

<hello@qingpei.me>

qingpei.me

2020 3 28

1

lrightWrite something.

normal **bold** *italic*

monospace **bold** *italic*

sans-serif **bold** *italic*

2

right

Let A be a ring $\neq 0$. Show that $A^m \cong A^n \Rightarrow m = n$ [Let \mathfrak{m} be a maximal ideal of A and let $\phi : A^m \rightarrow A^n$ be an isomorphism. Then $1 \otimes \phi : (A/\mathfrak{m}) \otimes A^m \rightarrow (A/\mathfrak{m}) \otimes A^n$ is an isomorphism between vector spaces of dimensions m and n over the field $k = A/\mathfrak{m}$. Hence $m = n$.] (Cf. Chapter 3, Exercise 15.) If $\phi : A^m \rightarrow A^n$ is surjective, then $m \geq n$. If $\phi : A^m \rightarrow A^n$ is injective, is it always the case that $m \leq n$?