,\phi_i)\, f(\theta_i,\phi_i;\theta_r,\phi_r)\cos\theta_i\ \underline{d\omega_i}$$

Convert from solid angle to theta-phi representation:

$$L^{surface}(\theta_r,\phi_r) \quad = \quad \int_{-\pi}^{\pi}\int_{0}^{\pi/2} L^{src}(\theta_i,\phi_i)\, f(\theta_i,\phi_i;\theta_r,\phi_r)\cos\theta_i\ \underline{\sin\theta_i d\theta_i d\phi_i}$$

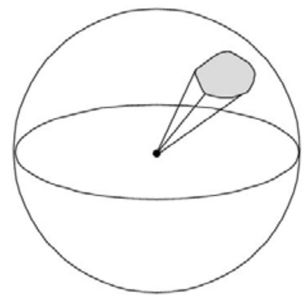
Angles and Solid Angles

- **Angle** $\theta = \frac{l}{r}$

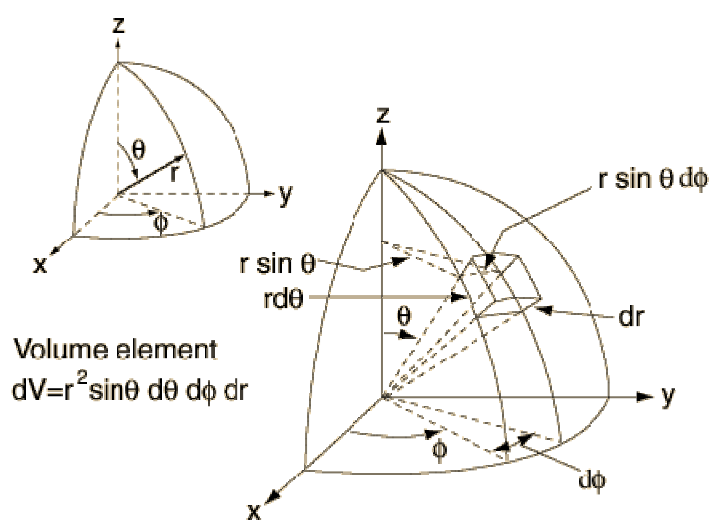
  $\Rightarrow$ circle has 2π radians

- **Solid angle** $\Omega = \frac{A}{R^2}$

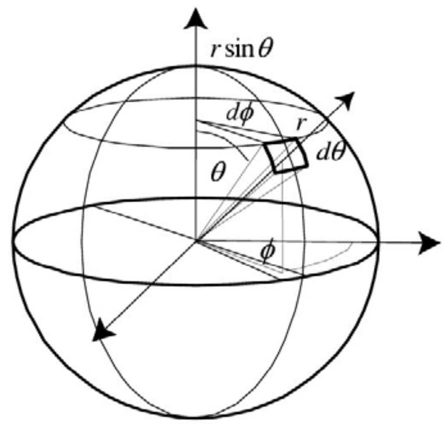
  $\Rightarrow$ sphere has 4π steradians



Differential Solid Angle and Spherical Polar Coordinates



Differential Solid Angles



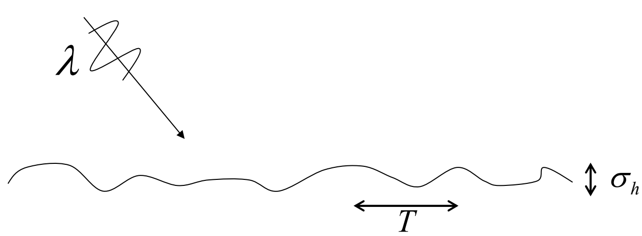
$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2\sin\theta\,d\theta\,d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

$$S = \int_0^{\pi}\int_0^{2\pi} \sin\theta\,d\theta\,d\phi = 4\pi$$

Reflectance Models

Reflection: An Electromagnetic Phenomenon



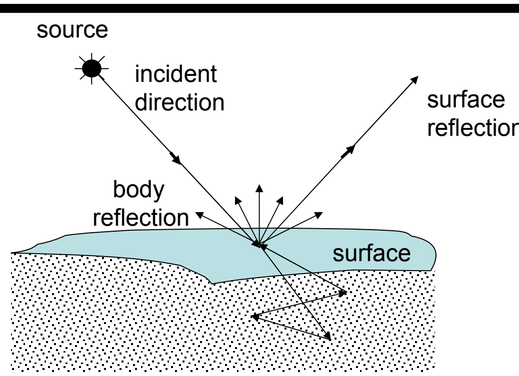
Two approaches to derive Reflectance Models:

- Physical Optics (Wave Optics)
- Geometrical Optics (Ray Optics)

Geometrical models are approximations to physical models
But they are easier to use!

Reflectance that Require Wave Optics

Mechanisms of Reflection



- Body Reflection:

  Diffuse Reflection
  Matte Appearance
  Non-Homogeneous Medium
  Clay, paper, etc

- Surface Reflection:

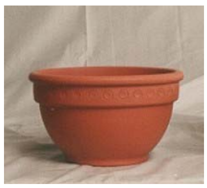
  Specular Reflection
  Glossy Appearance
  Highlights
  Dominant for Metals

Image Intensity = Body Reflection + Surface Reflection

Example Surfaces

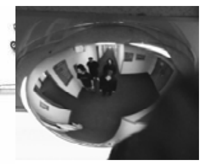
Body Reflection:

Diffuse Reflection
Matte Appearance
Non-Homogeneous Medium
Clay, paper, etc



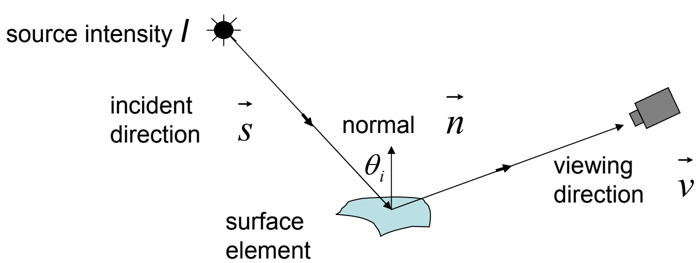
Surface Reflection:

Specular Reflection
Glossy Appearance
Highlights
Dominant for Metals



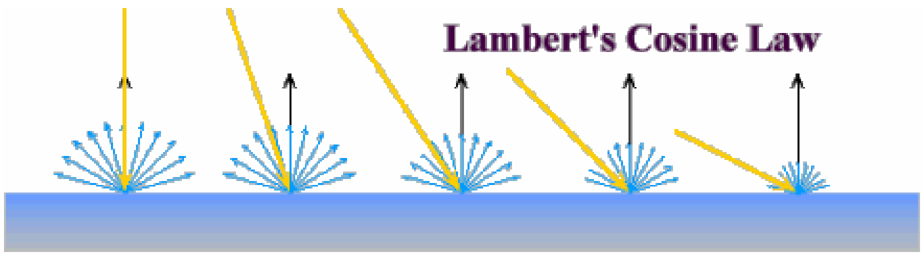
Many materials exhibit both Reflections:

Diffuse Reflection and Lambertian BRDF



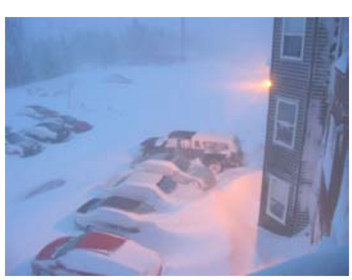
- Surface appears equally bright from ALL directions! (independent of $\vec{v}$)
- Lambertian BRDF is simply a constant : $f(\theta_i, \phi_i; \theta_r, \phi_r) \quad = \quad \dfrac{\rho_d}{\pi}$ albedo
- Surface Radiance : $L = \dfrac{\rho_d}{\pi} I \cos \theta_i \quad = \quad \dfrac{\rho_d}{\pi} I \;\vec{n} \cdot \vec{s}$ source intensity
- Commonly used in Vision and Graphics!

Diffuse Reflection and Lambertian BRDF



White-out: Snow and Overcast Skies


CAN'T perceive the shape of the snow covered terrain!



CAN perceive shape in regions
lit by the street lamp!!

WHY?

Diffuse Reflection from Uniform Sky

$$L^{surface}(\theta_r,\phi_r) \quad = \quad \int_{-\pi}^{\pi}\int_{0}^{\pi/2} L^{src}(\theta_i,\phi_i)\, f(\theta_i,\phi_i;\theta_r,\phi_r)\cos\theta_i\,\sin\theta_i d\theta_i d\phi_i$$

• Assume Lambertian Surface with Albedo = 1 (no absorption)

$$f(\theta_i,\phi_i;\theta_r,\phi_r) = \frac{1}{\pi}$$

• Assume Sky radiance is constant

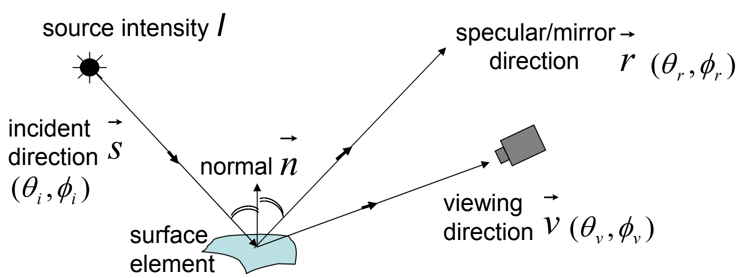
$$L^{src}(\theta_i,\phi_i) = L^{sky}$$

• Substituting in above Equation:

$$\boxed{L^{surface}(\theta_r,\phi_r) \quad = \quad L^{sky}}$$

Radiance of any patch is the same as Sky radiance !! (white-out condition)

Specular Reflection and Mirror BRDF



- Valid for very smooth surfaces.
- All incident light energy reflected in a SINGLE direction (only when $\vec{v}$ = $\vec{r}$).
- Mirror BRDF is simply a double-delta function :

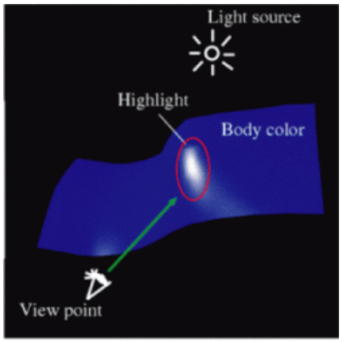
specular albedo

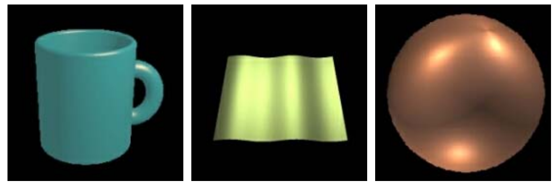
$$f(\theta_i,\phi_i;\theta_v,\phi_v) = \rho_s\ \delta(\theta_i-\theta_v)\ \delta(\phi_i+\pi-\phi_v)$$

- Surface Radiance : $L = I\ \rho_s\ \delta(\theta_i-\theta_v)\ \delta(\phi_i+\pi-\phi_v)$

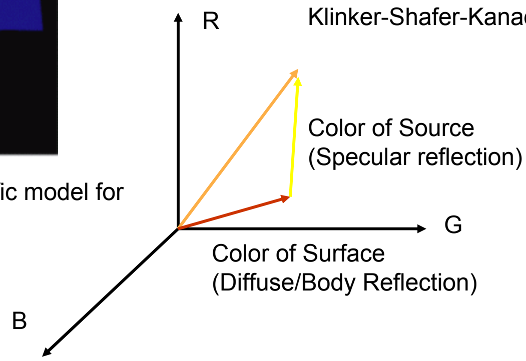
Combing Specular and Diffuse: Dichromatic Reflection

Observed Image Color = a x Body Color + b x Specular Reflection Color





Klinker-Shafer-Kanade 1988



Does not specify any specific model for Diffuse/specular reflection

Diffuse and Specular Reflection

diffuse specular diffuse+specular

Next Class

- Photometric Stereo
- Reading: Horn, Chapter 10.