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Design of Kronecker Product Beamformers with Cuboid Microphone Arrays

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Abstract

Microphone array beamforming has been widely used in a wide range of acoustic applications. To make it effective in suppressing noise, yet being able to preserve the fidelity and quality of broadband speech signals of interest, the beamformer needs to be designed with high spatial gain, consistent responses at different frequencies, and high robustness against array imperfections. A great deal of efforts have been devoted in the literature to achieving this goal, among which the Kronecker product beamforming method, developed recently for linear and rectangular microphone arrays, has demonstrated some interesting properties. In this work, we extend this approach to three-dimensional arrays. Focusing on cuboid shape of microphone arrays, we first decompose the global beamforming filter into a Kronecker product of two sub-beamforming filters. Algorithms are then developed to design each sub-beamforming filter so that the global beamformer has a high directivity factor and can be steered flexibly in the three-dimensional space.

Keywords: Microphone arrays, cuboid arrays, superdirective beamforming, Kronecker product.

1 INTRODUCTION

Microphone arrays can be used to solve many important acoustic problems in a wide range of applications. A critical component of a microphone array system is the so-called beamforming, which is basically a spatial filter applied to the array observation signals to estimate the signal of interest while suppressing noise and interferences [1–3]. While beamforming has been studied for many decades in the field of sensor arrays, extending those methods developed for narrowband applications to microphone arrays is not trivial as microphone array beamforming needs to suppress noise and meanwhile preserve the fidelity of the speech and acoustic signals of interest, which are broadband in nature and their frequencies span from 20 Hz to 20 kHz. Among many beamforming methods developed in the literature [5–10], the Kronecker product beamforming developed recently has demonstrated some promising results in terms of performance, robustness, and complexity [2, 11–20]. This method basically decomposes the global beamforming filter into the Kronecker product of two sub-filters. It has been demonstrated, based on linear and rectangular microphone arrays, that by optimizing these sub-filters, the global beamforming filter can be made to achieve high spatial gain and consistent responses over frequencies with high robustness [2, 21].

In this paper, we extend the Kronecker product beamforming from two-dimensional microphone arrays to a particular type of three-dimensional microphone arrays, i.e., uniform cuboid arrays. We first show how to decompose the global beamforming filter into the Kronecker product of two sub-beamforming filters: one corresponds to a linear array and the other corresponds to a rectangular grid array. We then derive algorithms on how to design those two sub-beamforming filters. It is shown that the developed Kronecker product beamformer can

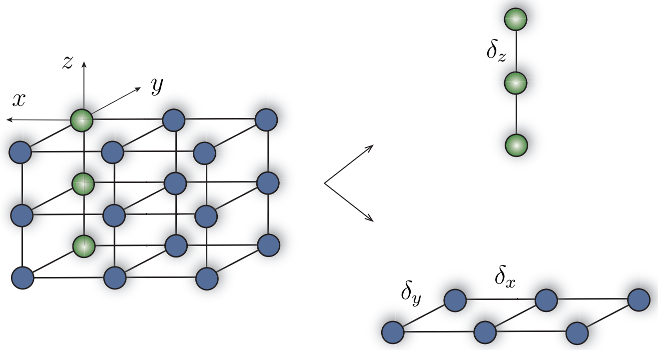

Figure 1. Illustration of the Kronecker product decomposition of a uniform cuboid array into a uniform rectangular subarray and a uniform linear subarray.

achieve a high directivity factor (DF) with a reasonable white noise gain (WNG), and its beampattern has good steering flexibility in the three-dimensional space.

2 SIGNAL MODEL, PROBLEM FORMULATION, AND PERFORMANCE MEASURES

Let us consider a uniform cuboid microphone array as illustrated in Fig. 1. We choose the sensor at the top-left inner corner as the origin of the Cartesian coordinate system. The array consists of M_z parallel uniform rectangular array along z (negative) axis and each rectangular array consists of M_x sensors along the x (negative) axis and M_y sensors along the y (negative) axis. The interelement spacing along each axis is denoted as δ_x, δ_y, and δ_z, respectively.

Assume that, in an anechoic acoustic environment, a far-field source signal (plane-wave) propagates from the direction $\{\theta, \phi\}$ at the speed of sound, i.e., $c = 340$ m/s, and impinges on the cuboid array, where the elevation angle θ $(0 \leq \theta \leq \pi)$ is measured downward from the z axis and the azimuth angle ϕ $(0 \leq \phi \leq 2\pi)$ is measured counterclockwise from the x axis. The steering matrix of size $M_x \times M_y$ corresponding to the rectangular array can be written as [2,7]

$$\mathbf{D}_{xy}(\omega, \theta, \phi) = \mathbf{d}_x(\omega, \theta, \phi)\, \mathbf{d}_y^T(\omega, \theta, \phi), \tag{1}$$

where

$$\mathbf{d}_x(\omega, \theta, \phi) = \begin{bmatrix} 1 & e^{-\jmath \frac{\omega \delta_x}{c} \sin\theta \cos\phi} & \cdots & e^{-\jmath \frac{(M_x - 1)\omega \delta_x}{c} \sin\theta \cos\phi} \end{bmatrix}^T, \tag{2}$$

$$\mathbf{d}_y(\omega, \theta, \phi) = \begin{bmatrix} 1 & e^{-\jmath \frac{\omega \delta_y}{c} \sin\theta \sin\phi} & \cdots & e^{-\jmath \frac{(M_y - 1)\omega \delta_y}{c} \sin\theta \sin\phi} \end{bmatrix}^T \tag{3}$$

are the steering vectors associated with the linear subarrays along the x and y axes, respectively, $\jmath$ is the imaginary unit with $\jmath^2 = -1$, $\omega = 2\pi f$ is the angular frequency, and $f > 0$ is the temporal frequency.

Applying the vectorization operation to (1), we can deduce that

$$\mathbf{d}_{xy}(\omega, \theta, \phi) = \text{vec}\left[\mathbf{D}_{xy}(\omega, \theta, \phi)\right] = \mathbf{d}_y(\omega, \theta, \phi) \otimes \mathbf{d}_x(\omega, \theta, \phi), \tag{4}$$

where $\otimes$ denotes the Kronecker product operation. Then, the steering matrix of size $(M_x M_y) \times M_z$ corresponding to the cuboid array can be written as

$$\begin{aligned}\mathbf{D}(\omega,\theta,\phi) &= \left[\begin{array}{cccc}\mathbf{d}_{xy}(\omega,\theta,\phi) & e^{-\jmath\frac{\omega\delta_z}{c}\cos\theta}\mathbf{d}_{xy}(\omega,\theta,\phi) & \cdots & e^{-\jmath\frac{(M_z-1)\omega\delta_z}{c}\cos\theta}\mathbf{d}_{xy}(\omega,\theta,\phi)\end{array}\right] \\ &= \mathbf{d}_{xy}(\omega,\theta,\phi)\,\mathbf{d}_z^T(\omega,\theta),\end{aligned} \tag{5}$$

where

$$\mathbf{d}_z(\omega,\theta) = \left[\begin{array}{cccc}1 & e^{-\jmath\frac{\omega\delta_z}{c}\cos\theta} & \ldots & e^{-\jmath\frac{(M_z-1)\omega\delta_z}{c}\cos\theta}\end{array}\right]^T \tag{6}$$

is the steering vector of length M_z associated with the linear subarray along the z axis $\left[\text{Note that } \mathbf{d}_z(\omega,\theta) \text{ is independent of the azimuth angle } \phi\right]$. Similarly, applying the vectorization operation to (5) yields

$$\mathbf{d}(\omega,\theta,\phi) = \text{vec}\left[\mathbf{D}(\omega,\theta,\phi)\right] = \mathbf{d}_z(\omega,\theta)\otimes\mathbf{d}_{xy}(\omega,\theta,\phi). \tag{7}$$

The conventional beamforming is performed by applying a complex-valued filter, $\mathbf{h}(\omega)$ of length $M = M_xM_yM_z$, to all the microphone observations to get an estimate of the desired signal [7]. Assume that the desired source is incident from the direction $\{\theta_\mathrm{d},\phi_\mathrm{d}\}$. The distortionless constraint in the desired look direction is required, i.e.,

$$\mathbf{h}^H(\omega)\,\mathbf{d}(\omega,\theta_\mathrm{d},\phi_\mathrm{d}) = 1, \tag{8}$$

where the superscript ^{H} is the conjugate-transpose operator.

Three important performance measures are commonly used to evaluate the performance of beamformers: the beampattern, the WNG, and the DF. The beampattern describes the sensitivity of the beamformer to a plane wave; it is defined as [3]

$$\mathcal{B}\left[\mathbf{h}(\omega),\theta,\phi\right] = \mathbf{h}^H(\omega)\,\mathbf{d}(\omega,\theta,\phi). \tag{9}$$

The WNG evaluates the sensitivity of the array to some of its imperfections; it is defined as [4]

$$\mathcal{W}\left[\mathbf{h}(\omega)\right] = \frac{\left|\mathbf{h}^H(\omega)\,\mathbf{d}(\omega,\theta_\mathrm{d},\phi_\mathrm{d})\right|^2}{\mathbf{h}^H(\omega)\,\mathbf{h}(\omega)}. \tag{10}$$

The DF quantifies how the microphone array performs in the presence of spatially diffuse noise; it is defined as [4]

$$\begin{aligned}\mathcal{D}\left[\mathbf{h}(\omega)\right] &= \frac{\left|\mathcal{B}\left[\mathbf{h}(\omega),\theta_\mathrm{d},\phi_\mathrm{d}\right]\right|^2}{\frac{1}{4\pi}\int_0^\pi\int_0^{2\pi}\left|\mathcal{B}\left[\mathbf{h}(\omega),\theta,\phi\right]\right|^2\sin\theta d\phi d\theta} \\ &= \frac{\left|\mathbf{h}^H(\omega)\,\mathbf{d}(\omega,\theta_\mathrm{d},\phi_\mathrm{d})\right|^2}{\mathbf{h}^H(\omega)\,\mathbf{\Gamma}(\omega)\,\mathbf{h}(\omega)},\end{aligned} \tag{11}$$

where

$$\mathbf{\Gamma}(\omega) = \frac{1}{4\pi}\int_0^\pi\int_0^{2\pi}\mathbf{d}(\omega,\theta,\phi)\,\mathbf{d}^H(\omega,\theta,\phi)\sin\theta d\phi d\theta. \tag{12}$$

3 KRONECKER PRODUCT BEAMFORMERS

As seen from (7), the steering vector of the cuboid array can be decomposed into the Kronecker product of the counterparts of two subarrays. Consequently, we propose to decompose the global beamforming filter as

$$\mathbf{h}(\omega) = \mathbf{h}_z(\omega)\otimes\mathbf{h}_{xy}(\omega), \tag{13}$$

where $\mathbf{h}_z(\omega)$ is the sub-beamforming filter of length M_z corresponding to the linear subarray along the z axis and $\mathbf{h}_{xy}(\omega)$ is the sub-beamforming filter of length $M_x M_y$ corresponding to the rectangular subarray along the $x-y$ plane.

Now, the beampattern of the Kronecker product beamformer can be written as

$$\begin{aligned}\mathcal{B}[\mathbf{h}(\omega), \theta, \phi] &= [\mathbf{h}_z(\omega) \otimes \mathbf{h}_{xy}(\omega)]^H [\mathbf{d}_z(\omega, \theta) \otimes \mathbf{d}_{xy}(\omega, \theta, \phi)] \\ &= \left[\mathbf{h}_z^H(\omega)\, \mathbf{d}_z(\omega, \theta)\right] \left[\mathbf{h}_{xy}^H(\omega)\, \mathbf{d}_{xy}(\omega, \theta, \phi)\right] \\ &= \mathcal{B}[\mathbf{h}_z(\omega), \theta] \times \mathcal{B}[\mathbf{h}_{xy}(\omega), \theta, \phi], \end{aligned} \tag{14}$$

where $\mathcal{B}[\mathbf{h}_z(\omega), \theta]$ and $\mathcal{B}[\mathbf{h}_{xy}(\omega), \theta, \phi]$ are the beampatterns corresponding to the linear and rectangular subarrays, respectively. It can be easily observed that the global beampattern of the cuboid array is the product of the individual beampatterns of the two sub-beamforming filters.

The distortionless constraint becomes

$$\left[\mathbf{h}_z^H(\omega)\, \mathbf{d}_z(\omega, \theta_\mathrm{d})\right] \left[\mathbf{h}_{xy}^H(\omega)\, \mathbf{d}_{xy}(\omega, \theta_\mathrm{d}, \phi_\mathrm{d})\right] = 1. \tag{15}$$

Let us set the distortionless constraint for each sub-beamforming filter as

$$\mathbf{h}_z^H(\omega)\, \mathbf{d}_z(\omega, \theta_\mathrm{d}) = \mathbf{h}_{xy}^H(\omega)\, \mathbf{d}_{xy}(\omega, \theta_\mathrm{d}, \phi_\mathrm{d}) = 1, \tag{16}$$

so that the global distortionless constraint in (15) is satisfied.

The WNG of the Kronecker product beamformer is deduced as

$$\begin{aligned}\mathcal{W}[\mathbf{h}(\omega)] &= \frac{\left|\left[\mathbf{h}_z^H(\omega)\, \mathbf{d}_z(\omega, \theta_\mathrm{d})\right] \left[\mathbf{h}_{xy}^H(\omega)\, \mathbf{d}_{xy}(\omega, \theta_\mathrm{d}, \phi_\mathrm{d})\right]\right|^2}{[\mathbf{h}_z(\omega) \otimes \mathbf{h}_{xy}(\omega)]^H [\mathbf{h}_z(\omega) \otimes \mathbf{h}_{xy}(\omega)]} \\ &= \frac{\left|\mathbf{h}_z^H(\omega)\, \mathbf{d}_z(\omega, \theta_\mathrm{d})\right|^2}{\mathbf{h}_z^H(\omega)\, \mathbf{h}_z(\omega)} \times \frac{\left|\mathbf{h}_{xy}^H(\omega)\, \mathbf{d}_{xy}(\omega, \theta_\mathrm{d}, \phi_\mathrm{d})\right|^2}{\mathbf{h}_{xy}^H(\omega)\, \mathbf{h}_{xy}(\omega)} \\ &= \mathcal{W}[\mathbf{h}_z(\omega)] \times \mathcal{W}[\mathbf{h}_{xy}(\omega)], \end{aligned} \tag{17}$$

where $\mathcal{W}[\mathbf{h}_z(\omega)]$ and $\mathcal{W}[\mathbf{h}_{xy}(\omega)]$ are the WNGs corresponding to the linear and rectangular subarrays, respectively. It is clear that the WNG of the cuboid array is the product of the WNGs of the two subarrays.

Now, we can design the two sub-beamforming filters individually, and thereby obtaining the global beamformer [22–24].

Apparently, there are different ways to design the sub-beamforming filters. For example, we may design a superdirective beamformer for the rectangular subarray, which is obtained by maximizing the DF while considering the distortionless constraint [2,25–27]. The result is as follows:

$$\mathbf{h}_{xy,\mathrm{SD}}(\omega) = \frac{\mathbf{\Gamma}_{xy}^{-1}(\omega)\, \mathbf{d}_{xy}(\omega, \theta_\mathrm{d}, \phi_\mathrm{d})}{\mathbf{d}_{xy}^H(\omega, \theta_\mathrm{d}, \phi_\mathrm{d})\, \mathbf{\Gamma}_{xy}^{-1}(\omega)\, \mathbf{d}_{xy}(\omega, \theta_\mathrm{d}, \phi_\mathrm{d})}, \tag{18}$$

where

$$\mathbf{\Gamma}_{xy}(\omega) = \begin{bmatrix} \mathbf{\Gamma}_1(\omega) & \mathbf{\Gamma}_2(\omega) & \cdots & \mathbf{\Gamma}_{M_y}(\omega) \\ \mathbf{\Gamma}_2(\omega) & \mathbf{\Gamma}_1(\omega) & \cdots & \mathbf{\Gamma}_{M_y-1}(\omega) \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{\Gamma}_{M_y}(\omega) & \mathbf{\Gamma}_{M_y-1}(\omega) & \cdots & \mathbf{\Gamma}_1(\omega) \end{bmatrix} \tag{19}$$

is a symmetric block Toeplitz matrix of size $(M_x M_y) \times (M_x M_y)$ and $\mathbf{\Gamma}_{m_y}(\omega),\ m_y = 1, 2, \ldots, M_y$ are symmetric Toeplitz matrices of size $M_x \times M_x$ whose (i,j)th element is defined as

$$\left[\mathbf{\Gamma}_{m_y}(\omega)\right]_{ij} = \mathrm{sinc}\left[\frac{\omega\sqrt{(i-j)^2\delta_x^2 + (m_y-1)^2\delta_y^2}}{c}\right], \tag{20}$$

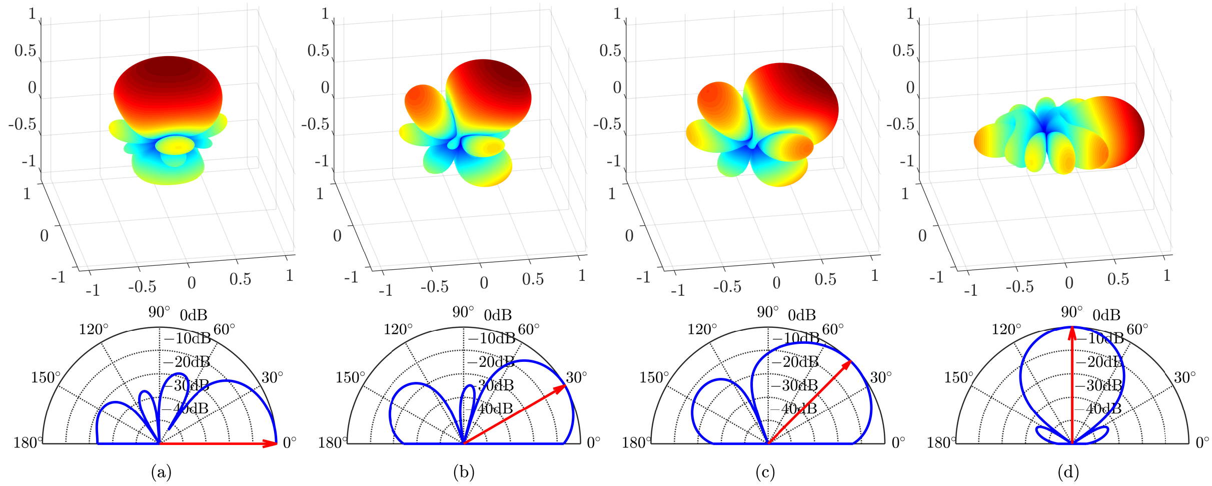

Figure 2. Beampatterns of the Kronecker product beamformer with a cuboid array for different values of θ_{d}: (a) $\theta_{\mathrm{d}} = 0°$, (b) $\theta_{\mathrm{d}} = 30°$, (c) $\theta_{\mathrm{d}} = 45°$, and (d) $\theta_{\mathrm{d}} = 90°$. The upper subplots show the three-dimensional beampatterns and the lower subplots show the corresponding two-dimensional beampatterns for $\phi = 0°$. Conditions of simulations: $M_x = M_y = 4$, $M_z = 8$, $\delta_x = 1$ cm, $\delta_y = \delta_z = 2$ cm, $\phi_{\mathrm{d}} = 0°$, and $f = 2$ kHz.

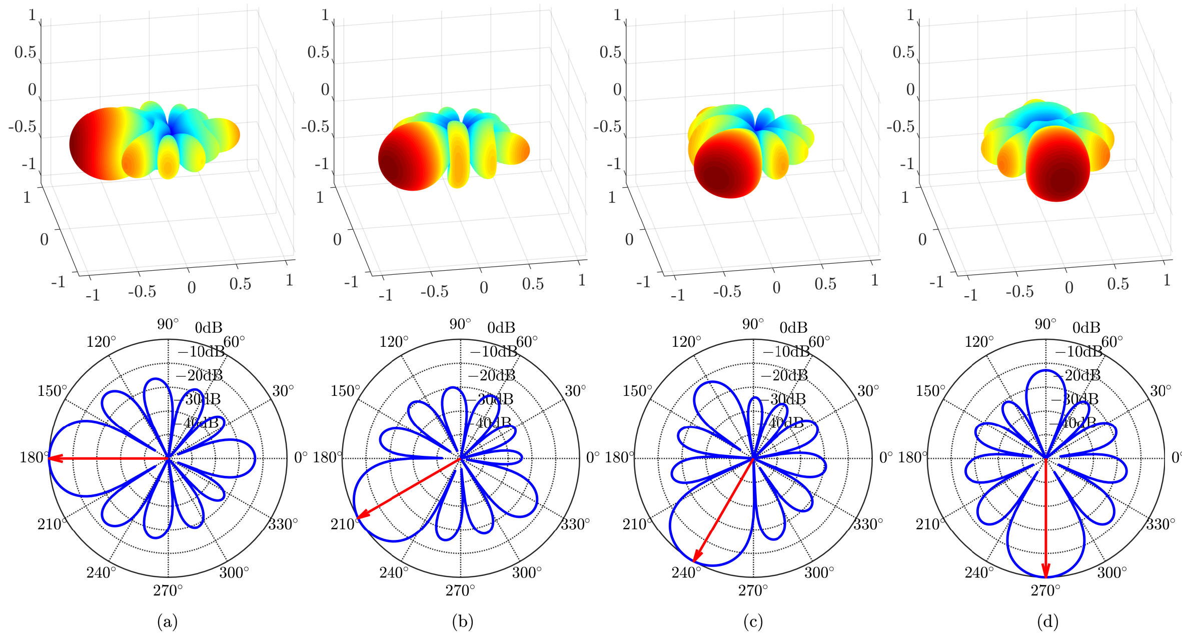

Figure 3. Beampatterns of the Kronecker product beamformer with a cuboid array for different values of ϕ_{d}: (a) $\phi_{\mathrm{d}} = 180°$, (b) $\phi_{\mathrm{d}} = 210°$, (c) $\phi_{\mathrm{d}} = 240°$, and (d) $\phi_{\mathrm{d}} = 270°$. The upper subplots show the three-dimensional beampatterns and the lower subplots show the two-dimensional beampatterns for $\theta = 90°$. Conditions of simulations: $M_x = M_y = 4$, $M_z = 8$, $\delta_x = 1$ cm, $\delta_y = \delta_z = 2$ cm, $\theta_{\mathrm{d}} = 90°$, and $f = 2$ kHz.

with $\mathrm{sinc}(x) = \sin x / x$, and the subscript $_{\mathrm{SD}}$ stands for "superdirective."

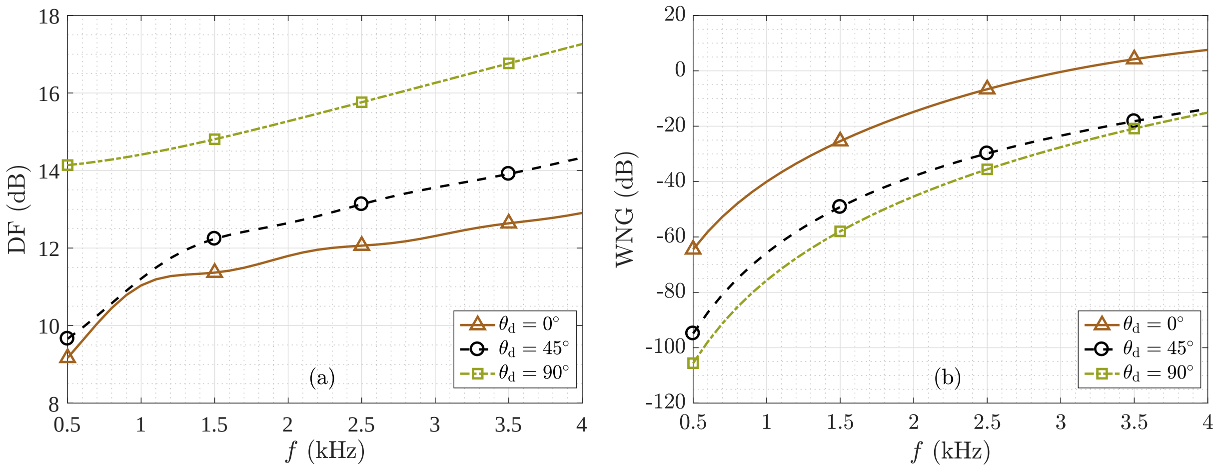

Figure 4. Performance of the Kronecker product beamformer with a cuboid array for different values of $\theta_{\rm d}$: (a) DF and (b) WNG. Conditions of simulations: $M_x = M_y = 4$, $M_z = 8$, $\delta_x = 1$ cm, $\delta_y = \delta_z = 2$ cm, and $\phi_{\rm d} = 0°$.

Superdirective beamformers generally have low WNG at low frequencies, leading to the so-called white noise amplification problem. To deal with this problem, let us consider the beamformer that has the largest WNG for the linear subarray, which corresponds to the delay-and-sum beamformer:

$$\mathbf{h}_{z,\rm DS}(\omega) = \frac{1}{M_z}\mathbf{d}_z(\omega, \theta_{\rm d}). \tag{21}$$

Thus, the global beamformer is obtained as

$$\mathbf{h}(\omega) = \mathbf{h}_{z,\rm DS}(\omega) \otimes \mathbf{h}_{xy,\rm SD}(\omega). \tag{22}$$

This beamformer can achieve a high value of DF while maintaining a reasonable level of WNG for robustness. Meanwhile, it has advantages in computational efficiency since the dimension of the matrix to invert is M_xM_y instead of $M_xM_yM_z$ [2]. Moreover, the designed beampattern can be steered to different desired directions in the three-dimensional space as will be shown in the following section.

4 SIMULATIONS

In this section, we examine the performance of the Kronecker product beamformer through simulations. We consider a uniform cuboid array with $M_x = 4$, $M_y = 4$, $M_z = 8$, $\delta_x = 1$ cm, $\delta_y = 2$ cm, and $\delta_z = 2$ cm.

Figure 2 plots the beampatterns of the proposed beamformer for $\theta_{\rm d} \in \{0°, 30°, 45°, 90°\}$, $\phi_{\rm d} = 0°$, and $f = 2$ kHz, where the upper subplots show the three-dimensional beampatterns and the lower subplots show the two-dimensional beampatterns for $\phi = 0°$. Figure 3 plots the beampatterns of the proposed beamformer for $\phi_{\rm d} \in \{180°, 210°, 240°, 270°\}$, $\theta_{\rm d} = 90°$, and $f = 2$ kHz, where the lower subplots show the two-dimensional beampatterns for $\theta = 90°$. Clearly, the Kronecker product beamformer can form a main beam to different directions, which implies that the proposed beamforming method has good steering flexibility in the three-dimensional space.

Figure 4 plots the WNG and the DF of the Kronecker product beamformer as a function of frequency, f, for $\theta_{\rm d} \in \{0°, 45°, 90°\}$, and $\phi_{\rm d} = 0°$. It is clearly seen that the proposed beamformer can achieve a high DF while maintaining a reasonable level of WNG. It is also observed that the DF increases while the WNG decreases as $\theta_{\rm d}$ increases.

5 CONCLUSIONS

We have proposed a Kronecker product beamforming method for cuboid microphone arrays. This method first decomposes the uniform cuboid array into linear and rectangular subarrays so that the steering vector of the cuboid array is written as a Kronecker product of the steering vectors of the two subarrays. The global beamforming filter is, in a similar manner, also decomposed as a Kronecker product of two sub-beamforming filters. As such, we can design the two sub-beamforming filters individually. In this way, the design of the global beamforming filter becomes very flexible since each sub-beamforming filter can be designed using different methods such as the superdirective beamformer, delay-and-sum beamformer, differential beamformer, and adaptive beamformer. Then we demonstrated a design of a global beamforming filter by combining superdirective and delay-and-sum beamformers. Simulation results verified that the designed beamformer can achieve high DF and a reasonable level of WNG while facilitating good steering flexibility in the three-dimensional space.

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