

Yutong Wang (001530602)  
**Program Structures & Algorithms**  
**FALL 2021**  
**Assignment 3**

- **Task**

Step 1:

(a) Implement height-weighted Quick Union with Path Compression. For this, you will flesh out the class UF\_HWQUPC. All you have to do is to fill in the sections marked with `// TO BE IMPLEMENTED ... // ...END IMPLEMENTATION`.

(b) Check that the unit tests for this class all work. You must show "green" test results in your submission (screenshot is OK).

Step 2:

Using your implementation of UF\_HWQUPC, develop a UF ("union-find") client that takes an integer value  $n$  from the command line to determine the number of "sites." Then generates random pairs of integers between 0 and  $n-1$ , calling `connected()` to determine if they are connected and `union()` if not. Loop until all sites are connected then print the number of connections generated. Package your program as a static method `count()` that takes  $n$  as the argument and returns the number of connections; and a `main()` that takes  $n$  from the command line, calls `count()` and prints the returned value. If you prefer, you can create a main program that doesn't require any input and runs the experiment for a fixed set of  $n$  values. Show evidence of your run(s).

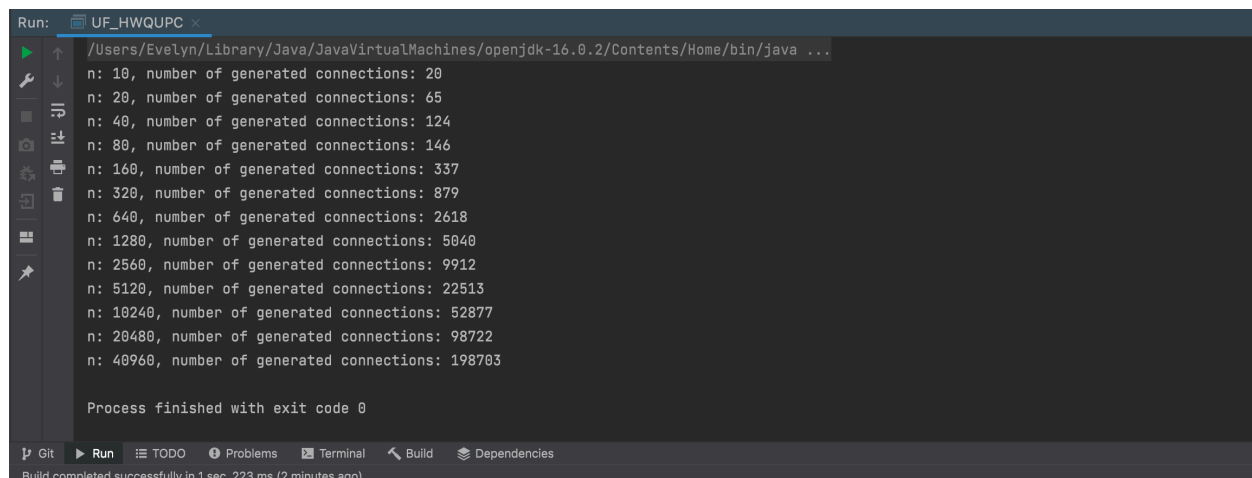
Step 3:

Determine the relationship between the number of objects ( $n$ ) and the number of pairs ( $m$ ) generated to accomplish this (i.e. to reduce the number of components from  $n$  to 1). Justify your conclusion in terms of your observations and what you think might be going on.

- **Relationship Conclusion:**

The relationship between the number of objects ( $n$ ) and the number of pairs ( $m$ ) generated to accomplish is  $N/2 * \ln(N)$ . We can see this result by comparing their graph.

- Output:



```
Run: UF_HWQUPC x
/Users/Evelyn/Library/Java/JavaVirtualMachines/openjdk-16.0.2/Contents/Home/bin/java ...
n: 10, number of generated connections: 20
n: 20, number of generated connections: 65
n: 40, number of generated connections: 124
n: 80, number of generated connections: 146
n: 160, number of generated connections: 337
n: 320, number of generated connections: 879
n: 640, number of generated connections: 2618
n: 1280, number of generated connections: 5040
n: 2560, number of generated connections: 9912
n: 5120, number of generated connections: 22513
n: 10240, number of generated connections: 52877
n: 20480, number of generated connections: 98722
n: 40960, number of generated connections: 198703

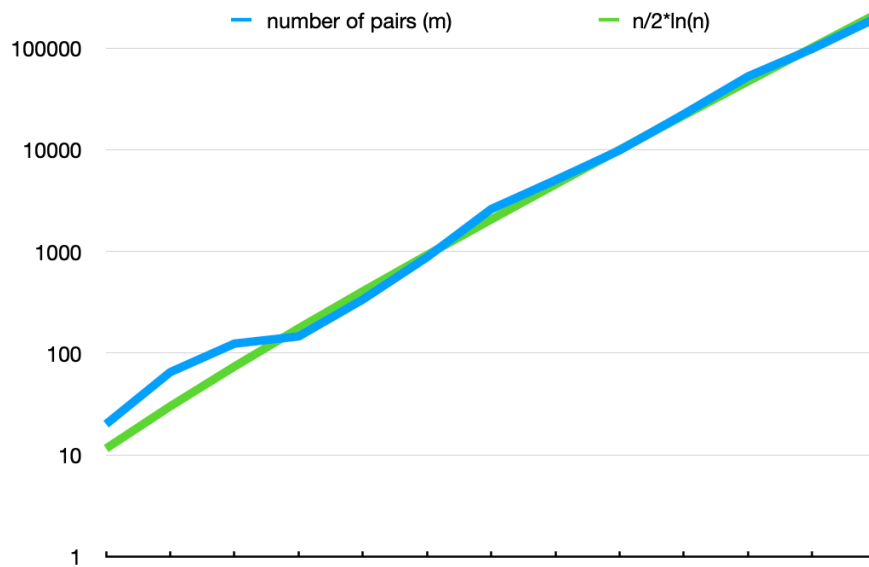
Process finished with exit code 0

Git Run TODO Problems Terminal Build Dependencies
Build completed successfully in 1 sec, 223 ms (2 minutes ago)
```

- Graphical Representation:

Table 1

number of objects (n)	number of pairs (m)	$n/2 \cdot \ln(n)$
10	20	11.5129254649702
20	65	29.9573227355399
40	124	73.7775890822787
80	146	175.281065386955
160	337	406.013905218706
320	879	922.931359327004
640	2618	2067.66981643319
1280	5040	4578.95382842474
2560	9912	10045.1360479662
5120	22513	21864.7288781659
10240	52877	47278.3713207987
20480	98722	101654.569770531
40960	198703	217504.79379893



- **Unit Test:**

