Project 1: Spline

Author: WangHaoZhe (522072910008)
Submit Date: 2023/4/30

Main Code

You can run this code section by section to test my each question.

You can also find my full code (so you don't have to copy my programs from this pdf) and the raw file (in Markdown) of this pdf on my Github: github.com/WangHaoZhe/Project1_Spline

```
% Project 1: Spline
% Author: WangHaoZhe 522072910008
% Create Date: 2023/3/18
% Submit Date: 2023/4/30

%% Question 1
% Curve 1
x1 = [1, 2, 5, 6, 7, 8, 10, 13, 17];
y1 = [3.0, 3.7, 3.9, 4.2, 5.7, 6.6, 7.1, 6.7, 4.5];
v11 = 1; v1n = -2/3;
% Curve 2
x2 = [17, 20, 23, 24, 25, 27, 27.7];
y2 = [4.5, 7.0, 6.1, 5.6, 5.8, 5.2, 4.1];
v21 = 3; v2n = -4;
% Curve 3
x3 = [27.7, 28, 29, 30];
y3 = [4.1, 4.3, 4.1, 3.0];
v31 = 1/3; v3n = -3/2;

% Plot spline curve
splineplot(x1,y1,v11,v1n);
splineplot(x2,y2,v21,v2n);
splineplot(x3,y3,v31,v3n);

%% Question 2
% Curve 1
coeff1 = splinecoeff(x1,y1,v11,v1n);
coeff2 = splinecoeff(x2,y2,v21,v2n);
coeff3 = splinecoeff(x3,y3,v31,v3n);

beziercurve(x1,y1,coeff1);
beziercurve(x2,y2,coeff2);
beziercurve(x3,y3,coeff3);

%% Question 3
img = imread('dog.jpg');
min_x = 0;
max_x = 52.5;
```

```
min_y = 0;
max_y = 10;
imagesc([min_x max_x], [min_y max_y], img);
hold on;
[points,v1,vn] = bezierdraw;
x = [points(:,1)]';
y = [points(:,2)]';
pause(3);
splineplot(x,y,v1,vn);
```

Question 1: Cubic Spline

Code

File Name: splinecoeff.m

```
% Program 3.5 Calculation of spline coefficients
% Calculates coefficients of cubic spline
% Input: x,y vectors of data points
%   plus two optional extra data v1, vn
% Output: matrix of coefficients b1,c1,d1;b2,c2,d2;...
function coeff=splinecoeff(x,y,v1,vn)
n=length(x);
A=zeros(n,n);             % matrix A is nxn
r=zeros(n,1);
for i=1:n-1               % define the deltas
    dx(i) = x(i+1)-x(i); dy(i)=y(i+1)-y(i);
end
for i=2:n-1               % load the A matrix
    A(i,i-1:i+1)=[dx(i-1) 2*(dx(i-1)+dx(i)) dx(i)];
    r(i)=3*(dy(i)/dx(i) - dy(i-1)/dx(i-1)); % right-hand side
end

% Set endpoint conditions
% Use only one of following 5 pairs:
%A(1,1) = 1;               % natural spline conditions
%A(n,n) = 1;

A(1,1)=2;r(1)=v1;        % curvature-adj conditions
A(n,n)=2;r(n)=vn;

%A(1,1:2)=[2*dx(1) dx(1)];r(1)=3*(dy(1)/dx(1)-v1); % clamped
%A(n,n-1:n)=[dx(n-1) 2*dx(n-1)];r(n)=3*(vn-dy(n-1)/dx(n-1));

%A(1,1:2)=[1 -1];         % parabol-term conditions, for n>=3
%A(n,n-1:n)=[1 -1];

%A(1,1:3)=[dx(2) -(dx(1)+dx(2)) dx(1)]; % not-a-knot for n>=4
%A(n,n-2:n)=[dx(n-1) -(dx(n-2)+dx(n-1)) dx(n-2)];

coeff=zeros(n,3);
```

```
coeff(:,2)=A\r;          % solve for c coefficients
for i=1:n-1              % solve for b and d
    coeff(i,3)=(coeff(i+1,2)-coeff(i,2))/(3*dx(i));
    coeff(i,1)=dy(i)/dx(i)-dx(i)*(2*coeff(i,2)+coeff(i+1,2))/3;
end
coeff=coeff(1:n-1,1:3);
```

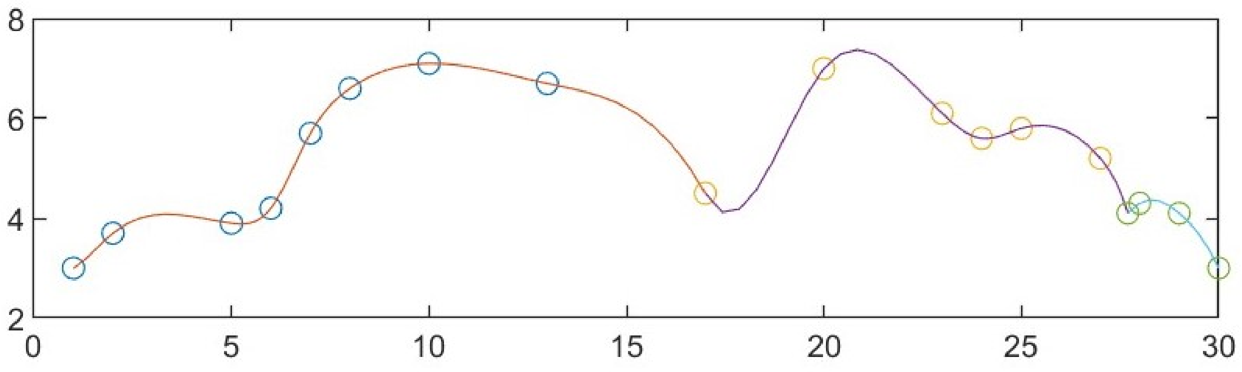
File Name: splineplot.m

```
% Plot cubic spline
% Input: x,y vectors of data points
% Output: a figure of spline curve
function [x1,y1]=splineplot(x,y,v1,vn)
n=length(x);
coeff=splinecoeff(x,y,v1,vn);
x1=[];y1=[];
for i=1:n-1
    xs=linspace(x(i),x(i+1),n+1);
    dx=xs-x(i);

    ys=coeff(i,3)*dx;
    ys=(ys+coeff(i,2)).*dx;
    ys=(ys+coeff(i,1)).*dx+y(i);

    x1=[x1;xs(1:n)'];
    y1=[y1;ys(1:n)'];
end
x1=[x1;x(end)];y1=[y1;y(end)];
plot(x,y,'o',x1,y1)
hold on;
```

Result



Question 2: Bezier Curve

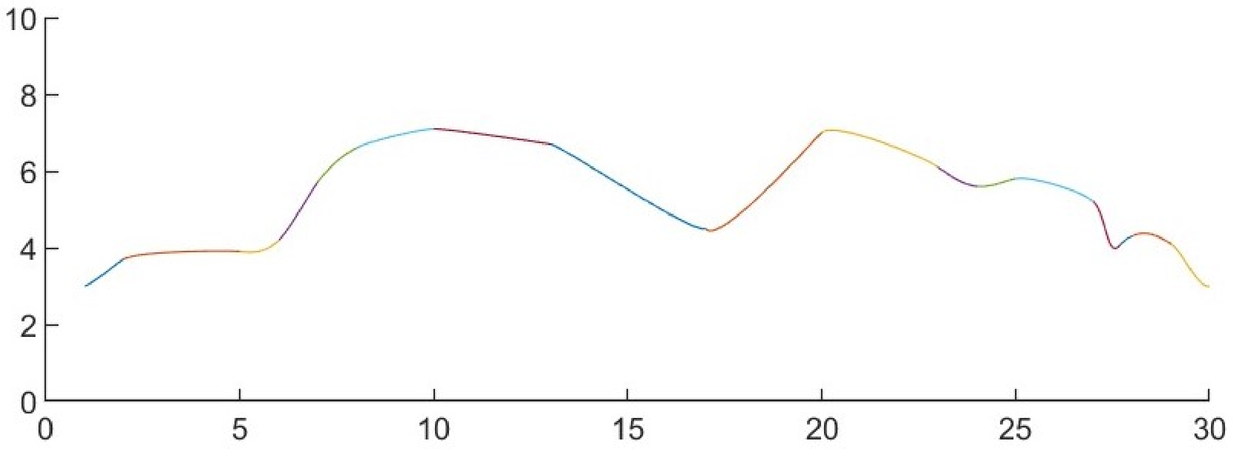
Code

File Name: beziercurve.m

```
% Plot Bezier curve
% Input: x,y vectors of data points, coefficient from cubic spline
% Output: a figure of Bezier curve
function beziercurve(x,y,coeff)
for i = 1:length(x)-1
    P0 = [x(i),y(i)];
    P1 = [x(i+1),y(i+1)];
    T0 = [1,coeff(i)];
    T1 = [1,coeff(i+1)];
    % Calculate Control Points: C0、C1、C2、C3
    x0 = P0(1);
    x1 = P1(1);
    y0 = P0(2);
    y1 = P1(2);
    dx0 = T0(1);
    dx1 = T1(1);
    dy0 = T0(2);
    dy1 = T1(2);
    C0 = [x0, y0];
    C3 = [x1, y1];
    C1 = C0 + [1/3*dx0, 1/3*dy0];
    C2 = C3 - [1/3*dx1, 1/3*dy1];

    % Plot Bezier Curve
    hold on;
    t = linspace(0,1,100);
    B = [0,0];
    for i=1:100
        B = [B;(1-t(1,i)).^3.*C0 + 3*(1-t(1,i)).^2.*t(1,i).*C1 + 3*(1-t(1,i)).*t(1,i).^2.*C2 + t(1,i).^3.*C3]; % 计算贝塞尔曲线上的点
    end
    plot(B(2:101,1), B(2:101,2));
    xlim([min(min(B)), max(max(B))]);
    ylim([min(min(B)), max(max(B))]);
end
hold on;
```

Result



Question 3: Freehand Draw

Code

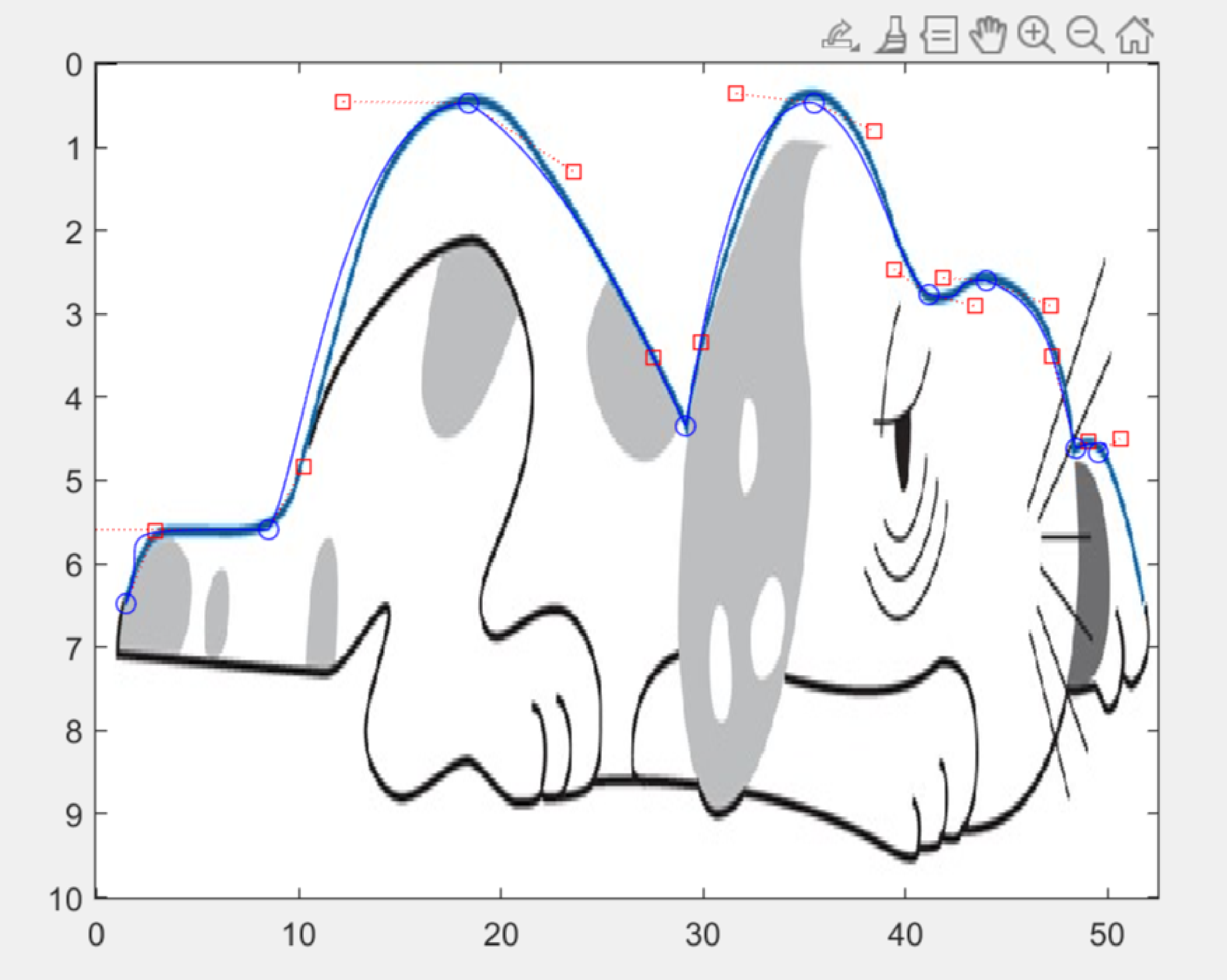
File Name: bezierdraw.m

```
%Program 3.7 Freehand Draw Program Using Bezier Splines
%Click in Matlab figure window to locate first point, and click
%     three more times to specify 2 control points and the next
%     spline point. Continue with groups of 3 points to add more
%     to the curve. Press return to terminate program.
function [points,v1,vn]=bezierdraw
plot([-1 1],[0,0],'k',[0 0],[-1 1],'k');hold on
t=0:.02:1;
[x,y]=ginput(1);            % get one mouse click
points=[x,y];
n=8;
for i=1:n
  [xnew,ynew] = ginput(3);  % get three mouse clicks
  if length(xnew) < 3
    break                   % if return pressed, terminate
  end
  x=[x;xnew];y=[y;ynew];    % plot spline points and control pts
  points=[points;x(4),y(4)];
  plot([x(1) x(2)],[y(1) y(2)],'r:',x(2),y(2),'rs');
  plot([x(3) x(4)],[y(3) y(4)],'r:',x(3),y(3),'rs');
  plot(x(1),y(1),'bo',x(4),y(4),'bo');
  bx=3*(x(2)-x(1)); by=3*(y(2)-y(1)); % spline equations ...
  cx=3*(x(3)-x(2))-bx;cy=3*(y(3)-y(2))-by;
  dx=x(4)-x(1)-bx-cx;dy=y(4)-y(1)-by-cy;
  xp=x(1)+t.*(bx+t.*(cx+t*dx));    % Horner's method
  yp=y(1)+t.*(by+t.*(cy+t*dy));
  plot(xp,yp,'b')             % plot spline curve
  x=x(4);y=y(4);            % promote last to first and repeat
  if(i==1)
      v1 = (by+2*cy+3*dy)/(bx+2*cx+3*dx);
  end
  if(i==n)
      vn = (by+2*cy+3*dy)/(bx+2*cx+3*dx);
  end
end
hold off
```

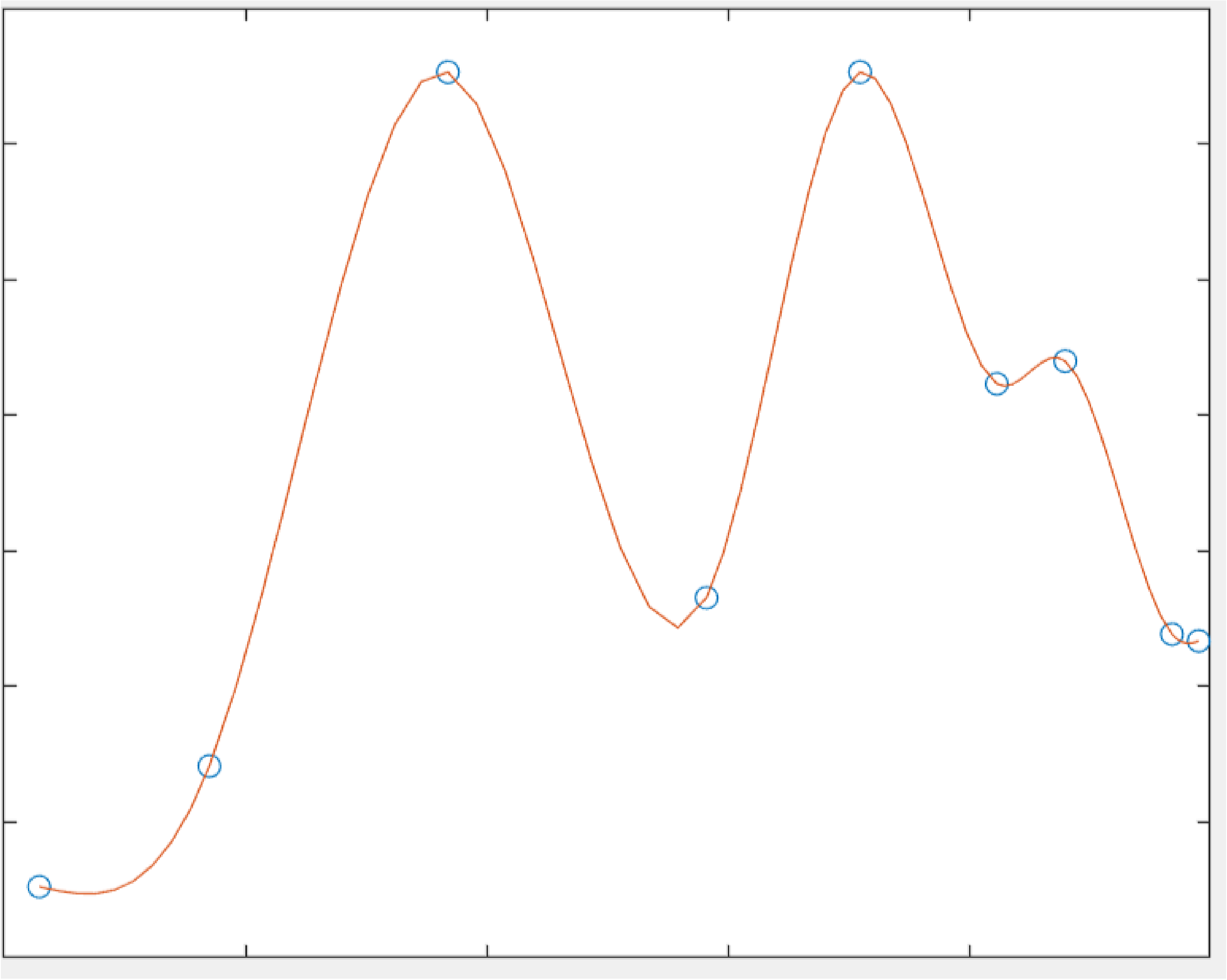
I modified the original program that was given in the assignment. This function can return the points and two slopes, so I was able to feed this data into the cubic spline function in question 1.

Result

Freehand Draw:



Cubic spline based on the freehand draw data:



I forgot to scale the picture so that the dog seems much fatter XD.

But I think the result is right mathematically and it's a big work to draw that dog, so I still want to keep that fat dog and put it into my project report...