Useful notation for the Q,r model

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Here we summary some notation and functions used by Hadley and Whitin (Section 4.7 for the analysis and Section 4.10 for a numerical example) and Federgruen and Zheng (OR, 1991), in their descriptions of the (Q, r)-inventory control rule. We refer to these places for further background.

The long-run average cost is given by

$$C(r,Q) = K\frac{\lambda}{Q} + \frac{1}{Q}\sum_{y=r+1}^{r+Q} G(y),$$

where

$$G(y) = \mathbb{E} f(y - X)$$

and $f(y)$ is a cost function when the inventory level is y and X is the leadtime demand. To capture the average inventory cost, take

$$f(y) = h[y]^+,$$

for the backorder cost per unit per unit time, let

$$f(y) = b[-y]^+.$$

The loss fraction is given by

$$f(y) = \mathbb{1}_{y\leq 0}.$$

To see this last equation, note that $\mathbb{P}(X \geq y)$ is the fraction of demand lost, by PASTA. Clearly, for this f,

$$\mathbb{P}(X \geq y) = \mathbb{P}(y - X \leq 0) = \mathbb{E}\mathbb{1}_{y-X\leq 0} = \mathbb{E} f(y - X) = G(y).$$

Thus, the cost per backorder becomes

$$f(y) = \lambda\pi 1\{y \leq 0\},$$

where π is the cost per backordered demand and λ the arrival rate. The total cost follows by summing all these costs to

$$f(y) = h[y]^+ + b[-y]^+ + \lambda\pi 1\{y \leq 0\}.$$

To compute the cost associated with each of the separate components, set $K = 0$ in $C(r,Q)$ above, since, for instance,the average holding costs should not include ordering costs.

Finally, we relate the notation of Federgruen and Zheng to the notation of Hadley and Whitin.

λ = Arrival rate of demand,

$L = \tau$ = The replenishment lead time,

X = The stochastic demand during the replenishment lead time,

F = The distribution of X,

Q = The ordering quantity,

r = Reorder level,

$A = K$ = Ordering cost,

π = Stockout cost per unit,

$\hat{\pi}$ = Stockout cost per unit per unit time,

C = Item Cost.

I = Inventory carring charge .

$h = C * I$ = Inventory cost per unit per unit time.