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Analysis and pinning control for passivity and synchronization of multiple derivative coupled reaction diffusion neural networks

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Abstract

In this paper, a class of multiple derivative coupled reaction-diffusion neural networks with and without parameter uncertainties is investigated. Firstly, we analyze the passivity and synchronization of the proposed network models and derive several criteria based on inequality techniques. Furthermore, a pinning control strategy is also developed to ensure that the proposed networks can achieve passivity and synchronization. Finally, a numerical example is presented to verify the effectiveness of the obtained criteria.

1. Introduction

In recent years, the dynamical behaviors of neural networks (NNs) have been paid more and more attention due to their extensive applications in image processing, pattern classification, optimization and so on [1–8]. As is well known, the movement of electrons in a nonuniform

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electromagnetic field can result in the phenomenon of reaction diffusion [9–11]. Therefore, reaction diffusion phenomenon should be taken into account in the neural networks. Up to now, the dynamical behaviors of reaction-diffusion neural networks (RDNNs) have attracted a great deal of attention of many researchers [12–18]. With the help of stochastic analysis and inequality techniques, Sheng et al. [12] not only discussed the passivity of the stochastic delayed RDNNs, but also considered the robust passivity in stochastic delayed RDNNs. Wang et al. [13] studied the passivity of the RDNNs, and analyzed the robust passivity for RDNNs with parameter uncertainties by emoloying some inequality techniques and Lyapunov functional method. In [14], the global exponential stability was considered for RDNNs with time-varying delays by employing the Wirtinger's inequality and a diffusion-dependent Lyapunov functional.

More recently, coupled RDNNs (CRDNNs), composed of multiple nonidentical or identical RDNNs, has been paid more and more attention in various fields due to their diversity of applications in harmonic oscillation generation, chaotic generator design and pattern recognition, etc.. Hence, the dynamical behaviors for CRDNNs have been studied by a large number of authors, especially the synchronization [19–22] and passivity [23–28]. Wang et al. [19] discussed the synchronization problem of CRDNNs with time delays based on the designed adaptive feedback controllers, and presented several sufficient conditions for ensuring the synchronization with the help of the LaSalle invariant principle. In [20], the authors discussed the pinning synchronization for CRDNNs with state coupling and spatial diffusion coupling, and some synchronization criteria were gained based on the inequality techniques and Lyapunov functional method. Huang et al. [25] proposed a nonlinear CRDNNs, and considered the pinning passivity and passivity problems for such network model. In [27], the authors researched the passivity of the CRDNNs with switching topology by exploiting the Lyapunov functional method and inequality techniques, and given several sufficient conditions to ensure the output-strictly passivity and input-strictly passivity. Unfortunately, in these existing results on the passivity of the CRDNNs, the input and output were required to have the same dimension [23–28]. As far as we know, the passivity for CRDNNs with different input and output dimensions was infrequently studied [29,30]. Wang et al. [29] discussed the passivity for two types of CRDNNs with different input and output dimensions, and analyzed the stability of the passive CRDNNs. In [30], some passivity criteria for the CRDNNs with undirected and directed topologies were gained based on the designed adaptive laws, and the relationship between the output strict passivity and synchronization was also studied.

Actually, different time derivatives of node may give rise to different changes of its neighbor nodes. Therefore, the synchronization and passivity problems for complex dynamical networks (CDNs) with derivative coupling have been studied by some researchers [31–34,36–38]. In [31], the authors considered the impulsive synchronization of the CDNs with derivative coupling, and gave several delay-dependent synchronization criteria by using the designed impulsive controller. Xu et al. [34] studied the adaptive synchronization for a class of uncertain complex networks with derivative coupling. However, most of the work on the dynamical behaviors of CDNs are based on the single weighted network models [31–34]. In fact, it is necessary to describe some real networks by multi-weighted network models, such as social networks, public traffic roads networks, etc. [35]. Therefore, some authors also have considered the problems of synchronization and passivity for CDNs with multiple derivative couplings [36–38]. In [36], Wang et al. discussed two kinds of CDNs with multiple derivative couplings, and researched the synchronization for these network models by designed the adaptive state feedback controllers. Wang et al. [37] investigated the output synchronization and adaptive control problems for CDNs with multiple derivative couplings, and presented several output synchronization criteria based on the matrix theory and inequality techniques. Evidently, it is also very significative to further research the problems of synchronization and passivity for CRDNNs with multiple derivative couplings. Regretfully, the synchronization and passivity for CRDNNs with multiple derivative couplings has not been studied.

Furthermore, the CRDNNs themselves cannot achieve desired dynamical behaviors (such as passivity and synchronization) in many circumstances. Generally speaking, a complex network consists of large number of interconnected nodes, thus it is impossible to design controller for every node. Consequently, some authors have developed several pinning control strategies for ensuring the passivity and synchronization of CRDNNs [20,25,39–43]. Yang et al. [39] studied the globally exponential synchronization in CRDNNs by using pinning-impulsive control strategy. In [43], the author discussed the μ-synchronization for a type of CRDNNs by utilizing the pinning control method. Obviously, it is very meaningful to further study the dynamical behaviors of multiple derivative coupled RDNNs (MDCRDNNs) by exploiting pinning control strategy. To the best of our knowledge, the pinning control for passivity and synchronization of MDCRDNNs has not yet been investigated.

In this paper, we focus on the analysis and pinning control for passivity and synchronization of MDCRDNNs. The main contributions in this paper are given as follows. First, by making use of some inequality techniques and Lyapunov functional, we analyze the passivity and synchronization of MDCRDNNs. Second, for the MDCRDNNs with parameter uncertainties, some passivity and synchronization criteria are also presented. Third, we give some sufficient conditions for ensuring the passivity and synchronization of these network models on the basis of the designed pinning controller.

Notations: Let $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}^{\epsilon}$ be the ϵ-dimensional Euclidean space and $\mathbb{R}^{z\times\eta}$ be the space of $z\times\eta$ real matrices. $\Omega = \{b = (b_1, b_2, \ldots, b_{\epsilon})^T \in \mathbb{R}^{\epsilon} \mid |b_{\rho}| < s_{\rho}, \rho = 1, 2, \ldots, \epsilon\}$ is the bounded domain in $\mathbb{R}^{\epsilon}$ with smooth boundary $\partial\Omega$, and $\triangle = \sum_{\rho=1}^{\epsilon} \frac{\partial^2}{\partial b_{\rho}^2}$. $\gamma_m(\cdot)$ and $\gamma_M(\cdot)$ denote the minimum and the maximum eigenvalue of the symmetric matrix. $\mathcal{D} = \{1, 2, \ldots, Z\}$ and $\mathcal{B} \subseteq \mathcal{D}\times\mathcal{D}$ represent the node set and the undirected edge set in the network, $\mathcal{Z}_m = \{n \in \mathcal{D} : (m, n) \in \mathcal{B}\}$.

2. Network model and preliminaries

2.1. Network model

The MDCRDNNs is described by:

$$\begin{aligned}\frac{\partial \chi_m(b,t)}{\partial t} &= H\triangle\chi_m(b,t) - P\chi_m(b,t) + Cf(\chi_m(b,t)) + J + Mu_m(b,t)\\ &\quad + \sum_{s=1}^{d}\sum_{n=1}^{Z} a_s F_{mn}^s \Gamma^s \dot{\chi}_n(b,t), \quad m = 1, 2, \ldots, Z, \end{aligned} \tag{1}$$

where $\chi_m(b,t) = (\chi_{m1}(b,t), \chi_{m2}(b,t), \ldots, \chi_{mz}(b,t))^T \in \mathbb{R}^z$ is the state vector of the node m; $u_m(b,t) \in \mathbb{R}^{\eta}$ is the external input of the node m; $0 < H = \mathrm{diag}(H_1, H_2, \ldots, H_z) \in \mathbb{R}^{z\times z}$; $0 < P = \mathrm{diag}(P_1, P_2, \ldots, P_z) \in \mathbb{R}^{z\times z}$; $f(\chi_m(b,t)) = (f_1(\chi_{m1}(b,t)), f_2(\chi_{m2}(b,t)), \ldots, f_z(\chi_{mz}(b,t)))^T \in \mathbb{R}^z$; $J = (J_1, J_2, \ldots, J_z)^T \in \mathbb{R}^z$; $C \in \mathbb{R}^{z\times z}$ and $M \in \mathbb{R}^{z\times\eta}$ are known matrices; $0 < a_s \in \mathbb{R}$ $(s = 1, 2, \ldots, d)$ represents the coupling strength; $0 < \Gamma^s = \mathrm{diag}(\Gamma_1^s, \Gamma_2^s, \cdots, \Gamma_z^s) \in \mathbb{R}^{z\times z}$ $(s = 1, 2, \cdots, d)$ denotes the inner coupling relationship; $F^s =$

$(F^s_{mn})_{Z\times Z} \in \mathbb{R}^{Z\times Z}$ $(s = 1, 2, \ldots, d)$ is the outer coupling matrix, where $\mathbb{R} \ni F^s_{mn}$ is defined as follows: if there is an edge between the node m and node $n(m\neq n)$, then $\mathbb{R} \ni F^s_{mn} = F^s_{nm} > 0$; otherwise, $\mathbb{R} \ni F^s_{mn} = F^s_{nm} = 0$ $(m \neq n)$; moreover, $F^s_{mm} = -\sum_{\substack{n=1\\ n\neq m}}^{Z} F^s_{mn}$.

The boundary value and initial value for network (1) are given as follows:

$$
\begin{aligned}
&\chi_m(b,t) = 0, && (b,t) \in \partial\Omega \times [0, +\infty),\\
&\chi_m(b,0) = \varphi_m(b), && b \in \Omega,
\end{aligned}
$$

in which $\varphi_m(b)(m = 1, 2, \ldots, z)$ is continuous and bounded on Ω.

In this paper, the network (1) is connected and the function $f_m(\cdot)(m = 1, 2, \ldots, z)$ satisfies the following condition:

$$
|f_m(\theta_1) - f_m(\theta_2)| \leqslant \Upsilon_m|\theta_1 - \theta_2| \tag{2}
$$

for any $\theta_1, \theta_2 \in \mathbb{R}$, where Υ_m is a positive constant. Let $\Upsilon = \text{diag}\,(\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_z) \in \mathbb{R}^{z\times z}$.

2.2. Definitions and Lemma

Definition 2.1 (see [44]). If there exist a non-negative function W and a constant matrix $K \in \mathbb{R}^{\beta\times\zeta}$ such that

$$
\int_{t_0}^{t_r}\int_\Omega y^T(b,t)Ku(b,t)dbdt \geqslant W(t_r) - W(t_0)
$$

for any $t_r, t_0 \in [0, +\infty)$ and $t_r \geq t_0$, the system with input $u(b,t) \in \mathbb{R}^\zeta$ and output $y(b,t) \in \mathbb{R}^\beta$ is passive.

Definition 2.2 (see [44]). If there exist a non-negative function W and two constant matrices $K \in \mathbb{R}^{\beta\times\zeta}$ and $\mathbb{R}^{\zeta\times\zeta} \ni A_1 > 0$ such that

$$
\int_{t_0}^{t_r}\int_\Omega y^T(b,t)Ku(b,t)dbdt \geqslant W(t_r) - W(t_0) + \int_{t_0}^{t_r}\int_\Omega u^T(b,t)A_1u(b,t)dbdt
$$

for any $t_r, t_0 \in [0, +\infty)$ and $t_r \geq t_0$, the system is input-strictly passive.

Definition 2.3 (see [44]). If there exist a non-negative function W and two constant matrices $K \in \mathbb{R}^{\beta\times\zeta}$ and $\mathbb{R}^{\beta\times\beta} \ni A_2 > 0$ such that

$$
\int_{t_0}^{t_r}\int_\Omega y^T(b,t)Ku(b,t)dbdt \geqslant W(t_r) - W(t_0) + \int_{t_0}^{t_r}\int_\Omega y^T(b,t)A_2y(b,t)dbdt
$$

for any $t_r, t_0 \in [0, +\infty)$ and $t_r \geq t_0$, the system is output-strictly passive.

Lemma 2.1 (see [44]). *The following inequality obviously holds:*

$$
\int_\Omega g^2(b)db \leqslant s_\rho^2\int_\Omega\left(\frac{\partial g(b)}{\partial b_\rho}\right)^2 db, \qquad \rho = 1, 2, \ldots, \epsilon,
$$

where $C^1(\Omega)\ni g(b)$ *is a real-valued function and* $g(b)|_{\partial\Omega=0}$.

3. Passivity and synchronization of MDCRDNNs

Defining $\bar{\chi}(b,t)=\frac{1}{Z}\sum_{m=1}^{Z}\chi_m(b,t)$, one gets

$$\begin{aligned}\frac{\partial\bar{\chi}(b,t)}{\partial t}&=\frac{1}{Z}\sum_{m=1}^{Z}H\triangle\chi_m(b,t)-\frac{1}{Z}\sum_{m=1}^{Z}P\chi_m(b,t)+\frac{1}{Z}\sum_{m=1}^{Z}Cf(\chi_m(b,t))+J\\&\quad+\frac{1}{Z}\sum_{m=1}^{Z}Mu_m(b,t)+\frac{1}{Z}\sum_{s=1}^{d}\sum_{n=1}^{Z}a_s\left(\sum_{m=1}^{Z}F_{mn}^{s}\right)\Gamma^s\dot{\chi}_n(b,t)\\&=H\triangle\bar{\chi}(b,t)-P\bar{\chi}(b,t)+\frac{1}{Z}\sum_{m=1}^{Z}Cf(\chi_m(b,t))+J+\frac{1}{Z}\sum_{m=1}^{Z}Mu_m(b,t).\end{aligned}$$

Selecting $e_m(b,t)=\chi_m(b,t)-\bar{\chi}(b,t)$, one obtains

$$\begin{aligned}\frac{\partial e_m(b,t)}{\partial t}&=H\triangle e_m(b,t)-Pe_m(b,t)+Cf(\chi_m(b,t))-\frac{1}{Z}\sum_{l=1}^{Z}Cf(\chi_l(b,t))\\&\quad+Mu_m(b,t)-\frac{1}{Z}\sum_{l=1}^{Z}Mu_l(b,t)+\sum_{s=1}^{d}\sum_{n=1}^{Z}a_sF_{mn}^{s}\Gamma^s\dot{e}_n(b,t).\end{aligned}\tag{3}$$

Choose the following output vector $y_m(b,t)\in\mathbb{R}^{\zeta}$ for the network (3):

$$y_m(b,t)=E_1e_m(b,t)+E_2u_m(b,t),$$

where $E_1\in\mathbb{R}^{\zeta\times z}$ and $E_2\in\mathbb{R}^{\zeta\times\eta}$.

Take

$$\begin{aligned}u(b,t)&=(u_1^T(b,t),u_2^T(b,t),\ldots,u_Z^T(b,t))^T,\\y(b,t)&=(y_1^T(b,t),y_2^T(b,t),\ldots,y_Z^T(b,t))^T,\\e(b,t)&=(e_1^T(b,t),e_2^T(b,t),\ldots,e_Z^T(b,t))^T.\end{aligned}$$

3.1. Passivity and synchronization analysis of MDCRDNNs

3.1.1. Passivity analysis

Theorem 3.1. *If there are matrices $K\in\mathbb{R}^{\zeta Z\times\eta Z}$ and $0<Q=\mathrm{diag}(q_1,q_2,\ldots,q_z)\in\mathbb{R}^{z\times z}$ satisfying*

$$\begin{pmatrix}W_1 & D_1\\ D_1^T & -(I_Z\otimes E_2^T)K-K^T(I_Z\otimes E_2)\end{pmatrix}\leqslant 0,\tag{4}$$

where $W_1=I_Z\otimes(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}QH-2QP+QCC^TQ+\Upsilon^2)$, $D_1=I_Z\otimes(QM)-(I_Z\otimes E_1^T)K$, *then the network (3) is passive.*

Proof. Select a Lyapunov functional for the network (3) as follows:

$$V_1(t)=\sum_{m=1}^{Z}\int_{\Omega}e_m^T(b,t)Qe_m(b,t)db-\sum_{s=1}^{d}a_s\int_{\Omega}e^T(b,t)[F^s\otimes(Q\Gamma^s)]e(b,t)db.$$

Then, we acquire

$$
\begin{aligned}
\dot{V}_1(t) = {} & 2\sum_{m=1}^{Z}\int_{\Omega} e_m^T(b,t)Q\bigg(H\triangle e_m(b,t) - Pe_m(b,t) + Cf(\chi_m(b,t)) \\
& - \frac{1}{Z}\sum_{l=1}^{Z} Cf(\chi_l(b,t)) + Mu_m(b,t) - \frac{1}{Z}\sum_{l=1}^{Z} Mu_l(b,t) + \sum_{s=1}^{d}\sum_{n=1}^{Z} a_s F_{mn}^s \Gamma^s \dot{e}_n(b,t)\bigg)db \\
& - 2\sum_{s=1}^{d} a_s \int_{\Omega} e^T(b,t)[F^s \otimes (Q\Gamma^s)]\dot{e}(b,t)db \\
= {} & 2\sum_{m=1}^{Z}\int_{\Omega} e_m^T(b,t)Q\bigg(H\triangle e_m(b,t) - Pe_m(b,t) + Cf(\chi_m(b,t)) \\
& -\frac{1}{Z}\sum_{l=1}^{Z} Cf(\chi_l(b,t)) + Mu_m(b,t) - \frac{1}{Z}\sum_{l=1}^{Z} Mu_l(b,t)\bigg)db \\
& + 2\sum_{s=1}^{d} a_s \int_{\Omega} e^T(b,t)[F^s \otimes (Q\Gamma^s)]\dot{e}(b,t)db \\
& - 2\sum_{s=1}^{d} a_s \int_{\Omega} e^T(b,t)[F^s \otimes (Q\Gamma^s)]\dot{e}(b,t)db \\
= {} & 2\sum_{m=1}^{Z}\int_{\Omega} e_m^T(b,t)Q\bigg(H\triangle e_m(b,t) - Pe_m(b,t) + Cf(\chi_m(b,t)) - Cf(\bar{\chi}(b,t)) \\
& + Cf(\bar{\chi}(b,t)) - \frac{1}{Z}\sum_{l=1}^{Z} Cf(\chi_l(b,t)) + Mu_m(b,t) - \frac{1}{Z}\sum_{l=1}^{Z} Mu_l(b,t)\bigg)db.
\end{aligned}
$$

Obviously,

$$
\begin{aligned}
\sum_{m=1}^{Z} e_m(b,t) &= \sum_{m=1}^{Z} (\chi_m(b,t) - \bar{\chi}(b,t)) \\
&= \sum_{m=1}^{Z}\bigg(\chi_m(b,t) - \frac{1}{Z}\sum_{n=1}^{Z}\chi_n(b,t)\bigg) \\
&= \sum_{m=1}^{Z}\chi_m(b,t) - \sum_{n=1}^{Z}\chi_n(b,t) \\
&= 0.
\end{aligned}
$$

Then, we can get

$$
\begin{aligned}
& \sum_{m=1}^{Z} e_m^T(b,t)Q\bigg(Cf(\bar{\chi}(b,t)) - \frac{1}{Z}\sum_{l=1}^{Z} Cf(\chi_l(b,t))\bigg) = 0, \\
& \sum_{m=1}^{Z} e_m^T(b,t)Q\bigg(\frac{1}{Z}\sum_{l=1}^{Z} Mu_l(b,t)\bigg) = 0.
\end{aligned} \tag{5}
$$

From Lemma 2.1, one obtains

$$\begin{aligned}
&\int_\Omega e_m^T(b,t)(QH)\Delta e_m(b,t)db \\
&= \sum_{i=1}^{z} q_i h_i \int_\Omega e_{mi}(b,t)\Delta e_{mi}(b,t)db \\
&= -\sum_{i=1}^{z}\sum_{\rho=1}^{\epsilon} q_i h_i \int_\Omega \left(\frac{\partial e_{mi}(b,t)}{\partial b_\rho}\right)^2 db \\
&\leqslant -\sum_{i=1}^{z}\sum_{\rho=1}^{\epsilon} \frac{q_i h_i}{s_\rho^2} \int_\Omega e_{mi}^2(b,t)db \\
&\leqslant -\sum_{\rho=1}^{\epsilon} \frac{1}{s_\rho^2} \int_\Omega e_m^T(b,t)(QH)e_m(b,t)db.
\end{aligned} \tag{6}$$

Furthermore,

$$2e_m^T(b,t)(QC)(f(\chi_m(b,t)) - f(\bar{\chi}(b,t))) \leqslant e_m^T(b,t)(QCC^TQ + \Upsilon^2)e_m(b,t). \tag{7}$$

By Eqs. (5)–(7), one gets

$$\begin{aligned}
\dot{V}_1(t) &\leqslant \sum_{m=1}^{Z}\int_\Omega e_m^T(b,t)\Big(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}QH - 2QP + QCC^TQ + \Upsilon^2\Big)e_m(b,t)db \\
&\quad + 2\sum_{m=1}^{Z}\int_\Omega e_m^T(b,t)QMu_m(b,t)db \\
&= \int_\Omega e^T(b,t)\Big[I_Z\otimes\Big(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}QH - 2QP + QCC^TQ + \Upsilon^2\Big)\Big]e(b,t)db \\
&\quad + 2\int_\Omega e^T(b,t)[I_Z\otimes(QM)]u(b,t)db.
\end{aligned} \tag{8}$$

Therefore,

$$\begin{aligned}
&\dot{V}_1(t) - 2\int_\Omega y^T(b,t)Ku(b,t)db \\
&\leqslant \int_\Omega e^T(b,t)\Big[I_Z\otimes\Big(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}QH - 2QP + QCC^TQ + \Upsilon^2\Big)\Big]e(b,t)db \\
&\quad - 2\int_\Omega [e^T(b,t)(I_Z\otimes E_1^T) + u^T(b,t)(I_Z\otimes E_2^T)]Ku(b,t)db + 2\int_\Omega e^T(b,t)[I_Z\otimes(QM)]u(b,t)db \\
&= \sigma^T(b,t)\begin{pmatrix} W_1 & D_1 \\ D_1^T & W_2 \end{pmatrix}\sigma(b,t),
\end{aligned}$$

where $W_2 = -(I_Z\otimes E_2^T)K - K^T(I_Z\otimes E_2)$, $\sigma(b,t) = (e^T(b,t), u^T(b,t))^T$.

By (4), one gets

$$2\int_\Omega y^T(b,t)Ku(b,t)db \geqslant \dot{V}_1(t).$$

Therefore,

$$\frac{\dot{V}_1(t)}{2} \leqslant \int_{\Omega} y^T(b,t)Ku(b,t)db.$$

Letting $V(t) = V_1(t)/2$, one has

$$\int_{t_0}^{t_r} \int_{\Omega} y^T(b,t)Ku(b,t)dbdt \geqslant V(t_r) - V(t_0)$$

for any $0 \leqslant t_0 \leqslant t_r \in \mathbb{R}$. □

Using the similar proof method as in Theorem 3.1, we can easily obtain the following conclusions.

Theorem 3.2. *If there are matrices* $\mathbb{R}^{\eta Z\times\eta Z} \ni A_1 > 0$, $K \in \mathbb{R}^{\zeta Z\times\eta Z}$ *and* $0 < Q = \operatorname{diag}(q_1, q_2, \ldots, q_z) \in \mathbb{R}^{z\times z}$ *satisfying*

$$\begin{pmatrix} W_1 & D_1 \\ D_1^T & A_1 - (I_Z \otimes E_2^T)K - K^T(I_Z \otimes E_2) \end{pmatrix} \leqslant 0,$$

where $W_1 = I_Z \otimes (-\sum_{\rho=1}^{\epsilon} \frac{2}{s_\rho^2} QH - 2QP + QCC^TQ + \Upsilon^2), D_1 = I_Z \otimes (QM) - (I_Z \otimes E_1^T)K,$ *then the network (3) is input-strictly passive.*

Theorem 3.3. *If there are matrices* $\mathbb{R}^{\zeta Z\times\zeta Z} \ni A_2 > 0$, $K \in \mathbb{R}^{\zeta Z\times\eta Z}$ *and* $0 < Q = \operatorname{diag}(q_1, q_2, \ldots, q_z) \in \mathbb{R}^{z\times z}$ *satisfying*

$$\begin{pmatrix} W_3 & D_2 \\ D_2^T & W_4 \end{pmatrix} \leqslant 0,$$

where $W_3 = I_Z \otimes (-\sum_{\rho=1}^{\epsilon} \frac{2}{s_\rho^2} QH - 2QP + QCC^TQ + \Upsilon^2) + (I_Z \otimes E_1^T)A_2(I_Z \otimes E_1), W_4 = -(I_Z \otimes E_2^T)K - K^T(I_Z \otimes E_2) + (I_Z \otimes E_2^T)A_2(I_Z \otimes E_2), D_2 = I_Z \otimes (QM) - (I_Z \otimes E_1^T)K + (I_Z \otimes E_1^T)A_2(I_Z \otimes E_2),$ *then the network (3) is output-strictly passive.*

3.1.2. Synchronization criterion

Definition 3.1. The network (1) with the input $u_m(b,t) = 0$ is synchronized if

$$\lim_{t\to+\infty} \left\| \chi_m(\cdot,t) - \frac{1}{Z}\sum_{n=1}^{Z} \chi_n(\cdot,t) \right\| = 0, \quad m = 1, 2, \ldots, Z.$$

Theorem 3.4. *The network (1) is synchronized if*

$$-\sum_{\rho=1}^{\epsilon} \frac{2}{s_\rho^2} QH - 2QP + QCC^TQ + \Upsilon^2 < 0. \tag{9}$$

Proof. Take the Lyapunov functional $V_1(t)$ as follows:

$$V_1(t) = \sum_{m=1}^{Z} \int_{\Omega} e_m^T(b,t)Qe_m(b,t)db - \sum_{s=1}^{d} a_s \int_{\Omega} e^T(b,t)[F^s \otimes (Q\Gamma^s)]e(b,t)db.$$

Then, we acquire

$$
\begin{aligned}
\dot{V}_1(t) &= 2\sum_{m=1}^{Z}\int_{\Omega} e_m^T(b,t)Q\bigg(H\triangle e_m(b,t) - Pe_m(b,t) + Cf(\chi_m(b,t)) \\
&\quad - \frac{1}{Z}\sum_{l=1}^{Z}Cf(\chi_l(b,t)) + \sum_{s=1}^{d}\sum_{n=1}^{Z}a_sF_{mn}^s\Gamma^s\dot{e}_n(b,t)\bigg)db \\
&\quad - 2\sum_{s=1}^{d}a_s\int_{\Omega}e^T(b,t)[F^s\otimes(Q\Gamma^s)]\dot{e}(b,t)db \\
&= 2\sum_{m=1}^{Z}\int_{\Omega} e_m^T(b,t)Q\bigg(H\triangle e_m(b,t) - Pe_m(b,t) + Cf(\chi_m(b,t)) - \frac{1}{Z}\sum_{l=1}^{Z}Cf(\chi_l(b,t))\bigg)db \\
&\quad + 2\sum_{s=1}^{d}a_s\int_{\Omega}e^T(b,t)[F^s\otimes(Q\Gamma^s)]\dot{e}(b,t)db - 2\sum_{s=1}^{d}a_s\int_{\Omega}e^T(b,t)[F^s\otimes(Q\Gamma^s)]\dot{e}(b,t)db \\
&= 2\sum_{m=1}^{Z}\int_{\Omega} e_m^T(b,t)Q\bigg(H\triangle e_m(b,t) - Pe_m(b,t) + Cf(\chi_m(b,t)) \\
&\quad - Cf(\bar{\chi}(b,t)) + Cf(\bar{\chi}(b,t)) - \frac{1}{Z}\sum_{l=1}^{Z}Cf(\chi_l(b,t))\bigg)db \\
&\leqslant \sum_{m=1}^{Z}\int_{\Omega} e_m^T(b,t)\bigg(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}QH - 2QP + QCC^TQ + \Upsilon^2\bigg)e_m(b,t)db \\
&\leqslant \gamma_M(W_1)\|e(\cdot,t)\|^2,
\end{aligned}
$$

where $W_1 = -\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}QH - 2QP + QCC^TQ + \Upsilon^2$.

On the basis of the definition of $V_1(t)$, we can get

$$
\delta_1\|e(\cdot,t)\|^2 \leqslant V_1(t) \leqslant \delta_2\|e(\cdot,t)\|^2, \tag{10}
$$

where $\delta_1 = \gamma_m(I_Z\otimes Q - \sum_{s=1}^{d}a_sF^s\otimes(Q\Gamma^s))$, $\delta_2 = \gamma_M(I_Z\otimes Q - \sum_{s=1}^{d}a_sF^s\otimes(Q\Gamma^s))$.

Then, one has

$$
\dot{V}_1(t) \leqslant \frac{\gamma_M(W_1)}{\delta_2}V_1(t). \tag{11}
$$

From Eqs. (10) and (11), we can get

$$
\|e(\cdot,t)\| \leqslant \sqrt{\frac{\delta_2}{\delta_1}}\|e(\cdot,0)\|e^{\frac{\gamma_M(W_1)t}{2\delta_2}}.
$$

Therefore, the network (1) is synchronized. □

3.2. Passivity and synchronization analysis of MDCRDNNs with parameter uncertainties

As everyone knows, the parameter fluctuations in neural networks are unavoidable, and these fluctuations are bounded in many circumstance. In this section, the fluctuation ranges of the parameters H, P and C are defined as follows:

$$\begin{cases} H_I : = \{H = \mathrm{diag}(h_r) : \underline{H} \leqslant H \leqslant \overline{H}, i.e., \underline{h}_r \leqslant h_r \leqslant \overline{h}_r, \\ \qquad r = 1, 2, \dots, z, \forall H \in H_I\}, \\ P_I : = \{P = \mathrm{diag}(p_r) : \underline{P} \leqslant P \leqslant \overline{P}, i.e., \underline{p}_r \leqslant p_r \leqslant \overline{p}_r, \\ \qquad r = 1, 2, \dots, z, \forall P \in P_I\}, \\ C_I : = \{C = (c_{rj})_{z\times z} : \underline{C} \leqslant C \leqslant \overline{C}, i.e., \underline{c}_{rj} \leqslant c_{rj} \leqslant \overline{c}_{rj}, \\ \qquad r, j = 1, 2, \dots, z, \forall C \in C_I\}. \end{cases} \tag{12}$$

Define

$$\hat{c}_{rj} = \max\{|\underline{c}_{rj}|, |\overline{c}_{rj}|\},$$
$$\phi = \sum_{r=1}^{z}\sum_{j=1}^{z}\hat{c}_{rj}^2.$$

3.2.1. Passivity analysis

Theorem 3.5. *If there are two matrices $K \in \mathbb{R}^{\zeta Z\times \eta Z}$ and $0 < Q = \mathrm{diag}(q_1, q_2, \dots, q_z) \in \mathbb{R}^{z\times z}$ satisfying*

$$\begin{pmatrix} W_5 & D_1 \\ D_1^T & -(I_Z \otimes E_2^T)K - K^T(I_Z \otimes E_2) \end{pmatrix} \leqslant 0, \tag{13}$$

where $W_5 = I_Z \otimes (-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}Q\underline{H} - 2Q\underline{P} + \phi Q^2 + \Upsilon^2)$, $D_1 = I_Z \otimes (QM) - (I_Z \otimes E_1^T)K$, then the network (3) with the parameter ranges defined by (12) is passive.

Proof. Select a Lyapunov functional for the network (3) as follows:

$$V_1(t) = \sum_{m=1}^{Z}\int_\Omega e_m^T(b,t)Qe_m(b,t)db - \sum_{s=1}^{d}a_s\int_\Omega e^T(b,t)[F^s\otimes(Q\Gamma^s)]e(b,t)db.$$

Then, we acquire

$$\begin{aligned} \dot{V}_1(t) &= 2\sum_{m=1}^{Z}\int_\Omega e_m^T(b,t)Q\Big(H\triangle e_m(b,t) - Pe_m(b,t) + Cf(\chi_m(b,t)) \\ &\quad - \frac{1}{Z}\sum_{l=1}^{Z}Cf(\chi_l(b,t)) + Mu_m(b,t) - \frac{1}{Z}\sum_{l=1}^{Z}Mu_l(b,t) + \sum_{s=1}^{d}\sum_{n=1}^{Z}a_sF_{mn}^s\Gamma^s\dot{e}_n(b,t)\Big)db \\ &\quad - 2\sum_{s=1}^{d}a_s\int_\Omega e^T(b,t)[F^s\otimes(Q\Gamma^s)]\dot{e}(b,t)db \\ &= 2\sum_{m=1}^{Z}\int_\Omega e_m^T(b,t)Q\Big(H\triangle e_m(b,t) - Pe_m(b,t) + Cf(\chi_m(b,t)) \\ &\quad - \frac{1}{Z}\sum_{l=1}^{Z}Cf(\chi_l(b,t)) + Mu_m(b,t) - \frac{1}{Z}\sum_{l=1}^{Z}Mu_l(b,t)\Big)db \\ &\quad + 2\sum_{s=1}^{d}a_s\int_\Omega e^T(b,t)[F^s\otimes(Q\Gamma^s)]\dot{e}(b,t)db - 2\sum_{s=1}^{d}a_s\int_\Omega e^T(b,t)[F^s\otimes(Q\Gamma^s)]\dot{e}(b,t)db \end{aligned}$$

$$= 2\sum_{m=1}^{Z}\int_{\Omega} e_m^T(b,t)Q\bigg(H\triangle e_m(b,t) - Pe_m(b,t) + Cf(\chi_m(b,t))$$
$$- Cf(\bar{\chi}(b,t)) + Cf(\bar{\chi}(b,t)) - \frac{1}{Z}\sum_{l=1}^{Z} Cf(\chi_l(b,t)) + Mu_m(b,t) - \frac{1}{Z}\sum_{l=1}^{Z} Mu_l(b,t)\bigg)db$$
$$\leqslant \int_{\Omega} e^T(b,t)\bigg[I_Z\otimes\bigg(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}QH - 2QP + QCC^TQ + \Upsilon^2\bigg)\bigg]e(b,t)db$$
$$+ 2\int_{\Omega} e^T(b,t)[I_Z\otimes(QM)]u(b,t)db$$
$$\leqslant \int_{\Omega} e^T(b,t)\bigg[I_Z\otimes\bigg(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}Q\underline{H} - 2Q\underline{P} + \phi Q^2 + \Upsilon^2\bigg)\bigg]e(b,t)db$$
$$+ 2\int_{\Omega} e^T(b,t)[I_Z\otimes(QM)]u(b,t)db. \quad (14)$$

Therefore,

$$\dot{V}_1(t) - 2\int_{\Omega} y^T(b,t)Ku(b,t)db$$
$$\leqslant \int_{\Omega} e^T(b,t)\bigg[I_Z\otimes\bigg(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}Q\underline{H} - 2Q\underline{P} + \phi Q^2 + \Upsilon^2\bigg)\bigg]e(b,t)db$$
$$- 2\int_{\Omega}[e^T(b,t)(I_Z\otimes E_1^T) + u^T(b,t)(I_Z\otimes E_2^T)]Ku(b,t)db + 2\int_{\Omega} e^T(b,t)[I_Z\otimes(QM)]u(b,t)db$$
$$= \sigma^T(b,t)\begin{pmatrix} W_5 & D_1 \\ D_1^T & W_2\end{pmatrix}\sigma(b,t),$$

where $W_2 = -(I_Z\otimes E_2^T)K - K^T(I_Z\otimes E_2)$, $\sigma(b,t) = (e^T(b,t), u^T(b,t))^T$.

By Eq. (13), one gets

$$2\int_{\Omega} y^T(b,t)Ku(b,t)db \geqslant \dot{V}_1(t).$$

Therefore,

$$\frac{\dot{V}_1(t)}{2} \leqslant \int_{\Omega} y^T(b,t)Ku(b,t)db.$$

Letting $V(t) = V_1(t)/2$, one has

$$\int_{t_0}^{t_r}\int_{\Omega} y^T(b,t)Ku(b,t)dbdt \geqslant V(t_r) - V(t_0)$$

for any $0 \leqslant t_0 \leqslant t_r \in \mathbb{R}$. □

Using the similar proof method as in Theorem 3.5, we can easily obtain the following conclusions.

Theorem 3.6. *If there are matrices* $\mathbb{R}^{\eta Z\times\eta Z} \ni A_1 > 0$, $K \in \mathbb{R}^{\zeta Z\times\eta Z}$ *and* $0 < Q = \mathrm{diag}(q_1, q_2, \ldots, q_z) \in \mathbb{R}^{z\times z}$ *satisfying*

$$\begin{pmatrix} W_5 & D_1 \\ D_1^T & A_1 - (I_Z\otimes E_2^T)K - K^T(I_Z\otimes E_2)\end{pmatrix} \leqslant 0,$$

where $W_5 = I_Z \otimes (-\sum_{\rho=1}^{\epsilon} \frac{2}{s_\rho^2} Q\underline{H} - 2Q\underline{P} + \phi Q^2 + \Upsilon^2), D_1 = I_Z \otimes (QM) - (I_Z \otimes E_1^T)K$, *then the network (3) with the parameter ranges defined by Eq. (12) is input-strictly passive.*

Theorem 3.7. *If there are matrices* $\mathbb{R}^{\zeta Z \times \zeta Z} \ni A_2 > 0$, $K \in \mathbb{R}^{\zeta Z \times \eta Z}$ *and* $0 < Q = \mathrm{diag}(q_1, q_2, \ldots, q_z) \in \mathbb{R}^{z \times z}$ *satisfying*

$$\begin{pmatrix} W_6 & D_2 \\ D_2^T & W_4 \end{pmatrix} \leqslant 0,$$

where $W_6 = I_Z \otimes (-\sum_{\rho=1}^{\epsilon} \frac{2}{s_\rho^2} Q\underline{H} - 2Q\underline{P} + \phi Q^2 + \Upsilon^2) + (I_Z \otimes E_1^T)A_2(I_Z \otimes E_1), W_4 = -(I_Z \otimes E_2^T)K - K^T(I_Z \otimes E_2) + (I_Z \otimes E_2^T)A_2(I_Z \otimes E_2), D_2 = I_Z \otimes (QM) - (I_Z \otimes E_1^T)K + (I_Z \otimes E_1^T) A_2(I_Z \otimes E_2)$, *then the network (3) with the parameter ranges defined by Eq. (12) is output-strictly passive.*

3.2.2. Synchronization criterion

Theorem 3.8. *The network (1) with the parameter ranges defined by Eq. (12) is synchronized if*

$$-\sum_{\rho=1}^{\epsilon} \frac{2}{s_\rho^2} Q\underline{H} - 2Q\underline{P} + \phi Q^2 + \Upsilon^2 < 0.$$

Proof. Take the Lyapunov functional $V_1(t)$ as follows:

$$V_1(t) = \sum_{m=1}^{Z} \int_\Omega e_m^T(b,t) Q e_m(b,t) db - \sum_{s=1}^{d} a_s \int_\Omega e^T(b,t)[F^s \otimes (Q\Gamma^s)]e(b,t)db.$$

Then, we acquire

$$\begin{aligned}
\dot{V}_1(t) = {} & 2\sum_{m=1}^{Z} \int_\Omega e_m^T(b,t) Q \Big(H\triangle e_m(b,t) - Pe_m(b,t) + Cf(\chi_m(b,t)) \\
& - \frac{1}{Z}\sum_{l=1}^{Z} Cf(\chi_l(b,t)) + \sum_{s=1}^{d}\sum_{n=1}^{Z} a_s F_{mn}^s \Gamma^s \dot{e}_n(b,t) \Big) db \\
& - 2\sum_{s=1}^{d} a_s \int_\Omega e^T(b,t)[F^s \otimes (Q\Gamma^s)]\dot{e}(b,t)db \\
= {} & 2\sum_{m=1}^{Z} \int_\Omega e_m^T(b,t) Q \Big(H\triangle e_m(b,t) - Pe_m(b,t) + Cf(\chi_m(b,t)) - \frac{1}{Z}\sum_{l=1}^{Z} Cf(\chi_l(b,t)) \Big) db \\
& + 2\sum_{s=1}^{d} a_s \int_\Omega e^T(b,t)[F^s \otimes (Q\Gamma^s)]\dot{e}(b,t)db - 2\sum_{s=1}^{d} a_s \int_\Omega e^T(b,t)[F^s \otimes (Q\Gamma^s)]\dot{e}(b,t)db \\
= {} & 2\sum_{m=1}^{Z} \int_\Omega e_m^T(b,t) Q \Big(H\triangle e_m(b,t) - Pe_m(b,t) + Cf(\chi_m(b,t)) \\
& - Cf(\bar{\chi}(b,t)) + Cf(\bar{\chi}(b,t)) - \frac{1}{Z}\sum_{l=1}^{Z} Cf(\chi_l(b,t)) \Big) db
\end{aligned}$$

$$\leqslant \sum_{m=1}^{Z}\int_{\Omega} e_m^T(b,t)\bigg(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}QH-2QP+QCC^TQ+\Upsilon^2\bigg)e_m(b,t)db$$
$$\leqslant \sum_{m=1}^{Z}\int_{\Omega} e_m^T(b,t)\bigg(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}Q\underline{H}-2Q\underline{P}+\phi Q^2+\Upsilon^2\bigg)e_m(b,t)db$$
$$\leqslant \gamma_M(W_5)\|e(\cdot,t)\|^2,$$

where $W_5=-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}Q\underline{H}-2Q\underline{P}+\phi Q^2+\Upsilon^2$.

On the basis of the definition of $V_1(t)$, we can get

$$\delta_1\|e(\cdot,t)\|^2\leqslant V_1(t)\leqslant \delta_2\|e(\cdot,t)\|^2, \tag{15}$$

where $\delta_1=\gamma_m(I_Z\otimes Q-\sum_{s=1}^{d}a_sF^s\otimes(Q\Gamma^s))$, $\delta_2=\gamma_M(I_Z\otimes Q-\sum_{s=1}^{d}a_sF^s\otimes(Q\Gamma^s))$.

Then, one has

$$\dot{V}_1(t)\leqslant \frac{\gamma_M(W_5)}{\delta_2}V_1(t). \tag{16}$$

From Eqs. (15) and (16), we can get

$$\|e(\cdot,t)\|\leqslant\sqrt{\frac{\delta_2}{\delta_1}}\|e(\cdot,0)\|e^{\frac{\gamma_M(W_5)t}{2\delta_2}}.$$

Therefore, the network (1) with the parameter ranges defined by Eq. (12) is synchronized. □

4. Pinning passivity and synchronization of MDCRDNNs

4.1. Network model

The network (1) under the pinning adaptive controller is described as follows:

$$\begin{aligned}\frac{\partial\chi_m(b,t)}{\partial t}&=H\triangle\chi_m(b,t)-P\chi_m(b,t)+Cf(\chi_m(b,t))+J+Mu_m(b,t)\\&\quad+\sum_{s=1}^{d}\sum_{n=1}^{Z}a_sF_{mn}^s\Gamma^s\dot{\chi}_n(b,t)+v_m(b,t),\ m=1,2,\ldots,\alpha,\\\frac{\partial\chi_m(b,t)}{\partial t}&=H\triangle\chi_m(b,t)-P\chi_m(b,t)+Cf(\chi_m(b,t))+J+Mu_m(b,t)\\&\quad+\sum_{s=1}^{d}\sum_{n=1}^{Z}a_sF_{mn}^s\Gamma^s\dot{\chi}_n(b,t),\ m=\alpha+1,\alpha+2,\ldots,Z,\end{aligned} \tag{17}$$

where

$$\begin{aligned}v_m(b,t)&=-\sum_{s=1}^{d}a_sw_m^s(t)\Gamma^s\bigg(\chi_m(b,t)-\frac{1}{Z}\sum_{m=1}^{Z}\chi_m(b,t)\bigg),\\\dot{w}_m^s(t)&=\beta_m^s\int_{\Omega}\bigg(\chi_m(b,t)-\frac{1}{Z}\sum_{m=1}^{Z}\chi_m(b,t)\bigg)^TQ\Gamma^s\bigg(\chi_m(b,t)-\frac{1}{Z}\sum_{m=1}^{Z}\chi_m(b,t)\bigg)db,\end{aligned} \tag{18}$$

in which $1 \leqslant \alpha < Z, \mathbb{R} \ni \beta_m^s > 0, \mathbb{R} \ni w_m^s(0) > 0,\ 0 < Q = \mathrm{diag}(q_1, q_2, \ldots, q_z) \in \mathbb{R}^{z\times z},\ \mathbb{R} \ni w_m^s(t) \equiv 0, m = \alpha+1, \alpha+2, \ldots, Z$.

Defining $\bar{\chi}(b,t) = \frac{1}{Z}\sum_{m=1}^{Z}\chi_m(b,t)$. Then, one has

$$\begin{aligned}\frac{\partial\bar{\chi}(b,t)}{\partial t} &= \frac{1}{Z}\sum_{m=1}^{Z}H\triangle\chi_m(b,t) - \frac{1}{Z}\sum_{m=1}^{Z}P\chi_m(b,t) + \frac{1}{Z}\sum_{m=1}^{Z}Cf(\chi_m(b,t)) + J\\ &\quad + \frac{1}{Z}\sum_{m=1}^{Z}Mu_m(b,t) + \frac{1}{Z}\sum_{s=1}^{d}\sum_{n=1}^{Z}a_s\left(\sum_{m=1}^{Z}F_{mn}^s\right)\Gamma^s\dot{\chi}_n(b,t) + \frac{1}{Z}\sum_{m=1}^{\alpha}v_m(b,t)\\ &= H\triangle\bar{\chi}(b,t) - P\bar{\chi}(b,t) + \frac{1}{Z}\sum_{m=1}^{Z}Cf(\chi_m(b,t)) + J + \frac{1}{Z}\sum_{m=1}^{Z}Mu_m(b,t) + \frac{1}{Z}\sum_{m=1}^{\alpha}v_m(b,t).\end{aligned}$$

Letting $e_m(b,t) = \chi_m(b,t) - \bar{\chi}(b,t)$, we have

$$\begin{aligned}\frac{\partial e_m(b,t)}{\partial t} &= H\triangle e_m(b,t) - Pe_m(b,t) + Cf(\chi_m(b,t)) - \frac{1}{Z}\sum_{l=1}^{Z}Cf(\chi_l(b,t))\\ &\quad + Mu_m(b,t) - \frac{1}{Z}\sum_{l=1}^{Z}Mu_l(b,t) + \sum_{s=1}^{d}\sum_{n=1}^{Z}a_sF_{mn}^s\Gamma^s\dot{e}_n(b,t)\\ &\quad + v_m(b,t) - \frac{1}{Z}\sum_{m=1}^{\alpha}v_m(b,t), m = 1, 2, \ldots, Z.\end{aligned} \tag{19}$$

Choose the following output vector $y_m(b,t) \in \mathbb{R}^{\zeta}$ for the network (19):

$$y_m(b,t) = E_1e_m(b,t) + E_2u_m(b,t),$$

where $E_1 \in \mathbb{R}^{\zeta\times z}$ and $E_2 \in \mathbb{R}^{\zeta\times\eta}$.

Take

$$\begin{aligned}u(b,t) &= (u_1^T(b,t), u_2^T(b,t), \ldots, u_Z^T(b,t))^T,\\ y(b,t) &= (y_1^T(b,t), y_2^T(b,t), \ldots, y_Z^T(b,t))^T,\\ e(b,t) &= (e_1^T(b,t), e_2^T(b,t), \ldots, e_Z^T(b,t))^T.\end{aligned}$$

4.2. Pinning passivity and synchronization of MDCRDNNs

4.2.1. Passivity criteria

Theorem 4.1. *If there are matrices $K \in \mathbb{R}^{\zeta Z\times\eta Z}$ and $0 < \hat{w}^s = \mathrm{diag}(\hat{w}_1^s, \hat{w}_2^s, \ldots, \hat{w}_\alpha^s, 0, \ldots, 0) \in \mathbb{R}^{Z\times Z}$ satisfying*

$$\begin{pmatrix}\hat{W}_1 & D_1\\ D_1^T & -(I_Z\otimes E_2^T)K - K^T(I_Z\otimes E_2)\end{pmatrix} \leqslant 0, \tag{20}$$

where $\hat{W}_1 = I_Z\otimes(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}QH - 2QP + QCC^TQ + \Upsilon^2) - 2\sum_{s=1}^{d}a_s\hat{w}^s\otimes(Q\Gamma^s), D_1 = I_Z\otimes(QM) - (I_Z\otimes E_1^T)K$, then the network (19) under the pinning adaptive controller (18) is passive.

Proof. Select a Lyapunov functional for the network (19) as follows:

$$V_2(t) = \sum_{m=1}^{Z} \int_\Omega e_m^T(b,t) Q e_m(b,t) - \sum_{s=1}^{d} a_s \int_\Omega e^T(b,t)[F^s \otimes (Q\Gamma^s)] e(b,t) db + \sum_{s=1}^{d} \sum_{m=1}^{\alpha} \frac{a_s}{\beta_m^s} (w_m^s(t) - \hat{w}_m^s)^2. \tag{21}$$

Then, we acquire

$$\begin{aligned}
\dot{V}_2(t) = {} & 2\sum_{m=1}^{Z} \int_\Omega e_m^T(b,t) Q \Big(H\triangle e_m(b,t) - P e_m(b,t) + Cf(\chi_m(b,t)) \\
& - \frac{1}{Z}\sum_{l=1}^{Z} Cf(\chi_l(b,t)) + Mu_m(b,t) - \frac{1}{Z}\sum_{l=1}^{Z} Mu_l(b,t) \\
& + \sum_{s=1}^{d}\sum_{n=1}^{Z} a_s F_{mn}^s \Gamma^s \dot{e}_n(b,t) - \sum_{s=1}^{d} a_s w_m^s(t) \Gamma^s e_m(b,t) - \frac{1}{Z}\sum_{m=1}^{\alpha} v_m(b,t) \Big) db \\
& - 2\sum_{s=1}^{d} a_s \int_\Omega e^T(b,t)[F^s \otimes (Q\Gamma^s)] \dot{e}(b,t) db + 2\sum_{s=1}^{d}\sum_{m=1}^{\alpha} \frac{a_s}{\beta_m^s} (w_m^s(t) - \hat{w}_m^s) \dot{w}_m^s(t) \\
= {} & 2\sum_{m=1}^{Z} \int_\Omega e_m^T(b,t) Q \Big(H\triangle e_m(b,t) - P e_m(b,t) + Cf(\chi_m(b,t)) \\
& - Cf(\bar{\chi}(b,t)) + Cf(\bar{\chi}(b,t)) - \frac{1}{Z}\sum_{l=1}^{Z} Cf(\chi_l(b,t)) \\
& + Mu_m(b,t) - \frac{1}{Z}\sum_{l=1}^{Z} Mu_l(b,t) - \frac{1}{Z}\sum_{m=1}^{\alpha} v_m(b,t) \Big) db \\
& + 2\sum_{s=1}^{d} a_s \int_\Omega e^T(b,t)[F^s \otimes (Q\Gamma^s)] \dot{e}(b,t) db \\
& - 2\sum_{s=1}^{d}\sum_{m=1}^{Z} a_s w_m^s(t) \int_\Omega e_m^T(b,t) Q\Gamma^s e_m(b,t) db \\
& - 2\sum_{s=1}^{d} a_s \int_\Omega e^T(b,t)[F^s \otimes (Q\Gamma^s)] \dot{e}(b,t) db \\
& + 2\sum_{s=1}^{d}\sum_{m=1}^{\alpha} a_s (w_m^s(t) - \hat{w}_m^s) \int_\Omega e_m^T(b,t) Q\Gamma^s e_m(b,t) db \\
= {} & 2\sum_{m=1}^{Z} \int_\Omega e_m^T(b,t) Q \Big(H\triangle e_m(b,t) - P e_m(b,t) + Cf(\chi_m(b,t)) \\
& - Cf(\bar{\chi}(b,t)) + Cf(\bar{\chi}(b,t)) - \frac{1}{Z}\sum_{l=1}^{Z} Cf(\chi_l(b,t))
\end{aligned}$$

$$
\begin{aligned}
&+ Mu_m(b,t) - \frac{1}{Z}\sum_{l=1}^{Z} Mu_l(b,t) - \frac{1}{Z}\sum_{m=1}^{\alpha} v_m(b,t)\Big)db \\
&- 2\sum_{s=1}^{d}\sum_{m=1}^{\alpha} a_s \hat{w}_m^s \int_\Omega e_m^T(b,t)Q\Gamma^s e_m(b,t)db \\
\leqslant &\int_\Omega e^T(b,t)\Big[I_Z\otimes\Big(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}QH - 2QP + QCC^TQ + \Upsilon^2\Big)\Big]e(b,t)db \\
&+ 2\int_\Omega e^T(b,t)[I_Z\otimes(QM)]u(b,t)db - 2\sum_{s=1}^{d} a_s\int_\Omega e^T(b,t)[\hat{w}^s\otimes(Q\Gamma^s)]e(b,t)db.
\end{aligned}
\tag{22}
$$

where $\mathbb{R}\ni\hat{w}_m^s = 0, m = \alpha+1, \alpha+2, \ldots, Z$.

Therefore,

$$
\begin{aligned}
&\dot{V}_2(t) - 2\int_\Omega y^T(b,t)Ku(b,t)db \\
&\leqslant \int_\Omega e^T(b,t)\Big[I_Z\otimes\Big(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}QH - 2QP + QCC^TQ + \Upsilon^2\Big) \\
&\quad - 2\sum_{s=1}^{d} a_s\hat{w}^s\otimes(Q\Gamma^s)\Big]e(b,t)db + 2\int_\Omega e^T(b,t)[I_Z\otimes(QM)]u(b,t)db \\
&\quad - 2\int_\Omega [e^T(b,t)(I_Z\otimes E_1^T) + u^T(b,t)(I_Z\otimes E_2^T)]Ku(b,t)db \\
&= \sigma^T(b,t)\begin{pmatrix}\hat{W}_1 & D_1 \\ D_1^T & W_2\end{pmatrix}\sigma(b,t),
\end{aligned}
$$

where $W_2 = -(I_Z\otimes E_2^T)K - K^T(I_Z\otimes E_2), \sigma(b,t) = (e^T(b,t), u^T(b,t))^T$.

By Eq. (20), one gets

$$
2\int_\Omega y^T(b,t)Ku(b,t)db \geqslant \dot{V}_2(t).
$$

Therefore,

$$
\frac{\dot{V}_2(t)}{2} \leqslant \int_\Omega y^T(b,t)Ku(b,t)db.
$$

Letting $\hat{V}(t) = V_2(t)/2$, one has

$$
\int_{t_0}^{t_r}\int_\Omega y^T(b,t)Ku(b,t)dbdt \geqslant \hat{V}(t_r) - \hat{V}(t_0)
$$

for any $0\leqslant t_0\leqslant t_r\in\mathbb{R}$. □

Using the similar proof method as in Theorem 4.1, we can easily obtain the following conclusions.

Theorem 4.2. *If there are matrices* $\mathbb{R}^{\eta Z\times\eta Z}\ni A_1>0, K\in\mathbb{R}^{\zeta Z\times\eta Z}$ *and* $0<\hat{w}^s=\mathrm{diag}(\hat{w}_1^s,\hat{w}_2^s,\dots,\hat{w}_\alpha^s,0,\dots,0)\in\mathbb{R}^{Z\times Z}$ *satisfying*

$$\begin{pmatrix}\hat{W}_1 & D_1\\ D_1^T & A_1-(I_Z\otimes E_2^T)K-K^T(I_Z\otimes E_2)\end{pmatrix}\leqslant 0,$$

where $\hat{W}_1=I_Z\otimes(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}QH-2QP+QCC^TQ+\Upsilon^2)-2\sum_{s=1}^{d}a_s\hat{w}^s\otimes(Q\Gamma^s), D_1=I_Z\otimes(QM)-(I_Z\otimes E_1^T)K$, *then the network (19) under the pinning adaptive controller (18) is input-strictly passive.*

Theorem 4.3. *If there are matrices* $\mathbb{R}^{\zeta Z\times\zeta Z}\ni A_2>0, K\in\mathbb{R}^{\zeta Z\times\eta Z}$ *and* $0<\hat{w}^s=\mathrm{diag}(\hat{w}_1^s,\hat{w}_2^s,\dots,\hat{w}_\alpha^s,0,\dots,0)\in\mathbb{R}^{Z\times Z}$ *satisfying*

$$\begin{pmatrix}\hat{W}_3 & D_2\\ D_2^T & W_4\end{pmatrix}\leqslant 0,$$

where $\hat{W}_3=I_Z\otimes(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}QH-2QP+QCC^TQ+\Upsilon^2)-2\sum_{s=1}^{d}a_s\hat{w}^s\otimes(Q\Gamma^s)+(I_Z\otimes E_1^T)A_2(I_Z\otimes E_1), W_4=-(I_Z\otimes E_2^T)K-K^T(I_Z\otimes E_2)+(I_Z\otimes E_2^T)A_2(I_Z\otimes E_2), \hat{D}_2=I_Z\otimes(QM)-(I_Z\otimes E_1^T)K+(I_Z\otimes E_1^T)A_2(I_Z\otimes E_2)$, *then the network (19) under the pinning adaptive controller (18) is output-strictly passive.*

4.2.2. Synchronization criteria

Theorem 4.4. *If there exists a matrix* $0<\hat{w}^s=\mathrm{diag}(\hat{w}_1^s,\hat{w}_2^s,\dots,\hat{w}_\alpha^s,0,\dots,0)\in\mathbb{R}^{Z\times Z}, s=1,\dots,d$, *such that*

$$W_1-2\sum_{s=1}^{d}a_s\hat{w}^s\otimes(Q\Gamma^s)<0,$$

where $W_1=I_Z\otimes(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}QH-2QP+QCC^TQ+\Upsilon^2)$, *the network (17) under the pinning adaptive controller (18) is synchronized.*

Proof. Take the Lyapunov functional $V_2(t)$ as follows:

$$\begin{aligned}V_2(t)=&\sum_{m=1}^{Z}\int_\Omega e_m^T(b,t)Qe_m(b,t)-\sum_{s=1}^{d}a_s\int_\Omega e^T(b,t)[F^s\otimes(Q\Gamma^s)]e(b,t)db\\&+\sum_{s=1}^{d}\sum_{m=1}^{Z}\frac{a_s}{\beta_m^s}(w_m^s(t)-\hat{w}_m^s)^2.\end{aligned}$$

Then, we acquire

$$\begin{aligned}\dot{V}_2(t)=&2\sum_{m=1}^{Z}\int_\Omega e_m^T(b,t)Q\bigg(H\triangle e_m(b,t)-Pe_m(b,t)+Cf(\chi_m(b,t))\\&-\frac{1}{Z}\sum_{l=1}^{Z}Cf(\chi_l(b,t))+\sum_{s=1}^{d}\sum_{n=1}^{Z}a_sF_{mn}^s\Gamma^s\dot{e}_n(b,t)\\&-\sum_{s=1}^{d}a_sw_m^s(t)\Gamma^se_m(b,t)-\frac{1}{Z}\sum_{m=1}^{\alpha}v_m(b,t)\bigg)db-2\sum_{s=1}^{d}a_s\int_\Omega e^T(b,t)[F^s\otimes(Q\Gamma^s)]\dot{e}(b,t)db\end{aligned}$$

$$
\begin{aligned}
&+2\sum_{s=1}^{d}\sum_{m=1}^{\alpha}\frac{a_s}{\beta_m^s}(w_m^s(t)-\hat{w}_m^s)\dot{w}_m^s(t)\\
&=2\sum_{m=1}^{Z}\int_{\Omega}e_m^T(b,t)Q\Big(H\triangle e_m(b,t)-Pe_m(b,t)+Cf(\chi_m(b,t))\\
&-Cf(\bar{\chi}(b,t))+Cf(\bar{\chi}(b,t))-\frac{1}{Z}\sum_{l=1}^{Z}Cf(\chi_l(b,t))-\frac{1}{Z}\sum_{m=1}^{\alpha}v_m(b,t)\Big)db\\
&+2\sum_{s=1}^{d}a_s\int_{\Omega}e^T(b,t)[F^s\otimes(Q\Gamma^s)]\dot{e}(b,t)db\\
&-2\sum_{s=1}^{d}\sum_{m=1}^{Z}a_sw_m^s(t)\int_{\Omega}e_m^T(b,t)Q\Gamma^se_m(b,t)db-2\sum_{s=1}^{d}a_s\int_{\Omega}e^T(b,t)[F^s\otimes(Q\Gamma^s)]\dot{e}(b,t)db\\
&+2\sum_{s=1}^{d}\sum_{m=1}^{\alpha}a_s(w_m^s(t)-\hat{w}_m^s)\int_{\Omega}e_m^T(b,t)Q\Gamma^se_m(b,t)db\\
&=2\sum_{m=1}^{Z}\int_{\Omega}e_m^T(b,t)Q\Big(H\triangle e_m(b,t)-Pe_m(b,t)+Cf(\chi_m(b,t))\\
&-Cf(\bar{\chi}(b,t))+Cf(\bar{\chi}(b,t))-\frac{1}{Z}\sum_{l=1}^{Z}Cf(\chi_l(b,t))-\frac{1}{Z}\sum_{m=1}^{\alpha}v_m(b,t)\Big)db\\
&-2\sum_{s=1}^{d}\sum_{m=1}^{\alpha}a_s\hat{w}_m^s\int_{\Omega}e_m^T(b,t)Q\Gamma^se_m(b,t)db\\
&\leqslant\int_{\Omega}e^T(b,t)\Big[I_Z\otimes\Big(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}QH-2QP+QCC^TQ+\Upsilon^2\Big)-2\sum_{s=1}^{d}a_s\hat{w}^s\otimes(Q\Gamma^s)\Big]e(b,t)db\\
&\leqslant\delta_3\|e(\cdot,t)\|^2,
\end{aligned}
\tag{23}
$$

where $\delta_3=\gamma_M(I_Z\otimes(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}QH-2QP+QCC^TQ+\Upsilon^2)-2\sum_{s=1}^{d}a_s\hat{w}^s\otimes(Q\Gamma^s))$.

Based on the definition of $V_2(t)$ and Eq. (23), we have $V_2(t)$ is bounded and non-increasing. Therefore, $\lim_{t\to+\infty}V_2(t)$ exists and $w_m^s(t)$ is bounded. Because $w_m^s(t)$ is monotonically increasing [see Eq. (18)], it is easy to infer that $w_m^s(t)$ asymptotically converges to a finite positive value. After that, we get that $\lim_{t\to+\infty}\int_{\Omega}e^T(b,t)[I_Z\otimes Q-\sum_{s=1}^{d}a_sF^s\otimes(Q\Gamma^s)]e(b,t)db$ exists and is a nonnegative real number. Suppose that

$$
\lim_{t\to+\infty}\int_{\Omega}e^T(b,t)\Big[I_Z\otimes Q-\sum_{s=1}^{d}a_sF^s\otimes(Q\Gamma^s)\Big]e(b,t)db=\wp>0.
$$

Therefore, there obviously exists $0<\Psi\in\mathbb{R}$ such that

$$
\int_{\Omega}e^T(b,t)\Big[I_Z\otimes Q-\sum_{s=1}^{d}a_sF^s\otimes(Q\Gamma^s)\Big]e(b,t)db>\frac{\wp}{2}
$$

for $t\geq\Psi$.

From Eq. (23), one gets

$$\dot{V}_2(t) < \frac{\delta_3\wp}{2\delta_4}, \quad t \geqslant \Psi, \tag{24}$$

where $\delta_4 = \gamma_M\Big(I_Z \otimes Q - \sum_{s=1}^{d} a_s F^s \otimes (Q\Gamma^s)\Big)$.

In light of Eq. (24), we can obtain

$$\begin{aligned} -V_2(\Psi) &\leqslant V_2(+\infty) - V_2(\Psi) \\ &= \int_{\Psi}^{+\infty} \dot{V}_2(t)dt \\ &\leqslant \int_{\Psi}^{+\infty} \frac{\delta_3\wp}{2\delta_4}dt \\ &= -\infty, \end{aligned}$$

which leads to an error. Therefore, we could derive that

$$\lim_{t\to+\infty} \int_{\Omega} e^T(b,t)\left[I_Z \otimes Q - \sum_{s=1}^{d} a_s F^s \otimes (Q\Gamma^s)\right]e(b,t)db = 0.$$

Evidently,

$$\lim_{t\to+\infty} \|e(\cdot,t)\| = 0.$$

Therefore, the network (17) under the pinning adaptive controller (18) is synchronized. □

4.3. Pinning passivity and synchronization of MDCRDNNs with parameter uncertainties

In this section, the fluctuation ranges of the parameters H, P and C are defined as follows:

$$\begin{cases} H_I := \{H = \mathrm{diag}(h_r) : \underline{H} \leqslant H \leqslant \overline{H}, i.e., \underline{h}_r \leqslant h_r \leqslant \overline{h}_r, \\ \qquad r = 1, 2, \dots, z, \forall H \in H_I\}, \\ P_I := \{P = \mathrm{diag}(p_r) : \underline{P} \leqslant P \leqslant \overline{P}, i.e., \underline{p}_r \leqslant p_r \leqslant \overline{p}_r, \\ \qquad r = 1, 2, \dots, z, \forall P \in P_I\}, \\ C_I := \{C = (c_{rj})_{z\times z} : \underline{C} \leqslant C \leqslant \overline{C}, i.e., \underline{c}_{rj} \leqslant c_{rj} \leqslant \overline{c}_{rj}, \\ \qquad r, j = 1, 2, \dots, z, \forall C \in C_I\}, \end{cases} \tag{25}$$

Define

$$\hat{c}_{rj} = \max\{|\underline{c}_{rj}|, |\overline{c}_{rj}|\},$$
$$\phi = \sum_{r=1}^{Z}\sum_{j=1}^{Z} \hat{c}_{rj}^2.$$

4.3.1. Passivity criteria

Theorem 4.5. *If there exist matrices $K \in \mathbb{R}^{\zeta Z\times\eta Z}$ and $0 < \hat{w}^s = \mathrm{diag}(\hat{w}_1^s, \hat{w}_2^s, \dots, \hat{w}_\alpha^s, 0, \dots, 0) \in \mathbb{R}^{Z\times Z}$ satisfying*

$$\begin{pmatrix} \hat{W}_5 & D_1 \\ D_1^T & -(I_Z \otimes E_2^T)K - K^T(I_Z \otimes E_2) \end{pmatrix} \leqslant 0, \tag{26}$$

where $\hat{W}_5 = I_Z \otimes (-\sum_{\rho=1}^{\epsilon} \frac{2}{s_\rho^2} Q\underline{H} - 2Q\underline{P} + \phi Q^2 + \Upsilon^2) - 2\sum_{s=1}^{d} a_s \hat{w}^s \otimes (Q\Gamma^s)$, $D_1 = I_Z \otimes (QM) - (I_Z \otimes E_1^T)K$, *then the network (19) with the parameter ranges defined by Eq.(25) is passive under the pinning adaptive controller (18).*

Proof. Select a Lyapunov functional for the network (19) as follows:

$$
\begin{aligned}
V_2(t) = &\sum_{m=1}^{Z} \int_\Omega e_m^T(b,t) Q e_m(b,t) - \sum_{s=1}^{d} a_s \int_\Omega e^T(b,t)[F^s \otimes (Q\Gamma^s)] e(b,t) db \\
&+ \sum_{s=1}^{d} \sum_{m=1}^{\alpha} \frac{a_s}{\beta_m^s} (w_m^s(t) - \hat{w}_m^s)^2. \qquad (27)
\end{aligned}
$$

Then, we acquire

$$
\begin{aligned}
\dot{V}_2(t) = &\, 2\sum_{m=1}^{Z} \int_\Omega e_m^T(b,t) Q \Big(H\triangle e_m(b,t) - P e_m(b,t) + Cf(\chi_m(b,t)) \\
&- \frac{1}{Z}\sum_{l=1}^{Z} Cf(\chi_l(b,t)) + M u_m(b,t) - \frac{1}{Z}\sum_{l=1}^{Z} M u_l(b,t) \\
&+ \sum_{s=1}^{d}\sum_{n=1}^{Z} a_s F_{mn}^s \Gamma^s \dot{e}_n(b,t) - \sum_{s=1}^{d} a_s w_m^s(t) \Gamma^s e_m(b,t) - \frac{1}{Z}\sum_{m=1}^{\alpha} v_m(b,t) \Big) db \\
&- 2\sum_{s=1}^{d} a_s \int_\Omega e^T(b,t)[F^s \otimes (Q\Gamma^s)] \dot{e}(b,t) db + 2\sum_{s=1}^{d}\sum_{m=1}^{\alpha} \frac{a_s}{\beta_m^s} (w_m^s(t) - \hat{w}_m^s) \dot{w}_m^s(t) \\
= &\, 2\sum_{m=1}^{Z} \int_\Omega e_m^T(b,t) Q \Big(H\triangle e_m(b,t) - P e_m(b,t) + Cf(\chi_m(b,t)) \\
&- Cf(\bar{\chi}(b,t)) + Cf(\bar{\chi}(b,t)) - \frac{1}{Z}\sum_{l=1}^{Z} Cf(\chi_l(b,t)) \\
&+ M u_m(b,t) - \frac{1}{Z}\sum_{l=1}^{Z} M u_l(b,t) - \frac{1}{Z}\sum_{m=1}^{\alpha} v_m(b,t) \Big) db \\
&+ 2\sum_{s=1}^{d} a_s \int_\Omega e^T(b,t)[F^s \otimes (Q\Gamma^s)] \dot{e}(b,t) db - 2\sum_{s=1}^{d}\sum_{m=1}^{Z} a_s w_m^s(t) \int_\Omega e_m^T(b,t) Q\Gamma^s e_m(b,t) db \\
&- 2\sum_{s=1}^{d} a_s \int_\Omega e^T(b,t)[F^s \otimes (Q\Gamma^s)] \dot{e}(b,t) db + 2\sum_{s=1}^{d}\sum_{m=1}^{\alpha} a_s (w_m^s(t) - \hat{w}_m^s) \int_\Omega e_m^T(b,t) Q\Gamma^s e_m(b,t) db \\
= &\, 2\sum_{m=1}^{Z} \int_\Omega e_m^T(b,t) Q \Big(H\triangle e_m(b,t) - P e_m(b,t) + Cf(\chi_m(b,t)) \\
&- Cf(\bar{\chi}(b,t)) + Cf(\bar{\chi}(b,t)) - \frac{1}{Z}\sum_{l=1}^{Z} Cf(\chi_l(b,t)) + M u_m(b,t) \\
&- \frac{1}{Z}\sum_{l=1}^{Z} M u_l(b,t) - \frac{1}{Z}\sum_{m=1}^{\alpha} v_m(b,t) \Big) db - 2\sum_{s=1}^{d}\sum_{m=1}^{\alpha} a_s \hat{w}_m^s \int_\Omega e_m^T(b,t) Q\Gamma^s e_m(b,t) db \\
\leqslant &\int_\Omega e^T(b,t) \Big[I_Z \otimes \Big(-\sum_{\rho=1}^{\epsilon} \frac{2}{s_\rho^2} QH - 2QP + QCC^T Q + \Upsilon^2 \Big) \Big] e(b,t) db
\end{aligned}
$$

$$
\begin{aligned}
&+ 2\int_{\Omega} e^T(b,t)[I_Z \otimes (QM)]u(b,t)db - 2\sum_{s=1}^{d} a_s \int_{\Omega} e^T(b,t)[\hat{w}^s \otimes (Q\Gamma^s)]e(b,t)db \\
&\leqslant \int_{\Omega} e^T(b,t)\Big[I_Z \otimes\Big(-\sum_{\rho=1}^{\epsilon} \frac{2}{s_\rho^2} Q\underline{H} - 2Q\underline{P} + \phi Q^2 + \Upsilon^2\Big)\Big]e(b,t)db \\
&+ 2\int_{\Omega} e^T(b,t)[I_Z \otimes (QM)]u(b,t)db - 2\sum_{s=1}^{d} a_s \int_{\Omega} e^T(b,t)[\hat{w}^s \otimes (Q\Gamma^s)]e(b,t)db,
\end{aligned}
\tag{28}
$$

where $\mathbb{R} \ni \hat{w}_m^s = 0, m = \alpha + 1, \alpha + 2, \ldots, Z$.

Therefore,

$$
\begin{aligned}
&\dot{V}_1(t) - 2\int_{\Omega} y^T(b,t)Ku(b,t)db \\
&\leqslant \int_{\Omega} e^T(b,t)\Big[I_Z \otimes\Big(-\sum_{\rho=1}^{\epsilon} \frac{2}{s_\rho^2} Q\underline{H} - 2Q\underline{P} + \phi Q^2 + \Upsilon^2\Big) \\
&- 2\sum_{s=1}^{d} a_s \hat{w}^s \otimes (Q\Gamma^s)\Big]e(b,t)db + 2\int_{\Omega} e^T(b,t)[I_Z \otimes (QM)]u(b,t)db \\
&- 2\int_{\Omega} \big[e^T(b,t)(I_Z \otimes E_1^T) + u^T(b,t)(I_Z \otimes E_2^T)\big]Ku(b,t)db \\
&= \sigma^T(b,t)\begin{pmatrix} \hat{W}_5 & D_1 \\ D_1^T & W_2 \end{pmatrix}\sigma(b,t),
\end{aligned}
$$

where $W_2 = -(I_Z \otimes E_2^T)K - K^T(I_Z \otimes E_2), \sigma(b,t) = (e^T(b,t), u^T(b,t))^T$.

By Eq. (26), one gets

$$
2\int_{\Omega} y^T(b,t)Ku(b,t)db \geqslant \dot{V}_2(t).
$$

Therefore,

$$
\frac{\dot{V}_2(t)}{2} \leqslant \int_{\Omega} y^T(b,t)Ku(b,t)db.
$$

Letting $\hat{V}(t) = V_2(t)/2$, one has

$$
\int_{t_0}^{t_r} \int_{\Omega} y^T(b,t)Ku(b,t)dbdt \geqslant \hat{V}(t_r) - \hat{V}(t_0)
$$

for any $0 \leqslant t_0 \leqslant t_r \in \mathbb{R}$. □

Using the similar proof method as in Theorem 4.5, we can easily obtain the following conclusions.

Theorem 4.6. *If there exist matrices* $\mathbb{R}^{\eta Z \times \eta Z} \ni A_1 > 0, K \in \mathbb{R}^{\zeta Z \times \eta Z}$ *and* $0 < \hat{w}^s = \mathrm{diag}(\hat{w}_1^s, \hat{w}_2^s, \ldots, \hat{w}_\alpha^s, 0, \ldots, 0) \in \mathbb{R}^{Z \times Z}$ *satisfying*

$$
\begin{pmatrix} \hat{W}_5 & D_1 \\ D_1^T & A_1 - (I_Z \otimes E_2^T)K - K^T(I_Z \otimes E_2) \end{pmatrix} \leqslant 0,
$$

where $\hat{W}_5 = I_Z \otimes (-\sum_{\rho=1}^{\epsilon} \frac{2}{s_\rho^2} Q\underline{H} - 2Q\underline{P} + \phi Q^2 + \Upsilon^2) - 2\sum_{s=1}^{d} a_s \hat{w}^s \otimes (Q\Gamma^s), D_1 = I_Z \otimes (QM) - (I_Z \otimes E_1^T)K$, *then the network (19) with the parameter ranges defined by Eq.(25) is input-strictly passive under the pinning adaptive controller (18).*

Theorem 4.7. *If there exist matrices* $\mathbb{R}^{\zeta Z\times\zeta Z} \ni A_2 > 0, K \in \mathbb{R}^{\zeta Z\times\eta Z}$ *and* $0 < \hat{w}^s = \mathrm{diag}(\hat{w}_1^s, \hat{w}_2^s, \dots, \hat{w}_\alpha^s, 0, \dots, 0) \in \mathbb{R}^{Z\times Z}$ *satisfying*

$$\begin{pmatrix} \hat{W}_6 & D_2 \\ D_2^T & W_4 \end{pmatrix} \leqslant 0,$$

where $\hat{W}_6 = I_Z \otimes (-\sum_{\rho=1}^{\epsilon} \frac{2}{s_\rho^2} Q\underline{H} - 2Q\underline{P} + \phi Q^2 + \Upsilon^2) - 2\sum_{s=1}^{d} a_s \hat{w}^s \otimes (Q\Gamma^s) + (I_Z \otimes E_1^T) A_2 (I_Z \otimes E_1), W_4 = -(I_Z \otimes E_2^T)K - K^T(I_Z \otimes E_2) + (I_Z \otimes E_2^T)A_2(I_Z \otimes E_2), D_2 = I_Z \otimes (QM) - (I_Z \otimes E_1^T)K + (I_Z \otimes E_1^T)A_2(I_Z \otimes E_2)$, *then the network (19) with the parameter ranges defined by Eq. (25) is output-strictly passive under the pinning adaptive controller (18).*

4.3.2. Synchronization criteria

Theorem 4.8. *If there exists a matrix* $0 < \hat{w}^s = \mathrm{diag}(\hat{w}_1^s, \hat{w}_2^s, \dots, \hat{w}_\alpha^s, 0, \dots, 0) \in \mathbb{R}^{Z\times Z}, s = 1, \dots, d$, *such that*

$$W_5 - 2\sum_{s=1}^{d} a_s \hat{w}^s \otimes (Q\Gamma^s) < 0,$$

where $W_5 = I_Z \otimes (-\sum_{\rho=1}^{\epsilon} \frac{2}{s_\rho^2} Q\underline{H} - 2Q\underline{P} + \phi Q^2 + \Upsilon^2)$, *the network (17) with the parameter ranges defined by Eq. (25) is synchronization under the pinning adaptive controller (18).*

Proof. Take the Lyapunov functional $V_2(t)$ as follows:

$$V_2(t) = \sum_{m=1}^{Z} \int_\Omega e_m^T(b,t) Q e_m(b,t) - \sum_{s=1}^{d} a_s \int_\Omega e^T(b,t)[F^s \otimes (Q\Gamma^s)]e(b,t)db$$
$$+ \sum_{s=1}^{d}\sum_{m=1}^{\alpha} \frac{a_s}{\beta_m^s}(w_m^s(t) - \hat{w}_m^s)^2.$$

Then, we acquire

$$\begin{aligned}
\dot{V}_2(t) &= 2\sum_{m=1}^{Z} \int_\Omega e_m^T(b,t) Q\Big(H\triangle e_m(b,t) - Pe_m(b,t) + Cf(\chi_m(b,t)) \\
&\quad - \frac{1}{Z}\sum_{l=1}^{Z} Cf(\chi_l(b,t)) + \sum_{s=1}^{d}\sum_{n=1}^{Z} a_s F_{mn}^s \Gamma^s \dot{e}_n(b,t) - \sum_{s=1}^{d} a_s w_m^s(t)\Gamma^s e_m(b,t) - \frac{1}{Z}\sum_{m=1}^{\alpha} v_m(b,t)\Big)db \\
&\quad - 2\sum_{s=1}^{d} a_s \int_\Omega e^T(b,t)[F^s \otimes (Q\Gamma^s)]\dot{e}(b,t)db + 2\sum_{s=1}^{d}\sum_{m=1}^{\alpha} \frac{a_s}{\beta_m^s}(w_m^s(t) - \hat{w}_m^s)\dot{w}_m^s(t) \\
&= 2\sum_{m=1}^{Z} \int_\Omega e_m^T(b,t) Q\Big(H\triangle e_m(b,t) - Pe_m(b,t) + Cf(\chi_m(b,t)) - Cf(\bar{\chi}(b,t)) + Cf(\bar{\chi}(b,t)) \\
&\quad - \frac{1}{Z}\sum_{l=1}^{Z} Cf(\chi_l(b,t)) - \frac{1}{Z}\sum_{m=1}^{\alpha} v_m(b,t)\Big)db + 2\sum_{s=1}^{d} a_s \int_\Omega e^T(b,t)[F^s \otimes (Q\Gamma^s)]\dot{e}(b,t)db
\end{aligned}$$

$$
\begin{aligned}
& - 2\sum_{s=1}^{d}\sum_{m=1}^{Z} a_s w_m^s(t)\int_\Omega e_m^T(b,t)Q\Gamma^s e_m(b,t)db - 2\sum_{s=1}^{d} a_s\int_\Omega e^T(b,t)[F^s\otimes(Q\Gamma^s)]\dot{e}(b,t)db \\
& + 2\sum_{s=1}^{d}\sum_{m=1}^{\alpha} a_s(w_m^s(t)-\hat{w}_m^s)\int_\Omega e_m^T(b,t)Q\Gamma^s e_m(b,t)db \\
= {} & 2\sum_{m=1}^{Z}\int_\Omega e_m^T(b,t)Q\Big(H\triangle e_m(b,t) - Pe_m(b,t) + Cf(\chi_m(b,t)) \\
& - Cf(\bar{\chi}(b,t)) + Cf(\bar{\chi}(b,t)) - \frac{1}{Z}\sum_{l=1}^{Z} Cf(\chi_l(b,t)) - \frac{1}{Z}\sum_{m=1}^{\alpha} v_m(b,t)\Big)db \\
& - 2\sum_{s=1}^{d}\sum_{m=1}^{\alpha} a_s\hat{w}_m^s\int_\Omega e_m^T(b,t)Q\Gamma^s e_m(b,t)db \\
\leqslant {} & \int_\Omega e^T(b,t)\Big[I_Z\otimes\Big(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}QH - 2QP + QCC^TQ + \Upsilon^2\Big) - 2\sum_{s=1}^{d} a_s\hat{w}^s\otimes(Q\Gamma^s)\Big]e(b,t)db \\
\leqslant {} & \int_\Omega e^T(b,t)\Big[I_Z\otimes\Big(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}Q\underline{H} - 2Q\underline{P} + \phi Q^2 + \Upsilon^2 - 2\sum_{s=1}^{d} a_s\hat{w}^s\otimes(Q\Gamma^s)\Big)\Big]e(b,t)db \\
\leqslant {} & \delta_5\|e(\cdot,t)\|^2,
\end{aligned}
\tag{29}
$$

where $\delta_5 = \gamma_M(I_Z\otimes(-\sum_{\rho=1}^{\epsilon}\frac{2}{s_\rho^2}Q\underline{H} - 2Q\underline{P} + \phi Q^2 + \Upsilon^2 - 2\sum_{s=1}^{d} a_s\hat{w}^s\otimes(Q\Gamma^s))$.

Based on the definition of $V_2(t)$ and Eq. (29), we have $V_2(t)$ is bounded and non-increasing. Therefore, $\lim_{t\to+\infty}V_2(t)$ exists and $w_m^s(t)$ is bounded. Because $w_m^s(t)$ is monotonically increasing [see Eq. (18)], it is easy to infer that $w_m^s(t)$ asymptotically converges to a finite positive value. After that, we get that $\lim_{t\to+\infty}\int_\Omega e^T(b,t)[I_Z\otimes Q - \sum_{s=1}^{d} a_sF^s\otimes(Q\Gamma^s)]e(b,t)db$ exists and is a nonnegative real number. Suppose that

$$
\lim_{t\to+\infty}\int_\Omega e^T(b,t)\Big[I_Z\otimes Q - \sum_{s=1}^{d} a_sF^s\otimes(Q\Gamma^s)\Big]e(b,t)db = \wp > 0.
$$

Therefore, there obviously exists $0 < \Psi \in \mathbb{R}$ such that

$$
\int_\Omega e^T(b,t)\Big[I_Z\otimes Q - \sum_{s=1}^{d} a_sF^s\otimes(Q\Gamma^s)\Big]e(b,t)db > \frac{\wp}{2}
$$

for $t \geq \Psi$.

From Eq. (29), we have

$$
\dot{V}_2(t) < \frac{\delta_5\wp}{2\delta_4}, \quad t \geqslant \Psi,
\tag{30}
$$

where $\delta_4 = \gamma_M(I_Z\otimes Q - \sum_{s=1}^{d} a_sF^s\otimes(Q\Gamma^s))$.

In light of Eq. (30), we can obtain

$$\begin{aligned}-V_2(\Psi) &\leqslant V_2(+\infty) - V_2(\Psi)\\ &= \int_{\Psi}^{+\infty} \dot{V}_2(t)dt\\ &\leqslant \int_{\Psi}^{+\infty} \frac{\delta_5\wp}{2\delta_4}dt\\ &= -\infty,\end{aligned}$$

which leads to an error. Therefore, we could derive that

$$\lim_{t\to+\infty}\int_{\Omega} e^T(b,t)\left[I_Z\otimes Q-\sum_{s=1}^{d}a_sF^s\otimes(Q\Gamma^s)\right]e(b,t)db=0.$$

Evidently,

$$\lim_{t\to+\infty}\|e(\cdot,t)\|=0.$$

Therefore, the network (17) with the parameter ranges defined by Eq. (25) is synchronization under the pinning adaptive controller (18). □

Remark 1. Up to now, some authors have studied the synchronization and passivity problems for CDNs with derivative coupling, and a number of meaningful results have been obtained [31–34,36–38]. Actually, many networks in the real life should be described by multi-weighted models of complex networks, such as social networks, public traffic roads networks, etc. [35]. Thus, it is more meaningful to study the CDNs with multiple derivative couplings [36–38]. Regretfully, the synchronization and passivity for CRDNNs with multiple derivative couplings have not yet been considered.

Remark 2. The main difficulty to deal with the MDCRDNNs comes from the multiple derivative couplings and the reaction-diffusion terms, which cannot be solved by those methods used in existing works. By making use of some inequality techniques and selecting appropriate Lyapunov functionals, several synchronization and passivity criteria for MDCRDNNs with and without parameter uncertainties are proposed in this paper.

Remark 3. As everyone knows, in many circumstances, the parameter fluctuations in reaction-diffusion neural networks are unavoidable but bounded due to the external disturbance and modeling inaccuracies. Therefore, the dynamical behaviors for reaction-diffusion neural networks with bounded parameter uncertainties have been studied extensively and deeply [12,13]. Consequently, we respectively study the robust passivity [see Theorems 3.5, 3.6, 3.7, 4.5, 4.6 and 4.7] and robust synchronization [see Theorems 3.8 and 4.8] of MDCRDNNs in this paper, in which the parameter fluctuations are bounded. From these theorems, we can clearly see that the obtained robust passivity and robust synchronization results are only related to the lower bound of the H and P since the H and P are diagonal matrices.

5. Numerical example

The MDCRDNNs with parameter uncertainties is given as follows:

$$
\begin{aligned}
\frac{\partial \chi_m(b,t)}{\partial t} &= H\triangle\chi_m(b,t) - P\chi_m(b,t) + Cf(\chi_m(b,t)) + J + Mu_m(b,t) \\
&\quad + \sum_{s=1}^{3}\sum_{n=1}^{5} a_s F_{mn}^{s}\Gamma^{s}\dot{\chi}_n(b,t) + v_m(b,t), \quad m = 1,2,3, \\
\frac{\partial \chi_m(b,t)}{\partial t} &= H\triangle\chi_m(b,t) - P\chi_m(b,t) + Cf(\chi_m(b,t)) + J + Mu_m(b,t) \\
&\quad + \sum_{s=1}^{3}\sum_{n=1}^{5} a_s F_{mn}^{1}\Gamma^{1}\dot{\chi}_n(b,t), \quad m = 4,5,
\end{aligned} \tag{31}
$$

where

$$
v_m(b,t) = -\sum_{s=1}^{3} a_s w_m^s(t)\Gamma^s\left(\chi_m(b,t) - \frac{1}{5}\sum_{m=1}^{5}\chi_m(b,t)\right),
$$

$$
\dot{w}_m^s(t) = \beta_m^s\int_{\Omega}\left(\chi_m(b,t) - \frac{1}{5}\sum_{m=1}^{5}\chi_m(b,t)\right)^T Q\Gamma^s\left(\chi_m(b,t) - \frac{1}{5}\sum_{m=1}^{5}\chi_m(b,t)\right)db, \quad m = 1,2,3,
$$

where $f_k(\xi) = \frac{|\xi+1|-|\xi-1|}{4}, k = 1,2,3, \Omega = \{b \mid -0.5 < b < 0.5\}, a_1 = 1, a_2 = 2, a_3 = 3$, $\beta_m^1 = 1, \beta_m^2 = 2, \beta_m^3 = 3, \Gamma^1 = \mathrm{diag}(0.5, 0.6, 0.5), \Gamma^2 = \mathrm{diag}(0.7, 0.4, 0.8), \Gamma^3 = \mathrm{diag}(0.5, 0.7, 0.8), Q = \mathrm{diag}(0.1, 0.2, 0.3), J = (0.5, 0.6, 0.7)^T$,

$$
M = \begin{pmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \\ 0.2 & 0.1 \end{pmatrix},
$$

$$
F^1 = \begin{pmatrix} -0.5 & 0.3 & 0 & 0 & 0.2 \\ 0.3 & -0.4 & 0.1 & 0 & 0 \\ 0 & 0.1 & -0.2 & 0.1 & 0 \\ 0 & 0 & 0.1 & -0.4 & 0.3 \\ 0.2 & 0 & 0 & 0.3 & -0.5 \end{pmatrix},
$$

$$
F^2 = \begin{pmatrix} -0.6 & 0.2 & 0 & 0 & 0.4 \\ 0.2 & -0.3 & 0.1 & 0 & 0 \\ 0 & 0.1 & -0.4 & 0.3 & 0 \\ 0 & 0 & 0.3 & -0.5 & 0.2 \\ 0.4 & 0 & 0 & 0.2 & -0.6 \end{pmatrix},
$$

$$
F^3 = \begin{pmatrix} -0.3 & 0.1 & 0 & 0 & 0.2 \\ 0.1 & -0.2 & 0.1 & 0 & 0 \\ 0 & 0.1 & -0.4 & 0.3 & 0 \\ 0 & 0 & 0.3 & -0.4 & 0.1 \\ 0.2 & 0 & 0 & 0.1 & -0.3 \end{pmatrix},
$$

$$
\begin{cases}
H_I := \{H = \mathrm{diag}(h_r) : 0.4 \leqslant h_1 \leqslant 0.5, 0.6 \leqslant h_2 \leqslant 0.7, 0.4 \leqslant h_3 \leqslant 0.5\}, \\
P_I := \{P = \mathrm{diag}(p_r) : 0.5 \leqslant p_1 \leqslant 0.6, 0.3 \leqslant p_2 \leqslant 0.4, 0.4 \leqslant p_3 \leqslant 0.5\}, \\
C_I := \{C = (c_{rj})_{3\times 3} : 0.2 \leqslant c_{11} \leqslant 1, 0.1 \leqslant c_{12} \leqslant 1.1, 0.3 \leqslant c_{13} \leqslant 1, 0.1 \\
\qquad \leqslant c_{21} \leqslant 1, 0.3 \leqslant c_{22} \leqslant 1, 0.2 \leqslant c_{23} \leqslant 1, 0.4 \leqslant c_{31} \leqslant 1, 0.3 \leqslant c_{32} \\
\qquad \leqslant 1, 0.2 \leqslant c_{33} \leqslant 1\}.
\end{cases}
$$

Apparently, $\phi = 0.81$ and function $f_k(\cdot)$ satisfies Lipschitz condition with $\Upsilon_k = 0.5$. The output vector $y_m(b,t) \in \mathbb{R}^3$ is selected as follows:

$$y_m(b,t) = E_1 e_m(b,t) + E_2 u_m(b,t), \qquad m = 1, 2, \ldots, 5,$$

where

$$E_1 = \begin{pmatrix} 0.7 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 0.7 \end{pmatrix}, \qquad E_2 = \begin{pmatrix} 0.2 & 0.3 \\ 0.1 & 0.2 \\ 0.1 & 0.4 \end{pmatrix}.$$

Case 1: By taking advantage of the MATLAB YALMIP Toolbox, the following matrices K and $\hat{w}^s (s = 1, 2, 3)$ that satisfy the condition of the Theorem 4.5 can be obtained:

$$K = \begin{pmatrix} 0.1245 & 0.2146 \\ 0.4676 & 0.8836 \\ 0.3516 & 1.0239 \end{pmatrix},$$

$$\hat{w}^1 = 10^8 \begin{pmatrix} 3.0260 & 0 & 0 & 0 & 0 \\ 0 & 3.0260 & 0 & 0 & 0 \\ 0 & 0 & 3.0260 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\hat{w}^2 = 10^8 \begin{pmatrix} 3.1668 & 0 & 0 & 0 & 0 \\ 0 & 3.1668 & 0 & 0 & 0 \\ 0 & 0 & 3.1668 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\hat{w}^3 = 10^8 \begin{pmatrix} 3.4029 & 0 & 0 & 0 & 0 \\ 0 & 3.4029 & 0 & 0 & 0 \\ 0 & 0 & 3.4029 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Therefore, the network (31) is passive.

Case 2: By taking advantage of the MATLAB YALMIP Toolbox, the following matrices A_1, K and $\hat{w}^s (s = 1, 2, 3)$ that satisfy the condition of the Theorem 4.6 can be obtained:

$$A_1 = I_5 \otimes \begin{pmatrix} 0.0207 & 0.0472 \\ 0.0472 & 0.1127 \end{pmatrix},$$

$$K = I_5 \otimes \begin{pmatrix} 0.1214 & 0.2099 \\ 0.4466 & 0.8452 \\ 0.3331 & 0.9738 \end{pmatrix},$$

$$\hat{w}^1 = 10^8 \begin{pmatrix} 2.6825 & 0 & 0 & 0 & 0 \\ 0 & 2.6825 & 0 & 0 & 0 \\ 0 & 0 & 2.6825 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\hat{w}^2 = 10^8 \begin{pmatrix} 3.0729 & 0 & 0 & 0 & 0 \\ 0 & 3.0729 & 0 & 0 & 0 \\ 0 & 0 & 3.0729 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\hat{w}^3 = 10^8 \begin{pmatrix} 3.4596 & 0 & 0 & 0 & 0 \\ 0 & 3.4596 & 0 & 0 & 0 \\ 0 & 0 & 3.4596 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Therefore, the network (31) is input-strictly passive.

Case 3: By taking advantage of the MATLAB YALMIP Toolbox, the following matrices A_2,K and $\hat{w}^s(s = 1, 2, 3)$ that satisfy the condition of the Theorem 4.7 can be obtained:

$$A_2 = I_5 \otimes \begin{pmatrix} 0.0978 & -0.0380 & 0.0220 \\ -0.0380 & 0.6429 & -0.0085 \\ 0.0220 & -0.0085 & 0.3823 \end{pmatrix},$$

$$K = I_5 \otimes \begin{pmatrix} 0.1138 & 0.1905 \\ 0.4460 & 0.8439 \\ 0.3287 & 0.9698 \end{pmatrix},$$

$$\hat{w}^1 = 10^8 \begin{pmatrix} 2.9041 & 0 & 0 & 0 & 0 \\ 0 & 2.9041 & 0 & 0 & 0 \\ 0 & 0 & 2.9041 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\hat{w}^2 = 10^8 \begin{pmatrix} 3.2313 & 0 & 0 & 0 & 0 \\ 0 & 3.2313 & 0 & 0 & 0 \\ 0 & 0 & 3.2313 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\hat{w}^3 = 10^8 \begin{pmatrix} 3.5406 & 0 & 0 & 0 & 0 \\ 0 & 3.5406 & 0 & 0 & 0 \\ 0 & 0 & 3.5406 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Therefore, the network (31) is output-strictly passive. The simulation results are displayed in Figs. 1 and 2.

Case 4: By taking advantage of the MATLAB YALMIP Toolbox, the following matrices $\hat{w}^s$ that satisfy the condition of Theorem 4.8 can be obtained:

$$\hat{w}^1 = 10^8 \begin{pmatrix} 2.7673 & 0 & 0 & 0 & 0 \\ 0 & 2.7673 & 0 & 0 & 0 \\ 0 & 0 & 2.7673 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

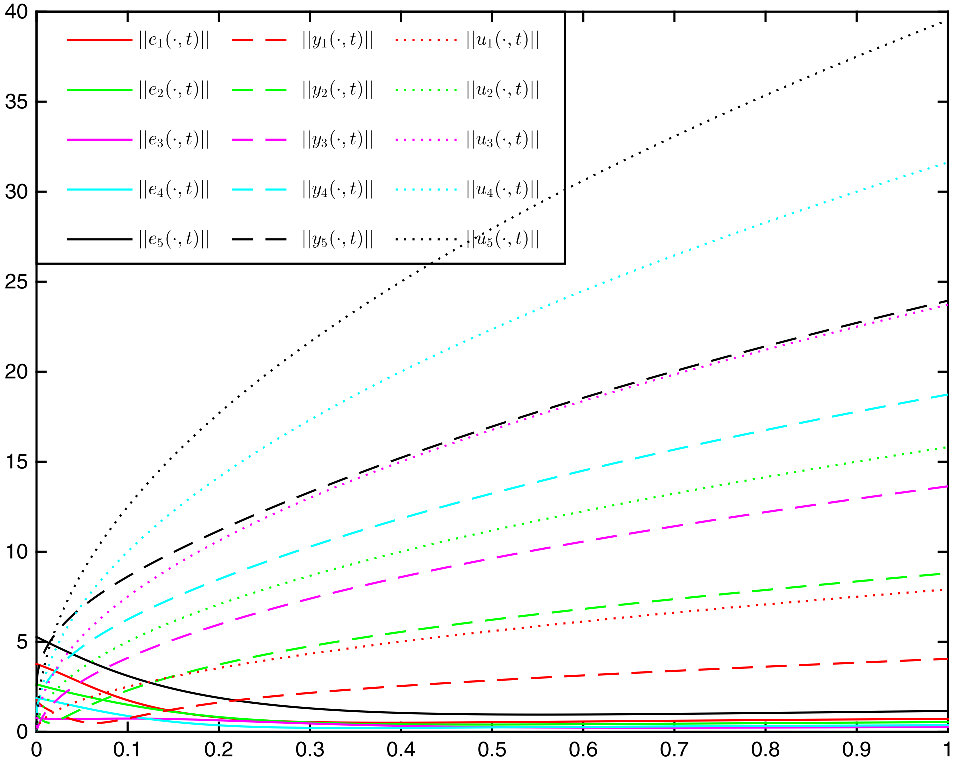

Fig. 1. $\|e_m(\cdot,t)\|$, $\|y_m(\cdot,t)\|$, $\|u_m(\cdot,t)\|$, $m = 1, 2, \ldots, 5$.

$$\hat{w}^2 = 10^8 \begin{pmatrix} 3.1990 & 0 & 0 & 0 & 0 \\ 0 & 3.1990 & 0 & 0 & 0 \\ 0 & 0 & 3.1990 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\hat{w}^3 = 10^8 \begin{pmatrix} 3.3655 & 0 & 0 & 0 & 0 \\ 0 & 3.3655 & 0 & 0 & 0 \\ 0 & 0 & 3.3655 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Therefore, the network (31) is synchronization. The simulation results are displayed in Figs. 3 and 4.

Remark 4. Figs. 1 and 2, respectively, show the changing processes of $\|e_m(\cdot,t)\|$, $\|y_m(\cdot,t)\|$, $\|u_m(\cdot,t)\|$, $m = 1, 2, \ldots, 5$, $w_1^s(t)$, $w_2^s(t)$ and $w_3^s(t)$, $s = 1, 2, 3$ of the network (31). For the network (31) with the input $u_m(b,t) = 0$, the evolution tendencies of $\|e_m(\cdot,t)\|$, $m = 1, 2, \ldots, 5$, $w_1^s(t)$, $w_2^s(t)$ and $w_3^s(t)$, $s = 1, 2, 3$ are displayed in Figs. 3 and 4. We can see clearly from Fig. 3 that $\|e_m(b,t)\|$, $m = 1, 2, \ldots, 5$ converge to 0. In Fig. 4, the coupling weights $w_1^s(t)$, $w_2^s(t)$ and $w_3^s(t)$, $s = 1, 2, 3$ asymptotically converge to some finite positive values.

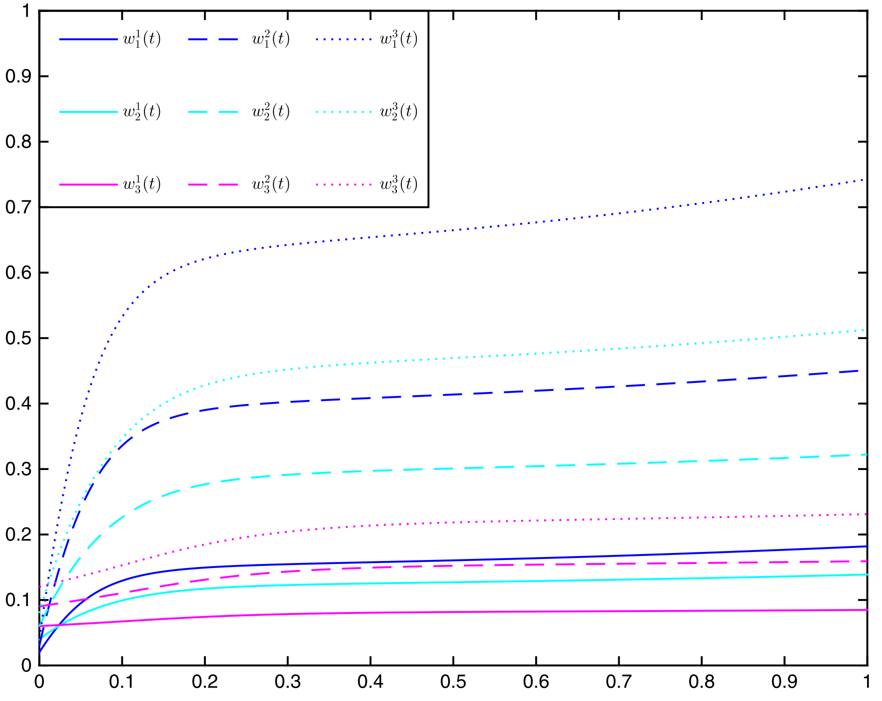

Fig. 2. $w_m^s(t)$, $s = 1, 2, 3$, $m = 1, 2, 3$.

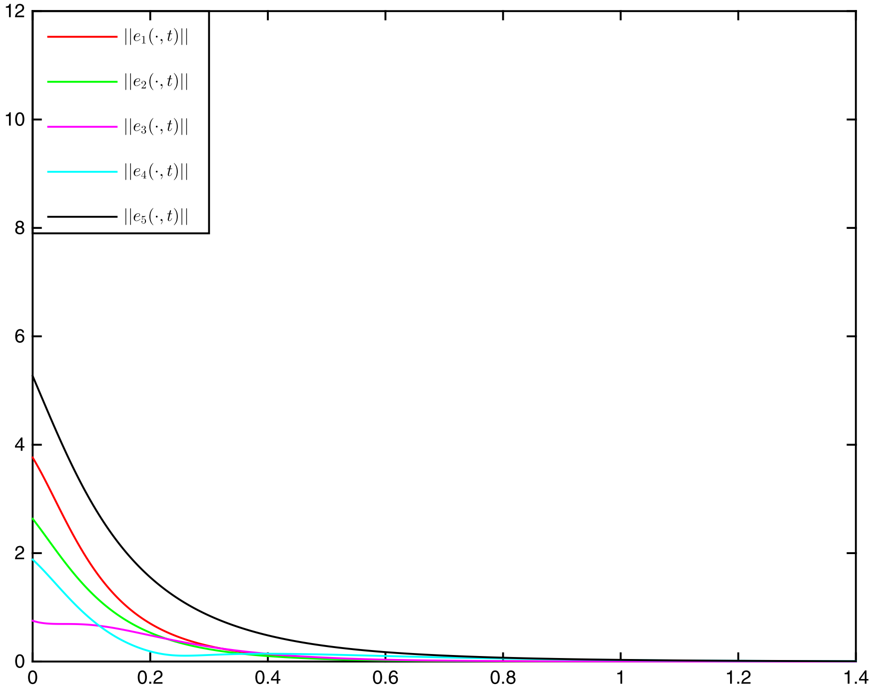

Fig. 3. $\|e_m(\cdot,t)\|$, $m = 1, 2, \ldots, 5$.

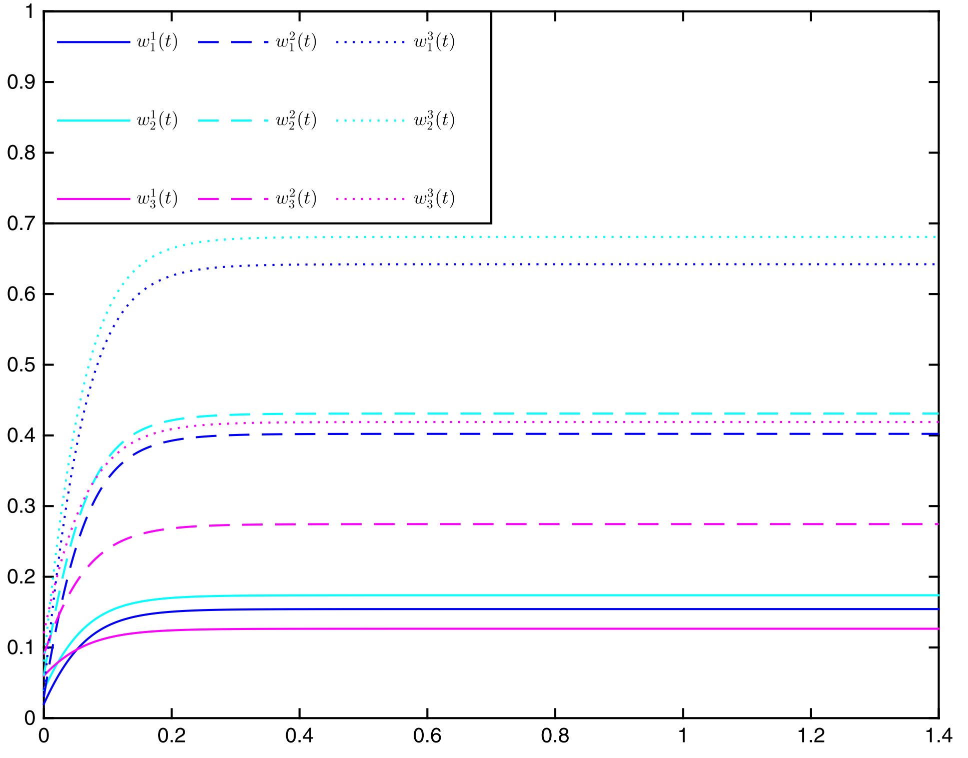

Fig. 4. $w_m^s(t), s = 1, 2, 3, m = 1, 2, 3$.

6. Conclusion

In this paper, a type of MDCRDNNs with and without parameter uncertainties have been discussed. With the help of the Lyapunov functionals, we have not only analyzed the passivity and synchronization of the MDCRDNNs, but also presented several criteria based on the designed pinning control strategy. In addition, the analysis and pinning control problems for passivity and synchronization of the MDCRDNNs with parameter uncertainties have been further considered. At last, a numeral example has been provided to demonstrate the correctness of these theoretical results. Actually, in most circumstances, MDCRDNNs are required to achieve synchronization in a finite time. Therefore, we shall further study the finite-time synchronization of MDCRDNNs in the future.

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