R02
Divide and Conquer
Peak Finding

50.004 Introduction to Algorithm

Gemma Roig
(slides adapted from Dr. Simon LUI)
ISTD, SUTD

Peak Finding

Peak Finding Problem (PFP): 1D array

- Consider an array A[1…n] :

10	13	5	8	3	2	1

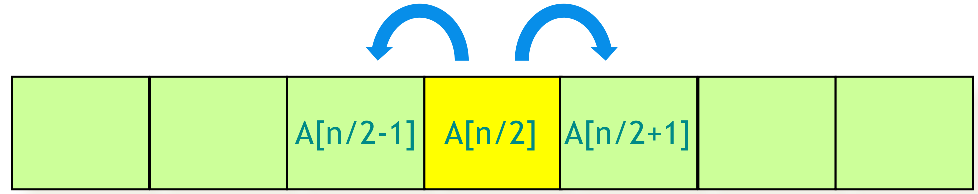
- An element A[i] is a *peak* if it is not smaller than all its neighbor(s)
  - if $i \neq 1, n$: $A[i] \geq A[i-1]$ and $A[i] \geq A[i+1]$
  - If $i=1$: $A[1] \geq A[2]$
  - If $i=n$: $A[n] \geq A[n-1]$
- Problem: find ***any*** peak.

Peak Finding

- Algorithm 1:
  - Scan the array from left to right
  - Compare each A[i] with its neighbors
  - Exit when found a peak
- Complexity:
  - Might need to scan all elements, so $T(n)=\Theta(n)$

1	2	4	8	9	12	21

Peak Finding



- Algorithm 2:
- Consider the middle element of the array and compare with neighbors
  - If $A[n/2-1]>A[n/2]$
    then search for a peak among $A[1] \ldots A[n/2-1]$
  - Else, if $A[n/2]<A[n/2+1]$
    then search for a peak among $A[n/2+1] \ldots A[n]$
  - Else $A[n/2]$ is a peak!
    (since $A[n/2-1] \leq A[n/2]$ and $A[n/2] \geq A[n/2+1]$)

Algorithm II: Complexity

(worse case)

Time needed to find peak in array of length n

Recursion

Time for comparing A[$n/2$] with **two** neighbors

- We have

$$T(n) = T(n/2) + \boxed{O(1)}$$

- Unraveling the recursion,

$$T(n) \le T(n/2) + c \le (T(n/2^2) + c) + c \le \ldots$$

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$$T(n) = T(n/2) + \boxed{O(1)}$$

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$$T(n) \le T(n/2) + c \le (T(n/2^2) + c) + c \le \ldots$$

$$\le T(\underbrace{n/2^{\log_2 n}}_{=1}) + \underbrace{c + c + \ldots + c}_{\log_2 n} = O(\log n)$$

- log n is much much better than n !

In class exercise

- Solve this with the Master Theorem

$$T(n) = T(n/2) \boxed{+ O(1)}$$

$$1)\text{if } f(n) = O(L^{1-\varepsilon}) = O(n^{\log_b a - \varepsilon})$$
$$\Rightarrow T(n) = \Theta(L) = \Theta(n^{\log_b a})$$

$$2)\text{ if } f(n) = \Theta(L) = \Theta(n^{\log_b a})$$
$$\Rightarrow T(n) = \Theta(L \log n) = \Theta(n^{\log_b a} \log n)$$

$$3)\text{ if } f(n) = \Omega(L^{1+\varepsilon}) = \Omega(n^{\log_b a + \varepsilon})$$
$$\Rightarrow T(n) = \Theta(f(n))$$

Divide and Conquer

- Very powerful design tool:
  - *Divide* input into multiple disjoint parts
  - *Conquer* each of the parts separately (using recursive call)

Peak finding: 2D

- Consider a square 2D array A[1…n, 1…n] :

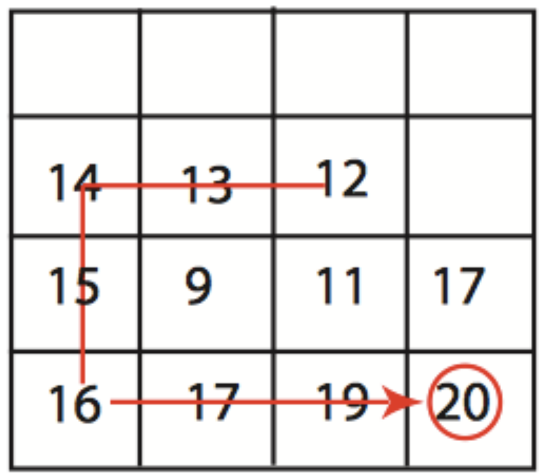
10	8	5
3	2	1
7	13	4
6	8	3

Size of problem = n
= # of rows or columns

- An element A[i] is a *2D peak* if it is not smaller than its (at most 4) neighbors.
- Problem: find any 2D peak.

2D Peak finding: Ideas?

- Algorithm 1: brute-force method (i.e. search for all square)



- Complexity = $\Theta(n^2)$

Algorithm 2: use the 1D algorithm

- Algorithm 2:
  - (a) For each column j, find its *global* maximum B[j]
  - (b) Apply 1D peak finder to find a peak of B[1…n] (say B[j])
- Running time ?

  …is $\Theta(n^2)$

- Proof of Algorithm Correctness (an informal way)
  - 12 is vertically the largest (trivial)
  - 12 is horizontally the largest, because
    - 12 is larger than 9 and 6
    - 9 and 6 are larger than all the elements in their column

A[][]

12	8	5
11	3	6
10	9	2
8	4	1

B[]

12	9	6

Algorithm 3: be "lazy" in the 1D algorithm

- Modify Algorithm 2.
- Use the 1D algorithm (P.5) to find the maximum in B[]

- Total time become O(n log n) !
  - We only need O(log n) entries of B[j]
  - Each B[j] can be computed in O(n)

A[][]

12	8	5
11	3	6
10	9	2
8	4	1

B[]

12	9	6

Exercise

- Find the complexity of the following formulas, using Master theorem

1. $T(n) = 3T(n/2) + n^2$

2. $T(n) = 4T(n/2) + n^2$

3. $T(n) = T(n/2) + 2^n$