AU7021 Optimization in Learning and Control (Fall 2021) **Homework: 4**

4. Solving Algorithm

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Problem 1

Consider the following LS problem

$$\min_x \quad \|Ax-b\|_2^2. \tag{1}$$

The data are generated by the following steps:

- randomly generate the true answer $\bar{x} \in R^n$ from $x_i \sim N(0,1)$;
- randomly generate $A \in R^{m\times n}$ from $a_{ij} \sim N(0,1)$;
- calculate $b = Ax + \varepsilon$ with $\varepsilon \sim N(0,\sigma^2)$, where the σ is set such that the signal-to-noise ratio is 20.

1. set $n = 50$ and $m = 200$ and implement GD algorithm to solve (1). You could try exact line search, backtracking line search, or fixed learning rate. Observe and discuss the solving procedure by plotting the objective value, $\|x^k - \bar{x}\|$, the difference to the pseudo-inverse result, and the length found step in each iteration.
2. modify your mode to SGD and change $n = 300, 500, 1000, 2000$ (m is fixed to 200). Try to use different starting points and report your result. Compare it with $\hat{x} = A^\top(AA^\top)^{-1}b$ and discuss the expected over-fitting phenomenon.

Answer.