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Adaptive passivity and synchronization of coupled reaction-diffusion neural networks with multiple state couplings or spatial diffusion couplings

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ABSTRACT

In this paper, two types of coupled reaction-diffusion neural networks with multiple state couplings or spatial diffusion couplings are presented. By selecting appropriate adaptive control schemes and employing inequality techniques, several passivity conditions for these network models are given. In addition, two sufficient conditions for ensuring the synchronization of the proposed network models are also established by exploiting the output-strictly passivity. Finally, we give two numerical examples to verify the effectiveness of the derived criteria.

1. Introduction

Recently, the dynamical behaviors of neural networks (NNs) have been widely studied in various fields due to their diversity of applications in optimization, pattern classification, image processing, signal processing, etc. [1–4]. As is well known, the phenomenon of reaction diffusion is caused by the movement of electrons in a nonuniform electromagnetic field, which is unavoidable [5–7]. Therefore, it is very important to consider the reaction-diffusion phenomenon in the neural networks. Until now, the dynamical behaviors of reaction-diffusion neural networks (RDNNs) have been studied extensively and deeply [8–13]. With the help of some inequality techniques and Lyapunov functional method, Wang et al. [8] not only analyzed the passivity of the RDNNs, but also considered the robust passivity under the circumstance that the parameter uncertainties appear in RDNNs. In [9], the robust passivity and passivity criteria were derived for stochastic RDNNs with time-varying delays by employing stochastic analysis and inequality techniques. In [10], the global exponential stability of RDNNs with time-varying delays was researched by exploiting the Wirtinger's inequality and a diffusion-dependent Lyapunov functional.

More recently, coupled RDNNs (CRDNNs) consisting of several identical or nonidentical RDNNs has attracted much attention in different research areas, such as harmonic oscillation generation, pattern recognition, and chaotic generator design. Therefore, the dynamical behaviors of CRDNNs have been extensively studied, in particular the synchronization [14–20] and passivity [21–25]. Wang et al. [14] respectively discussed the pinning synchronization for state coupled and spatial diffusion coupled CRDNNs, and several synchronization criteria for these networks were derived by exploiting several inequality techniques and Lyapunov functional method. In [15], the synchronization problem for CRDNNs with time delays was studied by exploiting the adaptive feedback control method, and the authors presented several sufficient conditions to ensure the synchronization based on the LaSalle invariant principle. In [19], several exponential synchronization criteria were obtained for a class of delayed CRDNNs by using the pinning impulsive control method. In [23], the authors considered the passivity of the CRDNNs with switching topology, several sufficient conditions were given to guarantee the output-strictly passivity and input-strictly passivity by using the Lyapunov functional method

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and inequality techniques. In [25], Huang et al. proposed a nonlinear CRDNNs, and the pinning passivity and passivity problems of such network model were studied. Unfortunately, in these existing results on the passivity of the CRDNNs, the input and output were required to have the same dimension [21–26]. As far as we know, passivity of CRDNNs with different input and output dimensions was seldom studied [27,28]. Wang et al. [27] discussed the passivity for two types of CRDNNs with different dimensions of input and output, and analyzed the internal stability of the CRDNNs. In [28], some passivity criteria for the CRDNNs with undirected and directed topologies were gained based on the designed adaptive laws, and the authors also studied the relationship between the output strict passivity and synchronization.

However, in the above-mentioned works about the dynamical behaviors of CRDNNs are all based upon the single weighted network models [14–28]. Actually, many networks in the real life should be modeled by multi-weighted models of complex network, examples are social networks, public traffic roads networks, etc. [29]. The dynamical behaviors for complex networks with multi-weights have been studied by some authors in recent years [30–36]. In [32], the authors gave some passivity criteria for complex networks with multi-weights by employing Lyapunov functional method and inequality techniques, and designed the pinning control strategy based on nodes and edges for ensuring the passivity, and the relationship between the output-strictly passivity and synchronization was also studied. Qin et al. [33] discussed the H_∞ synchronization and synchronization of the complex dynamical networks with multi-weights for switching and fixed topologies, and presented several H_∞ synchronization and synchronization criteria by employing Lyapunov functional method and inequality techniques. Regretfully, very few authors have considered the dynamical behaviors about the CRDNNs with multi-weights [37]. In [37], Wang et al. discussed the finite-time synchronization and finite-time passivity problems for CRDNNs with multiple state couplings and multiple delayed state couplings, and several sufficient conditions for ensuring the finite-time passivity or synchronization were given by selecting appropriate controllers. Particularly, no authors discussed the dynamical behaviors for CRDNNs with multiple spatial diffusion couplings. Therefore, it is very meaningful to further research the passivity and synchronization for CRDNNs with multiple state couplings or spatial diffusion couplings.

In this paper, we respectively study the passivity and synchronization of CRDNNs with multiple state couplings or spatial diffusion couplings. The main contributions of this paper are as follows. First, several passivity criteria for the multiple state coupled CRDNNs are obtained by employing some inequality techniques and the adaptive state feedback controller. Second, the synchronization problem of CRDNNs with multiple state couplings are studied based on the output-strictly passivity. Third, the adaptive passivity and synchronization problems for CRDNNs with multiple spatial diffusion couplings are also discussed.

Notations: $\mathbb{R}^\epsilon \ni \Omega = \{a = (a_1, a_2, \ldots, a_\epsilon)^T \mid \quad |a_h| < c_h, h = 1, 2, \ldots, \epsilon\}$. $\gamma_m(\cdot)$ and $\gamma_M(\cdot)$ denote the minimum and the maximum eigenvalue of the corresponding matrix. $\mathcal{D} = \{1, 2, \ldots, Z\}$ represents node set, $\mathcal{B} \subseteq \mathcal{D} \times \mathcal{D}$ represents undirected edge set. $\mathcal{Z}_p = \{q \in \mathcal{D} : (p, q) \in \mathcal{B}\}$.

2. Preliminaries

2.1. Definitions

Definition 2.1 (see [27]). If there exists a non-negative function W and a constant matrix $H \in \mathbb{R}^{\beta\times\zeta}$ such that

$$\int_{t_0}^{t_r}\int_\Omega y^T(a,t)Hu(a,t)dadt \geqslant W(t_r) - W(t_0)$$

for any $t_r, t_0 \in \mathbb{R}^+$ and $t_r \geq t_0$, the system with input $u(a,t) \in \mathbb{R}^\zeta$ and output $y(a,t) \in \mathbb{R}^\beta$ is passive.

Definition 2.2 (see [27]). If there exists a non-negative function W and two constant matrices $\mathbb{R}^{\zeta\times\zeta} \ni Q_1 > 0$ and $H \in \mathbb{R}^{\beta\times\zeta}$ such that

$$\int_{t_0}^{t_r}\int_\Omega y^T(a,t)Hu(a,t)dadt \geqslant W(t_r) - W(t_0) + \int_{t_0}^{t_r}\int_\Omega u^T(a,t)Q_1u(a,t)dadt$$

for any $t_r, t_0 \in \mathbb{R}^+$ and $t_r \geq t_0$, the system is input-strictly passive.

Definition 2.3 (see [27]). If there exists a non-negative function W and two constant matrices $\mathbb{R}^{\beta\times\beta} \ni Q_2 > 0$ and $H \in \mathbb{R}^{\beta\times\zeta}$ such that

$$\int_{t_0}^{t_r}\int_\Omega y^T(a,t)Hu(a,t)dadt \geqslant W(t_r) - W(t_0) + \int_{t_0}^{t_r}\int_\Omega y^T(a,t)Q_2y(a,t)dadt$$

for any $t_r, t_0 \in \mathbb{R}^+$ and $t_r \geq t_0$, the system is output-strictly passive.

2.2. Lemma

Lemma 2.1 (see [38]). *The following inequality obviously holds:*

$$\int_\Omega g^2(a)da \leqslant c_h^2\int_\Omega\left(\frac{\partial g(a)}{\partial a_h}\right)^2da, \qquad h = 1, 2, \ldots, \epsilon,$$

where $C^1(\Omega)\ni g(a)$ is a real-valued function and $g(a)|_{\partial\Omega=0}$.

3. Adaptive passivity and synchronization of CRDNNs with multiple state couplings

3.1. Network model

The network model considered in this section is described by:

$$\begin{aligned}\frac{\partial\chi_p(a,t)}{\partial t} &= M\triangle\chi_p(a,t) - A\chi_p(a,t) + Ef(\chi_p(a,t)) + J + Ku_p(a,t)\\ &\quad + \sum_{s=1}^{d}\sum_{q=1}^{Z} b_sB_{pq}^s\Gamma^s\chi_q(a,t) + \nu_p(a,t),\ p = 1, 2, \ldots, Z,\end{aligned} \tag{1}$$

where $\chi_p(a,t) = (\chi_{p1}(a,t), \chi_{p2}(a,t), \ldots, \chi_{pz}(a,t))^T \in \mathbb{R}^z$ is the state vector of node p; $u_p(a,t) \in \mathbb{R}^\eta$ is the external input of node p; $\nu_p(a,t) \in \mathbb{R}^z$ is the control input of node p; $0 < M = \mathrm{diag}(m_1, m_2, \ldots, m_z) \in \mathbb{R}^{z\times z}$; $0 < A = \mathrm{diag}(\Lambda_1, \Lambda_2, \ldots, \Lambda_z) \in \mathbb{R}^{z\times z}$; $\triangle = \sum_{h=1}^{\epsilon}(\partial^2/\partial a_h^2)$; $f(\chi_p(a,t)) = (f_1(\chi_{p1}(a,t)), f_2(\chi_{p2}(a,t)), \ldots, f_z(\chi_{pz}(a,t)))^T \in \mathbb{R}^z$; $J = (J_1, J_2, \ldots, J_z)^T \in \mathbb{R}^z$; $E \in \mathbb{R}^{z\times z}$ and $K \in \mathbb{R}^{z\times\eta}$ are two known matrices; $0 < b_s \in \mathbb{R}$ $(s = 1, 2, \ldots, d)$ represents the coupling strength; $0 < \Gamma^s \in \mathbb{R}^{z\times z}(s = 1, 2, \ldots, d)$ denotes the inner coupling relationship; $B^s = (B_{pq}^s)_{Z\times Z} \in \mathbb{R}^{Z\times Z}(s = 1, 2, \ldots, d)$ is the outer coupling matrix, where B_{pq}^s is defined as follows: if there is an edge between node p and node $q(p \neq q)$, then $B_{pq}^s = B_{qp}^s > 0$; otherwise, $B_{pq}^s = B_{qp}^s = 0$; and $B_{pp}^s = -\sum_{\substack{q=1\\q\neq p}}^{Z} B_{pq}^s$.

The boundary value and initial value for network (1) are given as follows:

$$\begin{aligned}&\chi_p(a,t) = 0, \qquad (a,t) \in \partial\Omega\times[0,+\infty),\\ &\chi_p(a,0) = \varphi_p(a), \qquad a \in \Omega,\end{aligned}$$

in which $\varphi_p(a)(p = 1, 2, \ldots, z)$ is continuous on Ω.

In this paper, the network (1) is connected and the function $f_p(\cdot)(p = 1, 2, \ldots, z)$ satisfies the following condition:

$$|f_p(\theta_1) - f_p(\theta_2)| \leqslant \mathcal{Q}_p|\theta_1 - \theta_2|$$

for any $\theta_1, \theta_2 \in \mathbb{R}$, where $\mathcal{Q}_p$ is a positive constant.

Defining $\bar{\chi}(a,t) = \frac{1}{Z}\sum_{p=1}^{Z}\chi_p(a,t)$, one gets

$$\begin{aligned}\frac{\partial\bar{\chi}(a,t)}{\partial t} &= M\triangle\bar{\chi}(a,t) - A\bar{\chi}(a,t) + \frac{1}{Z}\sum_{p=1}^{Z}Ef(\chi_p(a,t)) + J\\ &\quad + \frac{1}{Z}\sum_{p=1}^{Z}Ku_p(a,t) + \frac{1}{Z}\sum_{s=1}^{d}\sum_{q=1}^{Z}b_s\Big(\sum_{p=1}^{Z}B_{pq}^s\Big)\Gamma^s\chi_q(a,t)\\ &\quad + \frac{1}{Z}\sum_{p=1}^{Z}\upsilon_p(a,t)\\ &= M\triangle\bar{\chi}(a,t) - A\bar{\chi}(a,t) + \frac{1}{Z}\sum_{p=1}^{Z}Ef(\chi_p(a,t)) + J\\ &\quad + \frac{1}{Z}\sum_{p=1}^{Z}Ku_p(a,t) + \frac{1}{Z}\sum_{p=1}^{Z}\upsilon_p(a,t).\end{aligned}$$

Selecting $e_p(a,t) = \chi_p(a,t) - \bar{\chi}(a,t)$, one obtains

$$\begin{aligned}\frac{\partial e_p(a,t)}{\partial t} &= M\triangle e_p(a,t) - Ae_p(a,t) + Ef(\chi_p(a,t))\\ &\quad - \frac{1}{Z}\sum_{l=1}^{Z}Ef(\chi_l(a,t))\\ &\quad + Ku_p(a,t) - \frac{1}{Z}\sum_{l=1}^{Z}Ku_l(a,t) + \sum_{s=1}^{d}\sum_{q=1}^{Z}b_sB_{pq}^s\Gamma^s e_q(a,t)\\ &\quad + \upsilon_p(a,t) - \frac{1}{Z}\sum_{l=1}^{Z}\upsilon_l(a,t).\end{aligned}\tag{2}$$

Choose the following output vector $y_p(a,t)\in\mathbb{R}^{\zeta}$ for the network (2):

$$y_p(a,t) = P_1e_p(a,t) + P_2u_p(a,t),\tag{3}$$

where $P_1\in\mathbb{R}^{\zeta\times z}$ and $P_2\in\mathbb{R}^{\zeta\times\eta}$.

Take

$$\begin{aligned}u(a,t) &= (u_1^T(a,t), u_2^T(a,t), \ldots, u_Z^T(a,t))^T,\\ y(a,t) &= (y_1^T(a,t), y_2^T(a,t), \ldots, y_Z^T(a,t))^T,\\ e(a,t) &= (e_1^T(a,t), e_2^T(a,t), \ldots, e_Z^T(a,t))^T.\end{aligned}$$

Design the following adaptive state feedback controller for network (1):

$$\begin{aligned}\upsilon_p(a,t) &= \sum_{s=1}^{d}\sum_{q=1}^{Z}b_sG_{pq}^s(t)\Gamma^s\chi_q(a,t),\\ \dot{G}_{pq}^s(t) &= \alpha_{pq}^s\int_{\Omega}(\chi_p(a,t)-\chi_q(a,t))^T\Gamma^s(\chi_p(a,t)\\ &\quad - \chi_q(a,t))da,\ (p,q)\in\mathcal{B},\end{aligned}\tag{4}$$

where $p=1,2,\ldots,Z$; if $q\in\mathcal{Z}_p$, then $\mathbb{R}\ni\alpha_{pq}^s=\alpha_{qp}^s>0$ and $\mathbb{R}\ni G_{pq}^s(t)=G_{qp}^s(t)>0$; otherwise, $\mathbb{R}\ni\alpha_{pq}^s=\alpha_{qp}^s=0$ and $\mathbb{R}\ni G_{pq}^s(t)=G_{qp}^s(t)=0$; and $G_{pp}^s(t) = -\sum_{q=1, q\neq p}^{Z}G_{pq}^s(t)$.

3.2. Passivity criteria

Theorem 3.1. *If there is a matrix $H\in\mathbb{R}^{\zeta Z\times\eta Z}$ satisfying*

$$(I_Z\otimes P_2)^TH + H^T(I_Z\otimes P_2) > 0,$$

the network (2) is passive.

Proof. Define the following Lyapunov functional for the network (2):

$$\begin{aligned}V_1(t) &= \sum_{p=1}^{Z}\int_{\Omega}e_p^T(a,t)e_p(a,t)da\\ &\quad + \sum_{s=1}^{d}\sum_{p=1}^{Z}\sum_{q\in\mathcal{Z}_p}\frac{b_s}{2\alpha_{pq}^s}(G_{pq}^s(t)-\delta_{pq}^s)^2,\end{aligned}\tag{5}$$

where $\delta_{pq}^s=\delta_{qp}^s(p\neq q)$ are non-negative constants, and $\delta_{pq}^s = 0 (p\neq q)$ if and only if $G_{pq}^s(t)=0$.

Then, we acquire

$$\begin{aligned}\dot{V}_1(t) &= 2\sum_{p=1}^{Z}\int_{\Omega}e_p^T(a,t)\frac{\partial e_p(a,t)}{\partial t}da\\ &\quad + \sum_{s=1}^{d}\sum_{p=1}^{Z}\sum_{q\in\mathcal{Z}_p}\frac{b_s}{\alpha_{pq}^s}(G_{pq}^s(t)-\delta_{pq}^s)\dot{G}_{pq}^s(t)\\ &= 2\sum_{p=1}^{Z}\int_{\Omega}e_p^T(a,t)\Big[M\triangle e_p(a,t) - Ae_p(a,t)\\ &\quad + \sum_{s=1}^{d}\sum_{q=1}^{Z}b_sB_{pq}^s\Gamma^s e_q(a,t)\\ &\quad + Ku_p(a,t) - \frac{1}{Z}\sum_{l=1}^{Z}Ku_l(a,t) + Ef(\chi_p(a,t)) - Ef(\bar{\chi}(a,t))\\ &\quad + Ef(\bar{\chi}(a,t)) - \frac{1}{Z}\sum_{l=1}^{Z}Ef(\chi_l(a,t)) + \upsilon_p(a,t)\\ &\quad - \frac{1}{Z}\sum_{l=1}^{Z}\upsilon_l(a,t)\Big]da\\ &\quad + \sum_{s=1}^{d}\sum_{p=1}^{Z}\sum_{q\in\mathcal{Z}_p}b_s(G_{pq}^s(t)-\delta_{pq}^s)\\ &\quad \times\int_{\Omega}(\chi_p(a,t)-\chi_q(a,t))^T\Gamma^s(\chi_p(a,t)\\ &\quad - \chi_q(a,t))da.\end{aligned}\tag{6}$$

Obviously,

$$\begin{aligned}\sum_{p=1}^{Z}e_p(a,t) &= \sum_{p=1}^{Z}\Big(\chi_p(a,t) - \bar{\chi}(a,t)\Big)\\ &= \sum_{p=1}^{Z}\Big(\chi_p(a,t) - \frac{1}{Z}\sum_{q=1}^{Z}\chi_q(a,t)\Big)\\ &= \sum_{p=1}^{Z}\chi_p(a,t) - \sum_{q=1}^{Z}\chi_q(a,t)\\ &= 0.\end{aligned}$$

Then, we can get

$$\begin{aligned}\sum_{p=1}^{Z}e_p^T(a,t)\Big(Ef(\bar{\chi}(a,t)) - \frac{1}{Z}\sum_{l=1}^{Z}Ef(\chi_l(a,t))\Big) &= 0,\\ \sum_{p=1}^{Z}e_p^T(a,t)\Big(\sum_{l=1}^{Z}Ku_l(a,t)\Big) &= 0,\\ \sum_{p=1}^{Z}e_p^T(a,t)\Big(\sum_{l=1}^{Z}\upsilon_l(a,t)\Big) &= 0.\end{aligned}\tag{7}$$

From the boundary condition and Lemma 2.1, one obtains

$$2\int_{\Omega}e_p^T(a,t)M\Delta e_p(a,t)da$$

$$= 2\sum_{i=1}^{z} m_i \int_\Omega e_{pi}(a,t)\nabla\cdot\left(\frac{\partial e_{pi}(a,t)}{\partial a_h}\right)_{h=1}^{\epsilon} da$$
$$= 2\sum_{i=1}^{z} m_i \int_\Omega \nabla\cdot\left(e_{pi}(a,t)\frac{\partial e_{pi}(a,t)}{\partial a_h}\right)_{h=1}^{\epsilon} da$$
$$-2\sum_{i=1}^{z} m_i \int_\Omega \left(\frac{\partial e_{pi}(a,t)}{\partial a_h}\right)_{h=1}^{\epsilon}\cdot\nabla e_{pi}(a,t)da$$
$$= 2\sum_{i=1}^{z} m_i \int_{\partial\Omega} \left(e_{pi}(a,t)\frac{\partial e_{pi}(a,t)}{\partial a_h}\right)_{h=1}^{\epsilon} da \quad \text{(by Green formula)}$$
$$-2\sum_{i=1}^{z}\sum_{h=1}^{\epsilon} m_i \int_\Omega \left(\frac{\partial e_{pi}(a,t)}{\partial a_h}\right)^2 da$$
$$= -2\sum_{i=1}^{z}\sum_{h=1}^{\epsilon} m_i \int_\Omega \left(\frac{\partial e_{pi}(a,t)}{\partial a_h}\right)^2 da \qquad \text{(by boundary condition)}$$
$$\leqslant -2\sum_{i=1}^{z}\sum_{h=1}^{\epsilon} \frac{m_i}{c_h^2}\int_\Omega e_{pi}^2(a,t)da$$
$$= -\sum_{h=1}^{\epsilon}\frac{2}{c_h^2}\int_\Omega e_p^T(a,t)Me_p(a,t)da, \tag{8}$$

where $\left(\frac{\partial e_{pi}(a,t)}{\partial a_h}\right)_{h=1}^{\epsilon} = \left(\frac{\partial e_{pi}(a,t)}{\partial a_1}, \frac{\partial e_{pi}(a,t)}{\partial a_2}, \dots, \frac{\partial e_{pi}(a,t)}{\partial a_\epsilon}\right)^T$, "·" denotes the inner product, and $\nabla = \left(\frac{\partial}{\partial a_1}, \frac{\partial}{\partial a_2}, \dots, \frac{\partial}{\partial a_\epsilon}\right)$.

Let $\mathcal{Q} = \text{diag}\ (\mathcal{Q}_1^2, \mathcal{Q}_2^2, \dots, \mathcal{Q}_Z^2)$. Obviously

$$2e_p^T(a,t)E(f(\chi_p(a,t)) - f(\bar{\chi}(a,t))) \leqslant e_p^T(a,t)(EE^T + \mathcal{Q})e_p(a,t). \tag{9}$$

Define the matrix $\delta^s = (\delta_{pq}^s)_{Z\times Z}$, where $\delta_{pp}^s = -\sum_{q=1, q\neq p}^{Z}\delta_{pq}^s$. Then, we can easily derive

$$\sum_{s=1}^{d}\sum_{p=1}^{Z}\sum_{q\in\mathcal{Z}_p}(G_{pq}^s(t) - \delta_{pq}^s)(e_p(a,t) - e_q(a,t))^T\Gamma^s(e_p(a,t) - e_q(a,t))$$
$$= -2\sum_{s=1}^{d}\sum_{p=1}^{Z}\sum_{q=1}^{Z}(G_{pq}^s(t) - \delta_{pq}^s)e_p^T(a,t)\Gamma^s e_q(a,t). \tag{10}$$

By (4), (7)-(10), one gets

$$\dot{V}_1(t) = 2\sum_{p=1}^{Z}\int_\Omega e_p^T(a,t)\Big[M\triangle e_p(a,t) - Ae_p(a,t)$$
$$+\sum_{s=1}^{d}\sum_{q=1}^{Z} b_s B_{pq}^s\Gamma^s e_q(a,t)$$
$$+Ku_p(a,t) + Ef(\chi_p(a,t)) - Ef(\bar{\chi}(a,t))$$
$$+\sum_{s=1}^{d}\sum_{q=1}^{Z} b_s G_{pq}^s(t)\Gamma^s e_q(a,t)\Big]da$$
$$-2\sum_{s=1}^{d}\sum_{p=1}^{Z}\sum_{q=1}^{Z} b_s(G_{pq}^s(t) - \delta_{pq}^s)\int_\Omega e_p^T(a,t)\Gamma^s e_q(a,t)da$$
$$\leqslant \sum_{p=1}^{Z}\int_\Omega e_p^T(a,t)\Big(-\sum_{h=1}^{\epsilon}\frac{2}{c_h^2}M - 2A + EE^T + \mathcal{Q}\Big)e_p(a,t)da$$
$$+2\sum_{s=1}^{d}\sum_{p=1}^{Z}\sum_{q=1}^{Z} b_s\delta_{pq}^s\int_\Omega e_p^T(a,t)\Gamma^s e_q(a,t)da$$
$$+2\sum_{s=1}^{d}\sum_{p=1}^{Z}\sum_{q=1}^{Z} b_s B_{pq}^s\int_\Omega e_p^T(a,t)\Gamma^s e_q(a,t)da$$
$$+2\sum_{p=1}^{Z}\int_\Omega e_p^T(a,t)Ku_p(a,t)da$$
$$= \int_\Omega e^T(a,t)\Big[I_Z\otimes\Big(-\sum_{h=1}^{\epsilon}\frac{2}{c_h^2}M - 2A + EE^T + \mathcal{Q}\Big)$$
$$+2\sum_{s=1}^{d} b_s(B^s\otimes\Gamma^s)\Big]e(a,t)da$$
$$+2\sum_{s=1}^{d} b_s\int_\Omega e^T(a,t)(\delta^s\otimes\Gamma^s)e(a,t)da$$
$$+2\int_\Omega e^T(a,t)(I_Z\otimes K)u(a,t)da.$$

Therefore,

$$\dot{V}_1(t) - 2\int_\Omega y^T(a,t)Hu(a,t)da$$
$$= \int_\Omega e^T(a,t)\Big[I_Z\otimes\Big(-\sum_{h=1}^{\epsilon}\frac{2}{c_h^2}M - 2A + EE^T + \mathcal{Q}\Big)$$
$$+2\sum_{s=1}^{d} b_s(B^s\otimes\Gamma^s)\Big]e(a,t)da$$
$$+2\int_\Omega e^T(a,t)[(I_Z\otimes K) - (I_Z\otimes P_1^T)H]u(a,t)da$$
$$-\int_\Omega u^T(a,t)[(I_Z\otimes P_2^T)H + H^T(I_Z\otimes P_2)]u(a,t)da$$
$$+2\sum_{s=1}^{d} b_s\int_\Omega e^T(a,t)(\delta^s\otimes\Gamma^s)e(a,t)da$$
$$\leqslant \int_\Omega e^T(a,t)\Big\{I_Z\otimes\Big(-\sum_{h=1}^{\epsilon}\frac{2}{c_h^2}M - 2A + EE^T + \mathcal{Q}\Big)$$
$$+2\sum_{s=1}^{d} b_s(B^s\otimes\Gamma^s) + [(I_Z\otimes K) - (I_Z\otimes P_1^T)H]$$
$$\times[(I_Z\otimes P_2^T)H + H^T(I_Z\otimes P_2)]^{-1}[(I_Z\otimes K^T) - H^T(I_Z\otimes P_1)]\Big\}e(a,t)da$$
$$+2\sum_{s=2}^{d} b_s\int_\Omega e^T(a,t)(\delta^s\otimes\Gamma^s)e(a,t)da$$
$$+2b_1\int_\Omega e^T(a,t)(\delta^1\otimes\Gamma^1)e(a,t)da$$
$$\leqslant \int_\Omega e^T(a,t)\Big\{I_Z\otimes\Big(-\sum_{h=1}^{\epsilon}\frac{2}{c_h^2}M - 2A + EE^T + \mathcal{Q}\Big)$$
$$+2\sum_{s=1}^{d} b_s(B^s\otimes\Gamma^s) + [(I_Z\otimes K) - (I_Z\otimes P_1^T)H]$$
$$\times[(I_Z\otimes P_2^T)H + H^T(I_Z\otimes P_2)]^{-1}[(I_Z\otimes K^T) - H^T(I_Z\otimes P_1)]\Big\}e(a,t)da$$
$$+2b_1\int_\Omega e^T(a,t)(\delta^1\otimes\Gamma^1)e(a,t)da$$
$$= \int_\Omega e^T(a,t)(\Xi + 2b_1\delta^1\otimes\Gamma^1)e(a,t)da, \tag{11}$$

where $\Xi = I_Z\otimes[-2\sum_{h=1}^{\epsilon} M/c_h^2 - 2A + EE^T + \mathcal{Q}] + 2\sum_{s=1}^{d} b_s(B^s\otimes\Gamma^s) + [(I_Z\otimes K) - (I_Z\otimes P_1^T)H][(I_Z\otimes P_2^T)H + H^T(I_Z\otimes P_2)]^{-1}[(I_Z\otimes K^T) - H^T(I_Z\otimes P_1)]$.

On the other hand, there exists an orthogonal matrix $\mathcal{K} = (\kappa_1, \kappa_2, \dots, \kappa_Z) \in \mathbb{R}^{Z\times Z}$ such that $\mathcal{K}^T\delta^1\mathcal{K} = \Upsilon = \text{diag}\ (\Upsilon_1, \Upsilon_2, \dots, \Upsilon_Z)$, in which $0 = \Upsilon_1 > \Upsilon_2 \geqslant \Upsilon_3 \geqslant \dots \geqslant \Upsilon_Z$. Let $\varrho(a,t) = (\varrho_1^T(a,t), \varrho_2^T(a,t), \dots, \varrho_Z^T(a,t))^T = (\mathcal{K}^T\otimes I_z)e(a,t)$. Since $\kappa_1 = \frac{1}{\sqrt{Z}}(1,1,\dots,1)^T$, one has $\varrho_1(a,t) = (\kappa_1^T\otimes I_z)e(a,t) = 0$. Then, we can derive from (11) that

$$\begin{aligned}
&\dot{V}_1(t) - 2\int_\Omega y^T(a,t)Hu(a,t)da \\
&\leqslant \int_\Omega e^T(a,t)[2b_1(\mathcal{K}\otimes I_z)(\Upsilon\otimes\Gamma^1)(\mathcal{K}^T\otimes I_z)]e(a,t)da \\
&\quad + \int_\Omega e^T(a,t)\Xi e(a,t)da \\
&= \int_\Omega e^T(a,t)\Xi e(a,t)da + 2b_1\int_\Omega \varrho^T(a,t)(\Upsilon\otimes\Gamma^1)\varrho(a,t)da \\
&\leqslant \int_\Omega e^T(a,t)\Xi e(a,t)da + 2b_1\Upsilon_2\int_\Omega \varrho^T(a,t)(I_Z\otimes\Gamma^1)\varrho(a,t)da \\
&= \int_\Omega e^T(a,t)(\Xi + 2b_1\Upsilon_2 I_Z\otimes\Gamma^1)e(a,t)da.
\end{aligned}$$

By choosing δ_{pq} large enough such that

$$\gamma_M(\Xi) + 2b_1\Upsilon_2\gamma_m(\Gamma^1) \leqslant 0,$$

one has

$$\Xi + 2b_1\Upsilon_2 I_Z\otimes\Gamma^1 \leqslant 0.$$

Therefore,

$$\frac{\dot{V}_1(t)}{2} \leqslant \int_\Omega y^T(a,t)Hu(a,t)da.$$

Letting $V(t) = V_1(t)/2$, one has

$$\int_{t_0}^{t_r}\int_\Omega y^T(a,t)Hu(a,t)dadt \geqslant V(t_r) - V(t_0)$$

for any $0 \leqslant t_0 \leqslant t_r \in \mathbb{R}$. □

Theorem 3.2. *If there are two matrices $\mathbb{R}^{\eta Z\times\eta Z} \ni Q_1 > 0$ and $H \in \mathbb{R}^{\zeta Z\times\eta Z}$ satisfying*

$$(I_Z\otimes P_2)^T H + H^T(I_Z\otimes P_2) - Q_1 > 0,$$

the network (2) is input-strictly passive.

Theorem 3.3. *If there are two matrices $\mathbb{R}^{\zeta Z\times\zeta Z} \ni Q_2 > 0$ and $H \in \mathbb{R}^{\zeta Z\times\eta Z}$ satisfying*

$$(I_Z\otimes P_2)^T H + H^T(I_Z\otimes P_2) - (I_Z\otimes P_2)^T Q_2(I_Z\otimes P_2) > 0,$$

the network (2) is output-strictly passive.

3.3. Synchronization criteria

Definition 3.1. The network (1) with the input $u_p(a,t) = 0$ ($p = 1, 2, \ldots, Z$) is synchronized if

$$\lim_{t\to+\infty}\left\|\chi_p(\cdot,t) - \frac{1}{Z}\sum_{q=1}^{Z}\chi_q(\cdot,t)\right\| = 0, \quad p = 1, 2, \ldots, Z.$$

Theorem 3.4. *Under the adaptive state feedback controller (4), the network (1) can achieves synchronization if the network (2) is output-strictly passive with regard to the storage function $W(t) = V_1(t)/2$ and $P_1^T P_1 > 0$.*

Proof. If the network (2) under the adaptive state feedback controller (4) is output-strictly passive with regard to *W*(*t*), we have

$$\begin{aligned}
\frac{W(t+\phi) - W(t)}{\phi} &\leqslant \frac{\int_t^{t+\phi}\int_\Omega y^T(a,t)Hu(a,s)dads}{\phi} \\
&\quad - \frac{\int_t^{t+\phi}\int_\Omega y^T(a,t)Q_2 y(a,s)dads}{\phi}
\end{aligned}$$

for any $t \in \mathbb{R}^+$ and $\phi > 0$, where $\mathbb{R}^{\zeta Z\times\zeta Z} \ni Q_2 > 0$ and $H \in \mathbb{R}^{\zeta Z\times\eta Z}$.

Then, we can easily derive

$$\dot{W}(t) \leqslant \int_\Omega y^T(a,t)Hu(a,t)da - \int_\Omega y^T(a,t)Q_2 y(a,t)da.$$

Letting $u(a,t) = 0$, one gets

$$\dot{W}(t) \leqslant -\int_\Omega e^T(a,t)(I_Z\otimes P_1^T)Q_2(I_Z\otimes P_1)e(a,t)da.$$

Then, one has

$$\dot{W}(t) \leqslant -\gamma_m(P_1^T P_1)\gamma_m(Q_2)\|e(\cdot,t)\|^2. \tag{12}$$

By (12), we get that $\lim_{t\to+\infty}W(t)$ exists and $G_{pq}^s(t)((p,q)\in\mathcal{B})$ is bounded. Therefore, $\lim_{t\to+\infty}G_{pq}^s(t)$ exists based on (4). Then, it follows from (5) that

$$\lim_{t\to\infty}\|e(\cdot,t)\|^2 = \wp \geqslant 0.$$

Suppose that

$$\lim_{t\to\infty}\|e(\cdot,t)\|^2 = \wp > 0.$$

Then, there obviously exists $0 < S \in \mathbb{R}$ such that

$$\|e(\cdot,t)\|^2 > \wp/2, \quad t \geqslant S. \tag{13}$$

By (12) and (13), one has

$$\dot{W}(t) < -\frac{\gamma_m(P_1^T P_1)\gamma_m(Q_2)\wp}{2}, \quad t \geqslant S. \tag{14}$$

In light of (14), one derives

$$\begin{aligned}
-W(S) &\leqslant W(+\infty) - W(S) \\
&= \int_S^{+\infty}\dot{W}(t)dt \\
&< -\int_S^{+\infty}\frac{\gamma_m(P_1^T P_1)\gamma_m(Q_2)\wp}{2}dt \\
&= -\infty,
\end{aligned}$$

which results in a contradiction.

Therefore, $\lim_{t\to\infty}\|e(\cdot,t)\| = 0$. Namely, the network (1) can achieves synchronization under the adaptive state feedback controller (4).

Based on the Theorems 3.3 and 3.4, the following conclusion can be obtained. □

Corollary 3.1. *The network (1) under the adaptive controller (4) realizes synchronization if there exist $\mathbb{R}^{\zeta Z\times\zeta Z} \ni Q_2 > 0$ and $H \in \mathbb{R}^{\zeta Z\times\eta Z}$ satisfying*

$$\begin{pmatrix} (I_Z\otimes P_2)^T H + H^T(I_Z\otimes P_2) - (I_Z\otimes P_2)^T Q_2(I_Z\otimes P_2) & 0 \\ 0 & P_1^T P_1 \end{pmatrix} > 0.$$

4. Adaptive passivity and synchronization of CRDNNs with multiple spatial diffusion couplings

4.1. Network model

The network model considered in this section is described by:

$$\begin{aligned}
\frac{\partial\chi_p(a,t)}{\partial t} &= M\triangle\chi_p(a,t) - A\chi_p(a,t) + Ef(\chi_p(a,t)) + J + Ku_p(a,t) \\
&\quad + \sum_{s=1}^{d}\sum_{q=1}^{Z}\hat{b}_s\hat{B}_{pq}^s\hat{\Gamma}^s\Delta\chi_q(a,t) + \upsilon_p(a,t),\ p = 1, 2, \ldots, Z,
\end{aligned} \tag{15}$$

where $\chi_p(a,t), u_p(a,t), \upsilon_p(a,t), f(\cdot), M, A, E, J, K$ and Δ have the same meanings as these in Section 3, $\hat{b}_s, \hat{B}_{pq}^s$ and $\hat{\Gamma}^s$ satisfy the similar conditions as b_s, B_{pq}^s and Γ^s in Section 3.

In this section, the network (15) is connected. The boundary value and initial value for network (15) are given as follows:

$$\begin{aligned}
&\chi_p(a,t) = 0, && (a,t) \in \partial\Omega\times[0,+\infty), \\
&\chi_p(a,0) = \varphi_p(a), && x \in \Omega,
\end{aligned}$$

in which $\varphi_p(a)(p = 1, 2, \ldots, z)$ is continuous on Ω.

Remark 1. In reaction-diffusion networks, different diffusion of node has a great influence on other nodes. Therefore, we should consider the spatial diffusion coupling in reaction-diffusion networks [16]. On the other hand, there may exist several impact factors in reaction-diffusion networks. For instance, in food webs, we need consider the influences of some external factors, such as environmental changes, human activities and so on. Consequently, it is very significant to research the multiple spatial diffusion couplings in reaction-diffusion networks.

Remark 2. Recently, the dynamical behaviors of CRDNNs have attracted much attention because of their widespread applications in various fields such as harmonic oscillation generation, pattern recognition, chaotic generator design, and so on. In particular, the synchronization [14–20] and passivity [21–25] of CRDNNs have been extensively studied, and many meaningful results have been obtained. However, in these existing works [14–25], the network models with single weight were discussed. Regrettably, very few results on the multiple state coupled CRDNNs have been reported, especially the synchronization and passivity for CRDNNs with multiple spatial diffusion couplings have not yet been investigated.

Defining $\bar{\chi}(a,t)=\frac{1}{Z}\sum_{l=1}^{Z}\chi_l(a,t)$ and $e_p(a,t)=\chi_p(a,t)-\bar{\chi}(a,t)$, we have

$$\begin{aligned}\frac{\partial e_p(a,t)}{\partial t} &= M\triangle e_p(a,t)-Ae_p(a,t)+Ef(\chi_p(a,t))\\ &\quad -\frac{1}{Z}\sum_{l=1}^{Z}Ef(\chi_l(a,t))+Ku_p(a,t)-\frac{1}{Z}\sum_{l=1}^{Z}Ku_l(a,t)\\ &\quad +\sum_{s=1}^{d}\sum_{q=1}^{Z}\hat{b}_s\hat{B}^s_{pq}\hat{\Gamma}^s\Delta e_q(a,t)+\upsilon_p(a,t)\\ &\quad -\frac{1}{Z}\sum_{l=1}^{Z}\upsilon_l(a,t). \end{aligned}\tag{16}$$

Choose the following output vector $y_p(a,t)\in\mathbb{R}^{\zeta}$ for the network (16):

$$y_p(a,t)=P_1e_p(a,t)+P_2u_p(a,t),\tag{17}$$

where $P_1\in\mathbb{R}^{\zeta\times z}$ and $P_2\in\mathbb{R}^{\zeta\times\eta}$.

Take

$$\begin{aligned}u(a,t) &= (u_1^T(a,t),u_2^T(a,t),\ldots,u_Z^T(a,t))^T,\\ y(a,t) &= (y_1^T(a,t),y_2^T(a,t),\ldots,y_Z^T(a,t))^T,\\ e(a,t) &= (e_1^T(a,t),e_2^T(a,t),\ldots,e_Z^T(a,t))^T.\end{aligned}$$

Design the following adaptive state feedback controller for network (15):

$$\begin{aligned}\upsilon_p(a,t) &= \sum_{s=1}^{d}\sum_{q=1}^{Z}\hat{b}_sG^s_{pq}(t)\hat{\Gamma}^s\chi_q(a,t),\\ \dot{G}^s_{pq}(t) &= \alpha^s_{pq}\int_\Omega(\chi_p(a,t)-\chi_q(a,t))^T\hat{\Gamma}^s(\chi_p(a,t)\\ &\quad -\chi_q(a,t))da,\ (p,q)\in\mathcal{B},\end{aligned}\tag{18}$$

where $p=1,2,\ldots,Z$; if $q\in\mathcal{Z}_p$, then $\mathbb{R}\ni\alpha^s_{pq}=\alpha^s_{qp}>0$ and $\mathbb{R}\ni G^s_{pq}(t)=G^s_{qp}(t)>0$; otherwise, $\mathbb{R}\ni\alpha^s_{pq}=\alpha^s_{qp}=0$ and $\mathbb{R}\ni G^s_{pq}(t)=G^s_{qp}(t)=0$; and $G^s_{pp}(t)=-\sum_{\substack{q=1\\q\neq p}}^{Z}G^s_{pq}(t)$.

4.2. Passivity criteria

Theorem 4.1. *If there is a matrix $H\in\mathbb{R}^{\zeta Z\times\eta Z}$ satisfying*

$$\sum_{s=1}^{d}\hat{b}_s(\hat{B}^s\otimes\hat{\Gamma}^s)+I_Z\otimes M\geqslant 0,$$

$$(I_Z\otimes P_2)^TH+H^T(I_Z\otimes P_2)>0,$$

the network (16) is passive.

Proof. Define the same Lyapunov functional $V_1(t)$ as (5) for the network (16). Then, we have

$$\begin{aligned}\dot{V}_1(t) &= 2\sum_{p=1}^{Z}\int_\Omega e_p^T(a,t)\frac{\partial e_p(a,t)}{\partial t}da\\ &\quad +\sum_{s=1}^{d}\sum_{p=1}^{Z}\sum_{q\in\mathcal{Z}_p}\frac{\hat{b}_s}{\alpha^s_{pq}}(G^s_{pq}(t)-\delta^s_{pq})\dot{G}^s_{pq}(t)\\ &= 2\sum_{p=1}^{Z}\int_\Omega e_p^T(a,t)\Big[M\triangle e_p(a,t)-Ae_p(a,t)\\ &\quad +\sum_{s=1}^{d}\sum_{q=1}^{Z}\hat{b}_s\hat{B}^s_{pq}\hat{\Gamma}^s\Delta e_q(a,t)\\ &\quad +Ku_p(a,t)-\frac{1}{Z}\sum_{l=1}^{Z}Ku_l(a,t)+Ef(\chi_p(a,t))-Ef(\bar{\chi}(a,t))\\ &\quad +Ef(\bar{\chi}(a,t))-\frac{1}{Z}\sum_{l=1}^{Z}Ef(\chi_l(a,t))+\upsilon_p(a,t)\\ &\quad -\frac{1}{Z}\sum_{l=1}^{Z}\upsilon_l(a,t)\Big]da\\ &\quad -\sum_{s=1}^{d}\sum_{p=1}^{Z}\sum_{q\in\mathcal{Z}_p}\hat{b}_s(G^s_{pq}(t)-\delta^s_{pq})\\ &\quad \times\int_\Omega(\chi_p(a,t)-\chi_q(a,t))^T\hat{\Gamma}^s(\chi_p(a,t)\\ &\quad -\chi_q(a,t))da.\end{aligned}\tag{19}$$

According to Lemma 2.1, we can derive that

$$\begin{aligned}&\sum_{s=1}^{d}\sum_{p=1}^{Z}\sum_{q=1}^{Z}\hat{b}_s\hat{B}^s_{pq}\int_\Omega e_p^T(a,t)\hat{\Gamma}^s\Delta e_q(a,t)da\\ &\quad +\sum_{p=1}^{Z}\int_\Omega e_p^T(a,t)M\Delta e_p(a,t)da\\ &= \int_\Omega e^T(a,t)\Big[\sum_{s=1}^{d}\hat{b}_s(\hat{B}^s\otimes\hat{\Gamma}^s)+I_Z\otimes M\Big]\Delta e(a,t)da\\ &= -\sum_{h=1}^{\epsilon}\int_\Omega\left(\frac{\partial e(a,t)}{\partial a_h}\right)^T\Big[\sum_{s=1}^{d}\hat{b}_s(\hat{B}^s\otimes\hat{\Gamma}^s)+I_Z\otimes M\Big]\frac{\partial e(a,t)}{\partial a_h}da\\ &\leqslant -\sum_{h=1}^{\epsilon}\int_\Omega e^T(a,t)\frac{\sum_{s=1}^{d}\hat{b}_s(\hat{B}^s\otimes\hat{\Gamma}^s)+I_Z\otimes M}{c_h^2}e(a,t)da.\end{aligned}\tag{20}$$

By (4), (7)–(10), (20), one gets

$$\begin{aligned}\dot{V}_1(t) &= 2\sum_{p=1}^{Z}\int_\Omega e_p^T(a,t)\Big[M\triangle e_p(a,t)-Ae_p(a,t)\\ &\quad +\sum_{s=1}^{d}\sum_{q=1}^{Z}\hat{b}_s\hat{B}^s_{pq}\hat{\Gamma}^s\Delta e_q(a,t)\\ &\quad +Ku_p(a,t)+Ef(\chi_p(a,t))-Ef(\bar{\chi}(a,t))\\ &\quad +\sum_{s=1}^{d}\sum_{q=1}^{Z}\hat{b}_sG^s_{pq}(t)\hat{\Gamma}^se_q(a,t)\Big]da\\ &\quad -2\sum_{s=1}^{d}\sum_{p=1}^{Z}\sum_{q=1}^{Z}\hat{b}_s(G^s_{pq}(t)-\delta^s_{pq})\int_\Omega e_p^T(a,t)\hat{\Gamma}^se_q(a,t)da\end{aligned}$$

$$\leqslant \int_\Omega e^T(a,t)\Big[-2\sum_{h=1}^{\epsilon}\sum_{s=1}^{d}\frac{\hat{b}_s}{c_h^2}(\hat{B}^s\otimes\hat{\Gamma}^s+I_Z\otimes M)$$
$$+I_Z\otimes(-2A+EE^T+\mathcal{Q})\Big]e(a,t)da$$
$$+2\sum_{s=1}^{d}\hat{b}_s\int_\Omega e^T(a,t)(\delta^s\otimes\hat{\Gamma}^s)e(a,t)da$$
$$+2\int_\Omega e^T(a,t)(I_Z\otimes K)u(a,t)da,$$

where $\delta^s=(\delta^s_{pq})_{Z\times Z}\in\mathbb{R}^{Z\times Z}$.

Therefore,

$$\dot{V}_1(t)-2\int_\Omega y^T(a,t)Hu(a,t)da$$
$$\leqslant \int_\Omega e^T(a,t)\Big[-2\sum_{h=1}^{\epsilon}\sum_{s=1}^{d}\frac{\hat{b}_s}{c_h^2}(\hat{B}^s\otimes\hat{\Gamma}^s+I_Z\otimes M)$$
$$+I_Z\otimes(-2A+EE^T+\mathcal{Q})\Big]e(a,t)da$$
$$+2\int_\Omega e^T(a,t)[(I_Z\otimes K)-(I_Z\otimes P_1^T)H]u(a,t)da$$
$$-\int_\Omega u^T(a,t)[(I_Z\otimes P_2^T)H+H^T(I_Z\otimes P_2)]u(a,t)da$$
$$+2\sum_{s=1}^{d}\hat{b}_s\int_\Omega e^T(a,t)(\delta^s\otimes\hat{\Gamma}^s)e(a,t)da$$
$$\leqslant \int_\Omega e^T(a,t)\Big\{-2\sum_{h=1}^{\epsilon}\sum_{s=1}^{d}\frac{\hat{b}_s}{c_h^2}(\hat{B}^s\otimes\hat{\Gamma}^s+I_Z\otimes M)$$
$$+I_Z\otimes(-2A+EE^T+\mathcal{Q})+[(I_Z\otimes K)-(I_Z\otimes P_1^T)H]$$
$$\times[(I_Z\otimes P_2^T)H+H^T(I_Z\otimes P_2)]^{-1}[(I_Z\otimes K^T)-H^T(I_Z\otimes P_1)]\Big\}e(a,t)da$$
$$+2\sum_{s=2}^{d}\hat{b}_s\int_\Omega e^T(a,t)(\delta^s\otimes\hat{\Gamma}^s)e(a,t)da$$
$$+2\hat{b}_1\int_\Omega e^T(a,t)(\delta^1\otimes\hat{\Gamma}^1)e(a,t)da$$
$$\leqslant \int_\Omega e^T(a,t)\Big\{-2\sum_{h=1}^{\epsilon}\sum_{s=1}^{d}\frac{\hat{b}_s}{c_h^2}(\hat{B}^s\otimes\hat{\Gamma}^s+I_Z\otimes M)$$
$$+I_Z\otimes(-2A+EE^T+\mathcal{Q})+[(I_Z\otimes K)-(I_Z\otimes P_1^T)H]$$
$$\times[(I_Z\otimes P_2^T)H+H^T(I_Z\otimes P_2)]^{-1}[(I_Z\otimes K^T)-H^T(I_Z\otimes P_1)]\Big\}e(a,t)da$$
$$+2\hat{b}_1\int_\Omega e^T(a,t)(\delta^1\otimes\hat{\Gamma}^1)e(a,t)da$$
$$=\int_\Omega e^T(a,t)(\Xi^*+2\hat{b}_1\delta^1\otimes\hat{\Gamma}^1)e(a,t)da, \quad (21)$$

where $\Xi^*=-2\sum_{h=1}^{\epsilon}\sum_{s=1}^{d}\frac{\hat{b}_s}{c_h^2}(\hat{B}^s\otimes\hat{\Gamma}^s+I_Z\otimes M)+I_Z\otimes(-2A+EE^T+\mathcal{Q})+[(I_Z\otimes K)-(I_Z\otimes P_1^T)H][(I_Z\otimes P_2^T)H+H^T(I_Z\otimes P_2)]^{-1}[(I_Z\otimes K^T)-H^T(I_Z\otimes P_1)]$.

On the other hand, there exists an orthogonal matrix $\mathcal{K}=(\kappa_1,\kappa_2,\ldots,\kappa_Z)\in\mathbb{R}^{Z\times Z}$ such that $\mathcal{K}^T\delta^1\mathcal{K}=\Upsilon=\mathrm{diag}(\Upsilon_1,\Upsilon_2,\ldots,\Upsilon_Z)$, in which $0=\Upsilon_1>\Upsilon_2\geqslant\Upsilon_3\geqslant\cdots\geqslant\Upsilon_Z$. Let $\varrho(a,t)=(\varrho_1^T(a,t),\varrho_2^T(a,t),\ldots,\varrho_Z^T(a,t))^T=(\mathcal{K}^T\otimes I_z)e(a,t)$. Since $\kappa_1=\frac{1}{\sqrt{Z}}(1,1,\ldots,1)^T$, one has $\varrho_1(a,t)=(\kappa_1^T\otimes I_z)e(a,t)=0$. Then, we can derive from (21) that

$$\dot{V}_1(t)-2\int_\Omega y^T(a,t)Hu(a,t)da$$
$$\leqslant\int_\Omega e^T(a,t)[2\hat{b}_1(\mathcal{K}\otimes I_z)(\Upsilon\otimes\hat{\Gamma}^1)(\mathcal{K}^T\otimes I_z)]e(a,t)da$$
$$+\int_\Omega e^T(a,t)\Xi^*e(a,t)da$$
$$=\int_\Omega e^T(a,t)\Xi^*e(a,t)da+2\hat{b}_1\int_\Omega\varrho^T(a,t)(\Upsilon\otimes\hat{\Gamma}^1)\varrho(a,t)da$$
$$\leqslant\int_\Omega e^T(a,t)\Xi^*e(a,t)da+2\hat{b}_1\Upsilon_2\int_\Omega\varrho^T(a,t)(I_Z\otimes\hat{\Gamma}^1)\varrho(a,t)da$$
$$=\int_\Omega e^T(a,t)[\Xi^*+2\hat{b}_1\Upsilon_2 I_Z\otimes\hat{\Gamma}^1]e(a,t)da.$$

By choosing δ_{pq} large enough such that

$$\gamma_M(\Xi^*)+2\hat{b}_1\Upsilon_2\gamma_m(\hat{\Gamma}^1)\leqslant 0,$$

one has

$$\Xi^*+2\hat{b}_1\Upsilon_2 I_Z\otimes\hat{\Gamma}^1\leqslant 0.$$

Therefore,

$$\frac{\dot{V}_1(t)}{2}\leqslant\int_\Omega y^T(a,t)Hu(a,t)da.$$

Letting $V(t)=V_1(t)/2$, one has

$$\int_{t_0}^{t_r}\int_\Omega y^T(a,t)Hu(a,t)dadt\geqslant V(t_r)-V(t_0)$$

for any $0\leqslant t_0\leqslant t_r\in\mathbb{R}$. □

Theorem 4.2. *If there are two matrices $\mathbb{R}^{\eta Z\times\eta Z}\ni Q_1>0$ and $H\in\mathbb{R}^{\zeta Z\times\eta Z}$ satisfying*

$$\sum_{s=1}^{d}\hat{b}_s(\hat{B}^s\otimes\hat{\Gamma}^s)+I_Z\otimes M\geqslant 0,$$
$$(I_Z\otimes P_2)^TH+H^T(I_Z\otimes P_2)-Q_1>0,$$

the network (16) is input-strictly passive.

Theorem 4.3. *If there are two matrices $\mathbb{R}^{\zeta Z\times\zeta Z}\ni Q_2>0$ and $H\in\mathbb{R}^{\zeta Z\times\eta Z}$ satisfying*

$$\sum_{s=1}^{d}\hat{b}_s(\hat{B}^s\otimes\hat{\Gamma}^s)+I_Z\otimes M\geqslant 0,$$
$$(I_Z\otimes P_2)^TH+H^T(I_Z\otimes P_2)-(I_Z\otimes P_2)^TQ_2(I_Z\otimes P_2)>0,$$

the network (16) is output-strictly passive.

4.3. Synchronization criteria

Theorem 4.4. *Under the adaptive state feedback controller (18), the network (15) can achieve synchronization if the network (16) is output-strictly passive with regard to the storage function $W(t)=V_1(t)/2$ and $P_1^TP_1>0$.*

Proof. Using the similar proof method as in Theorem 3.4, we can easily obtain results.

Based on the Theorems 4.3 and 4.4, the following conclusion can be obtained. □

Corollary 4.1. *The network (15) under the adaptive controller (18) realizes synchronization if there exist $\mathbb{R}^{\zeta Z\times\zeta Z}\ni Q_2>0$ and $H\in\mathbb{R}^{\zeta Z\times\eta Z}$ satisfying*

$$\sum_{s=1}^{d}\hat{b}_s(\hat{B}^s\otimes\hat{\Gamma}^s)+I_Z\otimes M\geqslant 0,$$
$$\begin{pmatrix}(I_Z\otimes P_2)^TH+H^T(I_Z\otimes P_2)-(I_Z\otimes P_2)^TQ_2(I_Z\otimes P_2) & 0\\ 0 & P_1^TP_1\end{pmatrix}>0.$$

Remark 3. Comparing with the traditional complex network models, the main difficulty to deal with the CRDNNs comes from the reaction-diffusion terms, the multiple state couplings and the multiple spatial diffusion couplings. By selecting appropriate Lyapunov functionals and employing inequality techniques, several passivity and synchronization criteria for CRDNNs with multiple state couplings or spatial diffusion couplings are proposed in this paper.

Remark 4. More recently, CRDNNs have found successful applications in information processing, secure communication, cognition behaviors, and so on. As is well known, these applications heavily depend on the dynamical behaviors of CRDNNs, especially the synchronization [14–20] and passivity [21–25]. In the most of existing results on CRDNNs, they always suppose that the network model has the single weight. In fact, in order to get a more accurate description, we should consider the multiple state couplings or spatial diffusion couplings in CRDNNs. Therefore, we respectively study the passivity and synchronization of CRDNNs with multiple state couplings or spatial diffusion couplings in this paper, which not only generalize the existing synchronization and passivity results to some extent, but also can serve as a stepping stone to study the dynamical behaviors of CRDNNs with multiple spatial diffusion couplings.

5. Numerical examples

Example 5.1. The CRDNNs with multiple state couplings is given as follows:

$$\begin{aligned}\frac{\partial \chi_p(a,t)}{\partial t} &= M\triangle\chi_p(a,t) - A\chi_p(a,t) + Ef(\chi_p(a,t)) + J\\ &+2\sum_{q=1}^{5} B^1_{pq}\Gamma^1\chi_q(a,t) + Ku_p(a,t)\\ &+3\sum_{q=1}^{5} B^2_{pq}\Gamma^2\chi_q(a,t) + \nu_p(a,t)\\ &+4\sum_{q=1}^{5} B^3_{pq}\Gamma^3\chi_q(a,t),\end{aligned} \tag{22}$$

where $p = 1,2,\ldots,5$, $f_k(\xi) = \frac{|\xi+1|-|\xi-1|}{4}$, $k = 1,2,3$, $\Gamma^1 = \mathrm{diag}(0.5,0.6,0.5)$, $\Gamma^2 = \mathrm{diag}(0.7,0.4,0.8)$, $\Gamma^3 = \mathrm{diag}(0.5,0.7,0.8)$, $\Omega = \{a \mid -0.5 < a < 0.5\}$, $J = (0.6,0.4,0.8)^T$, $M = \mathrm{diag}(0.3,0.5,0.2)$, $A = \mathrm{diag}(0.7,0.9,0.8)$.

$$E = \begin{pmatrix} 0.4 & 0.7 & 0.5\\ 0.5 & 0.5 & 0.6\\ 0.2 & 0.3 & 0.7\end{pmatrix},\quad K = \begin{pmatrix} 0.4 & 0.3\\ 0.4 & 0.4\\ 0.5 & 0.3\end{pmatrix},$$

$$B^1 = \begin{pmatrix} -0.4 & 0.1 & 0 & 0.2 & 0.1\\ 0.1 & -0.4 & 0.1 & 0.1 & 0.1\\ 0 & 0.1 & -0.6 & 0 & 0.5\\ 0.2 & 0.1 & 0 & -0.6 & 0.3\\ 0.1 & 0.1 & 0.5 & 0.3 & -1\end{pmatrix},$$

$$B^2 = \begin{pmatrix} -0.5 & 0.2 & 0 & 0.2 & 0.1\\ 0.2 & -0.7 & 0.3 & 0.1 & 0.1\\ 0 & 0.3 & -0.8 & 0 & 0.5\\ 0.2 & 0.1 & 0 & -0.6 & 0.3\\ 0.1 & 0.1 & 0.5 & 0.3 & -1\end{pmatrix},$$

$$B^3 = \begin{pmatrix} -0.8 & 0.3 & 0 & 0.3 & 0.2\\ 0.3 & -0.7 & 0.1 & 0.2 & 0.1\\ 0 & 0.1 & -0.4 & 0 & 0.3\\ 0.3 & 0.2 & 0 & -0.6 & 0.1\\ 0.2 & 0.1 & 0.3 & 0.1 & -0.7\end{pmatrix}.$$

Apparently, function $f_k(\cdot)$ satisfies Lipschitz condition with $\mathcal{Q}_k = 0.5$.

The output vector $y_p(a,t) \in \mathbb{R}^{3\times 2}$ is selected as follows:

$$y_p(a,t) = P_1 e_p(a,t) + P_2 u_p(a,t), \qquad p = 1,2,\ldots,5,$$

where

$$P_1 = \begin{pmatrix} 0.1 & 0 & 0\\ 0 & 0.2 & 0\\ 0 & 0 & 0.1\end{pmatrix},\qquad P_2 = \begin{pmatrix} 0.2 & 0.2\\ 0.2 & 0.3\\ 0.3 & 0.2\end{pmatrix}.$$

Case 1: By taking advantage of the MATLAB YALMIP Toolbox, the following matrix H that satisfies the condition of Theorem 3.1 can be obtained:

$$H = 10^8 I_5 \otimes \begin{pmatrix} 0.5800 & 0.5800\\ -4.0599 & 5.5099\\ 5.5099 & -4.0599\end{pmatrix}.$$

Therefore, under the adaptive state feedback controller (4), the network (22) achieves passivity. The simulation results are displayed in Figs. 1 and 2.

Case 2: By taking advantage of the MATLAB YALMIP Toolbox, the following matrices H and Q_2 that satisfy the condition of Corollary 3.1 can be obtained:

$$H = 10^8 I_5 \otimes \begin{pmatrix} 0.7185 & 0.7185\\ -3.6033 & 5.3995\\ 5.3995 & -3.6032\end{pmatrix},$$

$$Q_2 = 10^8 I_5 \otimes \begin{pmatrix} 1.7827 & -0.0000 & -0.0000\\ -0.0000 & 1.7827 & 0.0000\\ -0.0000 & 0.0000 & 1.7827\end{pmatrix}.$$

Therefore, under the adaptive state feedback controller (4), the network (22) achieves synchronization. The simulation results are displayed in Figs. 3 and 4.

Example 5.2. The CRDNNs with multiple spatial diffusion couplings are given as follows:

$$\begin{aligned}\frac{\partial \chi_p(a,t)}{\partial t} &= M\triangle\chi_p(a,t) - A\chi_p(a,t) + Ef(\chi_p(a,t)) + J\\ &+ 0.3\sum_{q=1}^{5} \hat{B}^1_{pq}\hat{\Gamma}^1\Delta\chi_q(a,t) + Ku_p(a,t)\\ &+ 0.4\sum_{q=1}^{5} \hat{B}^2_{pq}\hat{\Gamma}^2\Delta\chi_q(a,t) + \nu_p(a,t)\\ &+ 0.5\sum_{q=1}^{5} \hat{B}^3_{pq}\hat{\Gamma}^3\Delta\chi_q(a,t),\end{aligned} \tag{23}$$

where $p = 1,2,\ldots,5$, $f_k(\xi) = \frac{|\xi+1|-|\xi-1|}{4}$, $k = 1,2,3$, $\hat{\Gamma}^1 = \mathrm{diag}(0.1,0.3,0.2)$, $\hat{\Gamma}^2 = \mathrm{diag}(0.2,0.3,0.2)$, $\hat{\Gamma}^3 = \mathrm{diag}(0.4,0.3,0.1)$, $\Omega = \{a \mid -0.5 < a < 0.5\}$, $J = (0.4,0.6,0.7)^T$, $M = \mathrm{diag}(0.7,0.6,0.6)$, $A = \mathrm{diag}(0.5,0.7,0.8)$.

$$E = \begin{pmatrix} 0.6 & 0.5 & 0.6\\ 0.5 & 0.3 & 0.7\\ 0.4 & 0.7 & 0.5\end{pmatrix},\quad K = \begin{pmatrix} 0.3 & 0.7\\ 0.5 & 0.3\\ 0.2 & 0.5\end{pmatrix},$$

$$\hat{B}^1 = \begin{pmatrix} -0.6 & 0.3 & 0 & 0.1 & 0.2\\ 0.3 & -0.8 & 0.2 & 0.1 & 0.2\\ 0 & 0.2 & -0.5 & 0 & 0.3\\ 0.1 & 0.1 & 0 & -0.5 & 0.3\\ 0.2 & 0.2 & 0.3 & 0.3 & -1\end{pmatrix},$$

$$\hat{B}^2 = \begin{pmatrix} -0.5 & 0.2 & 0 & 0.2 & 0.1\\ 0.2 & -0.8 & 0.2 & 0.3 & 0.1\\ 0 & 0.2 & -0.6 & 0 & 0.4\\ 0.2 & 0.3 & 0 & -0.7 & 0.2\\ 0.1 & 0.1 & 0.4 & 0.2 & -0.8\end{pmatrix},$$

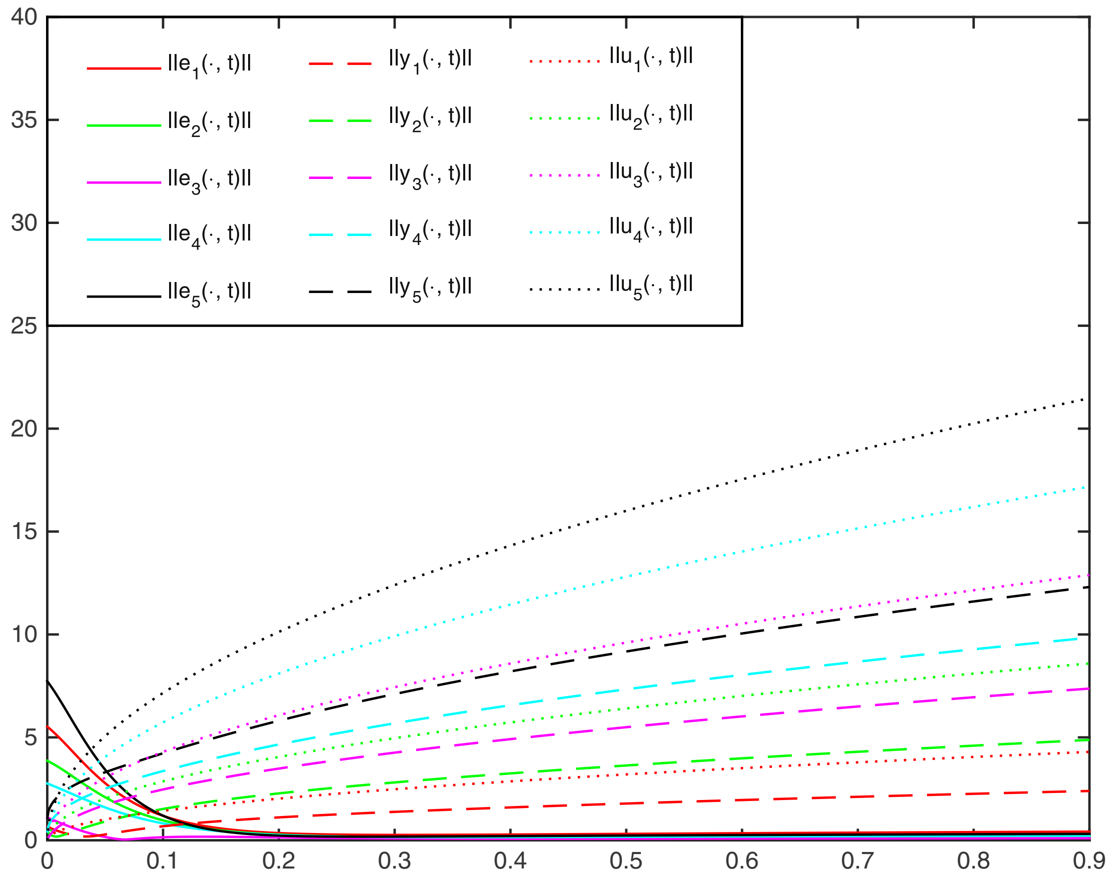

Fig. 1. $\|e_p(t)\|$, $\|y_p(t)\|$, $\|u_p(t)\|$, $p = 1, 2, \ldots, 5$.

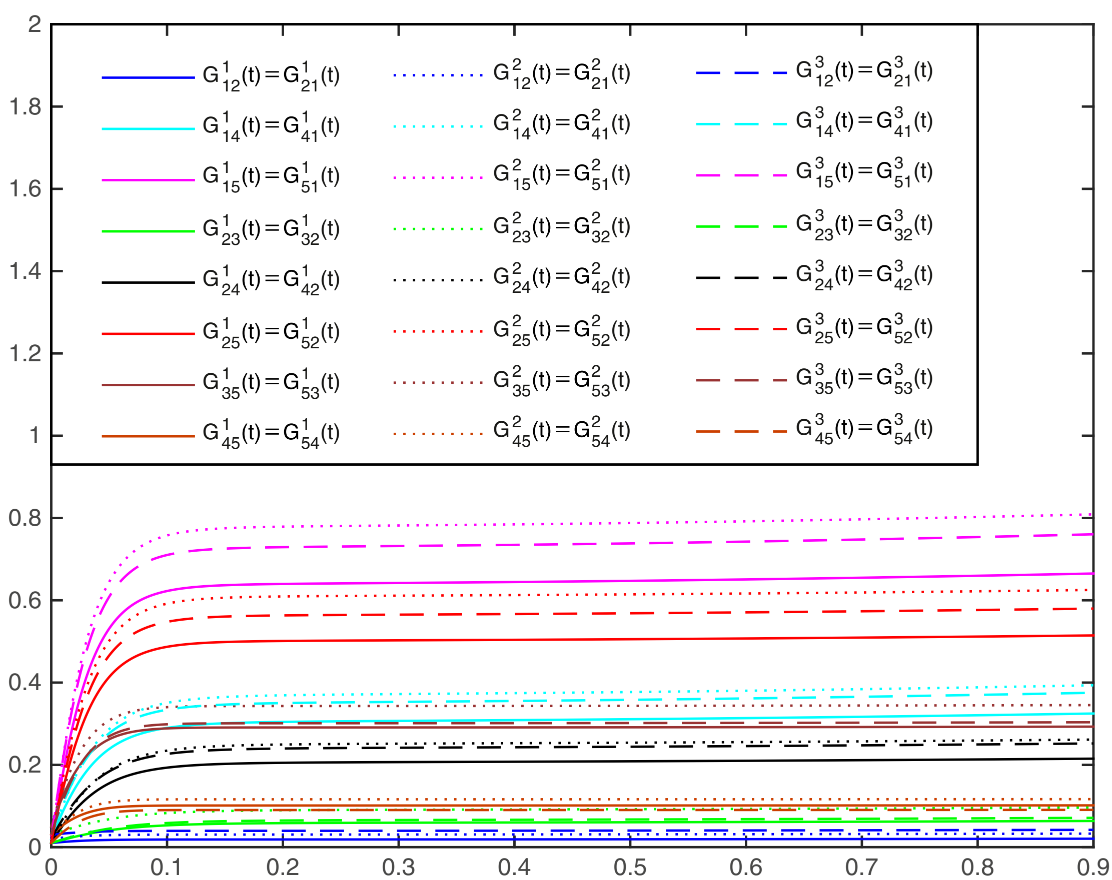

Fig. 2. Adaptive feedback gains.

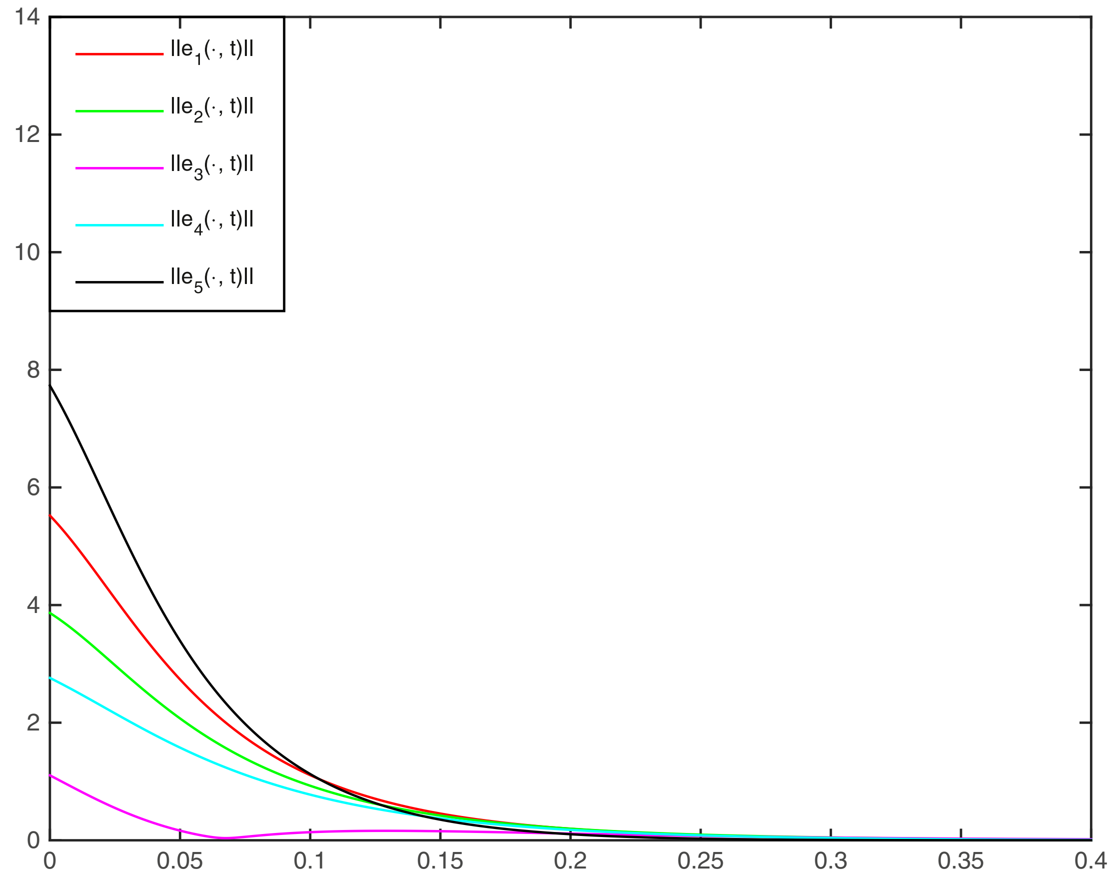

Fig. 3. $\|e_p(t)\|$, $p = 1, 2, \ldots, 5$.

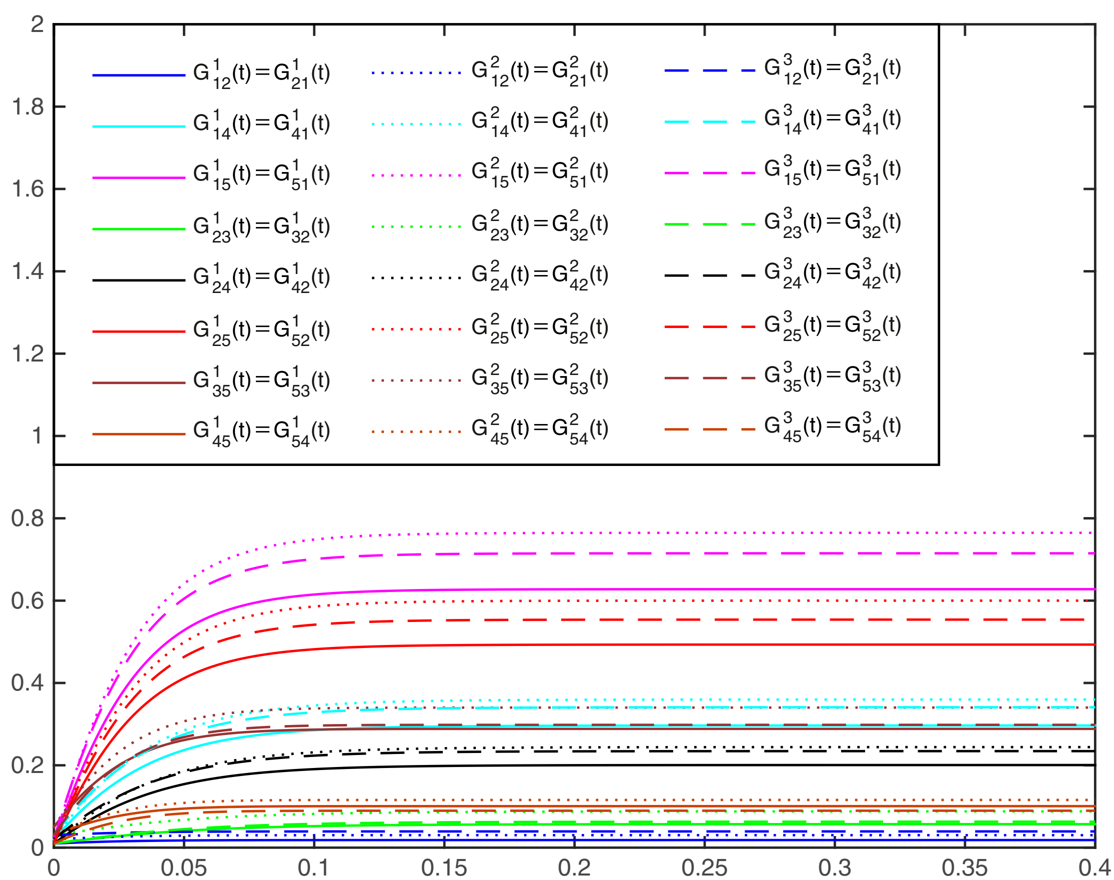

Fig. 4. Adaptive feedback gains.

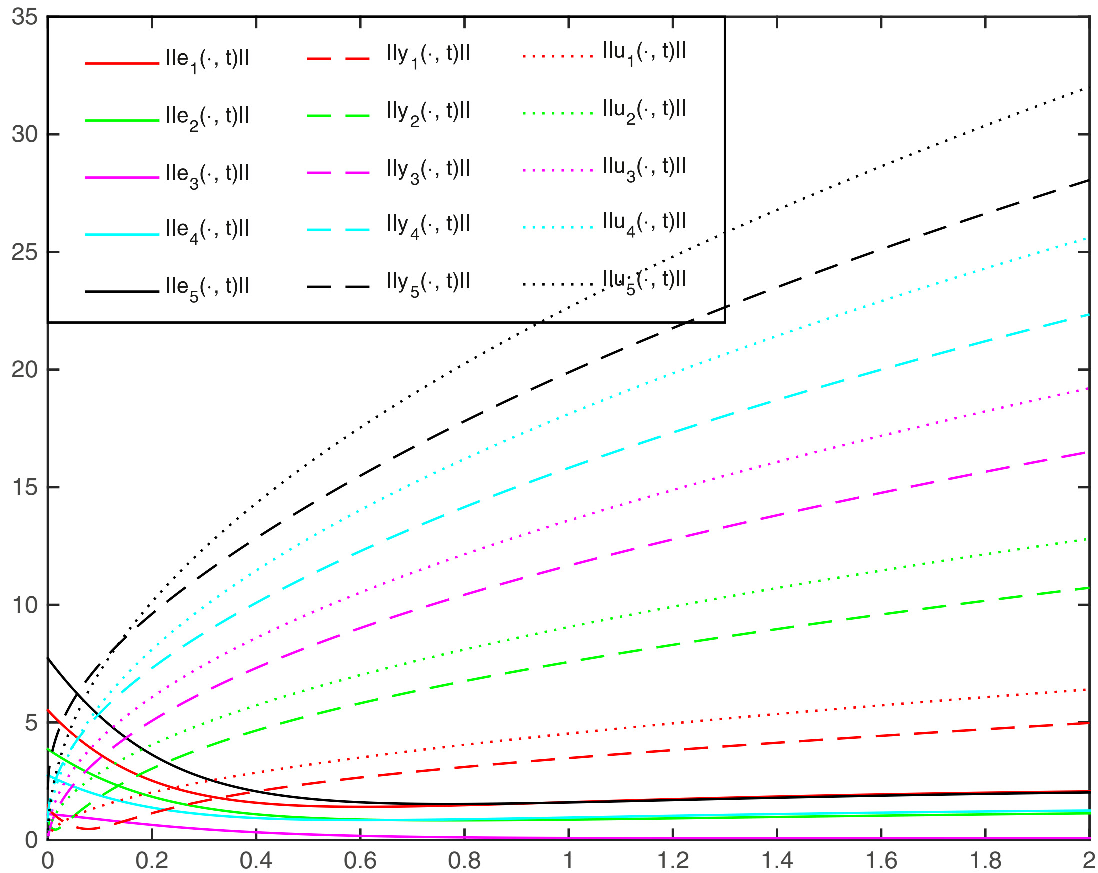

Fig. 5. $\|e_p(t)\|, \|y_p(t)\|, \|u_p(t)\|, p = 1, 2, \ldots, 5.$

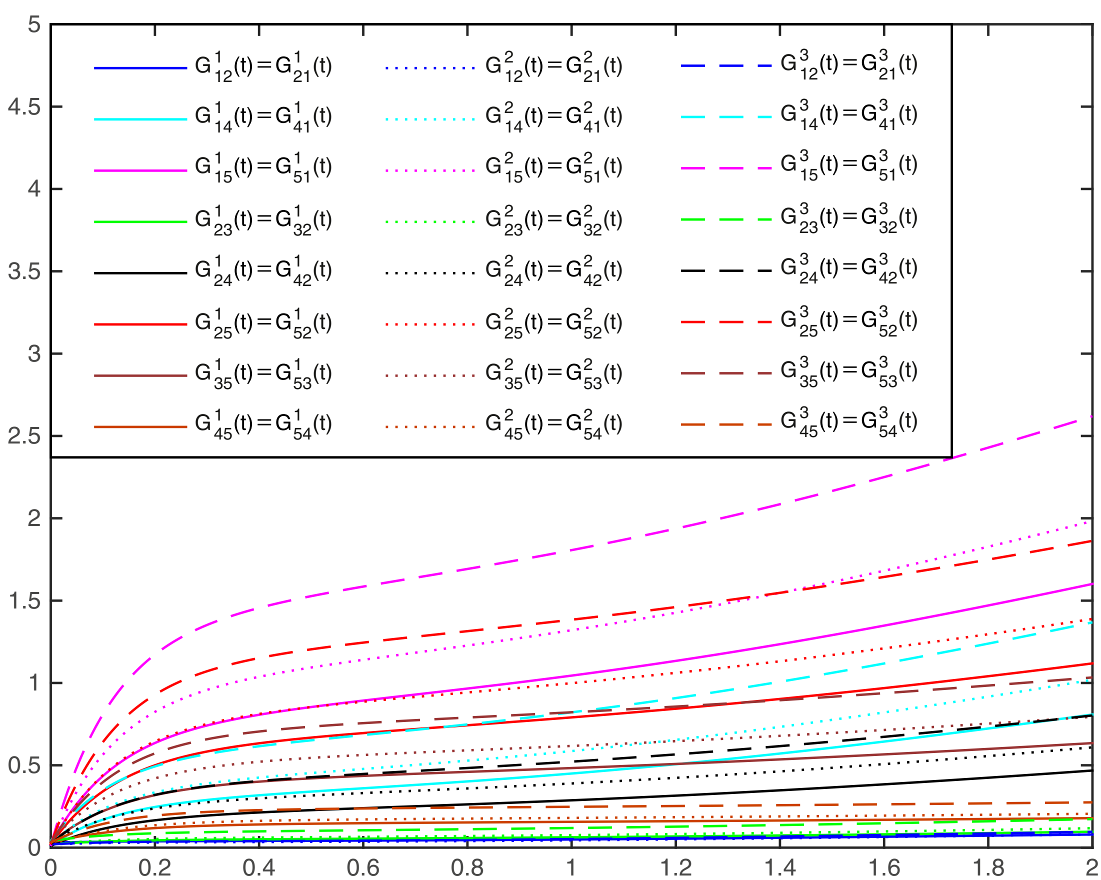

Fig. 6. Adaptive feedback gains.

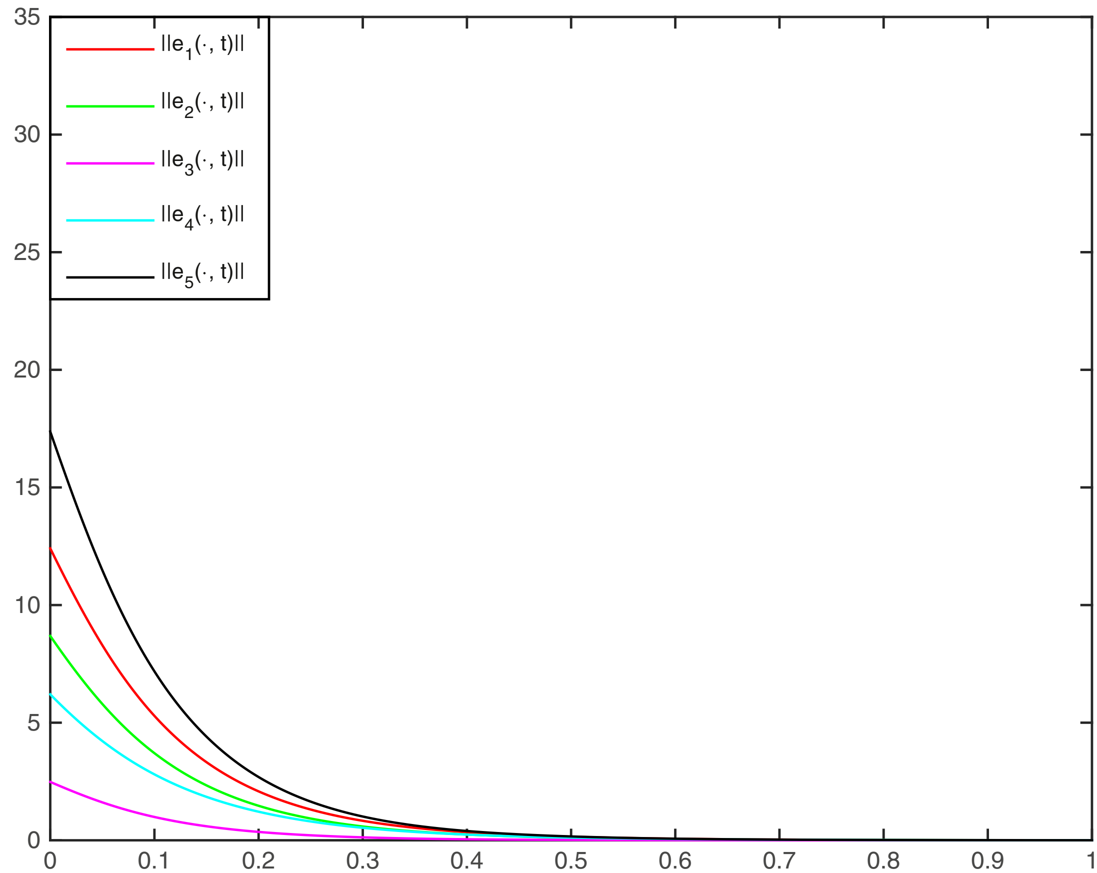

Fig. 7. $\|e_p(t)\|$, $p = 1, 2, \ldots, 5$.

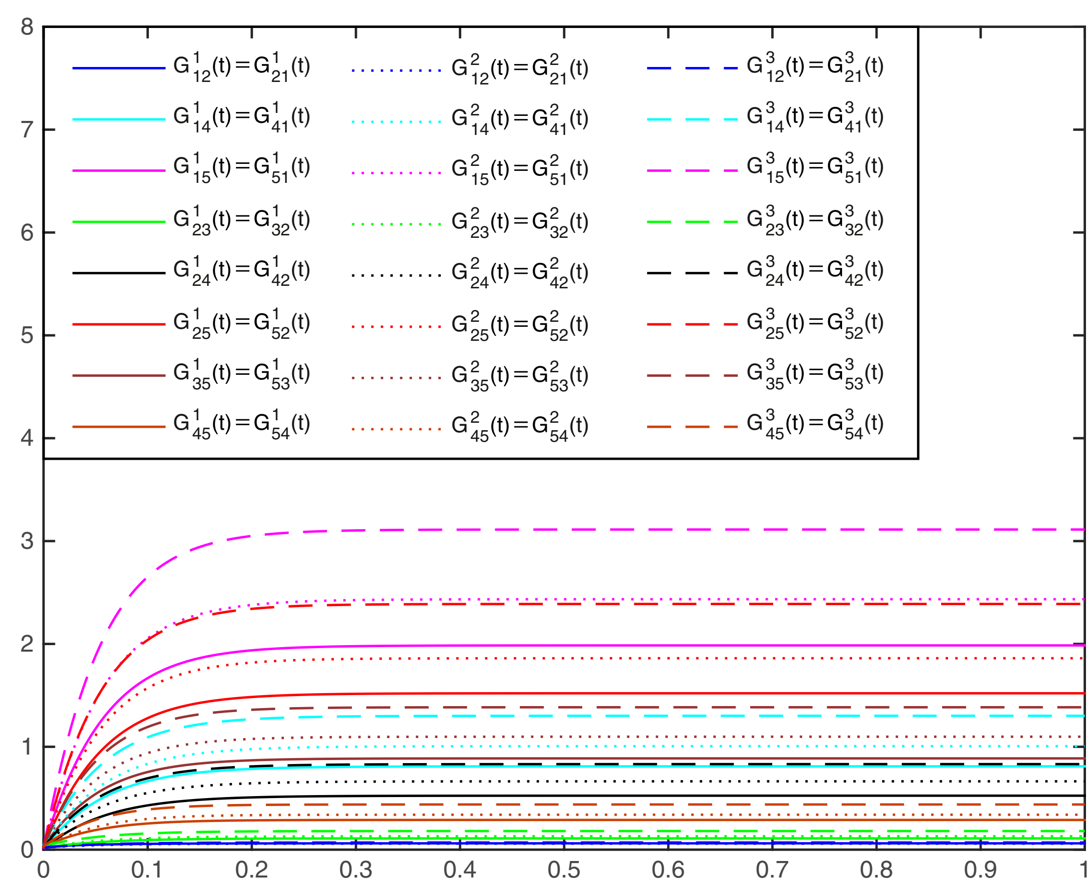

Fig. 8. Adaptive feedback gains.

$$\hat{B}^3 = \begin{pmatrix} -0.7 & 0.2 & 0 & 0.4 & 0.1 \\ 0.2 & -1 & 0.4 & 0.2 & 0.2 \\ 0 & 0.4 & -0.7 & 0 & 0.3 \\ 0.4 & 0.2 & 0 & -0.9 & 0.3 \\ 0.1 & 0.2 & 0.3 & 0.3 & -0.9 \end{pmatrix}.$$

Apparently, function $f_k(\cdot)$ satisfies Lipschitz condition with $\mathcal{Q}_k = 0.5$.

The output vector $y_p(a,t) \in \mathbb{R}^{3\times 2}$ is selected as follows:

$$y_p(a,t) = P_1 e_p(a,t) + P_2 u_p(a,t), \qquad p = 1,2,\dots,5,$$

where

$$P_1 = \begin{pmatrix} 0.3 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.3 \end{pmatrix}, \qquad P_2 = \begin{pmatrix} 0.4 & 0.2 \\ 0.3 & 0.5 \\ 0.4 & 0.3 \end{pmatrix}.$$

Case 1: By taking advantage of the MATLAB YALMIP Toolbox, the following matrix H that satisfies the condition of Theorem 4.1 can be obtained:

$$H = I_5 \otimes \begin{pmatrix} 62.1418 & -180.1078 \\ 136.7412 & 22.1211 \\ -163.8147 & 84.3807 \end{pmatrix}.$$

Therefore, under the adaptive state feedback controller (18), the network (23) achieves passivity. The simulation results are displayed in Figs. 5 and 6.

Case 2: By taking advantage of the MATLAB YALMIP Toolbox, the following matrices H and Q_2 that satisfy the condition of Corollary 4.1 can be obtained:

$$H = I_5 \otimes \begin{pmatrix} 55.0223 & -182.1712 \\ 138.2808 & 3.2040 \\ -168.9687 & 77.3546 \end{pmatrix},$$

$$Q_2 = I_5 \otimes \begin{pmatrix} -61.1501 & 127.0835 & 8.8332 \\ 127.0835 & -54.1311 & -114.8496 \\ 8.8332 & -114.8496 & -0.0123 \end{pmatrix}.$$

Therefore, under the adaptive state feedback controller (18), the network (23) achieves synchronization. The simulation results are displayed in Figs. 7 and 8.

6. Conclusion

In this paper, the CRDNNs with multiple state couplings or spatial diffusion couplings have been studied. By choosing appropriate adaptive state feedback controllers and making use of some inequality techniques, we have given several passivity conditions for these network models. Furthermore, two synchronization criteria for these networks also have been established by exploiting the obtained output strictly passivity results. Finally, two numeral examples have been provided to verify the effectiveness of the derived criterion. Obviously, in most circumstances CRDNNs are required to achieve synchronization in a finite time. In the future, we shall further study the finite-time synchronization problem for CRDNNs with multiple spatial diffusion couplings.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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