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A cash-constrained stochastic inventory model with consumer loans and supplier credits: the case of nanostores in emerging markets

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We consider a small traditional retailer that is managing its inventory under strict cash constraints, mainly because typically informal loans are offered to customers. These stores are widely present in emerging markets, and we refer to them as nanostores (also called 'mom-and-pop stores'). As the suppliers require immediate payments for goods delivered, a nanostore can only replenish products to the level for which it has on-hand cash available. To improve delivery efficiency, a supplier might offer a nanostore credit for its replenishments. However, currently, suppliers are often reluctant to do so as these nanostores quickly go bankrupt or disappear, hence defaulting on all outstanding credits. The objective of this paper is to determine when it is beneficial to offer supplier credits. We propose a multi-period, stochastic inventory model, and numerically compare scenarios with and without supplier credits. Our study shows that even in the presence of this risk, suppliers often have good incentives to provide these credits, even if interest is not incurred. For this to hold, the operations of the retailer should be (a little) profitable in the first place, for which we provide analytical conditions.

Keywords: inventory management; cash constraint; nanostore; supplier credit; customer loan; simulation

1. Introduction

The evaluation and optimisation of inventory policies for fast moving consumer goods (FMCG) has been well studied in the literature. In a typical setting, a retailer, such as a supermarket, is periodically replenished by its suppliers. Based on the sales in past periods, demand forecasts, and costs incurred, many papers provide replenishment policies, based on which expected costs for the retailer and the supplier, service levels and stock-out probabilities could be derived.

In most models, the financial position of the retailer at the moment of replenishment is not taken into consideration (e.g. Zipkin 2000; Axsäter 2006). If the retailer must pay on the receipt of the goods, the amount to be replenished is constrained by the retailer's amount of on-hand cash at that moment. Whereas large retailers often have contracts with their suppliers allowing them to pay later for the goods delivered, examples for pay-on-receipt are found for small, independent stores (Boulaksil et al. 2014). In emerging markets, a large part of consumer goods are sold by traditional, local retail stores, which we will refer to as *nanostores*, also known as 'mom-and-pop' stores or 'high frequency stores' (HFS). Fransoo, Blanco, and Mejia-Argueta (2017) estimate that there are around 50 million nanostores in developing countries, with a market share of around half of the total retail market in many countries. Martinez (2016) claims that nanostores represent 60 to 70% of the Latin American consumer packaged goods market. Moreover, these stores 'fit the needs of emerging consumers quite well' (cf. D'Andrea, Lopez-Aleman, and Stengel 2006); though prices might be higher at a nanostore compared to those found at a supermarket or hypermarket, these consumers may lack transportation means or may be short of cash for their purchases. Empirical studies on nanostores are found in D'Andrea, Lopez-Aleman, and Stengel (2006); Díaz, Lacayo, and Salcedo (2007); Martinez (2016) for Latin America; Lenartowicz and Balasubramanian (2009) for Brazil in particular; Boulaksil et al. (2014) for Morocco; Goldman (2001); Kalish, Roberts, and Gregory (2010) and Zhao et al. (2012) for China, just to mention a few.

Nanostores, often family owned and often operating on only a few square meters, can be opened with a low capital investment, and in many developing countries, rules and regulations provide little or no entry barrier to the market. Contrary to the streamlined and optimised business operations at multinational retailing corporations, and chains of hypermarkets, large department stores, and/or grocery stores, the operations of nanostores are far less sophisticated. Information systems for keeping track of inventory and cash positions, are either absent or not very advanced. Decisions, such as the amounts to replenish for products, are guesswork or left to the supplier. Moreover, these kind of small stores fulfil a role beyond retailing

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in their local community. The owner knows the regular customers, and can provide customers the option to pay later for their shopping. Boulaksil et al. (2014) show in an empirical study that this provisioning of small, informal credits is a common practice; in over 60% of sales transactions, customers rely on loans offered by the nanostore. No interest is incurred for these small informal loans, and there are no contracts involved. By providing these loans to its customers, the nanostore owner can increase sales. However, this postponement of payments impacts the on-hand cash available, which is a complicating factor in the operations of the nanostore. While the product has already been sold to the customer, (part of) the payment is received later, lowering the retailer's ability to pay for replenishments from its suppliers. Moreover, not all outstanding loans to consumers will eventually be paid back.

The suppliers, often large FMCG manufacturers, typically expect immediate payment for products delivered to a nanostore. This is because nanostores often pop-up and vanish quickly (Martinez 2016), either because of the low entry barriers, or due to unprofitable operations. Therefore, the supplier does not allow for any delay in payments. Upon a replenishment opportunity, the retailer will aim to stock up products to desired inventory levels. However, a limited amount of cash on hand might refrain him from doing so. The retailer could only buy the amount for which it has enough cash on hand, which might be lower than desired. The amount of cash that the retailer has available at a given moment in time, depends (among other factors) on past sales, the amount spent on past replenishments, loans issued to consumers in the past, and the amounts paid back by consumers on these loans. Therefore, the constraint on the amount that can be bought from the supplier depends heavily on the past operations of the nanostore.

Though more advanced strategies exist (Boulaksil and Belkora 2017), for many FMCGs it is a common practice that the supplier visits the nanostore with a (small) truck full of goods to replenish the retailer. This is done on a periodical base, such as daily, independent of any information from the store on sales or an expected amount to be replenished. The amount that the retailer can replenish is constrained by its on-hand cash at the moment the supplier is present. If on a visit of the supplier, none or a lower amount than requested of products are sold to the retailer due to lack of cash on-hand, this is referred to as an *empty delivery* or *partial delivery*. These are costly to the supplier, since the fixed cost and time investment to reach the nanostore have already been made. Therefore, to enable the retailer to acquire more (or at least any) items during the replenishment opportunity, the supplier might benefit from providing the retailer some credits, which we referred to as supplier credits. That is, the supplier provides the retailer a small loan to pay for the replenished items. In this way, the retailer can replenish its inventory to the desired level, while paying for it later, and the supplier can sell more products to the retailer on this run.

Though not currently a common practice, these supplier credits could create a win-win situation for the supplier and the retailer. Because the retailer is able to stock more using the supplier's credits, the availability of the products will be higher, and so will the sales. As the retailer's sales will be higher, so will the supplier's sales to the retailer. Hence, the supplier can profit from the credit it is providing. Moreover, in this way the supplier provides the retailer a better chance of economic survival.

Despite the potential benefits, it is a question under which conditions the supplier is indeed better off provisioning these credits to the retailer. Namely, the supplier must be careful in offering credits, as there is the risk that the nanostore will not pay back its loan. The operations of the nanostore might not be profitable in the first place. Moreover, the financial position of the nanostore might not be that prosperous, putting it at a relatively high risk of bankruptcy. Furthermore, the small retailer might suddenly close for other reasons, which could have a relative high probability, especially since initial investments have been relatively low. These discontinuations are frequent, according to Martinez (2016): 'most of the micro and small firms [...] in Latin America survive for less than a year'. Bankruptcy and the discontinuing of the retailer are the risks the supplier is facing (Ho et al. 2015). To protect itself from these risks, the supplier might charge a small amount of interest on the credits offered. Alternatively, the supplier should make sure that its benefits gained from the credits, summed over all the retailers it is supplying, are such that it can bear the risk of one or a small number of retailers not paying back in the long run.

We remark here that the supplier is usually the only option for the nanostore to get credit, since the nanostores often have no or poor access to traditional financing opportunities like loans from a bank. Even if they do, conditions might be poor (e.g. extremely high interest rates). For more detail on the impact of the difference between external financing and trade credits (see Kouvelis and Zhao 2012).

The operations of the nanostores are an example of a retailer's inventory management under cash-flow constraints. For its replenishment decisions, the retailer is limited by the amount of on-hand cash available. This amount depends on the operations (such as the amounts of sales, credits received, or loans issued) in previous periods. Consequently, the model becomes a stochastic process with a long memory, which is the intrinsic difficulty of these kind of models. This also means that these models are considerably different from inventory models with supply capacity constraints (see, e.g. Levi et al. 2008, for an overview), because of the dependency of the 'capacity constraint' on previous operations.

Though in a perfect capital market, operational and financial decision should not interfere with each other (cf. Modigliani and Miller 1958), the operations of small firms such as nanostores, which have no or limited access to credit, are in fact heavily affected by constraints resulting from their cash-flow. Therefore, it is essential to incorporate cash-flow into the model, and the decisions on operations and cash affect each other. The comparison of simultaneous, coordinated decision-making versus separate decision-making on operations and finance in such a setting, is studied in Li, Shubik, and Sobel (2013), showing that the differences in costs between both scenarios can be significant. Hu, Sobel, and Turcic (forthcoming) show that coordination leads to a higher market value for the equity of the retailer.

A cash-constrained inventory model that is related to the one presented in this paper, is studied in Chao, Chen, and Wang (2008) for a periodic, finite horizon model with stochastic demand. The goal is to maximise expected terminal wealth at the end of the planning period. The amount that can be replenished at decision epochs, is constrained by the on-hand cash. They allow for deposit interest on excess on-hand cash after replenishments has been paid for, and assumes stationary demand and deposit interest rate per period. The optimisation of the amount to be replenished in every period is studied, proving the optimal reordering policy to be a modified base-stock policy, which can be characterised by a single, dynamic threshold.

Katehakis, Melamed, and Shi (2016) consider a model similar to Chao, Chen, and Wang (2008), and add the possibility of the retailer taking out a loan (at some debit interest rate). Furthermore, they allow for non-stationary demand, and non-stationary deposit interest and loan interest rates. This makes the problem into a simultaneous operations-finance problem. They prove the optimal control policy, which can be characterised by two dynamic thresholds. Gong, Chao, and Simchi-Levi (2014) show for a similar setting how the optimal policy depends on the equity (sum of capital level and the value of on-hand inventory at purchasing cost) and characterise the optimal policy by four intervals on the equity level. On a related note, Babich and Sobel (2004) are interested in how the operational and financial decisions should be coordinated with the goal of a successful initial public offering (IPO), which they consider as a stopping time of the model, while Buzacott and Zhang (2004) show the importance of the joint production-financing decisions for a start-up setting to grow. For newsvendor type models, Dada and Hu (2008) and Kouvelis and Zhao (2011) derive results, including the insight that in the presence of interest on funds borrowed, the newsvendor procures amounts smaller than would otherwise be optimal. For a literature overview, see Birge (2014) and references therein. For a more elaborate discussion on trade credits and their impact on the supply chain as a whole, we refer the reader to Cai, Chen, and Xiao (2014); Seifert, Seifert, and Protopappa-Sieke (2013); Petersen and Rajan (1997); Shah and Cárdenas-Barrón (2015) and Van der Vliet, Reindorp, and Fransoo (2015), and the references therein. For more background on the management of cashflow (see Golden, Liberatore, and Lieberman 1979).

A way of providing trade credit is to allow a delay in payment after receipt of goods. Protopappa-Sieke and Seifert (2010) show the significance of payment delays, in a cash-constrained inventory model: an increase in upstream payment delays leads to a decline in the decreasing operational costs and favours the system's operations. Payment delays are also allowed in similar models considered in Chung (1998); Shinn and Hwang (2003); Moussawi-Haidar and Jaber (2013); Zeballos, Seifert, and Protopappa-Sieke (2013); Chen, Cárdenas-Barrón, and Teng (2014); Aljazzar, Jaber, and Moussawi-Haidar (2016); Seifert, Seifert, and Isaksson (2017).

Loans offered to customers are a common practice in many nanostores, and therefore including these is essential to our model. The literature on quantitative multi-period models for cash-constrained inventory management including this, however, seems to be limited. A model close to the one described in this paper is found in Luo and Shang (2014). They include both the offering of trade credit to customers, as well as the possibility of receiving trade credits from the supplier. They answer the question of how the firm should manage its inventory to minimise its total cost (inventory, purchasing, holding, back-order and a default penalty). In case, the period after which the supplier must be paid (the so-called payment period) is shorter than the period in which customers will pay back (the collection period), they prove that the optimal control policy can be described by two parameters, which is a hybrid of the classic base-stock policy and a cash-constrained base-stock policy. For the other case, when the collection period is longer than the payment period, they propose a heuristic policy depending on three parameters, and provide a lower bound to the optimal cost.

Also, the work of Bendavid, Herer, and Yücesan (2017) is closely related to the model proposed in this paper. Like Luo and Shang (2014), they study how inventory replenishments are affected by the payment and the collection periods. They, however, do not allow for unfulfilled payments, and they assume that demands that cannot be met directly, are back-ordered. They propose an algorithm, which is a simulation-based procedure, for determining the order-up-to-level. Furthermore, they discuss the impact of the variables, and they consider relaxing the constraint on the working capital. Biskup, Simons, and Jahnke (2003) consider the combination of customer loans and supplier credits from a different perspective. They derive the optimal production quantities in a production environment with set-up costs.

The setting of our work is most similar to those in Luo and Shang (2014) and Bendavid, Herer, and Yücesan (2017), however, the focus differs. While these two works investigate reorder policies for the retailer, we investigate whether the supplier should provide credits. The supplier, which we model explicitly, should consider that, in the setting of nanostores, the retailer has a high risk of bankruptcy or discontinuing its business, consequently defaulting on all outstanding loans. Furthermore, our model is based on empirical research on nanostores (cf. Boulaksil et al. 2014) and more realistic for such a retail setting: we take unsatisfied demand to be lost, incorporate that parts of customer loans are lost, and we assume an order-up-to policy. The latter more closely resembles practice, as typically operations at a nanostore are far from sophisticated and nanostores have very limited shelf-space per product. Furthermore, in practice, suppliers might often be hesitant to provide nanostores with trade credits, demanding immediate payment upon delivery of goods.

While there are worldwide millions of nanostores operating on a tight budget providing consumer loans, the current state of knowledge for these models seem to be scarce. There seems to be no clarity about whether and under which conditions the supplier should support the nanostore by providing it credits to be used for replenishments. Answering this question is the main goal of this study, and is particularly of interest to FMCG manufacturers in these markets. For this, we first develop a mathematical model of the retailer's cash and inventory position. We consider a periodic time model over a finite horizon, focusing on a single product, where the retailer has the opportunity to replenish at the start of every period to meet the stochastic customer demand it is facing. However, the amount to be replenished is constrained by the amount of on-hand cash at the start of the period. We incorporate in our model that only part of the sales results in cash received directly, whereas for the remainder, consumer loans are offered and paid back later (or not at all). We explicitly model these consumer loans into a cash-constrained retailer inventory model. We allow for the supplier to provide the retailer with a small loan to cover for a lack of cash to fully replenish its inventory. We focus on the comparison of the model without and with the supplier credits, measuring when the nanostore, when the supplier and when the system as a whole (nanostore and supplier) are better off under a setting with these credits. Benefits gained by either the supplier or the nanostore could be shared between both parties, which is a practice typically referred to as 'buyer-supplier coordination' (cf. Bensaou 1999). This coordination, however, is beyond the scope of this study. See Lee and Rhee (2010, 2011); Moon, Feng, and Ryu (2015) for more context on coordination in the setting of cash-limited inventory models.

The main contribution of this work is three-fold: (1) we develop a mathematical framework to model the setting of a nanostore operating under both customer loans and supplier credits. (2) We derive analytical conditions on whether the operations of the nanostore are profitable in the first place. (3) We discuss under which conditions the supplier should provide credits to the retailer, and show how these credits are beneficial to the retailer as well as to the supplier. In this way, we create insights into the operational characteristics of these systems. The main finding of our study is that, even in the presence of an elevated risk that the retailer discontinues its operations, if the operations of the retailer could be profitable in the first place, it is still beneficial for the supplier to provide trade credits to the retailer under specific circumstances.

The rest of this paper is organised as follows. We start by introducing the model in Section 2, where we discuss both the scenarios *without* and *with* supplier credits. We illustrate the dynamics and typical behaviour of the system in Section 3. For the non-salary case, we derive analytical expressions on whether the retailer's business is viable in Section 4. In Section 5, we show the impact of the probability that the retailer discontinues its store. The results of our numerical analysis are found in Section 6. Finally, we draw conclusions in Section 7. This section contains a discussion of our findings, managerial insights and directions for further research.

2. Model and notation

一个阶段为一天 单产品

We consider a small retailer and model its operations as a periodic review, finite horizon model, where the length of a period corresponds to a day. Let $t = 0, 1, 2, \ldots, T$ be the index for the period, where T is the length of the planning horizon. The retailer is visited every day at (approximately) the same time by a (small) van from the supplier filled with goods. This is the sole replenishment opportunity, and the start of a period coincides with this visit. That is, the replenishment takes place at the start of the period, after which demand is realised and satisfied if possible.

Focusing on a single product, the inventory level at the retailer upon the visit of the supplier's truck at the start of period t is denoted by I_t. The unit of inventory here and throughout the sequel is taken to be one consumer unit. The retailer has a target inventory level S, which can be calculated by using demand and a cycle service level τ or be the result of shelf space limitations. If $I_t < S$, the retailer aims to replenish $S - I_t$ units, however, its on-hand cash level might be insufficient to finance this transaction when the supplier requires a direct payment of the goods delivered.

补货水平 S 可以轻易算出或是既定的

We consider two scenarios: (1) the retailer only replenishes its inventory with the amount it can directly pay for, resulting in a partial delivery; (2) the supplier provides the retailer credits exactly enough for the retailer to replenish up till level S. We refer to the first scenario as the model *without* supplier credits, in which the nanostore is operating under cash-constraints, and the second as the model *with* supplier credits. Below we introduce both models in detail. After that, we discuss the relevant performance characteristics. All notation is summarised in Table 1.

根据是否包含商业信用，考虑两种情形

2.1 *Model without supplier's credits*

Assuming that the supplier is not willing to provide the retailer with any credits, upon a replenishment opportunity the retailer can only procure as many units as for which it has cash on-hand. The on-hand cash of the retailer at the start of period t is

Table 1. Notation.

t	Index of time period, $t = 1, \ldots, T$
T	Time horizon (number of periods in the horizon)
I_t	On-hand inventory at the start of period t
C_t	On-hand cash at the start of period t
B_t	Retailers outstanding loan at supplier at the end of period t
r_{year}, r	Yearly, resp. daily, interest rate charged by supplier
Q_t	Amount requested by the retailer for replenishment in period t
R_t	Amount replenished in period t
D_t, λ	Customers demand in period t, which is Poisson distributed with rate λ
F_t	Demand fulfilled in period t
S	Target order-up-to level
τ	Target cycle service level
c_S	Unit acquisition cost for the supplier
c_R	Unit acquisition cost for the retailer (equals unit selling price of the supplier)
p	Customer purchase price (equal unit selling price retailer)
θ	Retailer's profit margin (ratio of selling and acquisition price): $\theta = \frac{p}{c_R}$
γ	Fraction of fulfilled customers' demand paid immediately
ζ	Fraction of customers' loans that are never paid back
J	Number of periods over which customers pay back their credits
χ	Per period probability of the retailer discontinuing its operations
π^{max}	Expected monthly profit in the equivalent non-cash constrained model
π^{target}, β	Target monthly salary level, which can be expressed as a percentage β of π^{max}
$\pi_m, \pi_\varphi^{tot}, \pi_\psi^{tot}$	Salary in month m, $m = 1, \ldots, \lfloor T/30 \rfloor$, and the sum over all full months within the planning horizon in the scenarios without and with supplier credits
φ_t^R, ψ_t^R	Retailers net wealth (excluding salaries) at time t in the scenarios without and with supplier credits, respectively
φ^R, ψ^R	Increase in retailers net wealth (excluding salaries) over planning horizon in the scenarios without and with supplier credits, respectively
$\varphi_t^S, \psi_t^S, \varphi^S, \psi^S$	Idem ditto for supplier
Δ_R, Δ_S	Difference in increase of the retailers net wealth (excluding salaries), resp. suppliers net wealth, over the planning horizon, for the scenario *without* supplier credits minus that for the scenarios *with* supplier credits
Δ_π	Likewise for retailers salaries
Δ_{TOT}	Sum of Δ_R, Δ_π, and Δ_S

各阶段进货价，销售价格都是恒定的

用 C_t 表示阶段 t 的初始资金水平

denoted by C_t. The store wishes a replenishment of $Q_t = S - I_t$, however, with a unit price of c_R, it can procure at most $\lfloor C_t/c_R \rfloor$ units. Therefore, R_t, the amount replenished by the supplier to the retailer at the start of time period t, is

$$R_t = \min\left(\left\lfloor \frac{C_t}{c_R} \right\rfloor, S - I_t\right), \tag{1}$$

where the supplier's truck is assumed to have ample units available. The inventory level immediately after replenishment is $I_t + R_t$.

The retailer faces a daily customer demand of D_t units in period t, these being independent and identically distributed (i.i.d.) random variables. We assume that D_t follows a Poisson distribution with mean $\lambda > 0$. Unmet demand is lost and hence F_t, the fulfilled demand in period t, is given by: lost sale 的情况

$$F_t = \min(D_t, I_t + R_t). \tag{2}$$

The retailer offers informal credit to customers, by allowing them to pay for (part of) their purchases at a later moment. The fraction of demand paid immediately is denoted by $\gamma \in [0, 1]$. With a consumer purchasing price denoted by p (where $p > c_R$ to avoid trivialities and denoting $\theta = p/c_R$), the cash immediately generated from sales in period t equals $p\gamma F_t$. The remainder will be paid by customers over a time period of the next J days. However, not all outstanding balances will be paid back by the customers. Denote by $\zeta \in [0, 1]$ the fraction of the credits offered that are never paid back. Hence, in total, the retailer will receive $p(1-\gamma)(1-\zeta)F_t$ later for the sales in period t, which we assume to be paid back uniformly over the next J days.

有一部分的顾客立即付款，另一部分的顾客延迟付款，还有一部分顾客后来不要了

延迟支付期随机均匀分布

The dynamics of the on-hand inventory of the retailer are as follows:

$$I_{t+1} = \max(0, I_t + R_t - D_t). \tag{3}$$

The on-hand cash position is updated according to the amount replenished, the customers demand fulfilled (of which only a part is directly paid for), and repayments of customers on their outstanding credits (and possibly the end-of-month salary, discussed further onwards). When one notes that during period t the retailer receives repayments on the outstanding customers' loans of periods $\{t-J, t-J+1, \ldots, t-1\}$, the following expression provides the on-hand cash at the end of period t (i.e. just before the start of period $t+1$):

$$C_{t+1}^{-} = C_t - c_R R_t + p\gamma F_t + p(1-\gamma)(1-\zeta)\sum_{j=1}^{J}\frac{F_{t-j}}{J}. \tag{4}$$

At the end of each month (30 periods), the retailer prunes away part of its profits, for paying overheads such as rent and wages, and to support the owner and his family in their costs of living. We refer to the amount taken as the monthly salary, denoted by π_m in month m, and model them as we observed in real life. For this monthly salary, a target amount is set, denoted by π^{target}. However, whether this amount can indeed be taken at the end of the month, depends on the cash position at the moment, that is, it depends on the performance of the business.

This target amount π^{target} is based on the profit the store is expected to make in a setting that is identical to that of the nanostore, but differs only in one aspect, namely the cash constraint when replenishing. In this corresponding setting *without* a cash constraint, inventory is replenished at the start of every period up till level S, and all demands (up to S) are fulfilled during a period. For this model, it is straightforward to derive the expected monthly profit π^{max}:

$$\pi^{max} = 30\,(p - c_R)\sum_{x=0}^{\infty}\min\{x, S\}e^{-\lambda}\lambda^x/x!, \tag{5}$$

The summation term expresses the expected amount sold per period (the minimum of the demand and S, where the demand follows a Poisson distribution with parameter λ), the profit per sale is $p - c_R$, and there are 30 periods in a month.

Now, the target amount for the monthly salary π^{target} is expressed as a fraction $\beta \in [0, 1]$ of the expected profit in the corresponding model π^{max}:

每隔 30 天从现有资金中扣掉一些作为工资

$$\pi^{target} = \beta\,\pi^{max}. \tag{6}$$

However, the amount taken cannot be more than the cash on-hand at the end of the month. Therefore, the cash position is updated as follows:

$$C_{t+1} = \begin{cases} \max\{0, C_{t+1}^{-} - \pi^{target}\} & \text{if } t \mod 30 = 0, \\ C_{t+1}^{-} & \text{otherwise,} \end{cases} \tag{7}$$

and π_m, the salary actually taken in month m, $m = 1, \ldots, \lfloor T/30 \rfloor$, is

$$\pi_m = \min\{C_{30m}^{-}, \pi^{target}\}. \tag{8}$$

For the retailer, the total amount of salaries taken over the entire planning horizon is

$$\pi_{\varphi}^{tot} = \sum_{m=1}^{\lfloor T/30 \rfloor}\pi_m, \tag{9}$$

where here, and throughout the sequel, the φ will be used in reference to the scenario without supplier credits.

We define the net wealth at time $t \in \{0, 1, \ldots, T\}$ of the retailer (R), and that of the supplier (S), respectively, by:

$$\varphi_t^R = c_R I_t + C_t + p(1-\zeta)(1-\gamma)\sum_{j=1}^{J}\frac{J-j+1}{J}F_{t-j}, \qquad \varphi_t^S = (c_R - c_S)\sum_{i=1}^{t}R_i.$$

也考虑了供应商的收益

Finally, we consider the increase in net wealth over the planning horizon for both the retailer and the supplier. For this, we subtract the starting wealth (at $t = 0$) from the terminal wealth (at $t = T$), where $\varphi_0^R = c\,I_0 + C_0$ and using that $\varphi_0^S = 0$:

$$\varphi^R = \varphi_T^R - \varphi_0^R, \qquad \varphi^S = \varphi_T^S.$$

2.2 *Model with supplier's credits*

In the model presented in the previous section, upon the replenishment opportunity at the start of the day, the retailer can only purchase the amount for which it can directly pay for. Another scenario is that the supplier provides the retailer with credit, such that the retailer can stock its inventory to its desired stock level S, even if the on-hand cash is insufficient.

The supplier may have good reasons for doing so because (1) it sells more directly to the retailer, (2) it will sell more in consecutive periods, as the retailer is selling more to its customers, and (3) it keeps the nanostore financially healthier, letting it survive (longer). On top of that, the supplier can possibly earn debit interests on the outstanding credits. However, there is a risk to the supplier that should not be underestimated, namely the fact that at any moment the retailer can discontinue its business, in which case, the supplier loses all its outstanding balances. Such a discontinuation can occur because of a lack of profitability of the retailer, or for other reasons.

We model the supplier credits as follows. Upon the replenishment opportunity at the start of period t, the retailer requests the amount $S - I_t$. If its on-hand cash is sufficiently large to pay the supplier immediately, that is, if $C_t \geq c_R(S - I_t)$, it will do so. Also, it will pay back the supplier any outstanding loans, increased by the interest rate, up to the remaining on-hand cash available. Otherwise, the supplier will provide the retailer with a loan that is exactly enough for the retailer to replenish its inventory up to level S. This amount is $c_R(S - I_t) - C_t$, and is added to the possibly already outstanding credits, which in turn are increased by the interest rate.

The replenished amount in this scenario always equals $S - I_t$, and the inventory level immediately after replenishment is S. We have:

$$R_t = S - I_t, \tag{10}$$

$$F_t = \min(D_t, S), \tag{11}$$

$$I_{t+1} = \max(0, S - D_t). \tag{12}$$

Denote the outstanding supplier credits at the end of period t by B_t. The yearly interest rate charged by the supplier is denoted by r_{year}, which results in a daily interest rate r of $r = (r_{year})^{1/360}$ (assuming 360 days per year). In this way, the retailer's cash position at the end of period t becomes:

$$C^-_{t+1} = \max\Big\{C_t - (1+r)B_{t-1} - c_R\, R_t, 0\Big\} + p\,\gamma\, F_t + p\,(1-\gamma)(1-\zeta)\sum_{j=1}^{J}\frac{F_{t-j}}{J}, \tag{13}$$

$$B_t = -\min\Big\{C_t - (1+r)B_{t-1} - c_R\, R_t, 0\Big\}, \tag{14}$$

That is, from the on-hand cash at the start of period t, firstly the outstanding supplier credits at the end of the previous period (which is B_{t-1}) are paid back with interest, and the replenishments are paid for. This, however, can only be done up to C_t, which cannot become negative. The deficit, if any, becomes the new loan at the supplier (which is B_t). After that, the cash position is increased by the customers' payments and repayments for sales in this and previous periods.

The retailers target monthly salary π^{target} remains unchanged (cf. (6)) , as does the recurrence relation of C_{t+1} (cf. (7)) and the monthly salaries π_m (cf. (8)). Now, using ψ here and throughout the sequel in reference to the scenario with supplier credits, we define:

$$\pi^{tot}_{\psi} = \sum_{m=1}^{\lfloor T/30 \rfloor} \pi_m, \tag{15}$$

Similar to φ^R_t and φ^S_t, we now have the net wealth at time $t \in \{0, 1, \ldots, T\}$ of the retailer (R), and that of the supplier (S), respectively, are given by:

$$\psi^R_t = c_R\, I_t + C_t + p\,(1-\zeta)(1-\gamma)\sum_{j=1}^{J}\frac{J-j+1}{J}\,F_{t-j} - B_{t-1},$$

$$\psi^S_t = -B_t + (c_R - c_S)\sum_{i=1}^{t} R_t + r\sum_{i=1}^{t-1} B_i,$$

where we have considered the supplier credits B_t, and

$$\psi^R = \psi^R_T - \psi^R_0, \qquad \psi^S = \psi^S_T.$$

2.3 *Performance characteristics*

Recall that the main aim of this study is to answer the question under which conditions it is beneficial for the supplier to provide supplier credits to the nanostore, and what the impact is on performance characteristics. Recall that we used the notation φ in reference to the scenario *without* supplier credits (that is, the scenario in which the nanostore is operating under strict cash constraints), and ψ for the scenario *with* these credits. Define the differences by:

$$\Delta_R = \psi^R - \varphi^R, \qquad \Delta_\pi = \pi_\psi^{tot} - \pi_\varphi^{tot},$$
$$\Delta_S = \psi^S - \varphi^S, \qquad \Delta_{TOT} = \Delta_R + \Delta_\pi + \Delta_S.$$

For all four indicators, it holds that if a value is *positive*, the model *with* supplier credits outperforms the model *without*. For example, if $\Delta_R > 0$, the nanostore is better off if it receives supplier credits, however, if $\Delta_R < 0$, the nanostore will in expectation achieve a higher net wealth when operating under cash-constraints. Similarly, if $\Delta_S > 0$, the supplier is better off by providing credits to the nanostore, whereas if $\Delta_S < 0$, it can better refrain from doing so. As we will see in the numerical study (see Section 3), the retailer and supplier are not necessarily both better off under the same circumstances.

Values for I_0, C_0, and B_0 (i.e. the initial on-hand inventory position, on-hand cash position, and outstanding supplier loans) are required to initialise the model. Though different choices can be made here, throughout the sequel we assume $I_0 = S$ and $C_0 = B_0 = 0$ unless stated otherwise. Furthermore, we will assume no outstanding consumer loans before $t = 0$, that is, $F_t = 0$ for all $t < 0$.

Though the model can be described as a Markov model, due to the high dimension of the state-space (namely, $J + 2$ dimensions), the model becomes intractable to analyse analytically. Therefore, we must resort to numerical analysis by using simulations.

3. Examples

To create a better understanding of the dynamics and behaviour of the models presented, we start by presenting numerical examples. Consider the following setting, where the values have been chosen close to what we observed in real-life:

Example 1 Consider an example with the following parameters: $T = 360$, $\lambda = 5$, $S = I_0 = 10$, $C_0 = 0$, $\gamma = 0.6$, $\zeta = 0.005$, $p = 1.12$, $c_R = 1$ (hence, $\theta = 1.12$), $J = 30$, $\beta = 0.7$, and $r_{year} = 0.05$.

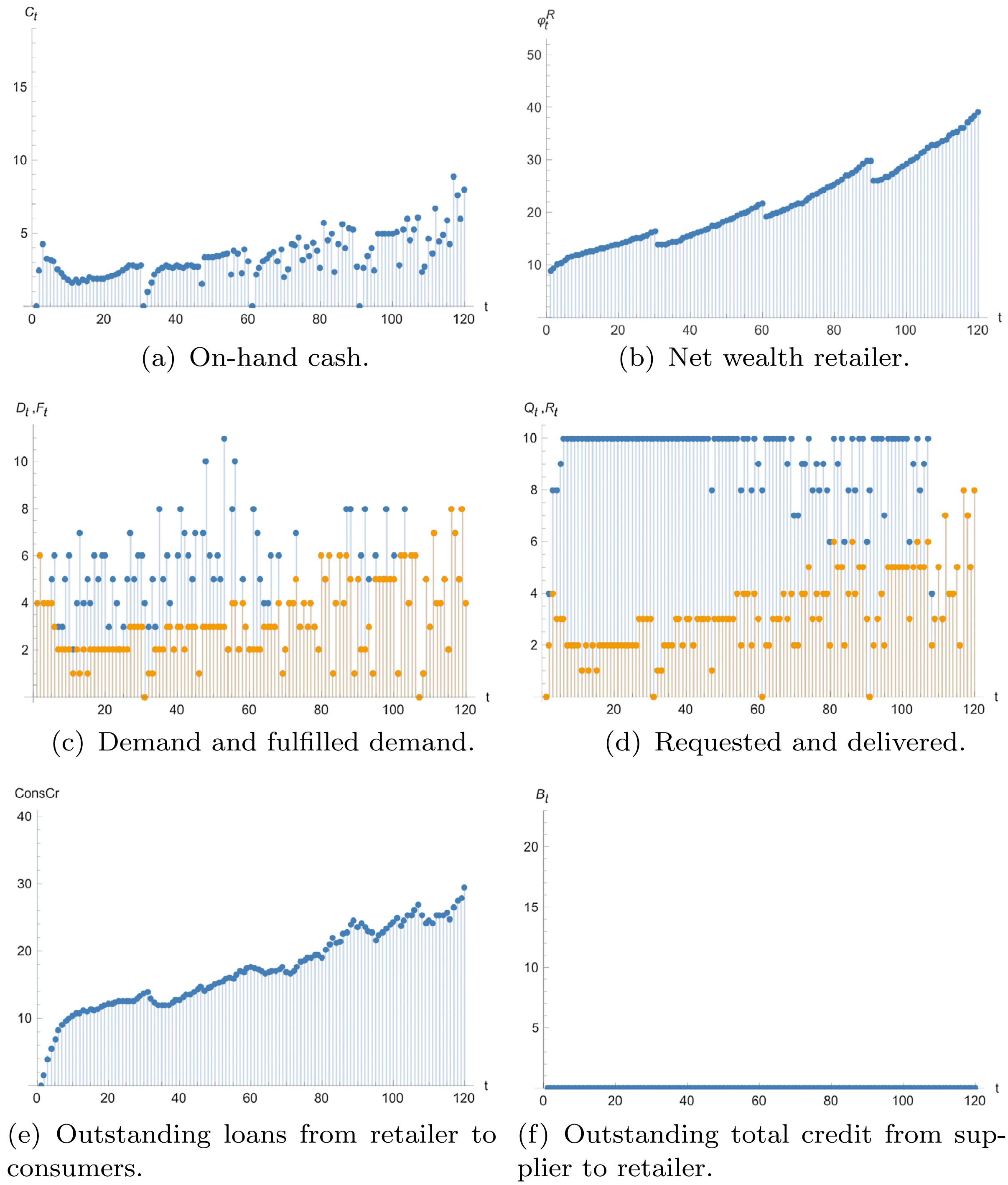
The sample paths of a typical run for the first four months (till $t = 120$) are shown in Figure 1 for the case *without* supplier credits, and in Figure 2 *with* supplier credits.

Focusing first on the case *without* supplier credits, we see that the cash position of the retailer, especially at the start, is rather volatile. In the first three-and-a-half month, the amounts it is requesting of the supplier cannot be paid for, therefore the amounts delivered are much lower (Figure 1(c)). This is directly reflected in the lost sales it is facing: typically, the customer demand is larger than can be fulfilled (Figure 1(d)).

In this setting, the retailer is making enough profit on its sales so that in the long run, even taking its salary after every 30 days (which is clearly visible on days 30, 60 and 90, see Figures 1(b) and 2(b)), still it is increasing its net wealth over time. Therefore, eventually, the model is going to behave (almost) identically to a non-cash-constrained model, since the retailer's on-hand cash will become sufficient to run the business. In this example, this happens from approximately three-and-a-half months onward (see again Figure 1(c) and (d)). We will further discuss under which parameter the retailer's business is profitable in Section 4. Note that after taking out salary, the retailer struggles for a few days to replenish its inventory: on days 30, 60 and 90, R_t drops to zero, and is small the next few days (see Figure 1(d)).

Initially, since sales are increasing, so are the outstanding loans to customers (Figure 2(e)). Though not shown in the graph, this will level off in the long run. To understand this, note that when the system behaves as a non-cash-constrained model, the demand distribution will be equal to that of the fulfillment distribution, namely a Poisson distribution truncated at S.

The model *with* supplier credits show a very different course. Using the suppliers credits, the retailer always gets the amount delivered that it is requesting. Consequently, it has always full shelves, resulting in no lost sales (unless the realised demand exceeds S). Its on-hand cash position is improving more quickly, as is its net wealth. Figure 2(f) shows that initially, the retailer heavily relies on supplier's credits, till approximately two-and-a-half months. From this point onwards, the system is again almost identical to a non-cash-constrained model, with the exception that after taking out salaries at the end of month 3, the retailer falls back on the credits for only a few days. In this scenario, the outstanding consumer loans are already levelling off, with the variability only depending on the variability in demand.

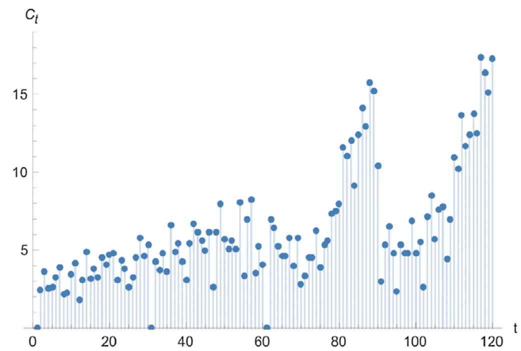


(a) On-hand cash. (b) Net wealth retailer.

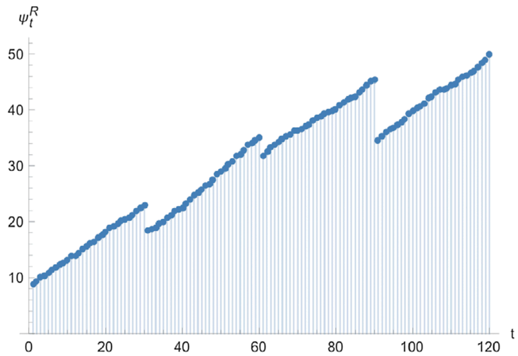
(c) Demand and fulfilled demand. (d) Requested and delivered.

(e) Outstanding loans from retailer to consumers. (f) Outstanding total credit from supplier to retailer.

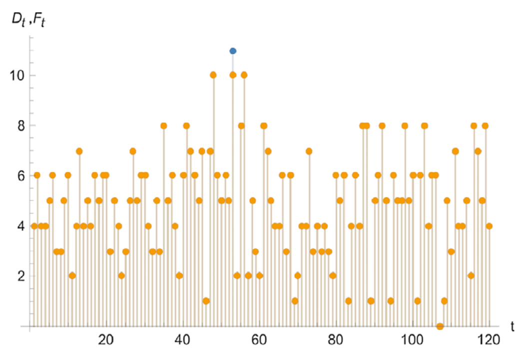
Figure 1. Sample paths for Example 1 when no credits are offered by the supplier.



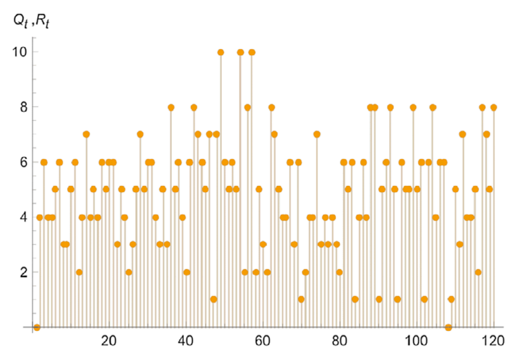
(a) On-hand cash.



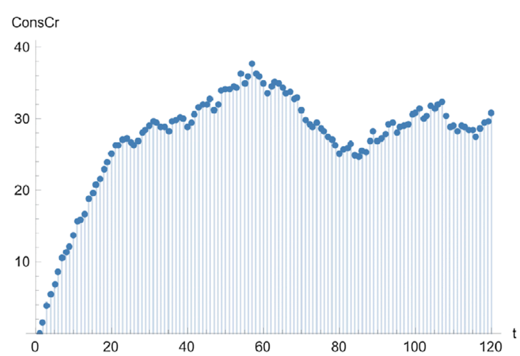
(b) Net wealth retailer.



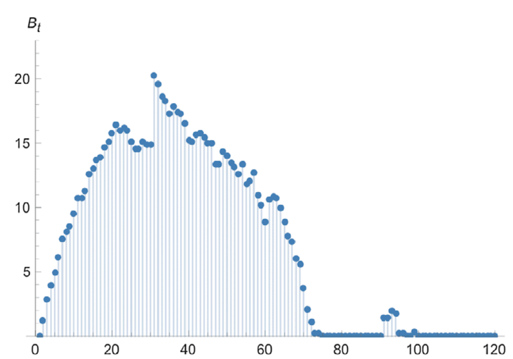
(c) Demand and fulfilled demand.



(d) Requested and delivered.



(e) Outstanding loans from retailer to consumers.



(f) Outstanding total credit from supplier to retailer.

Figure 2. Sample paths for Example 1 with supplier credits offered by the supplier.

Comparing both scenarios, the nanostore is much better off with supplier credits. Not only is its net wealth increasing more rapidly, it also takes higher salaries out at the end of each month. Also, the supplier is better off, since it is selling many more items to the retailer, making more profit (assuming $c_S < c_R$).

4. Equilibrium conditions

We derive analytical equilibrium conditions (switching curves) on the parameters γ, ζ, p, and c_R that determines whether the retailer's operations are viable in the first place. This provides direct insights into the profitability of the nanostore. Firstly, we do so for the case where the retailer does not take out any salaries at the end of the month. Then, we take these salaries into consideration. However, in this case, we can only solve these conditions analytically for the case with supplier credits.

4.1 *Conditions for non-salary model*

We consider the setting in which the retailer does not prune away any of its profits at the end of the month, that is, it does not take any salaries: by setting $\beta = 0$, one has $\pi_m \equiv 0$ for all m.

Without supplier credits, in the long run as t grows large, the net wealth of the retailer φ_t^R will either grow to infinity, or decline to 0, whereas only for a special parameter setting it will remain stationary over time. Below, we derive this parameter setting as a function of the acquisition price c_R, the selling price p, and the parameters γ and ζ. This expression acts as a switching curve: on one side, the nanostore's net wealth will increase in the long run, whereas on the other side, it will decrease to zero in the long run.

Note that if the amount of cash at the nanostore is steadily increasing over time, in the long run of the model, the cash constraint becomes irrelevant, and the amounts delivered will become equal to the demand distribution (truncated at S).

Denote by $\bar{F}$ the average amount fulfilled to customers during one period, assuming $\bar{F} > 0$. The average profit per period is $\bar{F}(p - c_R)$. A fraction $(1-\zeta)(1-\gamma)$ of the sales are eventually never paid by the consumers, therefore, the average loss by the retailer per period is $\bar{F}\, p\,(1-\gamma)\,\zeta$. Equating these and solving for ζ yields the value for ζ for the retailer to break-even. Expressed in γ and $\theta = p/c_R$, it is given by:

$$\zeta_{eq}(\gamma, \theta) = \frac{\theta - 1}{\theta\,(1-\gamma)}. \tag{16}$$

This $\zeta_{eq}(\gamma, \theta)$ is a switching curve: for all $\zeta > \zeta_{eq}$ the nanostore's net wealth will increase in the long run, whereas for all $\zeta < \zeta_{eq}$ it will decrease. Solving for γ, respectively θ, the equilibrium condition of Equation (16) can also be expressed as

$$\gamma_{eq}(\zeta, \theta) = \frac{1 - \theta\,(1-\zeta)}{\theta\,\zeta}, \qquad \theta_{eq}(\gamma, \zeta) = \frac{1}{1 - \zeta\,(1-\gamma)}.$$

Figure 3 shows some switching curves ζ_{eq} as a function of θ for different values of γ. For example, if there is a 15% profit margin on each sold item ($\theta = 1.15$) and 40% of all sales are paid by customers immediately ($\gamma = 0.4$), the retailer can only afford that at most 21.7% of its loans to customers never be paid back (i.e. $\zeta_{eq} = 0.217$). A larger ζ will result in a non-profitable business. If, at 15% profit, 90% is paid in cash immediately, then no matter the percentage of customers defaulting ($\zeta_{eq} = 1.3 > 1$), the retailer's business is profitable in the long run. In fact, if 90% is paid immediately, the retailer only needs an 11.1% profit margin per product to operate in a profitable way.

From (16), one can also derive a few special cases:

(1) If $\theta = 1$ (i.e. $p = c_R$, zero profit per item) then $\zeta_{eq} = 0$: all customer loans need to be paid back for the retailer not to end up at zero cash;
(2) If $\gamma \uparrow 1$, i.e. if there are no loans offered to consumers, $\zeta_{eq} \to \infty$: as no customer loans are given out, the value of ζ becomes irrelevant, and hence can become arbitrarily large;
(3) For $\zeta_{eq} \uparrow 1$, one requires $\theta = 1/\gamma$: the amount directly paid by the customer should equal the retailers acquisition price (the rest is set away as customer loans but never paid back).

Reconsider Example 1, where we now let $\beta = 0$, $p = 1.01$, $c_R = 1$ (hence, $\theta = 1.01$) and vary the values of γ and ζ. Figure 4(a) shows the net wealth increase of the retailer φ^R for this setting, as a function of both γ and ζ, where we take the average performance characteristics over 10 replications. The plot shows lines on which the value of φ^R is the same. The line where $\varphi^R = 0$ is the function $\zeta_{eq}(\gamma, \theta)$, which is drawn in bold. This illustrates its nature as a switching curve: the value of φ^R is positive in the area to the left of ζ_{eq}, equals zero on ζ_{eq}, and is negative in the area to the right of ζ_{eq}.

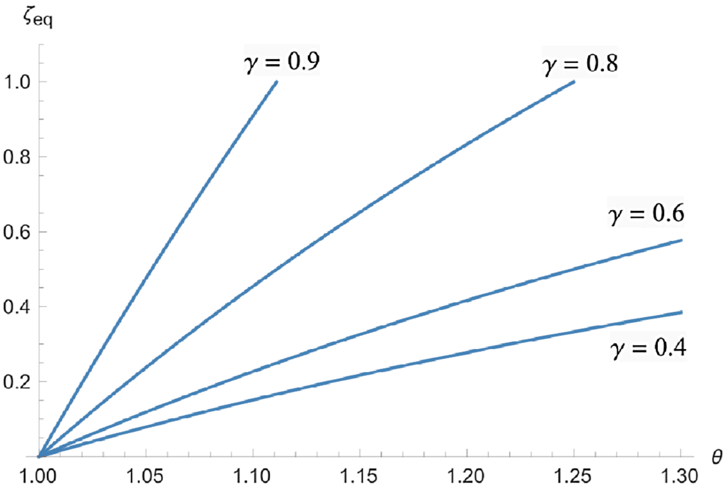


Figure 3. Switching curves $\zeta_{eq}(\gamma, \theta)$ of (16) for the profit rate $\theta = p/c_R$ (selling price over acquisition price) for four values of γ (fraction of sales immediately paid in cash): 0.4, 0.6, 0.8, and 0.9.

The switching curve $\zeta_{eq}(\gamma, \theta)$ of (16) remains valid in the scenario *with* supplier credits as long as $r = 0$. Namely, when supplier credits are incorporated, one would have to solve:

$$\bar{F}(p - c_R) = \bar{F}\, p\,(1-\gamma)\,\zeta + r\bar{B},$$

where $\bar{B}$ denotes the long-run average per period outstanding supplier credit. This $\bar{B}$ is unknown, but cancels out when $r = 0$, and therefore the equality is solved by (16). Though both equilibriums for the scenarios without and with supplier credits coincide, the values of φ^R and ψ^R will differ for all cases where $\zeta \neq \zeta_{eq}$. This is illustrated in Figure 4(b). When compared to Figure 4(a), the graphs $\varphi^R = 0$ and $\psi^R = 0$ (both cf. Equation (16)) coincide, however, to the left (right) of this line, ψ^R is more rapidly increasing (decreasing) than φ^R. One can conclude that the supplier credits increase the rate at which net wealth of the retailer is increasing or decreasing: if the retailer's net wealth is increasing over time, the use of supplier credits results in a faster increase; if the retailer's net wealth is decreasing over time, the use of supplier credits results in a faster decrease.

For $r > 0$, there is a slight shift in the switching curve determining whether the retailers operations are profitable. This is shown in Figure 4(c), where $r_{year} = 0.05$. For comparison, ζ_{eq} is drawn as well. The equilibrium $\psi^R = 0$ is now slightly shifted to the left compared to ζ_{eq}. That is, settings of (γ, ζ) for which φ^R and ψ^R were small or zero, now result in $\psi^R < 0$, that is, in a loss in the long run. In order to still achieve $\psi^R > 0$, either ζ needs to be (enough) smaller or γ needs to be (enough) larger, or both. One can explain this, since a part of the retailer's wealth has to be used to pay for the interest on the supplier's credit, and therefore, in order to be profitable in the long run, the retailer needs to make more money.

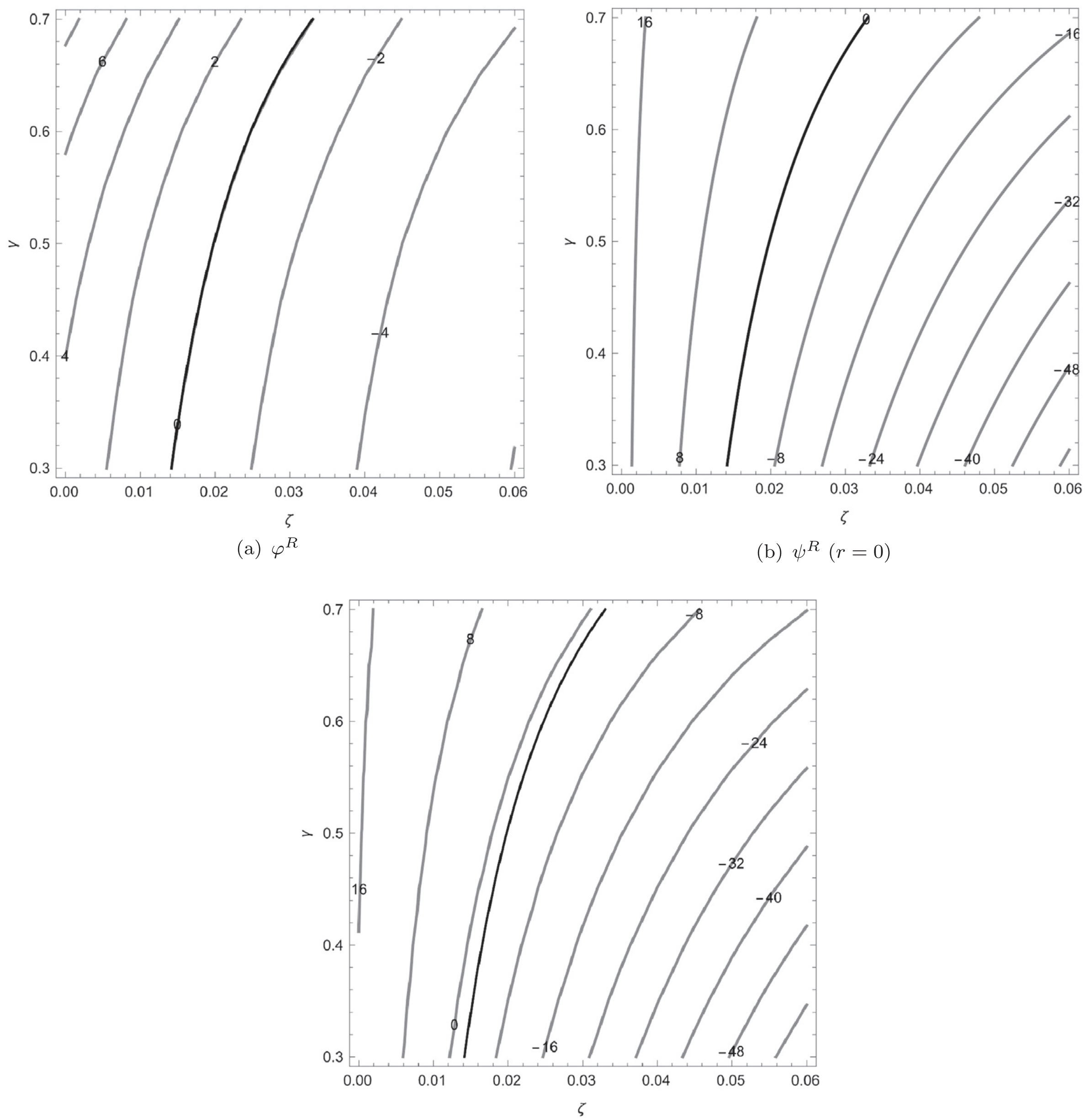
Furthermore, we learn from Figure 4(c), that the resulting loss in ψ^R in the long run is further 'inflated' by the presence of a positive interest rate: the values become negative more quickly. When the ψ_t^R are tending to become negative, the supplier's loan will tend to increase, so will the interests, resulting in a larger loss for the retailer.

4.2 *Conditions including salary*

When including the salaries that the retailer takes out at the end of the month, in order to find similar equilibrium conditions as in the previous sections, one would have to solve

$$30\,\bar{F}(p - c_R) = 30\,\bar{F}\, p\,(1-\gamma)\,\zeta + \bar{\pi},$$

where $\bar{\pi}$ denotes the long-run average of the monthly salary. This expression holds for the scenarios without supplier credits; and for the scenario with supplier credits if $r = 0$. The first scenario mentioned, however, cannot be solved, since both $\bar{F}$ and $\bar{\pi}$ are unknown. For the latter, however, we are able to solve the equality. For this, one needs to use that $\bar{F}$ is the same as the average sales that is used in $\bar{\pi}$ (recall Equations (5) and (6)). Solving this equality yields:



(a) φ^R

(b) ψ^R $(r = 0)$

(c) ψ^R $(r > 0)$

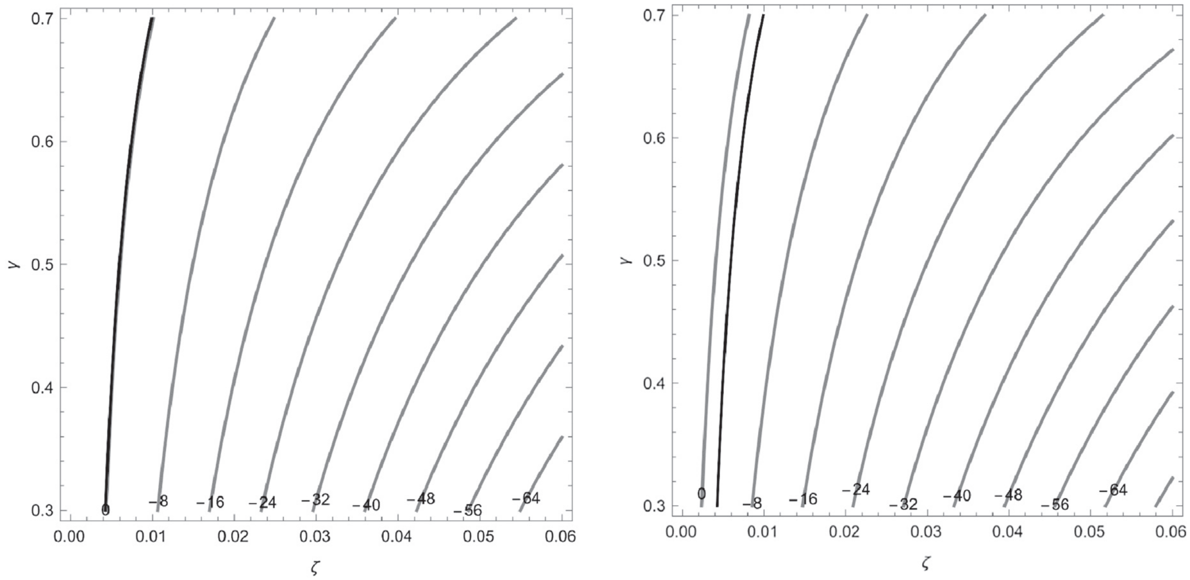
Figure 4. Increase in retailer's net wealth for Example 1, for the case without retailer's salaries, where on the horizontal axis ζ is varied from 0 to 0.06, and on the vertical axis γ is varied from 0.3 to 0.7. Thick black line: ζ_{eq}.

$$\zeta_{eq}^{\pi}(\gamma, \theta) = \frac{(1-\beta)(\theta-1)}{\theta(1-\gamma)}. \tag{17}$$

This $\zeta_{eq}^{\pi}(\gamma, \theta)$ is again a switching curve, as $\zeta_{eq}(\gamma, \theta)$ is.

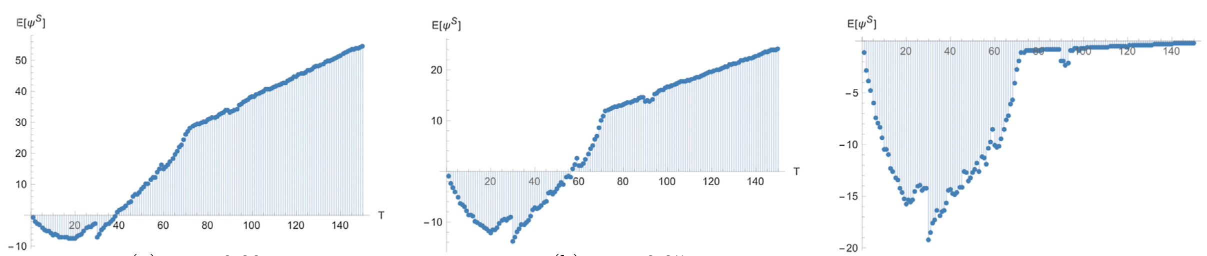
The ζ_{eq}^{π} for the same setting as in Figure 4 is shown in Figure 5(a), again together with lines that depict equal values of ψ^R. Though there is a large shift in these lines, one should keep in mind that the nanostore now is taking salaries at the end of each month.

When r becomes positive, a similar shift as to the non-salary case is observed, see Figure 5(b). When one would attempt to depict φ^R similarly as in Figure 4(a) for this setting, all values will be negative; the graph of $\varphi^R = 0$ starts at approximately $(\zeta, \gamma) = (0, 0.88)$ and is increasing in ζ.



(a) ψ^R $(r = 0)$ (b) ψ^R $(r > 0)$

Figure 5. Increase in retailer's net wealth for Example 1 for the case including retailer's salaries ($\beta = 0.7$), where on the horizontal axis ζ is varied from 0 to 0.06, and on the vertical axis γ is varied from 0.3 to 0.7. Thick black line: ζ_{eq}^{π}.



(a) $c_S = 0.80$ (b) $c_S = 0.85$ (c) $c_S = 0.89$

Figure 6. Sample paths of E[ψ^S] for T, when $\chi = 0.0025$.

Comparing the salary and non-salary cases, one can conclude that when the nanostore takes salaries, the equilibrium for the retailer's net wealth shifts substantially to the left. That is, a much smaller ζ is required in order to stay profitable.

5. Retailer discontinuing its store

In the previous section, we have seen when the operations of the retailer are profitable in the first place. These results partly answer the question when it would be beneficial for the supplier to provide the nanostore with credits. However, those results did not include the possibility of the retailer discontinuing its store, which is common practice for nanostores (Martinez 2016). We show how the probability of this happening, impacts the decision of the supplier to provide credits. We assume that for every time period, there is a certain, given probability that the nanostores ceases to exist. Given this, we derive a trade-off value with the per item profit for the supplier.

To show the impact of the retailer discontinuing its operations, we assume that in every period, there is an i.i.d. per period probability $\chi \in [0, 1)$ that the retailer ceases to exist, without any upfront warning to the supplier. In such a case, the retailer defaults on its outstanding loan, which is a loss for the supplier: it loses all its outstanding credits and possible future interests. Note that $\chi = 0.002$ results in $(1-\chi)^{360} \approx 0.49$, that is, almost a 50-50 survival probability of the nanostore over the time period of a year; for $\chi = 0.0025$ this survival probability is about 40%.

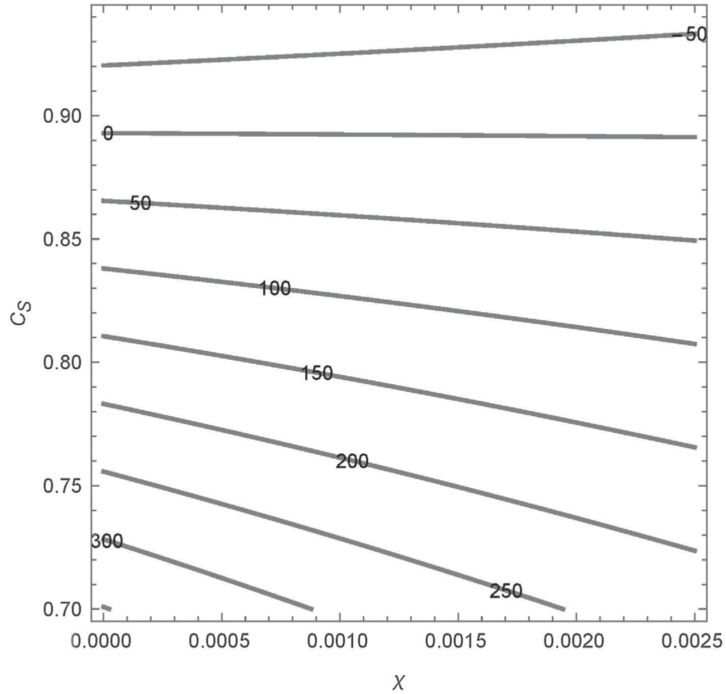


Figure 7. $E[\psi^S]$ as a function of both χ and c_S.

In this scenario, over a time horizon T, the expected terminal wealth of the supplier is given by

$$E[\psi^S] = \sum_{t=1}^{T} \chi(1-\chi)^{t-1}\psi_t^S + (1-\chi)^T \psi_{T+1}^S,$$

Note that the number of periods of operation of the shop follows a geometric distribution.

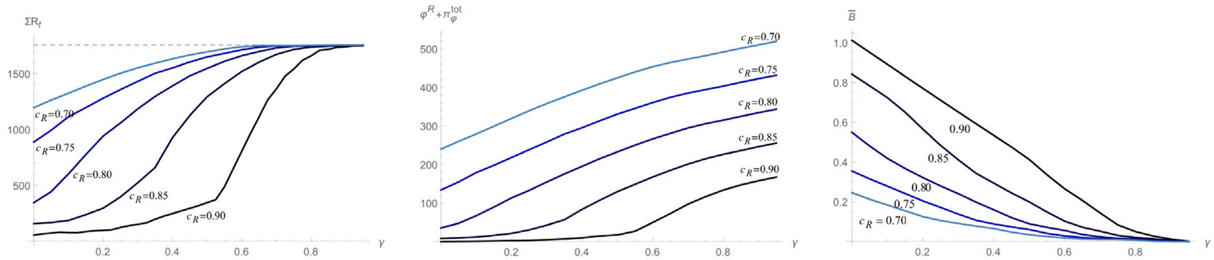
Typical sample paths of $E[\psi^S]$ for the first five months are show in Figure 6, where we have taken $\chi = 0.0025$ and $c_S = 0.80$, 0.85 and 0.89 (acquisition price for the supplier, does not affect the nanostore). Initially, the nanostore heavily uses the supplier credits (recall a similar setting in Figure 2(f)), and the supplier faces the risk of an expected loss if the retailer discontinues in this time period. For $c_S = 0.80$, this time period is approximately the first 40 days, for $c_S = 0.85$, approx. 60 days, and for $c_S = 0.89$ approx. 150 to 160 days. That is, one can state that, for $c_S = 0.80$, it holds that if the planning horizon of interest for the supplier is smaller (longer) than 40 days, the supplier has in expectation a net loss (profit), and should not (should) provide the retailer with credits. A similar statement can be made for $c_S = 0.85$ and $c_S = 0.89$. In the figures, also note the impact of the retailer taking out salaries after 30, 60, and 90 days.

Whereas Figure 6 shows the situation for three parameter settings, $E[\psi^S]$ is depicted in Figure 7 as a function of both χ and c_S, for $T = 360$. The graphs depict instances where $E[\psi^S]$ takes the same value. The graph for $E[\psi^S] = 0$ provides us a switching curve: above it, the supplier has a negative expected terminal wealth, whereas below, it expects to make a profit. Note that the graphs are not that sensitive towards χ, particularly around $E[\psi^S] = 0$, which occurs when c_S is close to $c_R \approx 0.893$. As the supplier's profit margin is small here, the supplier gains little from selling extra products, which it facilitates by providing credits. When the supplier's profit margin increases (i.e. when c_S decreases), the supplier's expected terminal wealth increases more rapidly, and is more impacted by a larger value of χ.

The main conclusion that can be drawn is, that if the retailer runs a profitable business in the first place (for which conditions are derived in Section 4), and the planning horizon of the supplier is large enough, the supplier is better off (in expectation) by supplying trade credits, even if the probability χ is large.

6. Numerical results

In this section, we perform a numerical analysis into the impact of the parameters. First, we consider the total amounts replenished, and how this depends on the retailer's profit margin. Second, we further analyse the outstanding supplier



(a) Total replenishments over the planning horizon *without* supplier credits. Dashed graph: total replenishments *with* supplier credits.

(b) Total profit retailer (net wealth increase plus total salaries).

(c) Average outstanding supplier credits.

Figure 8. Impact of the retailer's profit ratio on total replenishments, profits, and outstanding supplier credits.

credits. Third, we analyse under which circumstances supplier credits are beneficial. For this, we focus on three parameters: consumers' term of payment J, supplier credit interest rate r_{year}, fraction of consumer loans not paid back ζ, and the retailer's purchasing price c_R. We relate each of these to the fraction of consumer taking out a loan, γ.

For all results presented in this section, unless stated otherwise, we use the parameters $\lambda = 5$, $J = 30$, $\beta = 0.7$, $p = 1$, $c_R = 0.85$ (hence, $\theta \approx 1.18$), $c_S = 0.5$, $\zeta = 0.1$, and $r_{year} = 0.1$. The target order-up-to level is $S = 8$, which is the smallest S such that a target cycle service level of $\tau = 0.9$ is achieved. The time horizon is taken to be one year ($T = 360$), and we fix the consumer purchase price (i.e. the retailer's selling price) to $p = 1$. Initialisation values are $I_0 = S$, $C_0 = B_0 = 0$, and $F_t = 0$ for $t < 0$. The results are obtained by simulation where the number of replications equals 20.

6.1 *Replenishments*

When the nanostore does not have access to supplier credits, the amount it can replenish in every period is constrained by its on-hand cash. Therefore, the total amount the nanostore replenishes over the planning horizon, $\sum_{t=1}^{T} R_t$, depends on its cash position, the latter being affected by many parameters, including the retailers profit margin per product. In Figure 8(a), the total replenishments for the scenario without supplier credits are shown, as a function of $\gamma \in [0, 1]$, for different values of $c_R \in \{0.70, 0.75, 0.80, 0.85, 0.90\}$. Recall that $p = 1$. We added the graph of the total replenishments in the scenario with supplier credits (intermittent line), which is constant in γ and does not depend on c_R. For the same setting, Figure 8(b) shows the total profit the retailer is making (sum of net wealth increase and total salaries).

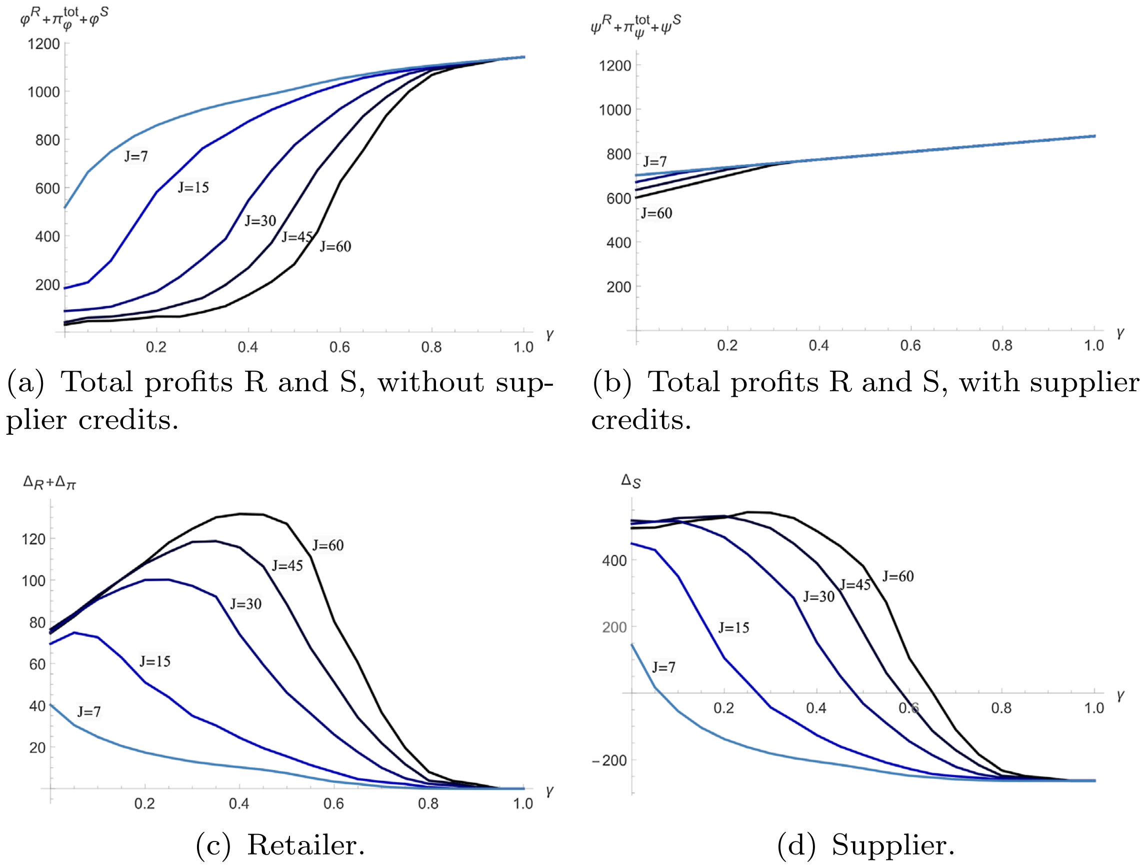
Regarding the replenishments, as expected, in the limit of $\gamma \to 1$, the amounts replenished will end up being equal to the amount in the supplier credits scenario. Furthermore, it is clear that the larger the profit margin (i.e. the smaller c_R), the larger the total quantity the retailer is able to replenish. This heavily impacts its profits, see Figure 8(b). The combination of the per-product profit and the amounts replenished (and hence, sold) result in the fact that for a c_R of 0.90 (a profit margin of 11 per cent) in combination with a small γ, the nanostore's business is hardly profitable; only from γ is 0.55 onward, the profit is picking up as a function of γ. Also for smaller values of c_R, the retailer's profit is largely impacted by the fraction of the customers buying on loans.

6.2 *Outstanding supplier credits*

When considering the scenario in which the supplier provides credits, a relevant question is how much credit the supplier is providing to the retailer. Figure 8(c) sheds insights into this question, showing the average amount of outstanding supplier credits over the planning horizon: $\bar{B} = \frac{1}{T} \sum_{t=1}^{T} B_t$, Here, $\bar{B}$ is plotted as a function of $\gamma \in [0, 1]$, for different values of $c_R = \in \{0.70, 0.75, 0.80, 0.85, 0.90\}$ (recall $p = 1$). When customers primarily pay directly (γ close to 1), the outstanding credits are small. However, the smaller γ becomes, the larger $\bar{B}$, even more when the per-product profit is low. This is in line with the previous insights.

6.3 *Net wealth increase*

In order to investigate whether supplier credits benefit the retailer and the supplier itself, we focus on the difference in the net wealth increase under the scenario with and the scenario without these credits. Recall that the four indicators Δ_R, Δ_π, Δ_S,



(a) Total profits R and S, without supplier credits.

(b) Total profits R and S, with supplier credits.

(c) Retailer.

(d) Supplier.

Figure 9. Total profits (of retailer and supplier combined), and differences in net wealth increase, for the scenarios without and with supplier credits.

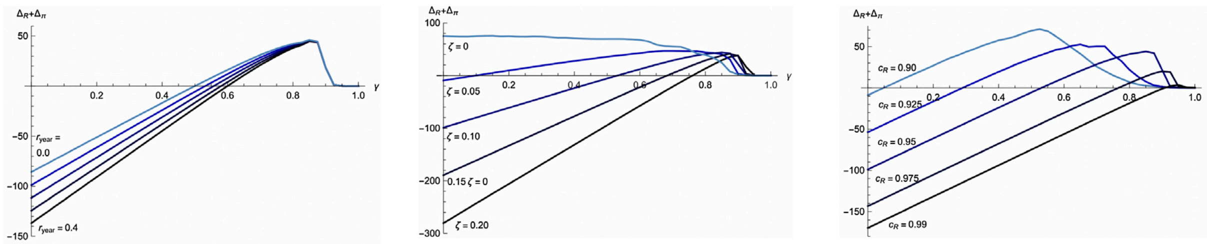
and Δ_{TOT} are defined such that if its value is positive, the model *with* supplier credits outperforms the model *without* these credits, and if negative, vice versa. By a study into the parameters, we derive under which circumstances either of these is the case. For this, we focus on parameters that have a larger impact, namely the payback period of customer loans (J), the interest rate on supplier credits (r_{year}), the fraction of consumer loans offered that are never paid back (ζ), and the retailer's purchasing price (c_R). All four are studied in interaction with the fraction paid directly by consumers (γ).

6.3.1 *Payback period*

Customers pay back their outstanding loans over a period of J days. This payback period, in particular when it is large, has a big impact on the nanstore's operations. There is a strong connection of J with γ, the percentage of customers using loans. In Figure 9, the totals of the net wealth increase in the nanostore and the supplier combined, is shown, for the scenarios without (9(a)) and with (9(b)) supplier credits. In the plots, γ ranges from 0 to 1, that is, from all customers buying on loans to no customers loans being used. Five values of J are chosen, ranging from one week to two months: $J \in \{7, 15, 30, 45, 60\}$.

Clearly, when supplier credits are in use (9(b)), there is little influence of J and γ on the total profits (sum of the retailer, including salaries, and supplier): a larger γ leads to larger total profits, but the difference is small, and only when γ is close to 0, there is a visible influence of J: a shorter payback period leads to slightly larger profits. However, in the scenario without credits (9(a)) both parameters have a big impact. When the retailer is self-financing, it clearly profits more from a shorter payback period. From approximately $\gamma = 0.8$ onward, the differences however, are small. Also, the larger the fraction paid directly, the larger the profits made. When J is large and γ small, the total profits are barely positive.

For the same setting, Figure 9(c) and (d) shows the *difference* between the scenarios with and without supplier credits, where the results are split out for the retailer (9(c): net wealth increase plus salaries) and the supplier (9(d): net wealth increase only). Positive (negative) values reflect that the net wealth increase is larger (smaller) in the scenario with credits compared with the scenario without. Figure 9(c) shows that the retailer is always better off with supplier credits, that is, for all values of $\gamma \in [0, 1]$ and all values of J depicted. It sincerely profits from the credits offered by the supplier, despite the interest it



(a) Supplier's interest rate $r_{year} \in \{0, 0.1, 0.2, 0.3, 0.4\}$. (b) Fraction of consumer loans not paid back $\zeta \in \{0, 0.05, 0.10, 0.15, 0.20\}$. (c) Retailer's purchasing price $c_R \in \{0.90, 0.925, 0.95, 0.975, 0.99\}$.

Figure 10. Impact of three parameters on the differences in the retailer's net wealth increase (including salaries), between the scenarios without and with supplier credits.

is paying over the outstanding balance. The larger J, the larger the extra net wealth increase for the retailer under supplier credits. However, again from approximately $\gamma = 0.8$ onward, the difference is small.

Figure 9(d) shows that for the supplier, it is a different story. For small values of γ, providing the retailer with credits is beneficial for the supplier itself. However, from a certain value of γ onward, the supplier's net wealth increase is better off not providing these credits. That is, there exists a critical value of γ, which depends on J, such that for γ smaller than this critical value, the supplier should offer credits to the retailer, but for γ larger than this critical value, it could better refrain from doing so. Note that the larger the J, the larger the critical value of γ. This might be counter-intuitive, as one would expect that the larger the J the more the nanostore needs credit, and the more beneficial it is for the supplier to offer it.

6.3.2 *Interest rate*

The interest rate on the supplier credits has an impact on whether it can be profitable for the retailer to make use of these credits. The higher the interest rate, the smaller the benefits that can be gained. This is shown in Figure 10(a), where the difference in net wealth increase (including salaries) is shown for the retailer, for five values of the interest rate r_{year}, ranging from 0 up to 40% of interest per year. For this graph, we have set $c_R = 0.95$, hence, the retailer's profit margin is small.

Note that the behaviour of the graphs is distinctively different for values of γ from 0.85 onwards. This follows as this is the equilibrium value such that the retailer's operations are viable in the first place in the case without supplier credits. Namely, given the parameters, $\gamma = 0.85$ is the solution of Equation (17). This holds exactly if $r_{year} = 0$, and seems to be an accurate approximation for positive values of r_{year}.

For γ small, the retailer is better off *without* supplier credits, as it is only accumulating credits. From a certain value of γ onwards, the retailer profits from the credits. The critical value of γ from where on this is the case, depends on the interest rate. In this setting, this value is $\gamma \approx 0.5$ for $r_{year} = 0$, whereas for $r_{year} = 0.4$, only from $\gamma \approx 0.6$ onwards, the retailer should use supplier credits.

6.3.3 *Consumer loans not paid back*

The profit the retailer is making on a product depends on the amount that a consumer in the end has paid for it, after (not) paying back any consumer loans. Recall that a fraction ζ of the outstanding consumer loans is never paid back. This parameter ζ has an impact on whether supplier credits provide benefits to the retailer, that is, on whether the retailer reaches a larger net wealth. For $\zeta \in \{0, 0.05, 0.10, 0.15, 0.20\}$, Figure 10(b) shows how $\Delta_R + \Delta_\pi$ varies as a function of γ, where again $c_R = 0.95$.

For $\zeta = 0$, the use of supplier credits is always the preferred option for the retailer. For a slightly larger ζ, namely 0.05, this still holds, unless γ is close to 0, but the extra benefits gained are much smaller. When ζ further increases, the critical value of γ from which values onwards supplier credits are beneficial, keeps increasing, until $\gamma \approx 0.76$ for $\zeta = 0.2$.

6.3.4 *Retailer's acquisition price*

The profit the retailer is making on a product depends on the acquisition price c_R (since the consumer price is taken to be fixed: $p = 1$). The per-product profit margin has an impact on whether supplier credits provide benefits to the retailer, which is particularly of interest when the profit margin is small. For five values of the acquisition price $c_R \in \{0.90, 0.925, 0.95, 0.975, 0.99\}$, Figure 10(c) shows how $\Delta_R + \Delta_\pi$ as a function of γ.

From the graph it is clear that in this setting, when $c_R = 0.90$, the retailer is almost always better with supplier credits, except when γ is close to zero. When c_R is larger, that is, if the profit margin is small, γ needs to be substantially large in order for the retailer to benefit from the supplier credits, where the critical value of γ depends on c_R.

7. Conclusion

In this section, we provide a discussion of the results, as well as managerial insights, and directions for further research.

7.1 *Discussion*

We studied a cash-constrained inventory model, where a small traditional retailer needs to immediately pay its supplier for goods delivered while it offers loans to its customers. These kinds of nanostores are common in emerging markets. In the light of these stores often closing soon after their start, suppliers are typically reluctant to provide these stores with credits to finance their replenishments.

Our study reveals interesting, non-trivial insights into when the supplier's credits should be offered. In general, the supplier's credits improve the cash position of the nanostore and result in a higher service level for the nanostore and supplier. Also, when the supplier offers credits, this is typically only for a limited duration. Eventually, the supplier's credits improve the nanostore's cash position sufficiently such that supplier credits are not needed anymore, provided it is running its business in a way that is profitable in the first place. Although the nanostore offers loans to its customers and loses some of them, the cash generated from the sales becomes at some point in time sufficient to get the needed amounts replenished without the supplier's credits.

However, as the nanostores is losing some of the loans offered to customers, its business might be losing money in the long run. In this case, it is not wise for the supplier to offer credits. The nanostore should keep its credit losses limited. This is rather counter-intuitive, as one would expect that the supplier's credits are needed when the nanostore's loss to consumers credit is high, as the need for cash would be high (see Figure 4 showing the nanostore's net wealth as a function of γ and ζ). We find that the supplier's credits have two effects. They either further raise or reduce the nanostore's wealth, depending on whether the retailer was already making a profit or loss without the supplier's credits. Hence, the supplier's credits have an amplifying effect. The insight here is also that supplier's credits should only be offered when the nanostore's operations are profitable, even if the profits are minor. Otherwise, the small retailer will not be able to pay back the loans. The interest rate that the supplier charges on the outstanding loans has only a minor effect on this.

In our numerical analysis, we also took into consideration the risk of a nanostore going bankrupt (or discontinuing its operations for other reasons). In such a case, the supplier will lose all outstanding credits. We find that the expected benefits for the supplier are only negative in the short-term. After some time (see Figure 6), the benefits of providing the credits, such as higher sales, outweigh the risk of the nanostore going bankrupt, especially when the unit profit of the supplier is sufficiently high (See Figure 7). The results also show that the bankruptcy probability has a bigger impact when the unit profit of the supplier increases. We modelled the bankruptcy as an exogenous event with a certain independent probability, though in practice, it might depend on the cash position. Basically, the cash position is a random variable that (disregarding taking salaries out) will eventually grow either to infinity or to zero. The crucial parameters that determine in which of the two directions the cash position evolves, are γ and ζ. We present an equilibrium value for ζ (Equation (16)). The more customers pay immediately (γ), the smaller the effect of ζ. Since in reality customers delay their payments, ζ is a critical factor that determines whether the nanostore goes bankrupt or not.

Nanostores offer informal loans to their customers, which are paid back after a certain period of time (payback period). Our results show that the benefits of the supplier's credit are smaller when the payback period is short (see Figure 9). If the nanostore's customers quickly pay back their loans, then the advantages of supplier's credits are limited. However, it is beneficial (for the supplier and the nanostore) to offer supplier credits when the nanostore faces a long payback period, especially when the fraction of customers who take a loan is large.

The nanostore's unit profit per product plays an important role regarding profits for the nanostore and the supplier (see Figure 8(b) and (c)). For products with a high profit margin, smaller loans will be taken out from the supplier, and the average size of the loans is decreasing when γ is small.

7.2 *Managerial insights*

The results of our paper contain substantial managerial insights that are useful for decision makers at the supplier and nanostore levels. We will take a high level view to our results and present the five most important managerial implications:

First, our study shows that for most settings, the supplier's credits are beneficial for both parties. For the nanostore, it means that the quantities replenished become larger, and hence close to (or equal to) the desired quantities. The supplier also benefits from replenishments of larger quantities. If the nanostore can set a maximum limit for the outstanding loans to the customers (and hence also limit the loss on customer loans), then supplier credits are beneficial even for products with a small profit margin. However, if the nanostore loses a substantial part of the loans, then the nanostore should limit the consumer loans to make supplier credits beneficial and to limit the bankruptcy risk.

Second, we find that the supplier credits are often only needed for a relatively short period of time. After a short period of time, the supplier's credits are only occasionally needed and the nanostore can mostly replenish sufficiently without the supplier's credits.

Third, the supplier's credits basically inflate the nanostore's performance (in terms of the net wealth). In other words, if a nanostore was initially performing well, then the suppliers' credit will boost the nanostore's performance positively, and eventually also the supplier's. The opposite also holds.

Fourth, our study provides exact expressions for the critical values of the model parameters that determine when supplier's credits should be offered (or not) in case the interest rate is zero. If the interest rate is substantially large, then our numerical results show that the switching curves shift accordingly.

Fifth, in practice, suppliers are often reluctant to offer credits due to the high bankruptcy risk of the nanostores. Our study shows that the supplier will only lose if the nanostore goes bankrupt in the short-term. We also show that if the supplier's unit profit is sufficiently large, then the nanostore's bankruptcy risk becomes almost irrelevant.

These are very helpful insights for decision-makers at both parties' levels to decide on offering (and accepting) credits.

7.3 *Further research directions*

In our paper, we made several modelling choices to stay close to reality (cf. Boulaksil et al., 2014). However, the model has some limitations and can be extended in several ways.

First, we assumed that γ and ζ are constant parameters. In practice, whether and when customers take out loans and pay them back will heavily depend on their own cash position. Towards the end of the month, before people typically receive their salaries, customers might be in more need of credit, while directly at the start of the month customers might be more likely to pay directly, and pay back outstanding loans. To incorporate this, the γ and ζ should become time-dependent, that is, dependent on, for example, the day of the month. This would lead to non-stationary customer loans over time.

Second, in practice, at some point, the nanostore owner might be refusing to offer loans to some customers when they have built up a large amount of loans and are not (or hardly) paying back. From a mathematical modelling perspective, this is a challenging extension, as this would require to model the outstanding loan to each individual customer. Similarly, including an upper bound for the supplier's credit would be interesting to study. In our model, this upper bound does not exist and it is not trivial how to set this upper bound. The upper bound will likely be dependent on the outstanding credits, the probability of a nanostore going bankrupt, and the retailer's historical payback behaviour.

Third, a similar reasoning can hold for the supplier in deciding whether and how much credit it would provide the nanostore. One could include factors like the number of years that the nanostore has been in operation, the order quantities from the nanostore, the shelf space the nanostore has allocated for the products of the supplier, whether the supplier has the impression that the nanostore is doing well financially, how much credit has been offered to the nanostore in the past, how well the nanostore has done before in repaying credits, etc. These factors are not straightforward to model, but some of these ideas could be interesting to incorporate in a future study.

Fourth, the paper can be extended by considering a multi-product inventory system. In that case, an additional level of complexity is added to the problem, especially if the products are supplied by different suppliers. It is likely that not all the suppliers will be offering credits. In that case, a substantial imbalance can result between the service levels of different products and consequently impact the nanostore's cash position. This will obviously impact the nanostore's ability to pay back the credits, but it is not obvious whether supplier credits will be beneficial in this case, because the credit offered by one supplier can be used to pay another supplier. Hence, a complex situation arises that needs to be further studied. Similarly, a one-supplier, multi-nanostore setting would raise the question to which nanostore the supplier should offer credits, as the cash-constraint will in this case shift to the supplier.

Fifth, one could consider the extension to a problem in which in every period the decision on whether supplier credits should be used, is optimised. In the current model, this decision is only made once: either, the supplier never provides any credits, or the supplier will always provide the exact amount the retailer is short of, such as to fully replenish up till its target inventory level. Adding this decision in every period will create a more interesting yet more complicated problem, as in every period a trade-off has to be made on whether supplier credits should be offered and are worth being accepted.

Despite these interesting possible extensions, we believe that the insights gained in our paper provide valuable managerial insights.

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