Big O

A review of concepts
of asymptotic notation

Notation	Name	Intuition	Informal definition: for sufficiently large n...	Formal Definition
$f(n) \in O(g(n))$	Big Omicron; Big O; Big Oh	f is bounded above by g (up to constant factor) asymptotically	$\|f(n)\| \leq g(n) \cdot k$ for some positive *k*	$\exists k > 0\ \exists n_0\ \forall n > n_0\ \|f(n)\| \leq \|g(n) \cdot k\|$ or $\exists k > 0\ \exists n_0\ \forall n > n_0\ f(n) \leq g(n) \cdot k$
$f(n) \in \Omega(g(n))$	Big Omega	**Two definitions :** Number theory: f is not dominated by g asymptotically Complexity theory: f is bounded below by g asymptotically	Number theory: $f(n) \geq g(n) \cdot k$ for infinitely many values of *n* and for some positive *k* Complexity theory: $f(n) \geq g(n) \cdot k$ for some positive *k*	Number theory: $\exists k > 0\ \forall n_0\ \exists n > n_0\ g(n) \cdot k \leq f(n)$ Complexity theory: $\exists k > 0\ \exists n_0\ \forall n > n_0\ g(n) \cdot k \leq f(n)$
$f(n) \in \Theta(g(n))$	Big Theta	f is bounded both above and below by g asymptotically	$g(n) \cdot k_1 \leq f(n) \leq g(n) \cdot k_2$ for some positive k_1, k_2	$\exists k_1 > 0\ \exists k_2 > 0\ \exists n_0\ \forall n > n_0$ $g(n) \cdot k_1 \leq f(n) \leq g(n) \cdot k_2$
$f(n) \in o(g(n))$	Small Omicron; Small O; Small Oh	f is dominated by g asymptotically	$\|f(n)\| \leq k \cdot \|g(n)\|$, for every fixed positive number k	$\forall k > 0\ \exists n_0\ \forall n > n_0\ \|f(n)\| \leq k \cdot \|g(n)\|$
$f(n) \in \omega(g(n))$	Small Omega	f dominates g asymptotically	$\|f(n)\| \geq k \cdot \|g(n)\|$, for every fixed positive number k	$\forall k > 0\ \exists n_0\ \forall n > n_0\ \|f(n)\| \geq k \cdot \|g(n)\|$
$f(n) \sim g(n)$	On the order of	f is equal to g asymptotically	$f(n)/g(n) \to 1$	$\forall \varepsilon > 0\ \exists n_0\ \forall n > n_0\ \left\| \frac{f(n)}{g(n)} - 1 \right\| < \varepsilon$

http://en.wikipedia.org/wiki/Big_O_notation

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$f(n) \in \Omega(g(n))$	Big Omega	**Two definitions :** Number theory: f is not dominated by g asymptotically Complexity theory: f is bounded below by g asymptotically	Number theory: $f(n) \geq g(n) \cdot k$ for infinitely many values of n and for some positive k Complexity theory: $f(n) \geq g(n) \cdot k$ for some positive k
$f(n) \in \Theta(g(n))$	Big Theta	f is bounded both above and below by g asymptotically	$g(n) \cdot k_1 \leq f(n) \leq g(n) \cdot k_2$ for some positive k_1, k_2

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$$f(n) \in O(g(n))$$

$$|f(n)| \leq g(n) \cdot k$$

$$f(n) \in \Omega(g(n))$$

$$f(n) \geq g(n) \cdot k$$

$$f(n) \in \Theta(g(n))$$

$$g(n) \cdot k_1 \leq f(n) \leq g(n) \cdot k_2$$

$O($

$\leq$

$\geq$

Ω

$f(n) \in \Theta(g(n))$

$g(n) \cdot k_1 \leq f(n) \leq g(n) \cdot k_2$

O	$\leq$
Ω	$\geq$
$f(n) \in \Theta(g(n))$	$g(n) \cdot k_1 \leq f(n) \leq g(n) \cdot k_2$

O	$\leq$
Ω	$\geq$
Θ	$O + \Omega$

$$f(x) = x$$

$$g(x) = x + 2$$

$$f \in O(g)?$$

$$f(x) = x$$

$$g(x) = x + 2$$

$f \in O(g)$?

$g \in \Omega(f)$?

$O + \Omega$

$$f(x) = x$$

$$g(x) = x + 2$$

$f \in O(g)?$

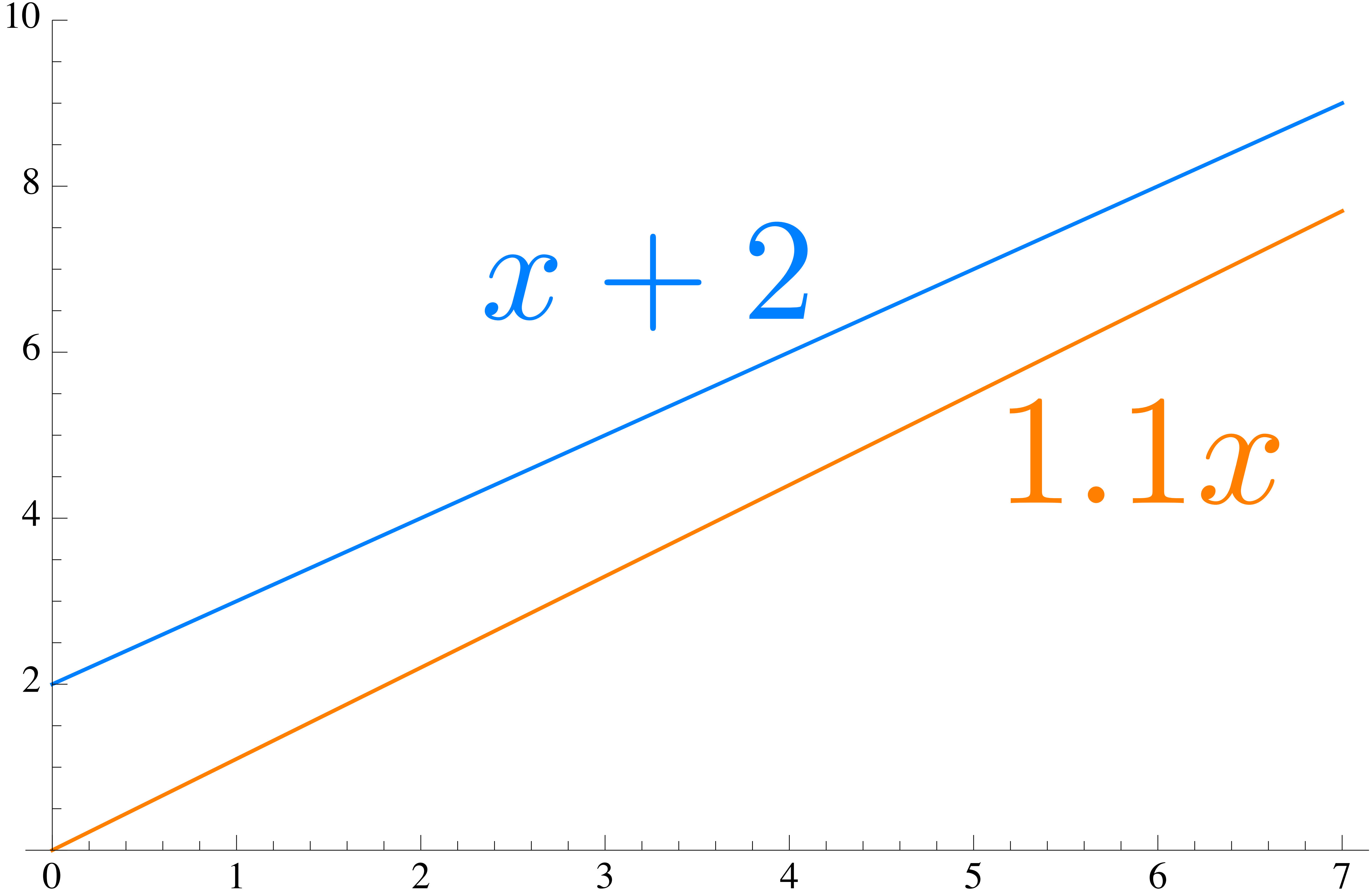
$g \in \Omega(f)?$

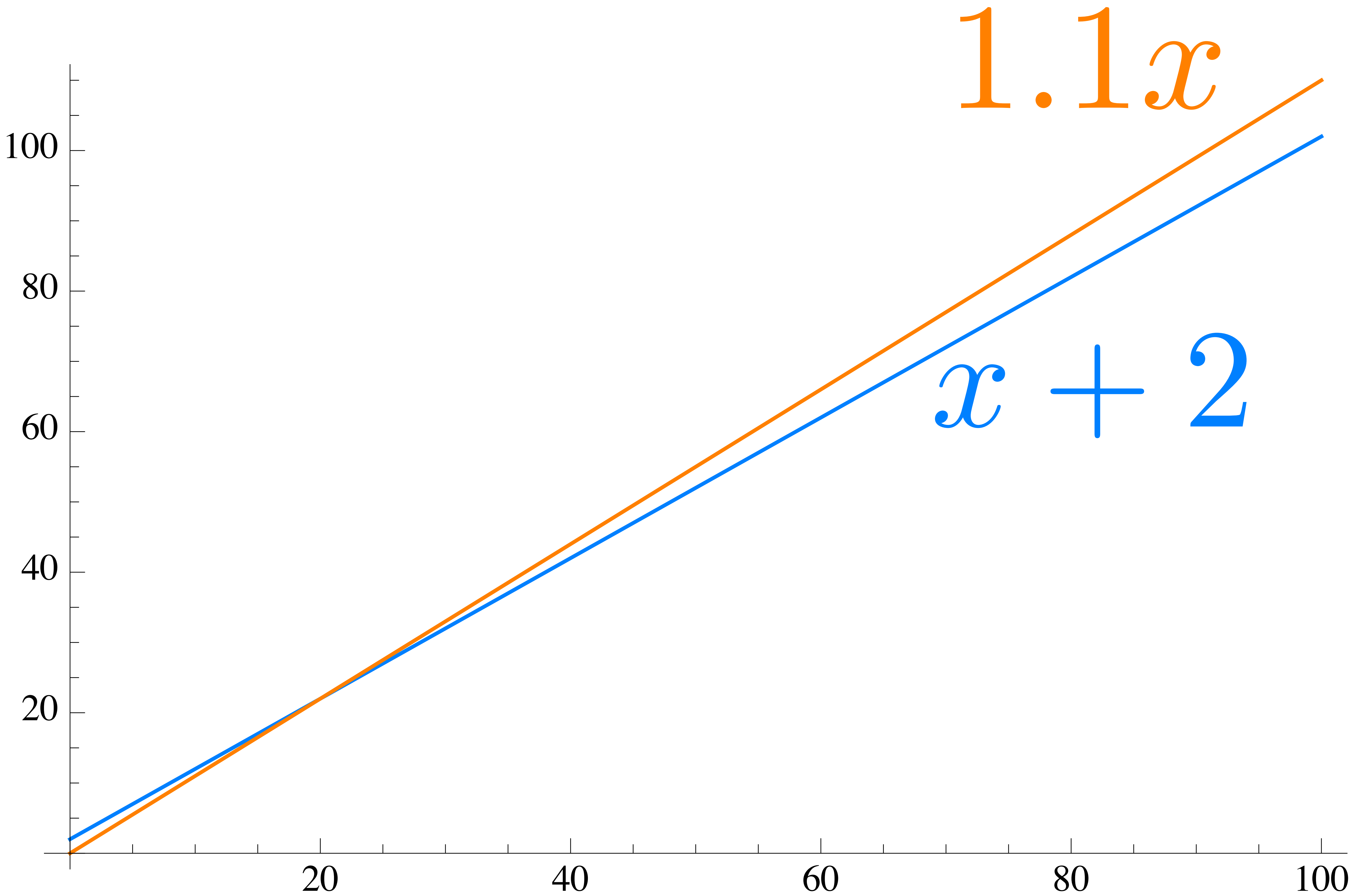
$g \in O(f)?$

$O + \Omega$

$$x + 2 \leq x \cdot ?$$

$g \in O(f)?$


$x + 2$
$1.1x$
10
8
6
4
2
0
1
2
3
4
5
6
7



$1.1x$
$x+2$
100
80
60
40
20
20
40
60
80
100

$$f \in O(g)?$$

$$f(x) = x^2$$

$$g(x) = x^2 + 2$$

$$h(x) = x^2 + 3x + 2$$

$$f \in O(g)?$$

$$f \in O(h)?$$

$$f(x) = x^2$$

$$g(x) = x^2 + 2$$

$$h(x) = x^2 + 3x + 2$$

$$f \in O(g)?$$

$$f(x) = x^2 \qquad f \in O(h)?$$

$$g(x) = x^2 + 2$$

$$h(x) = x^2 + 3x + 2 \qquad f \in \Omega(g)?$$

$$f \in O(g)?$$

$$f(x) = x^2$$

$$f \in O(h)?$$

$$g(x) = x^2 + 2$$

$$f \in \Omega(g)?$$

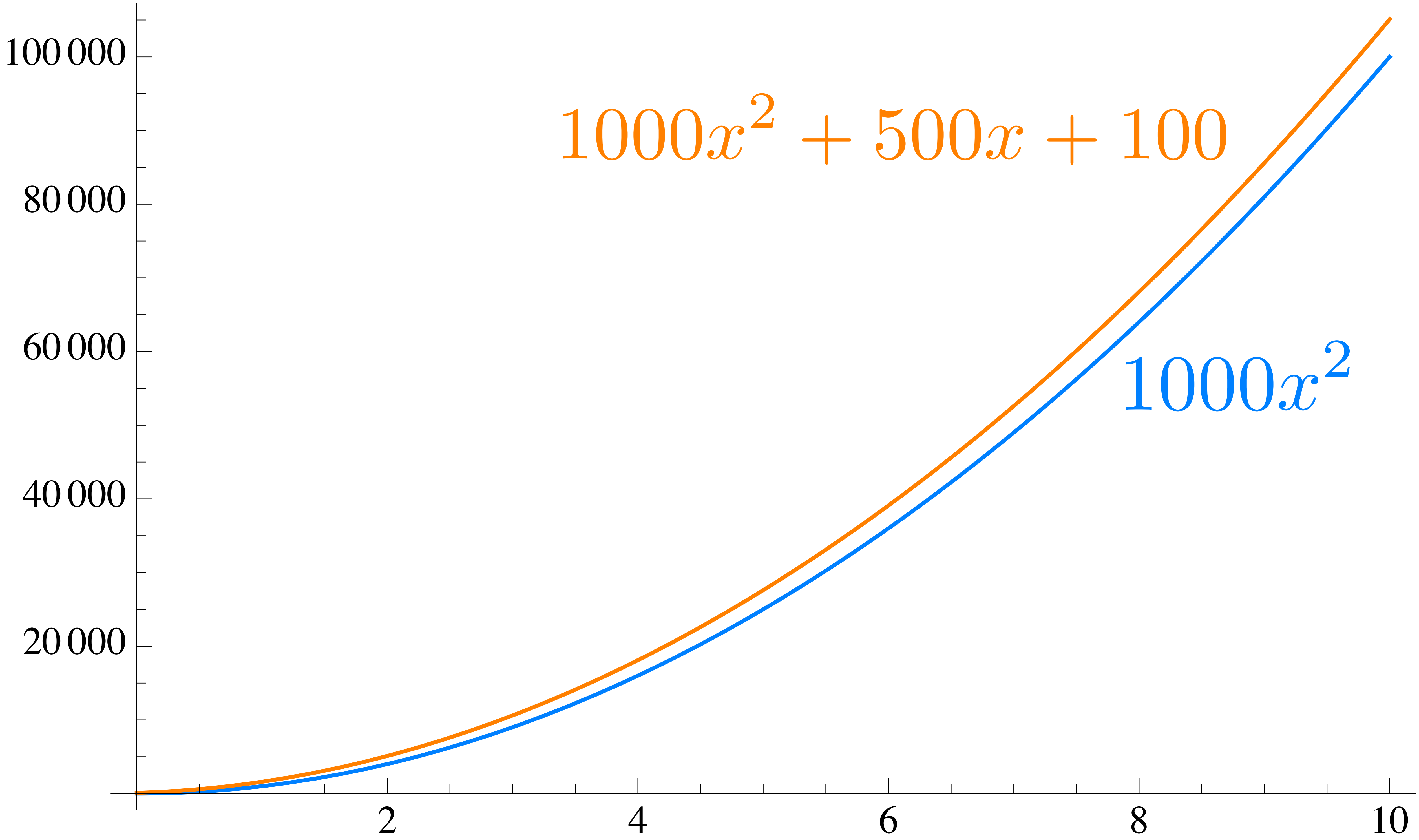
$$h(x) = x^2 + 3x + 2$$

$$f \in \Omega(h)?$$

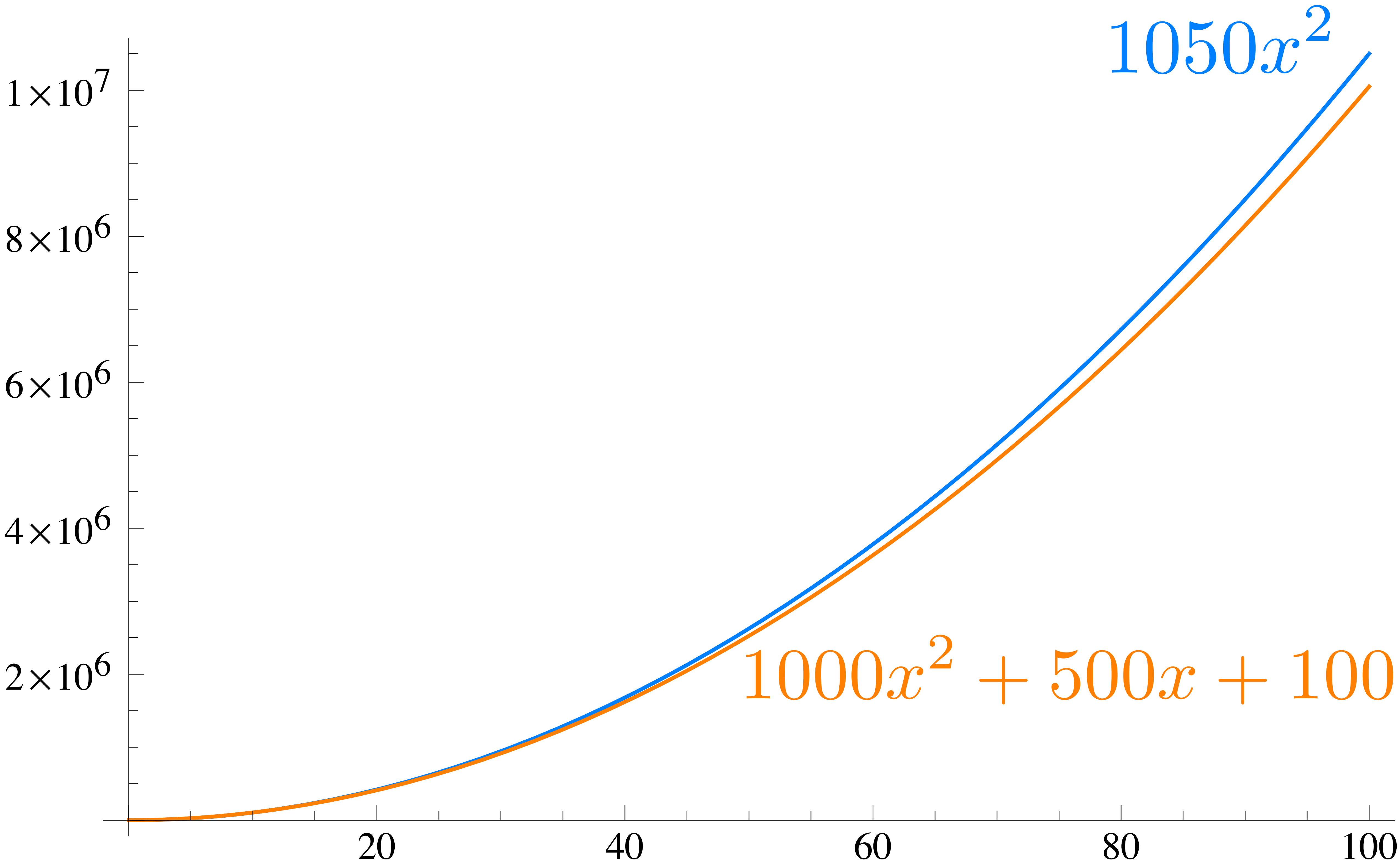
$$f \in \Omega(g)?$$

$$f(x) = x^2$$

$$g(x) = 1000x^2 + 500x + 100$$



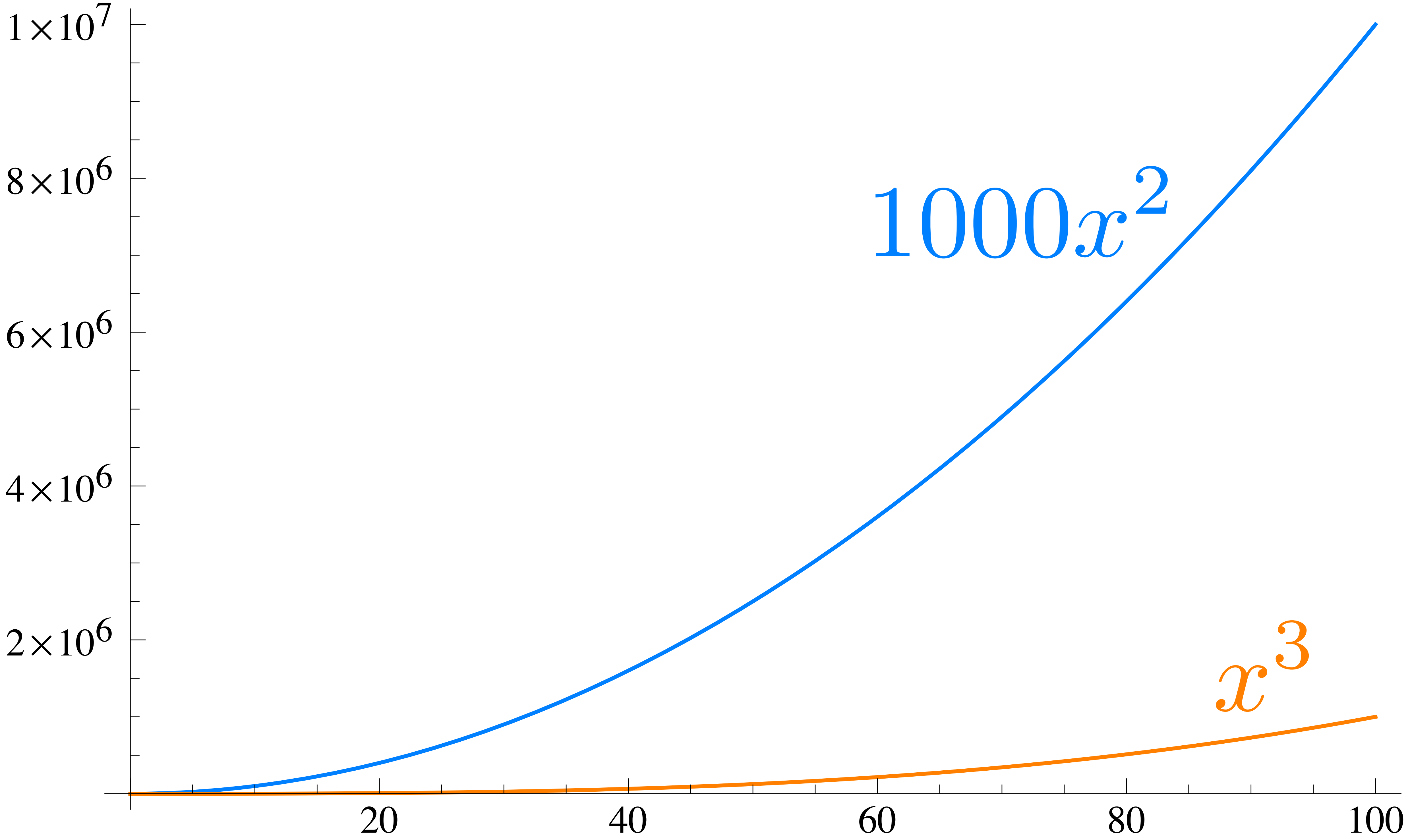
100 000
80 000
60 000
40 000
20 000
2
4
6
8
10
$1000x^2 + 500x + 100$
$1000x^2$

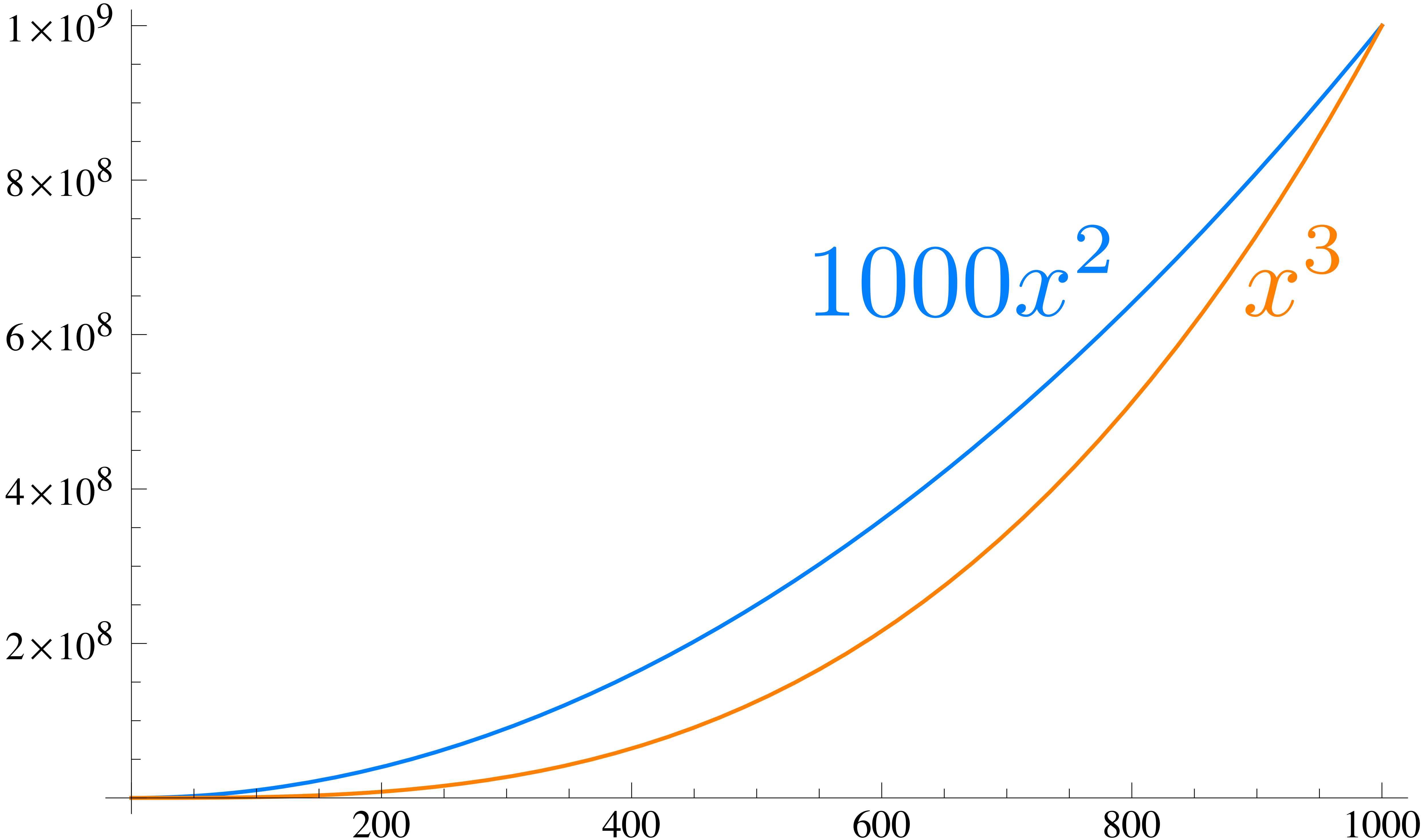


$1050x^2$
$1000x^2 + 500x + 100$
1×10^7
8×10^6
6×10^6
4×10^6
2×10^6
20
40
60
80
100

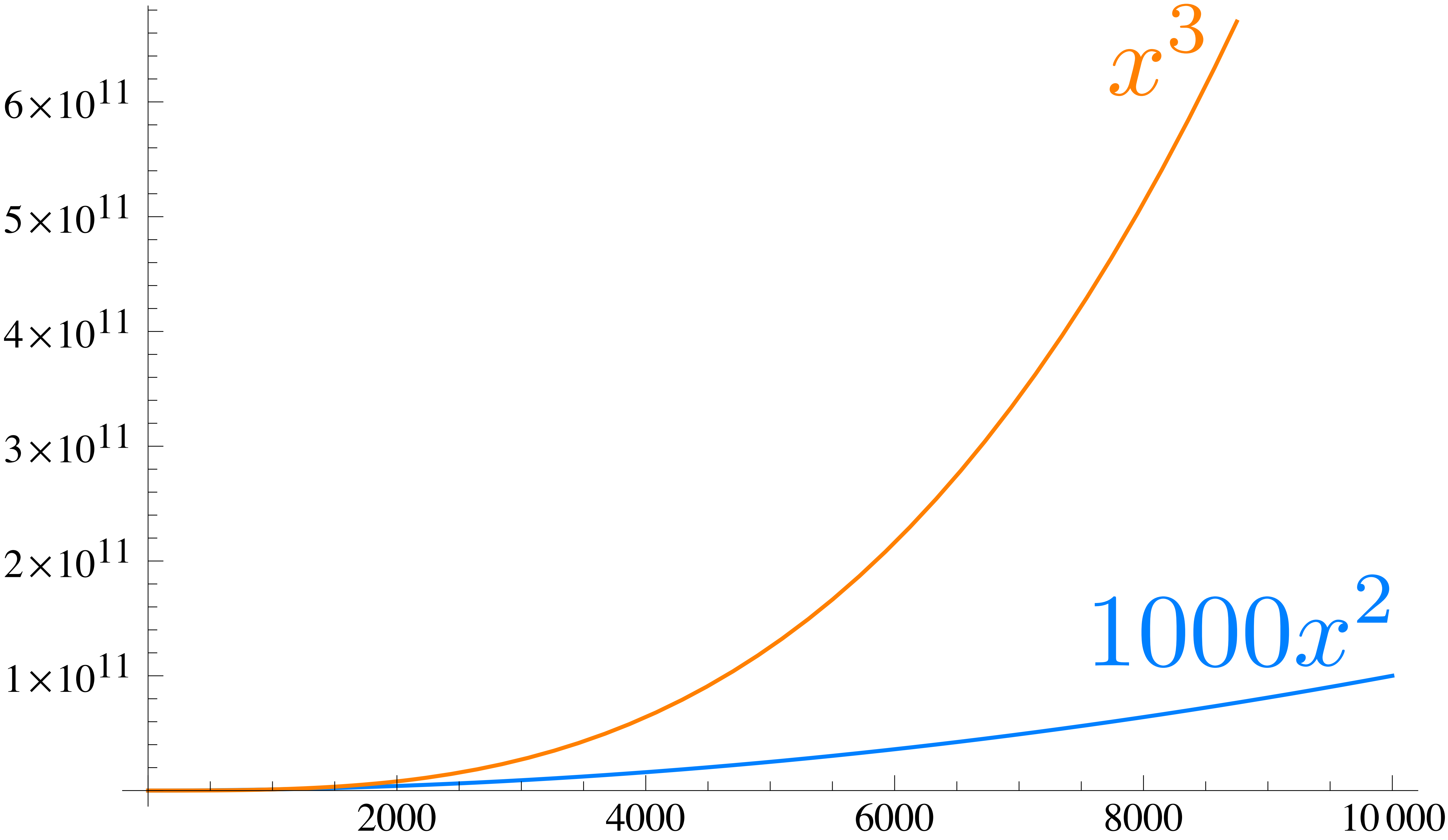
$$1000x^2 + 500x + 100 \in \Theta(x^2)$$

$$1000x^2 \in \Theta(x^3)?$$


1×10^7
8×10^6
6×10^6
4×10^6
2×10^6
20
40
60
80
100
$1000x^2$
x^3



1×10^9
8×10^8
6×10^8
4×10^8
2×10^8
200
400
600
800
1000
$1000x^2$
x^3


x^3
$1000x^2$
6×10^{11}
5×10^{11}
4×10^{11}
3×10^{11}
2×10^{11}
1×10^{11}
2000
4000
6000
8000
10 000

$$g \in O(f)?$$

$$f(x) = 1000x^2$$

$$g(x) = x^3$$

$$g \in O(f)?$$

$$g \in \Omega(f)?$$

$$f(x) = 1000x^2$$

$$g(x) = x^3$$

$$f(x) = 1000x^2$$

$$g(x) = x^3$$

$g \in O(f)$?

$g \in \Omega(f)$?

$f \in O(g)$?

$$f(x) = 1000x^2$$

$$g(x) = x^3$$

$$g \in O(f)?$$

$$g \in \Omega(f)?$$

$$f \in O(g)?$$

$$f \in \Omega(g)?$$

$$f(x) = 1000x^2$$

$$g(x) = x^3$$

$$g \in O(f)?$$

$$g \in \Omega(f)?$$

$$f \in O(g)?$$

$$f \in \Omega(g)?$$

$$f \in \Theta(g)?$$

$$f(x) = 1000x^2$$

$$g(x) = x^3$$

$g \in O(f)$?

$g \in \Omega(f)$?

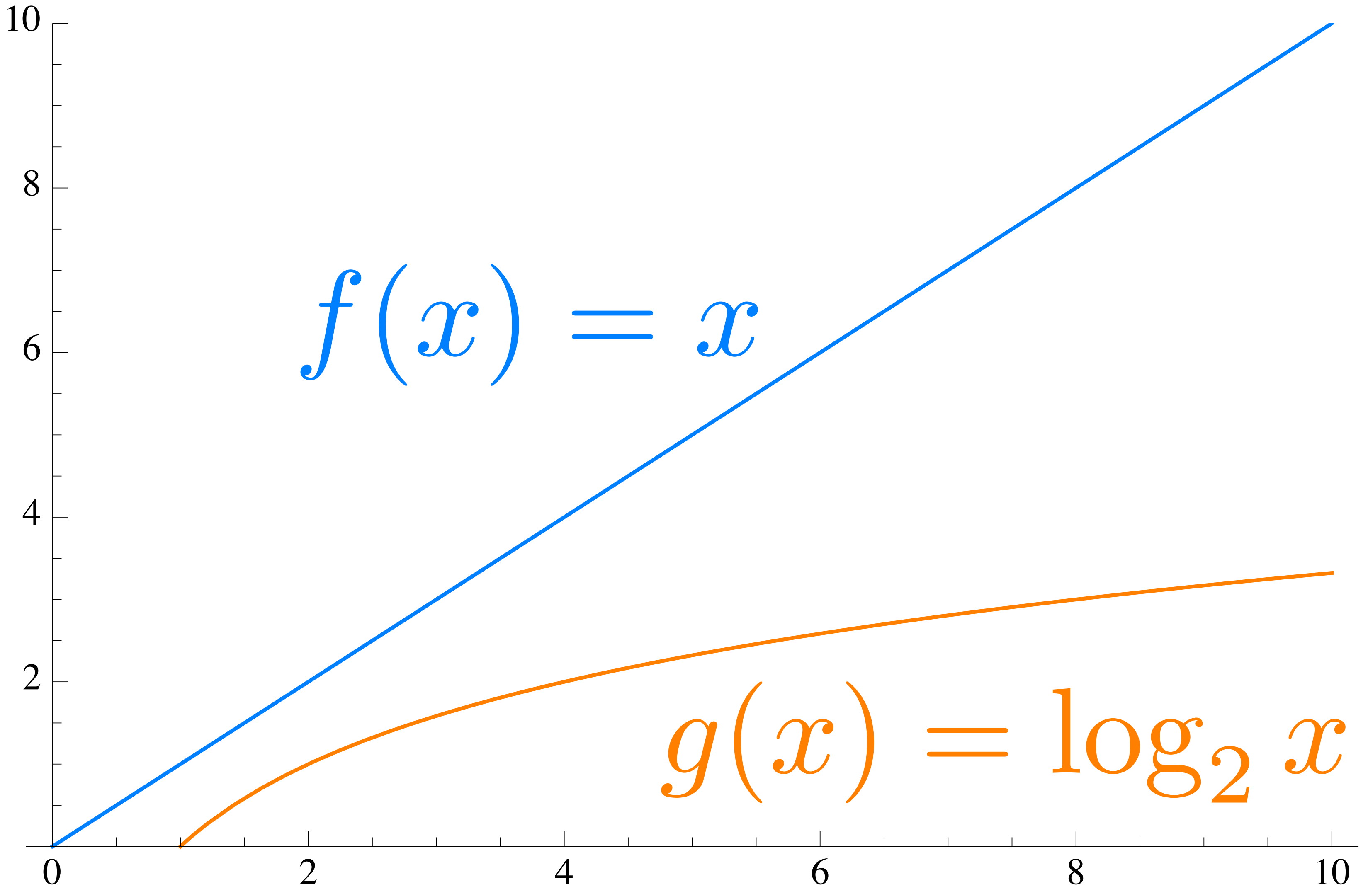
$f \in O(g)$?

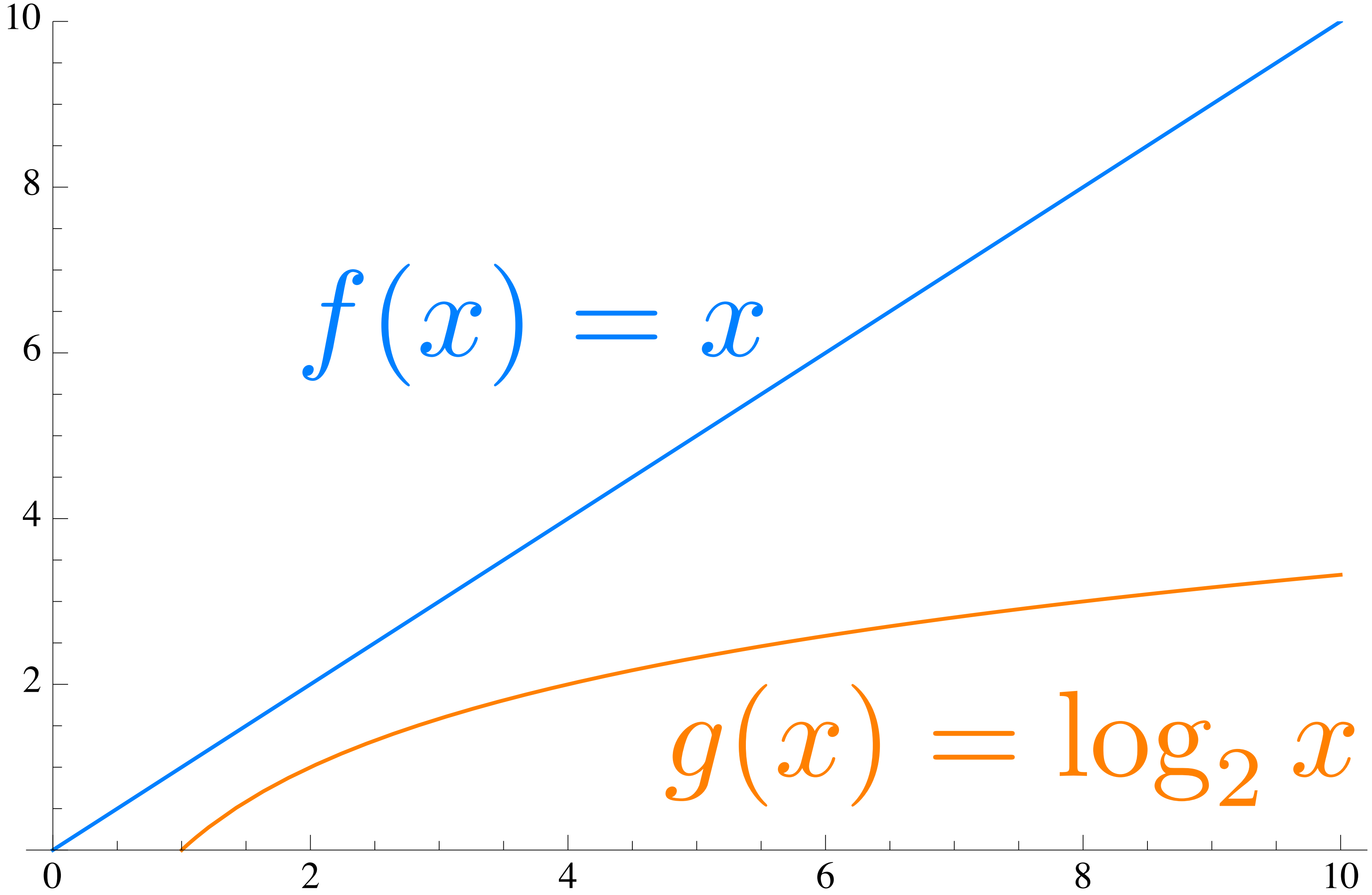
$f \in \Omega(g)$?

$f \in \Theta(g)$?

$g \in \Theta(f)$?

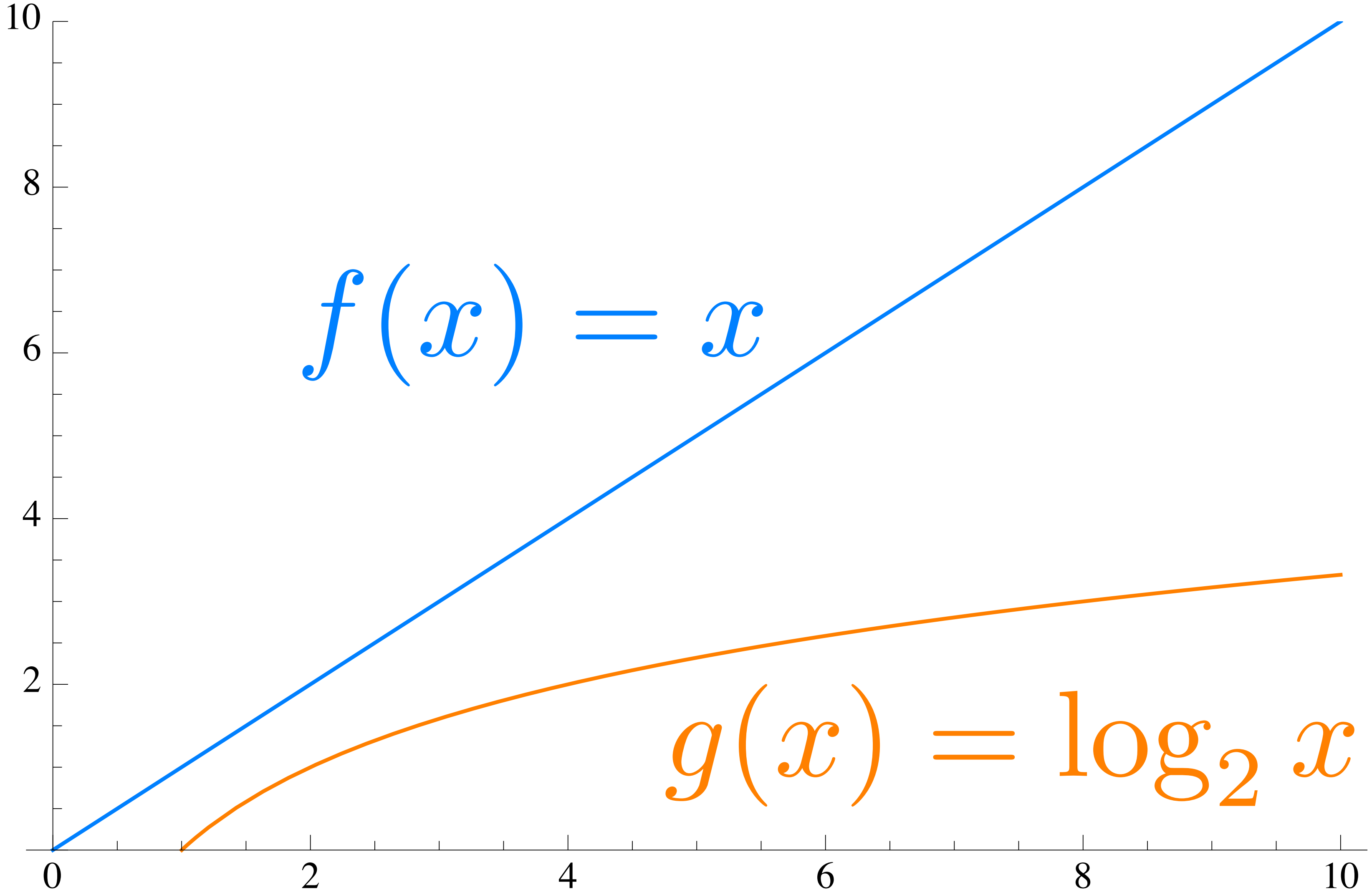
$$f \in O(g)?$$





$f \in O(g)?$

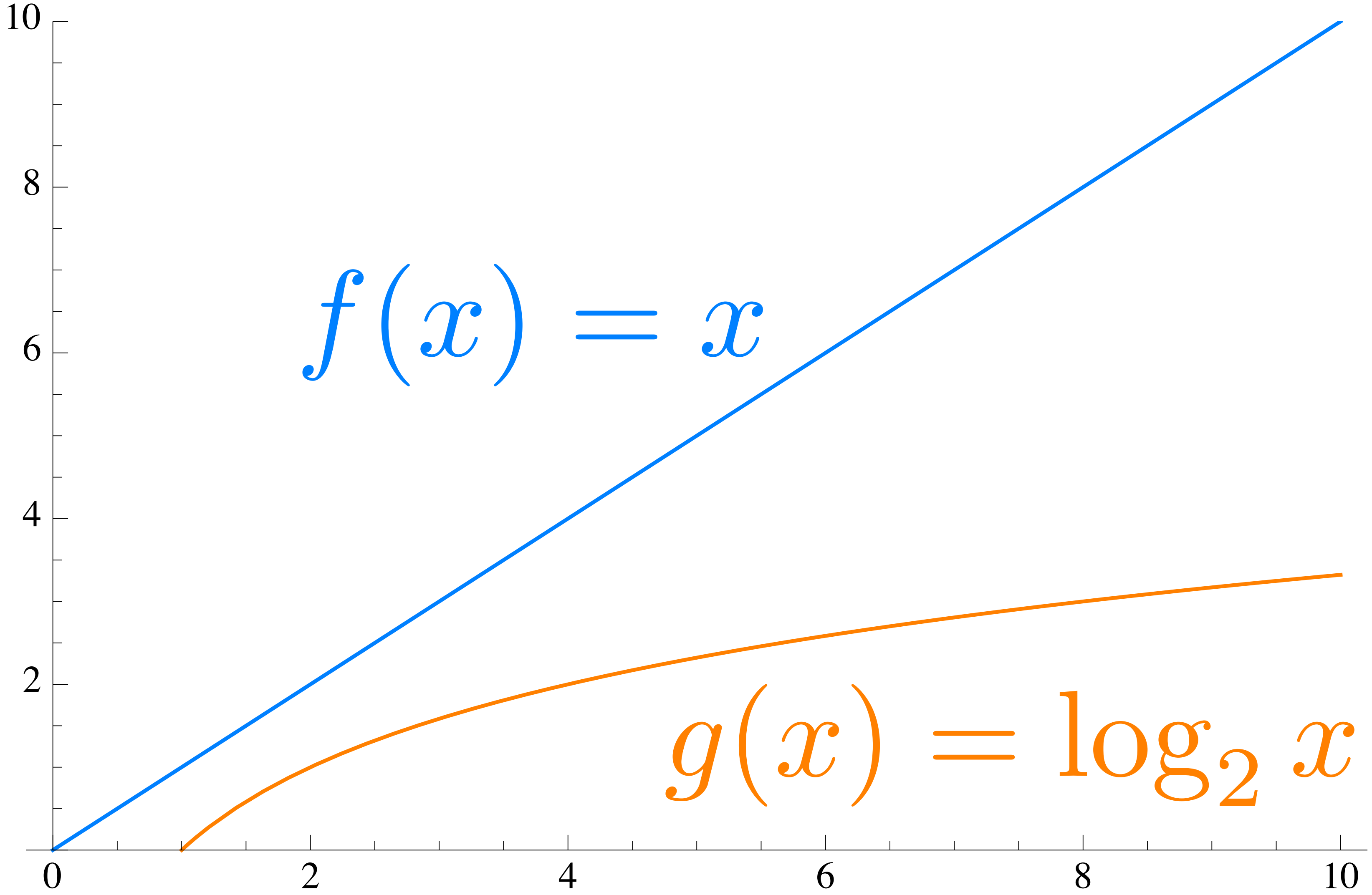
$f \in \Omega(g)?$



$f \in O(g)$?

$f \in \Omega(g)$?

$g \in O(f)$?

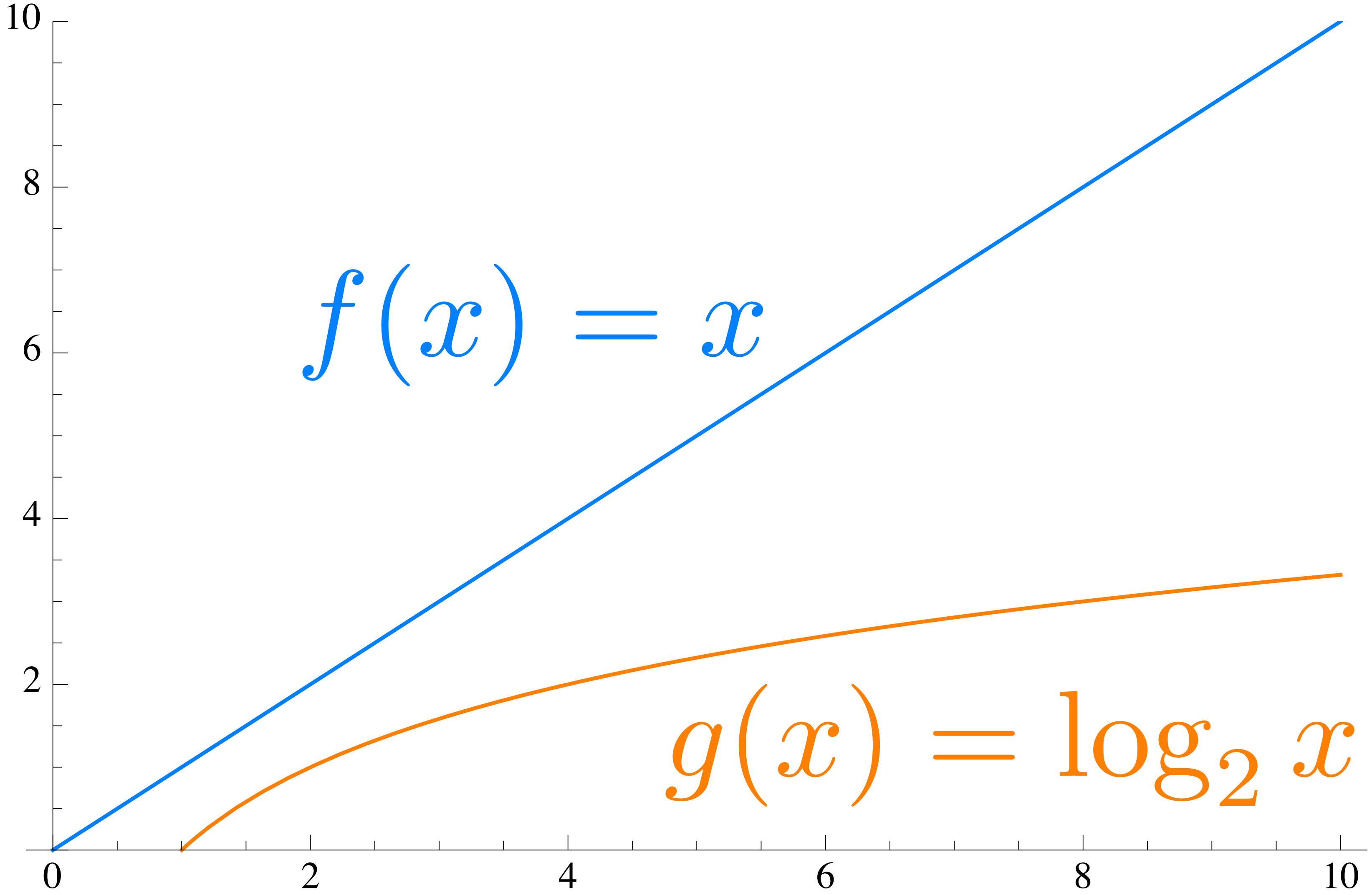


$f \in O(g)$?

$f \in \Omega(g)$?

$g \in O(f)$?

$g \in \Omega(f)$?



$f \in O(g)$?

$f \in \Omega(g)$?

$g \in O(f)$?

$g \in \Omega(f)$?

$f \in \Theta(g)$?

one last exercise

$$f \in O(g)?$$

$$f(n) = n \log_2 n$$

$$g(n) = n^2$$

$$f \in O(g)?$$

$$f \in \Omega(g)?$$

$$f(n) = n \log_2 n$$

$$g(n) = n^2$$

$$f(n) = n \log_2 n$$

$$g(n) = n^2$$

$f \in O(g)$?

$f \in \Omega(g)$?

$g \in O(f)$?

$$f(n) = n \log_2 n$$

$$g(n) = n^2$$

$f \in O(g)$?

$f \in \Omega(g)$?

$g \in O(f)$?

$g \in \Omega(f)$?

$$f(n) = n \log_2 n$$

$$g(n) = n^2$$

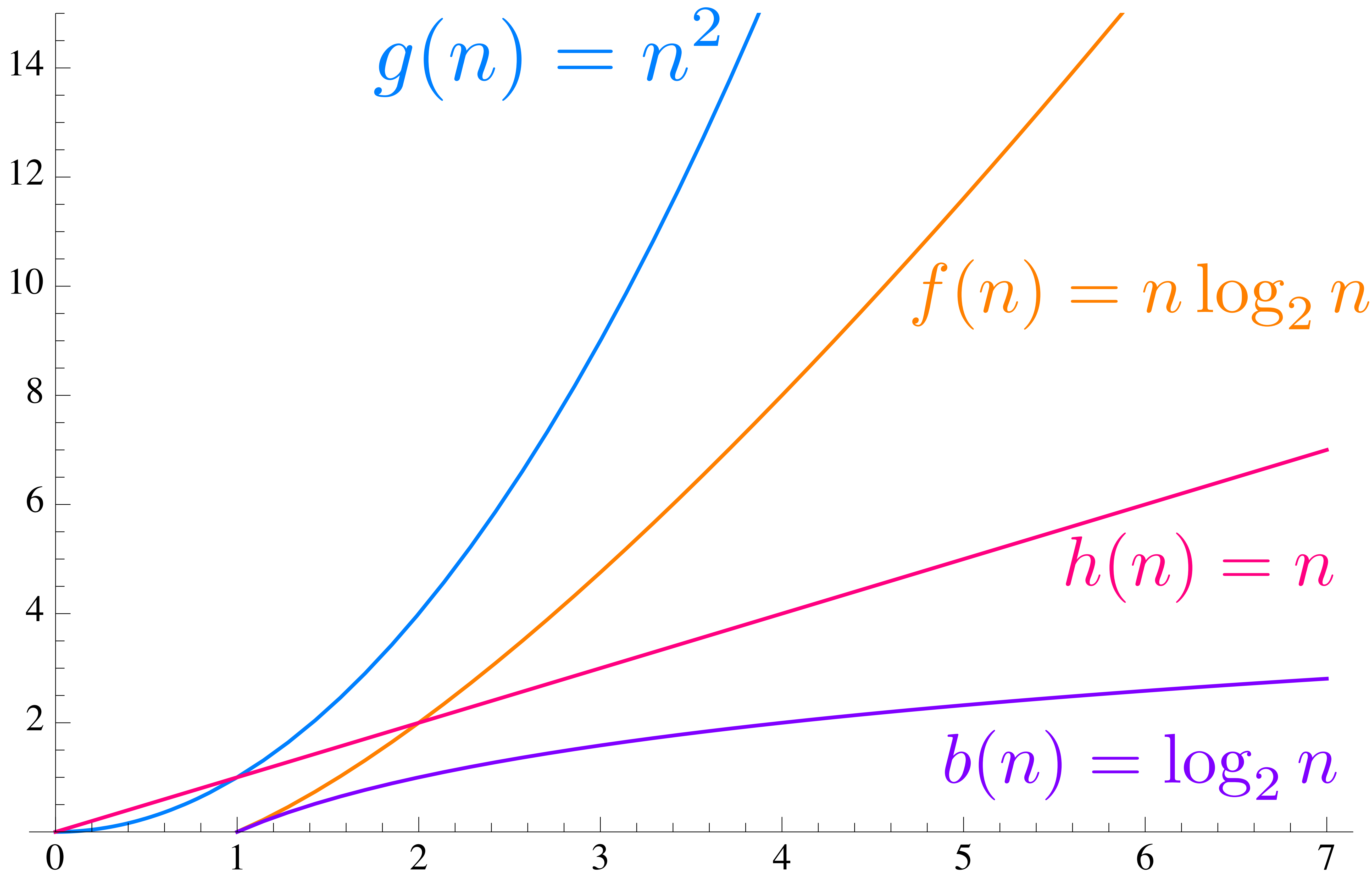
$f \in O(g)$?

$f \in \Omega(g)$?

$g \in O(f)$?

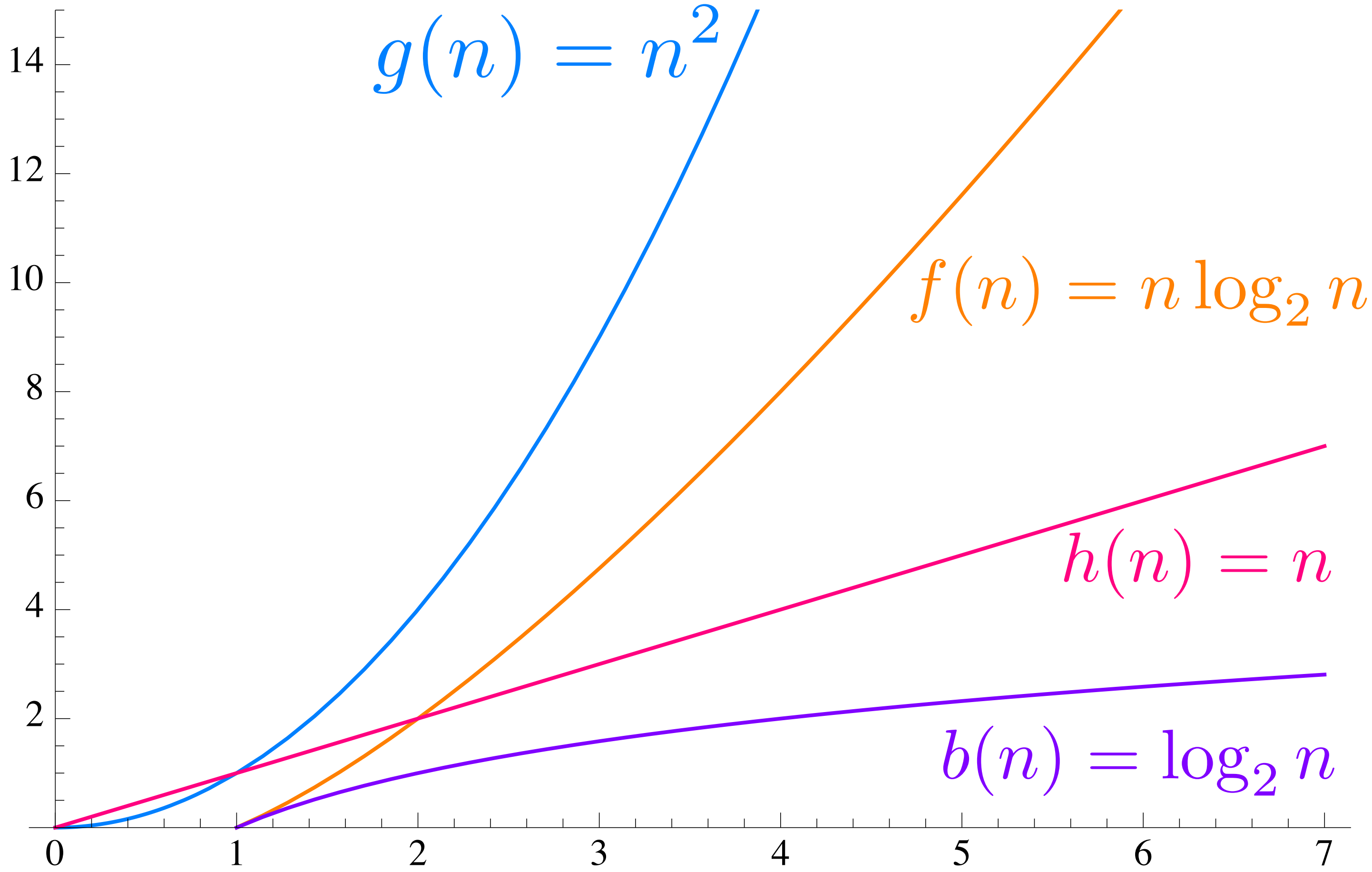
$g \in \Omega(f)$?

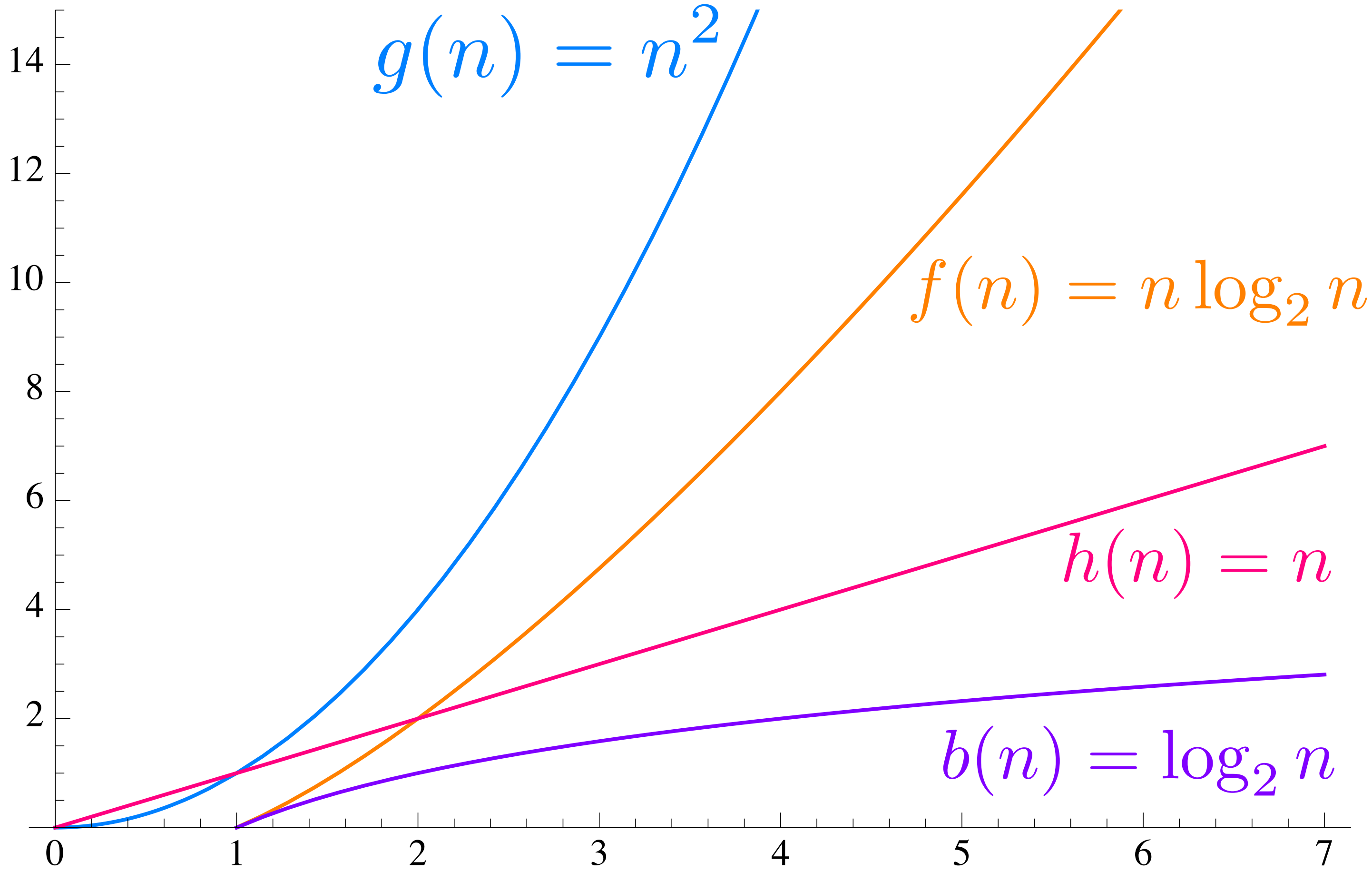
$f \in \Theta(g)$?



$g(n) = n^2$
$f(n) = n \log_2 n$
$h(n) = n$
$b(n) = \log_2 n$
14
12
10
8
6
4
2
0
1
2
3
4
5
6
7

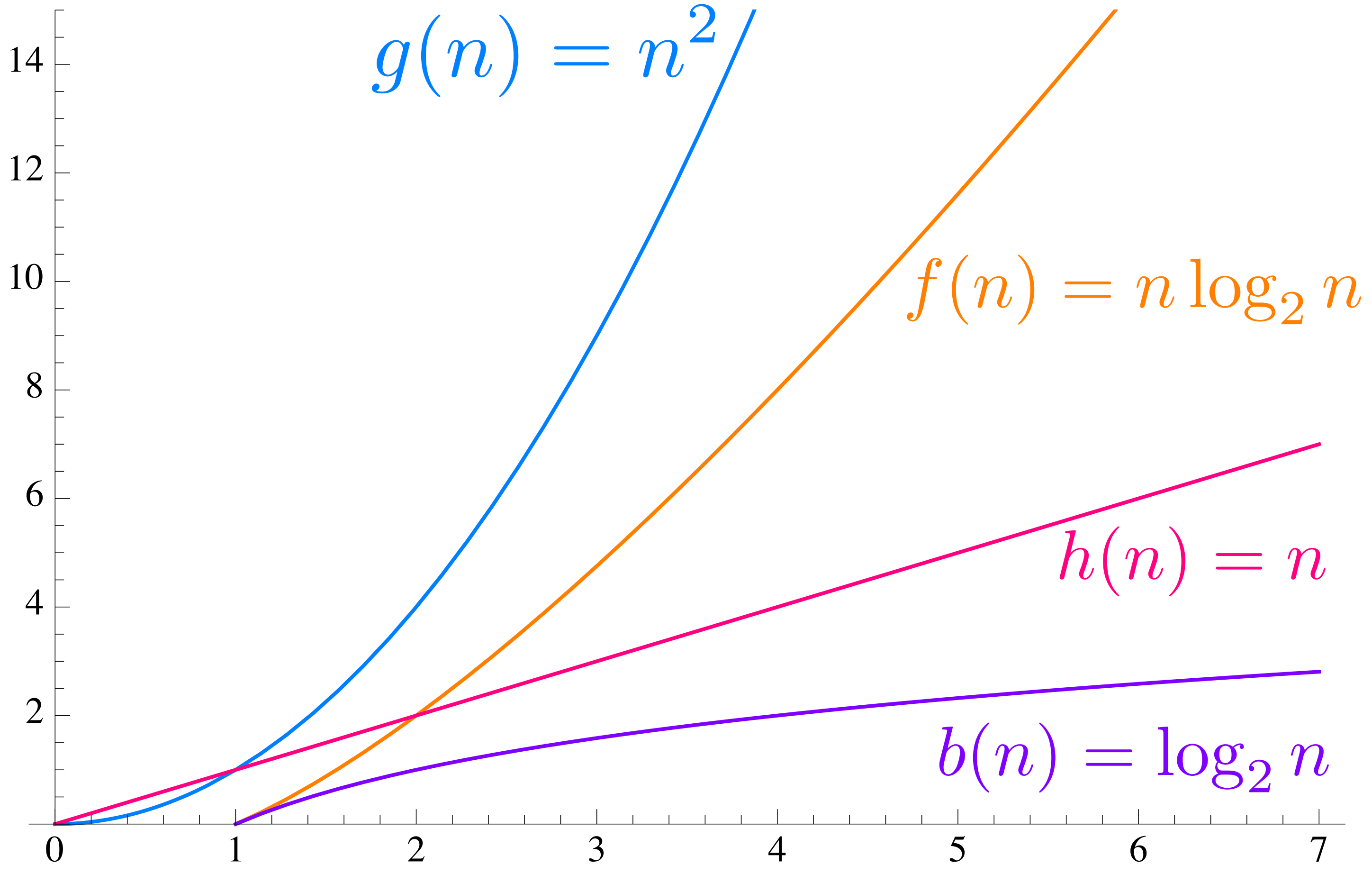
$$n^2 \in O(100 \cdot n \log_2 n)?$$





$n^2 \in O(100 \cdot n \log_2 n)$?

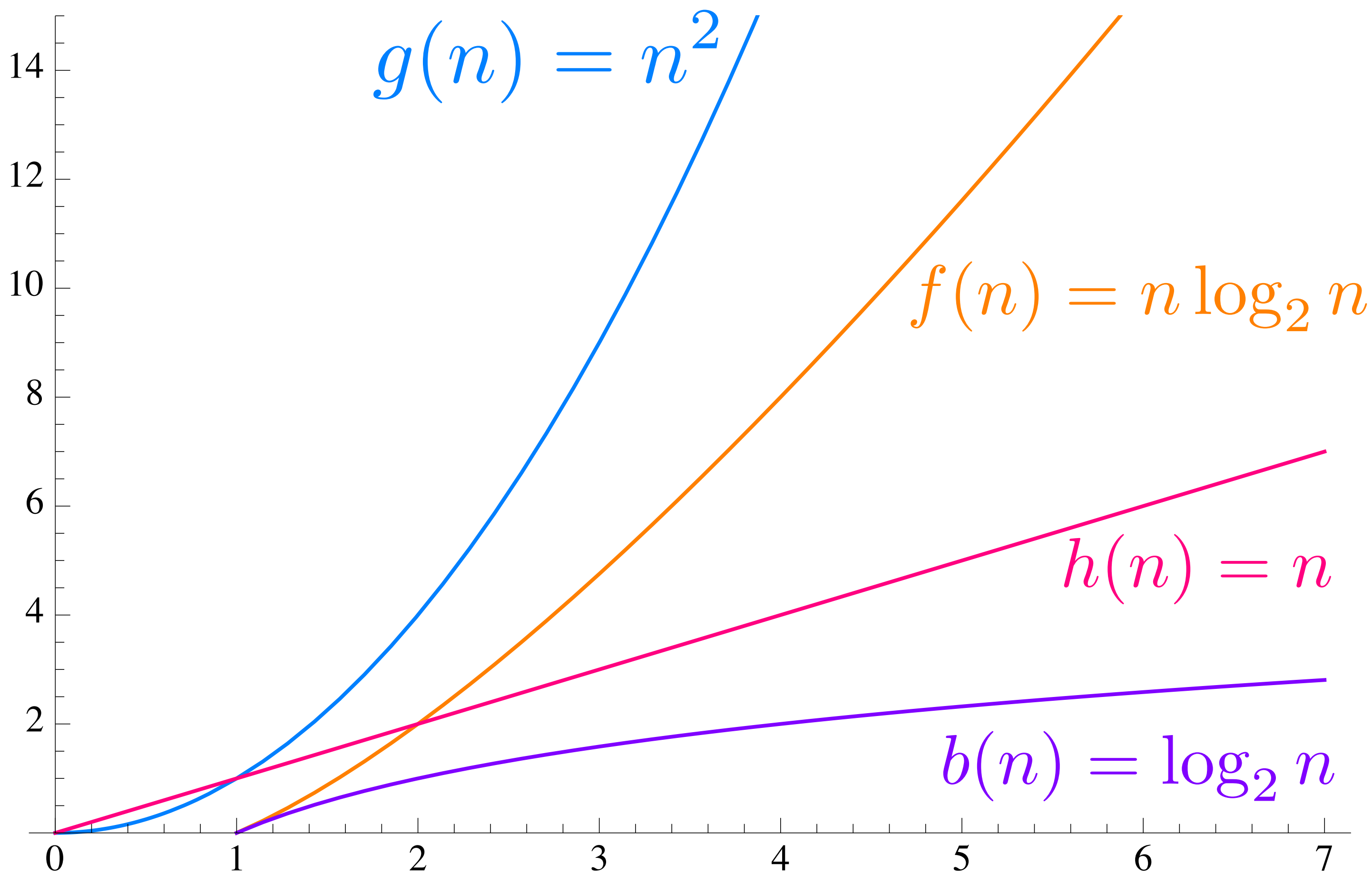
$n^2 \in \Omega(n \log_2 n)$?



$$n^2 \in O(100 \cdot n \log_2 n)?$$

$$n^2 \in \Omega(n \log_2 n)?$$

$$\log_2 n \in O(n \log_2 n)?$$

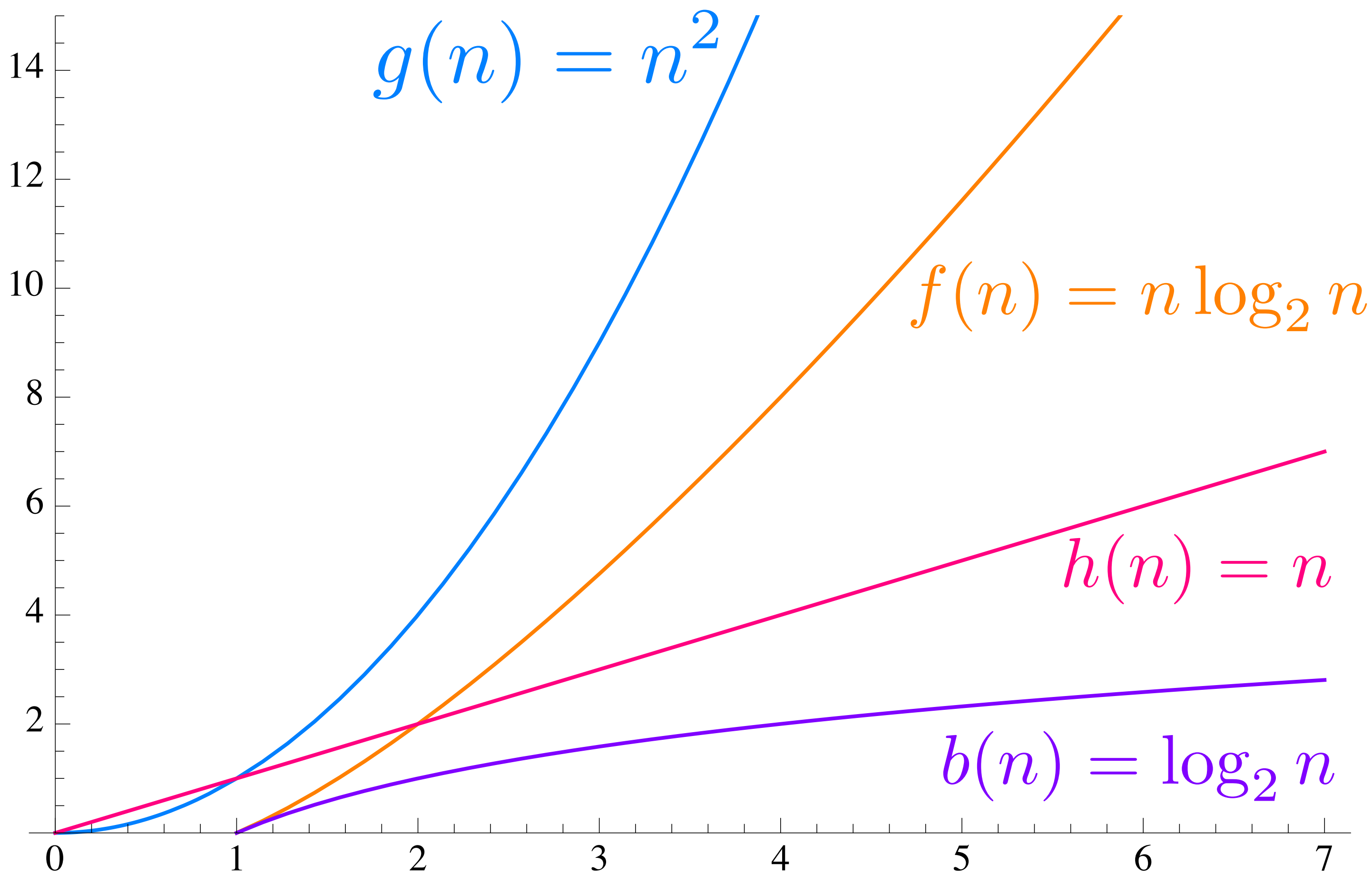


$$n^2 \in O(100 \cdot n \log_2 n)?$$

$$n^2 \in \Omega(n \log_2 n)?$$

$$\log_2 n \in O(n \log_2 n)?$$

$$\log_2 n \in \Theta(n \log_2 n)?$$



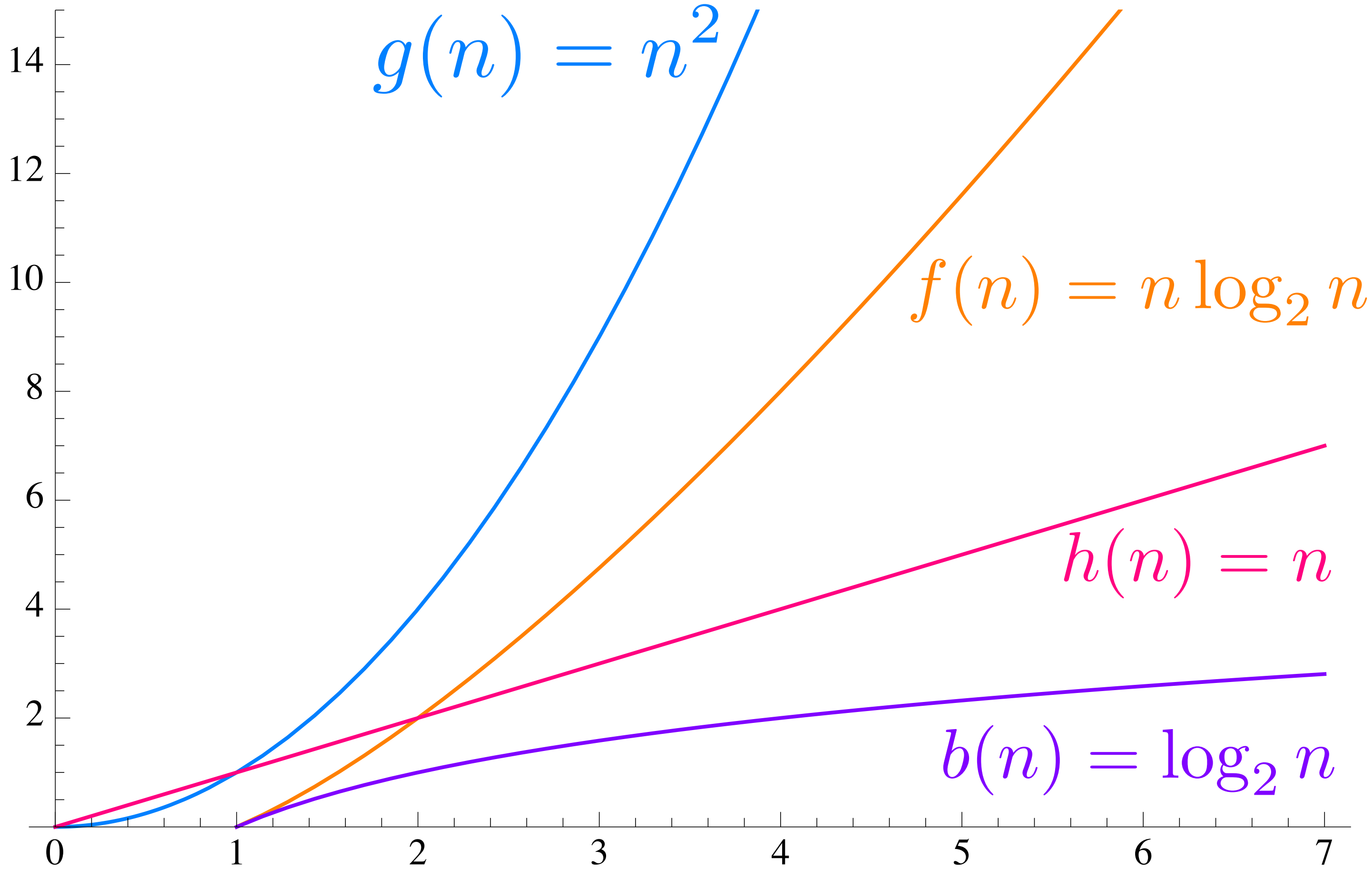
$$n^2 \in O(100 \cdot n \log_2 n)?$$

$$n^2 \in \Omega(n \log_2 n)?$$

$$\log_2 n \in O(n \log_2 n)?$$

$$\log_2 n \in \Theta(n \log_2 n)?$$

$$n \in O(\log_2 n + n)?$$



$$n^2 \in O(100 \cdot n \log_2 n)?$$

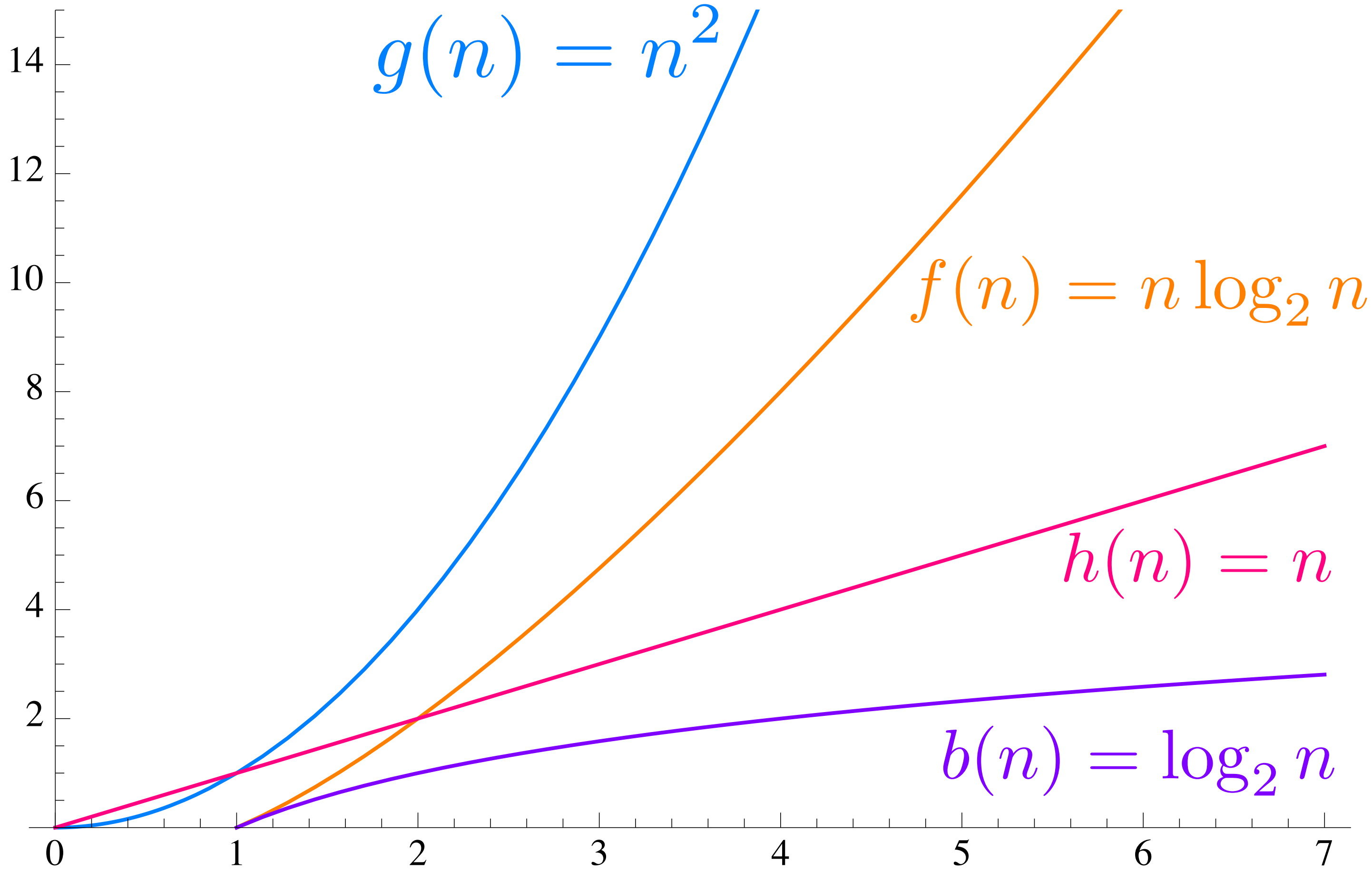
$$n^2 \in \Omega(n \log_2 n)?$$

$$\log_2 n \in O(n \log_2 n)?$$

$$\log_2 n \in \Theta(n \log_2 n)?$$

$$n \in O(\log_2 n + n)?$$

$$n^2 \in \Omega(n^2 + n)?$$



$$n^2 \in O(100 \cdot n \log_2 n)?$$

$$n^2 \in \Omega(n \log_2 n)?$$

$$\log_2 n \in O(n \log_2 n)?$$

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$$n \in O(\log_2 n + n)?$$

$$n^2 \in \Omega(n^2 + n)?$$

$$n^2 \in \Theta(n^2 + n)?$$