## 10/12 Approximate inference

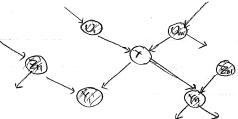
Last class: 1> Rejection sampling (slow)

2, Likelihood weighting (faster)

3> Markov chain Monte Carlo (MCMC) (fastest)

Def = Markov blanket Bx of node X consists of parents, children, and spouses of X

Thm: X is conditionally independent of all nodes outside Bx given nodes in Bx.



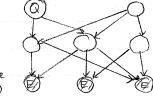
## MCMC STMulation

Query nodes Q

Evidence nodes E

How to estimate P(Q=q|E=e)?

query



evidence. (shaded)

- \* To estimate P(Q=q(E=e):
- fix evidence nodes to observed values.
- · initialize non-evidence nodes at random.
- · repeat N times:
  - pick non evidence node X € E at random.
  - use Bayes rule to compute P(XIBx) where Bx is fixed to current
  - resample X from P(X=x | Bx)
- \*Count # samples N(q) where Q=q
- \* Estimate P(Q=q | E=e) = N(q)/N.

Converges in limit  $N \rightarrow \infty$  to correct answer.

\* Key difference between likelihood weighting (LW) and MCMO:  $LW > \text{sample non-evidence nodes from } \left( \frac{P(X \mid Pa(X))}{P(X \mid Bx)} \right)$ 

Learning

\* BN = DAG + CPTs not always available from experts.

How to learn from examples?

\* Maximum likelihood (ML) estimation.

- Simplest form of learning in BNs.

- choose ("estimate") model (DAG+CPTs) to maximize

P( observed data | DAG + CPTs )

likelihood.

Case I known structure of DAG, lookup tables for CPTs, complete data

- DAG is fixed over some known, finite set of discrete nodes fX1, X2, ..., Xnt
- CPTs enumerate  $P(X_i = x \mid pa(X_i) = Ti)$  as look up table parent configuration.
- Data is T complete instantiations of nodes in BN.

Ex:



			1
example #	×	X2	×3
1	(	0	1
2.	0	Ð	٥
3	O	ı	O
,			
7	ı	0	(

Jargon: "complete data", "fully observed", "no hidden nodes", "fully visible". More generally, denote data as  $\int_{0}^{\infty} (x_{1}^{(t)}, x_{2}^{(t)}, \dots, x_{n}^{(t)}) f_{t=1}^{T}$ .

\* IID assumption

Samples are independently identically distributed from joint distribution  $P(X_1, X_2, \dots, X_n)$  of BN.

\* Probability of IID data set.

 $P(data) = \prod_{t=1}^{T} P(X_1 = X_1^{(t)}, X_2 = X_2^{(t)}, \dots, X_n = X_n^{(t)})$  due to IID assumption.

Coproduct over rows)

Probability of t-th example:

product rule

$$P(X_1 = X_1^{(t)}, X_2 = X_2^{(t)}, \dots, X_n = X_n^{(t)}) = \prod_{i=1}^{n} P(X_i = X_i^{(t)} | X_i = X_i^{(t)}, X_2 = X_2^{(t)}, \dots$$

$$\cdots$$
,  $\times_{\tilde{i}-1} = \times_{\tilde{i}-1}^{c(i)}$ 

$$= \prod_{i=1}^{n} P(X_i = x_i^{(t)} | pa(X_i) = pa_i^{(t)})$$

conditional independence from DAG.

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* log-likelihood &
 R = log P(DATA)
      = \log \prod_{t=1}^{T} P(X_{i}^{(t)}, X_{2}^{(t)}, \dots, X_{N}^{(t)}) \quad \text{IID}
= \log \prod_{t=1}^{T} \prod_{r=1}^{T} P(X_{i}^{(t)} \mid pa_{i}^{(t)}) \quad \text{due to prod rule } \mathcal{A} \subset I
= \sum_{t=1}^{T} \sum_{r=1}^{N} \log P(X_{i}^{(t)} \mid pa_{i}^{(t)})
= \sum_{r=1}^{N} \sum_{t=1}^{T} \log P(X_{i}^{(t)} \mid pa_{i}^{(t)}) \quad \text{Swapping order of summation}.
Let count (X_i = x, pa_i = TC) denote # examples for which X_i = x and pa_i = TC
 \mathcal{R} = \sum_{i=1}^{n} \sum_{x} \sum_{t} count(X_i = x, pa_i = t) log P(X_i = x | pa_i = t)
possible \qquad possible values \qquad unknowns to be optimized values of X; of pa(X_i) properties of data \qquad (learned from data)
 Write \mathcal{L} = \sum_{i = 1}^{\infty} \mathcal{L}_{i\pi} where \mathcal{L}_{i\pi} = \sum_{i = 1}^{\infty} count(X_i = x, pa_i = \pi) \log P(X_i = x | pa_i = \pi)
From decomposition \mathcal{L} = \sum_{i,n} \mathcal{L}_{in}, we can independently optimize CPT entries
 at each node in BN and for each parent configuration of that node.
* ML estimation
 For each node Xi, for each row IT of CPT, maximize Rin subject to:
     (i) \gtrsim P(X=x|pai=\pi)=1
     (ii) P(X_i = x \mid pa_i = \pi v) \ge 0
* Shorthand notation at node i, row Tr of CPT:
  let Cx = count (X_i = a, pa_i = Tr)
  let px = P(Xi = x | pai = Tt)
 how to maximize \( \int \text{Cx log Pa} \) Subject to \( \int \text{Epa} = 1 \) \( \text{Pa} \ge 2 \)
 Solution: Pa = Ca/SiCB
 ML Solution: P_{ML}(X_1 = x \mid Pa_x = \pi t) = count(X_1 = x, Pa_1 = \pi t)
                                                              \sum_{i=1}^{\infty} count(X_i = x', pa_i = \pi)
                                                             = count (X_i = x, pa_i = \tau_c) count (pa_i = \tau_c)
* Properties
- Asymptotically correct
   P_{ML}(X_1, X_2, \dots, X_n) \longrightarrow P(X_1, X_2, \dots, X_n) as T \rightarrow \infty.
- Problematic in non-asymptotic regime. (of "sparse" data)
   P_{ML}(X_i = x_i \mid pa_i = TC) = \int_{-\infty}^{\infty} O \ \text{if } \ count(X_i = x_i, pa_i = TC) = O

undefined \ \text{if } \ count(pa_i = TC) = O.
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