Cost Equations for Q,r model

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Here we summarize some notation and functions used by Hadley and Whitin (Section 4.7 for the analysis and Section 4.10 for a numerical example) and Federgruen and Zheng (OR, 1991) in their descriptions of the (Q,r)-inventory control rule. We refer to these places for further background.

The long-run average cost is given by

$$C(r,Q) = K\frac{\lambda}{Q} + \frac{1}{Q}\sum_{y=r+1}^{r+Q} G(y),$$

where

$$G(y) = \mathbb{E} f(y-X)$$

and $f(y)$ is a cost function when the inventory level is y and X is the leadtime demand. To capture the average inventory cost, take

$$f(y) = h[y]^+,$$

for the backorder cost per unit per unit time, let

$$f(y) = b[-y]^+.$$

The loss fraction is given by

$$f(y) = \mathbb{1}_{y\leq 0}.$$

To see this last equation, note that $\mathbb{P}(X \geq y)$ is the fraction of demand lost, by PASTA. Clearly, for this f,

$$\mathbb{P}(X\geq y) = \mathbb{P}(y-X\leq 0) = \mathbb{E}\mathbb{1}_{y-X\leq 0} = \mathbb{E} f(y-X) = G(y).$$

Thus, the cost per backorder becomes

$$f(y) = \lambda\pi 1\{y\leq 0\},$$

where π is the cost per backordered demand and λ the arrival rate. The total cost follows by summing all these costs to

$$f(y) = h[y]^+ + b[-y]^+ + \lambda\pi 1\{y\leq 0\}.$$

To compute the cost associated with each of the separate components, set $K = 0$ in $C(r,Q)$ above, since, for instance,the average holding costs should not include ordering costs.

We next relate the notation of Federgruen and Zheng to the notation of Hadley and Whitin.

$$
\begin{aligned}
\lambda &= \text{Arrival rate of demand,}\\
L = \tau &= \text{The replenishment lead time,}\\
X &= \text{The stochastic demand during the replenishment lead time,}\\
F &= \text{The distribution of } X,\\
Q &= \text{The ordering quantity,}\\
r &= \text{Reorder level,}\\
A = K &= \text{Ordering cost,}\\
\pi &= \text{Stockout cost per unit,}\\
\hat{\pi} &= \text{Stockout cost per unit per unit time,}\\
C &= \text{Item Cost.}\\
I &= \text{Inventory carring charge .}\\
h = C * I &= \text{Inventory cost per unit per unit time.}
\end{aligned}
$$

Finally we have some remarks on two formulae of Hadley and Whitin. (In our numerical work we got other values than theirs. . .)

1. The formula Eq. 4.93 in Hadley and Whitin for the average holding cost uses Q^* but it should be $(Q^* + 1)/2$.

2. The value of $\pi E(Q,r) = 12.341$ as reported by Hadley and Whitin in Eq. 4.94, is not the same as our value. However, our value is the same as the one computed by the methods presented in Factory Physics.

Some comments are in order here.