

S&DS 365 / 665
Intermediate Machine Learning

Convolutional Neural Networks

September 25

Yale

Reminders

- Assignment 1 due Wednesday
- Assignment 2 posted Wednesday
- Quiz 2 solutions/scores released (avg. 81%)

Outline for CNNs

- Mechanics of convolutional networks
- Filters and pooling and flattening
- Example: Brain scans of mice
- Assignment 2: Convnets for MNIST
- Other examples

From last September

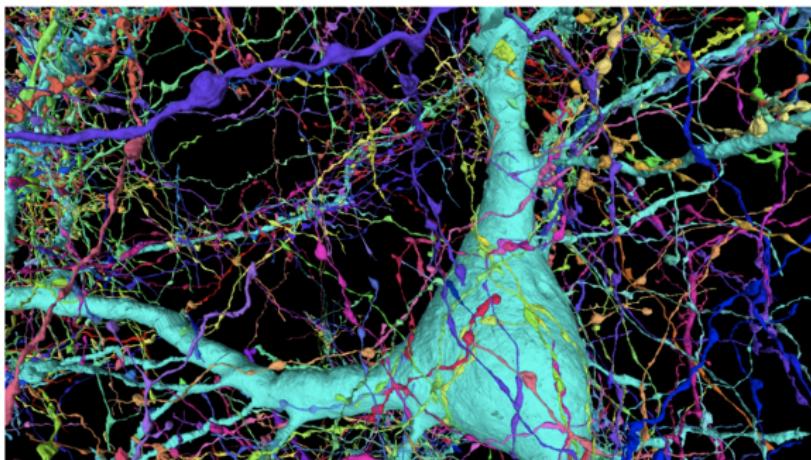
nature > technology features > article

TECHNOLOGY FEATURE | 20 September 2022

Five ways deep learning has transformed image analysis

From connectomics to behavioural biology, artificial intelligence is making it faster and easier to extract information from images.

Sandeep Ravindan



Neurons (teal) can have thousands of connections to other cells. Credit: Lichtman Lab, Harvard/Jain Group, Google

Notes

- Animations from tutorial “Let’s Code Convolutional Neural Network in plain NumPy: Mysteries of Neural Networks” by Piotr Skalski
 - <https://towardsdatascience.com/lets-code-convolutional-neural-network-in-plain-numpy-ce48e732f5d5>
- To see the animations, view this pdf using Adobe Acrobat

Convolution

Mathematically, a *convolution* sweeps a function $K(u)$ over the values of another function f :

Continuous:

$$K * f(x) = \int K(x - u) f(u) du$$

- Same operation we saw with smoothing kernels
- If K is a Gaussian bump, this blurs or smooths out the values.
- For CNNs, K will have small support, encode features of the input

Convolution

Mathematically, a *convolution* sweeps a function $K(u)$ over the values of another function f :

Discrete:

$$K * f(x_j) = \sum_i K(x_j - x_i) f(x_i)$$

- Same operation we saw with smoothing kernels
- If K is a Gaussian bump, this blurs or smooths out the values.
- For CNNs, K will have small support, encode features of the input

Examples

convolve_demo.ipynb gives examples



Principles

Some of the ideas behind convolutional neural networks (CNNs):

- ① Parameter tying: Let neurons share the same weights
- ② Kernel learning: The kernels should not be fixed, but rather *learned* from the data
- ③ Invariance: Doesn't matter where features are located

CNNs are most commonly used for images; can be used whenever data have some kind of spatial structure (e.g. DNA)

Tensors

A tensor is a multidimensional array*—like a matrix, but with more than just a pair of indices for row and column

In Python, tensors are represented in `numpy` using the type `numpy.ndarray` (“*n*-dimensional array”)

TensorFlow also has built-in types to represent tensors

* In mathematics, there is a little more structure to tensors.

Tensors

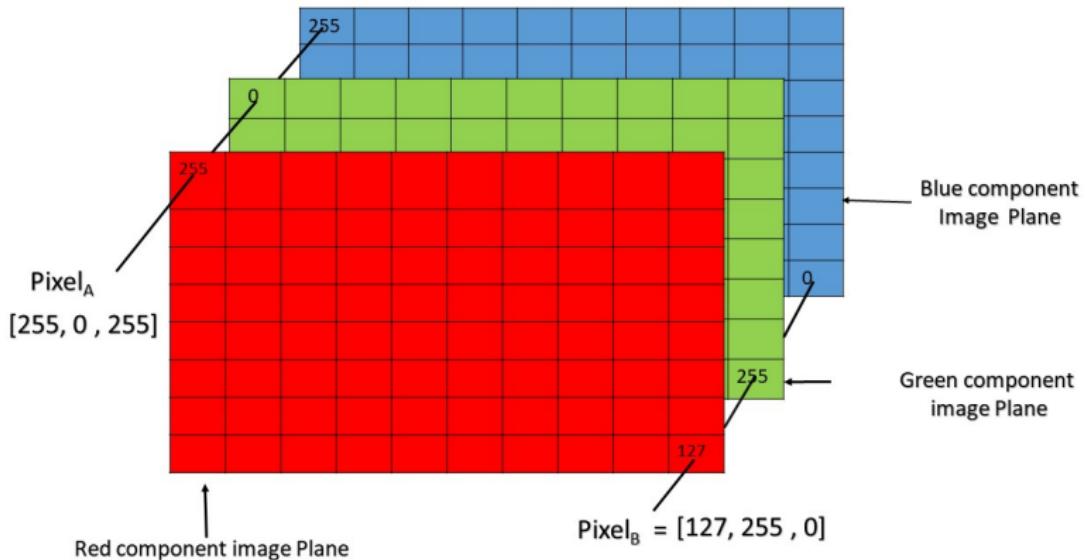
```
x = np.array(np.arange(24)).reshape(2, 3, 4)
print(x)
print(x[:, :, 0])

[[[ 0   1   2   3]
 [ 4   5   6   7]
 [ 8   9  10  11]]

[[12  13  14  15]
 [16  17  18  19]
 [20  21  22  23]]]

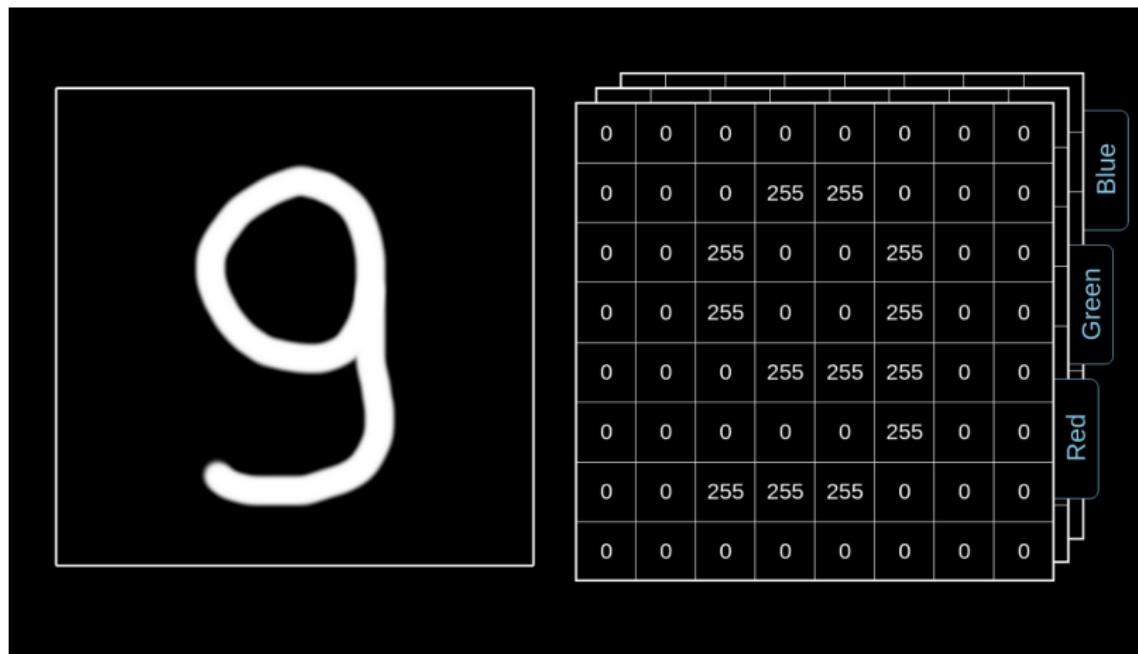
[[ 0   4   8]
 [12  16  20]]
```

Images as tensors: Channels



Pixel of an RGB image are formed from the corresponding pixel of the three component images

Images as tensors: Channels



Convolutional layer

- In a convolutional layer, a set of *kernels* are “swept over” or convolved over the input
- The kernels are themselves tensors with entries that are trained weights
- The output after convolving the kernel over the input tensor is called a *feature map*
- A convolutional layer is equivalent to a standard fully connected layer, but where the weights are constrained to be sparse and shared (or “tied”) across different neurons

Convolutional layer

Adding a nonlinearity

- So far everything is linear
- An activation function is used to add a nonlinearity
- Each filter also has an intercept (a single number), added before applying the activation function
- The same activation function is used for all of the kernels in a layer

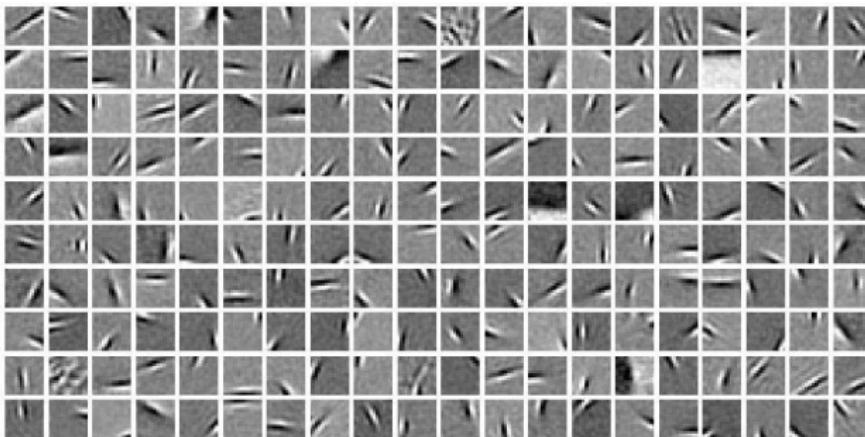
Pooling

- The point of a convolutional layer is to learn “features” of the input
- We don’t care too much exactly where they appear
- *Pooling* the feature map reduces the dimension and encodes the approximate location of features
- Two common types: max pooling and average pooling
- When using max pooling, to update weights in backpropagation, the locations of the features can be noted

Pooling

How features are built up

- Filters in the first layer may look like this:



- Filters in second layer can then capture more complex shapes

How features are built up

- Each kernel has a weight for each spatial position and channel
- Features from previous layers are recorded in the channels
- So the next layer can have a kernel that says

“Oh, I see that feature 17 (a horizontal edge) from the previous layer is active in position (1,1), feature 42 (a vertical edge) is active in position (1,2)” to capture “corners”

- This enables compound features to be built up

Pooling and backpropagation

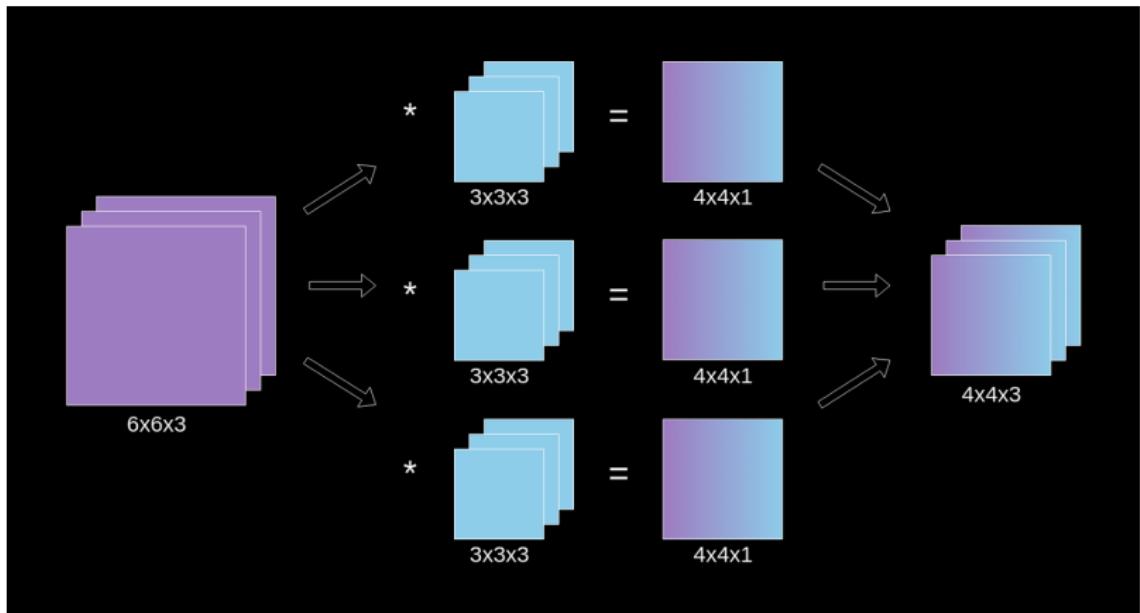
- A forward pass through the layers is used to make predictions, and store intermediate results to be used later
- A backward pass propagates the errors in prediction back through the layers, and computes changes to the weights for the gradient step

Forward and backward passes

Applying kernels across channels

- Each kernel (filter) has the same number of channels as the input tensor
- A kernel is applied at each location by multiplying component-wise and then summing
- The feature map for each kernel is then one channel in the output tensor
- So, the number of channels in the output is the number of kernels

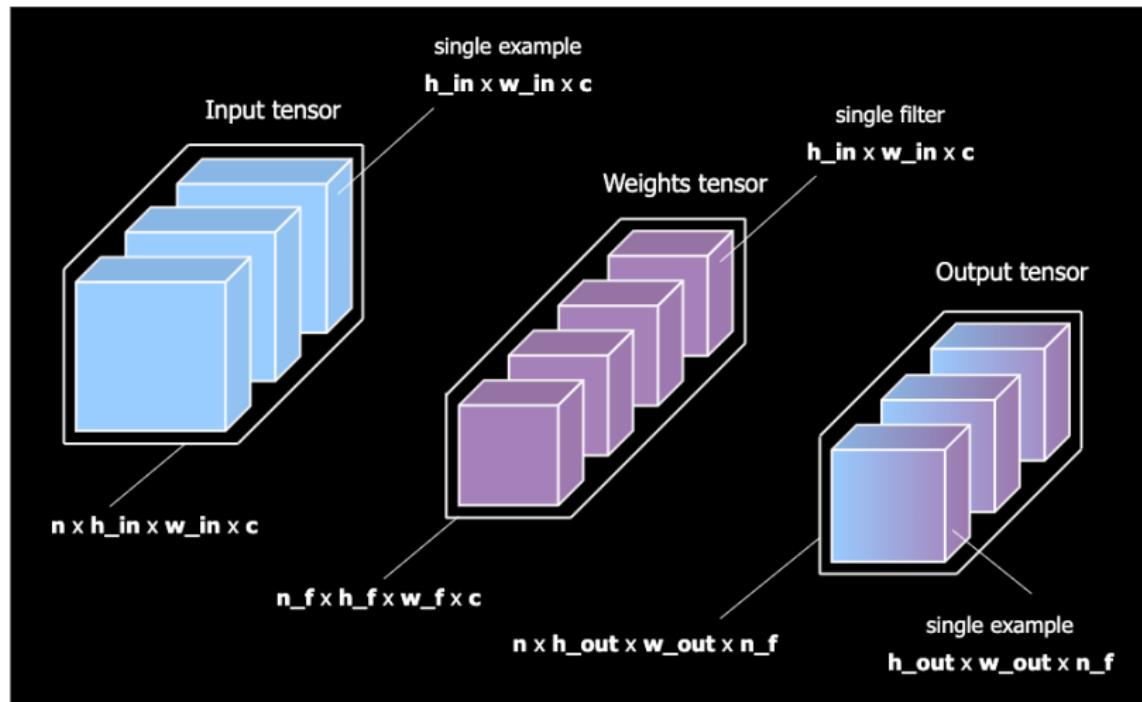
Applying kernels across channels



Convolutional layer: Tensor shapes

- Data points are processed in “mini-batches” of n items
- The first dimension of the input and output tensors at each layer is always n
- The last dimension of the output is the number of kernels (filters)

Convolutional layer: Tensor shapes



Can you find the typo in this figure?

Dropout

- “Dropout” is a type of regularization that is easy to incorporate
- Idea: Randomly zero out entries in a tensor
- Helps guard against overfitting
- Usual form:
 - ▶ Independently zero out each entry with probability ε
 - ▶ Divide by $(1 - \varepsilon)$

Equivalent to a type of ridge penalization: <https://arxiv.org/abs/1307.1493>, and approximate Bayesian averaging: <http://proceedings.mlr.press/v48/gal16.pdf>

Dropout

This illustrates something a bit different than the standard dropout

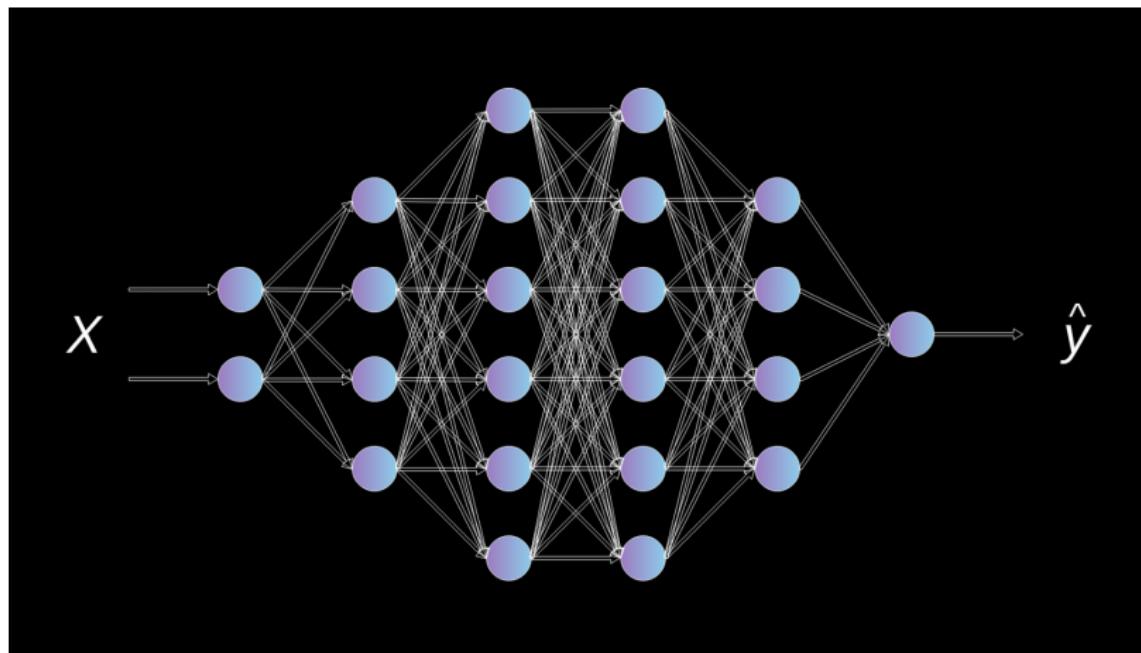
Flattening

- After one or more convolutional layers, we usually add “dense” layers
- These are the standard types of layers in a neural network, with each output neuron connected to each input neuron, with a tuneable weight
- In CNNs, the convolutional layers build up a set of “features” and then these features are used to make a prediction

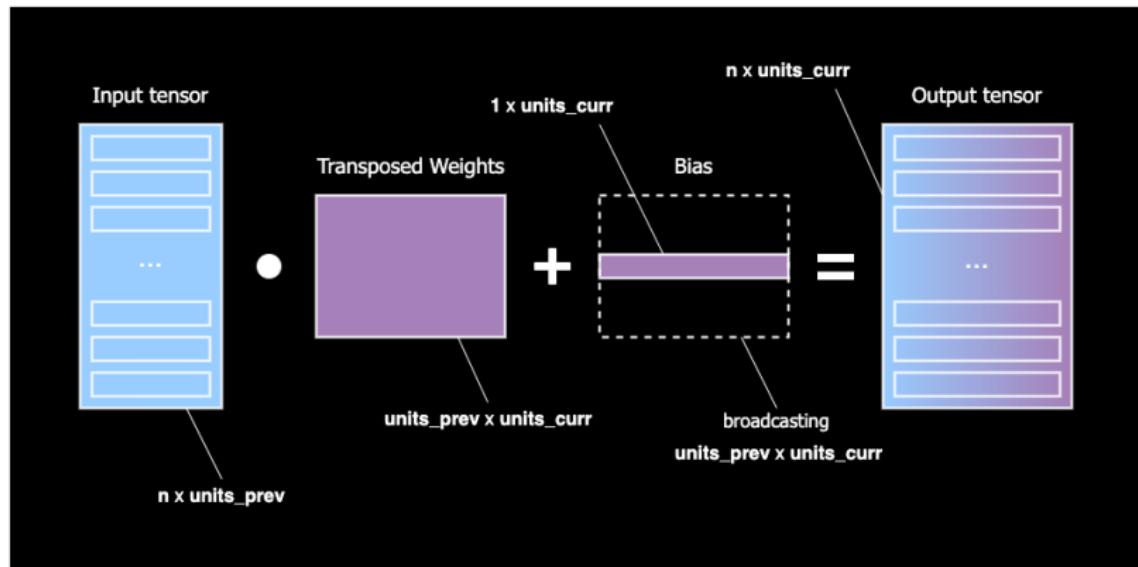
Flattening

This is the same as the `numpy.flatten()` operation

Dense layers



Dense operations

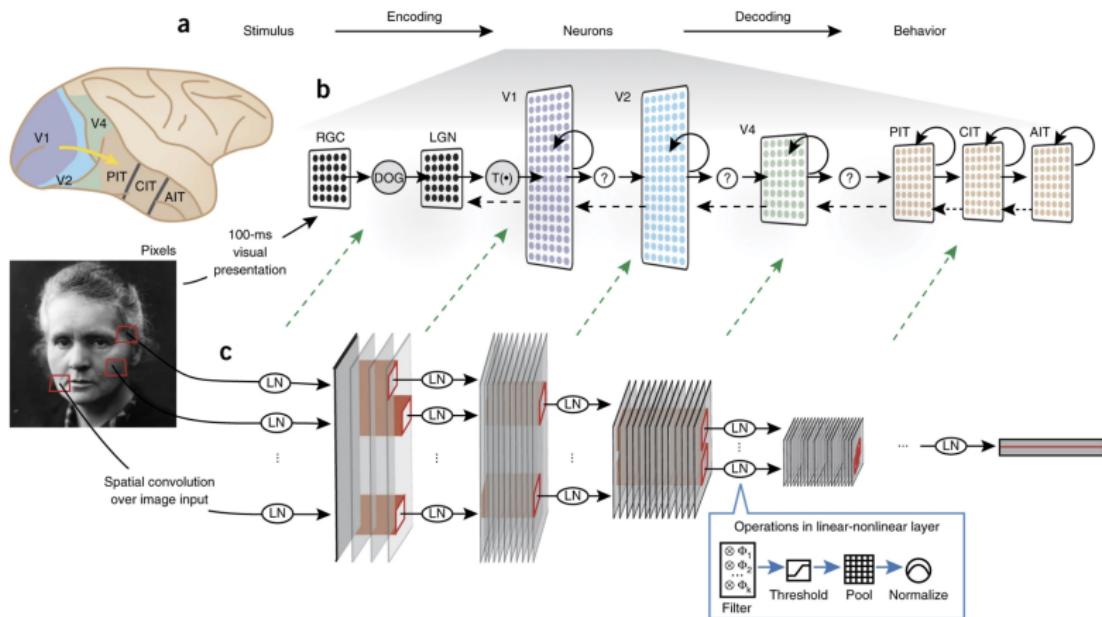


Again, this is not showing the use of an activation function

CNNs: Biological analogies

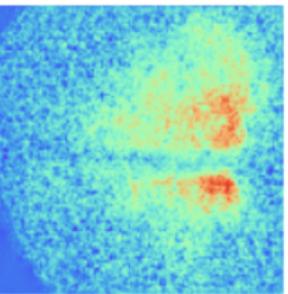
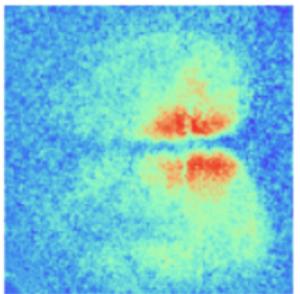
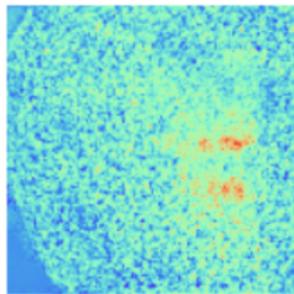
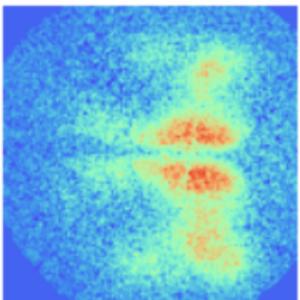
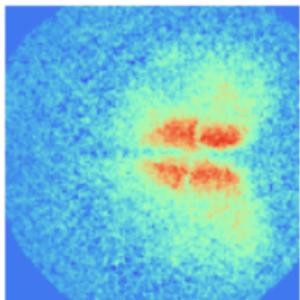
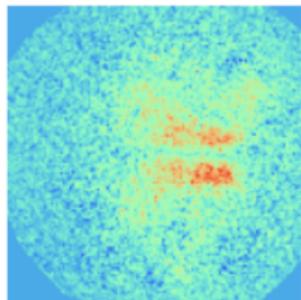
- It has been argued that common architectures for CNNs used for image processing mirror the organization and function of visual cortex in many species

CNNs: Biological analogies



Using goal-driven deep learning models to understand sensory cortex, Yamins and DiCarlo, 2016,
<https://www.nature.com/articles/nn.4244>

Task: Real or Fake?



Task: Real or Fake?

- The real images are optical images of neural activity at the surface of the cortex in transgenic mice
- The fake images were generated by a “deconvolutional” neural network trained as part of a GAN

Example CNN

```
model = models.Sequential()
model.add(layers.Conv2D(32, (5, 5), activation='relu', input_shape=(128, 128, 1)))
model.add(layers.MaxPooling2D((4, 4)))
model.add(layers.Flatten())
model.add(layers.Dense(2))
model.summary()
```

Model: "sequential"

Layer (type)	Output Shape	Param #
conv2d (Conv2D)	(None, 124, 124, 32)	832
max_pooling2d (MaxPooling2D)	(None, 31, 31, 32)	0
flatten (Flatten)	(None, 30752)	0
dense (Dense)	(None, 2)	61506

Total params: 62,338
Trainable params: 62,338
Non-trainable params: 0

Example CNN

First layer:

- Input: 128×128 images, 1 channel
- First layer: 32 filters, each of size 5×5
- Output of first layer: tensor of shape $124 \times 124 \times 32$
- Number of parameters: $(5 \times 5 + 1) \times 32 = 832$
- Activation function: ReLU
- Output of first layer: tensor of shape $124 \times 124 \times 32$

Example CNN

Second layer:

- Maximum of 4×4 regions in feature map
- Output: Tensor of shape $31 \times 31 \times 32$ since $124/4 = 31$
- Number of parameters: Zero!

Example CNN

Third layer:

- Flattened version of previous layer
- Number of neurons: $31 \times 31 \times 32 = 30,752$
- Number of parameters: Zero!

Example CNN

Final layer:

- Dense connections
- Two neurons in output (“class 0” and “class 1”)
- Number of parameters: $(30,752 + 1) \times 2 = 61,506$

Example CNN

Overall:

- Number of parameters: $832 + 61,506 = 62,338$
- Most come from dense layer (98.7%)

A model with two convolutional layers

```
model = models.Sequential()
model.add(layers.Conv2D(32, (5, 5), activation='relu', input_shape=(128, 128, 1)))
model.add(layers.MaxPooling2D((4, 4)))
model.add(layers.Conv2D(32, (2, 2), activation='relu'))
model.add(layers.MaxPooling2D((4,4)))
model.add(layers.Flatten())
model.add(layers.Dense(128, activation='relu'))
model.add(layers.Dense(2))
model.summary()
```

Model: "sequential_11"

Layer (type)	Output Shape	Param #
<hr/>		
conv2d_20 (Conv2D)	(None, 124, 124, 32)	832
<hr/>		
max_pooling2d_20 (MaxPooling)	(None, 31, 31, 32)	0
<hr/>		
conv2d_21 (Conv2D)	(None, 30, 30, 32)	4128
<hr/>		
max_pooling2d_21 (MaxPooling)	(None, 7, 7, 32)	0
<hr/>		
flatten_11 (Flatten)	(None, 1568)	0
<hr/>		
dense_17 (Dense)	(None, 128)	200832
<hr/>		
dense_18 (Dense)	(None, 2)	258
<hr/>		
Total params: 206,050		
Trainable params: 206,050		
Non-trainable params: 0		

Summary for today

- A convolutional layer applies a set of kernels (filters) across the input
- Each kernel is a tensor of smaller dimension than the input
- The values of the kernels are learned by backpropagation
- Successive convolutional layers build increasingly rich features
- The convolutional layers define a set of features that are then used to make predictions in one or more dense layers
- CNNs are attractive whenever the data have spatial structure where the same features might appear at many different locations