Strategy Space Collapse: Experiment and Theory

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Finding Nash equilibrium is the first task when facing a game. During a Nash equilibrium finding process, the dominated strategies will be eliminated, and the strategy space will collapse. Conceptually, as illustrated in figure 1(a), starting from randomly selection among available strategies, the strategy space will collapse due to successive eliminating dominated strategy (SEDS), and then fall into persistent cycles if possible, or converge to Nash equilibrium at last. This is a panorama of the Nash Equilibrium finding.

In theory, every step in this process has been well studied, including equilibrium existence and selection, stability and cycle, and SEDS [2] (or namely, strategy space collapse). In experiment, scientists have been able to quantitatively verify Nash equilibrium and cycles too[1, 4]. However, to the best of our knowledge, for space collapse, the only empirical result is about whether the dominated strategies would be deleted from the game [3]. What happening during the space collapse is rare known, and there is a lack of observations that could connect theory and experiment here. In this work, we focus on the strategy space collapse, by asking following two questions:

- In a laboratory experiment, how to detect the phenomena during the strategy space collapses?
- Under the current game theory framework, how to realize the observed experimental phenomena?

To detect the strategy space collapse during the successive eliminating dominated strategy, we conduct three matrix forms of the Von Neumann 3-Card Porker game experiments. In addition to the Nash distribution and social cycling in long run, we observed the pulse signals from dominated strategies before their extinction. The result shows that, all these observations, including Nash distribution, social cycling and the pulse signal, can be well explained by evolutionary game dynamics simultaneously and quantitatively. It can be seen that SEDS or the strategy space collapse processes is an area worthy of further exploration.

1 Method

Experiment We designed and conducted a 3-card Von Neumann Poker game to investigate the experimental phenomena and theoretical interpretation, which allows

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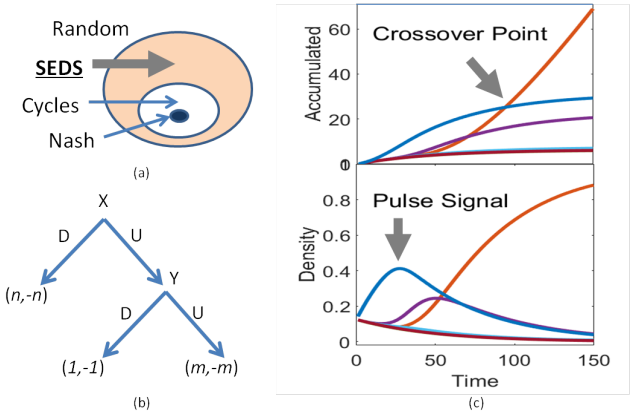

Figure 1: **Conceptual figure:** (a) The diagram of Finding Nash equilibrium. Strategy space collapse is a consequence of successive eliminating dominated strategy (SEDS). (b) Parameter (m, n) of the John von Neumann 3-card poker game experiment. (c) Pulse signal and crossover points in strategy evolution time series.

for the simultaneous research of the panorama — the collapse, cycles and solution concepts. The experiments include three (controlled parameters) treatment, in which (m,n) is set to (2,1), (3,2) and (4,3) respectively. Each treatment has 12 session, and each session has fixed-pair two human subjects playing 1000 repeated zero-sum game. The diagram can be seen in fig.1(b).

Dynamics Model In this work, a Logit dynamics, with noise parameter λ and proposed parameter time step Δt, is used to simulate the iterated game and realize the expected phenomena.

2 Findings and Arguments

Result 1: Finding Nash distribution & Persistent Cycle The distributions of the strategies used meet the Nash predictions in all of the treatment. The persistent cycles are observed in the first treatment, but not in the 2nd and 3rd treatments, which also consists with theoretical prediction of game dynamics model. [1, 4]

Result 2: Pulse signal during space collapse As shown in figure 1(c),**Pulse signals** could be observed. Here, the pulsing signal comes from the dominated strategy, which would be eliminated in long run but observed as the strongest signal during the collapse process. As a consequence, in accumulation curves of strategy used, there exist **crossover points** between the dominated/dominating strategy(See fig.2). Meanwhile, the order of SEDS can be identified, which do not reject the iterated eliminating dominated strategy hypothesis. In the experiment settings, the interval of time SEDS costs is [150 350] rounds depending on the parameters of the game treatments.

Result 3: Logit dynamics model, with noise parameter ($\lambda = 50$) and time step ($\triangle t = 0.02$), can **simultaneously** capture: (1) the pulsing signals of the dominated strategy and the crossover of the dominating/dominated strategies, (2) the cyclic behaviour

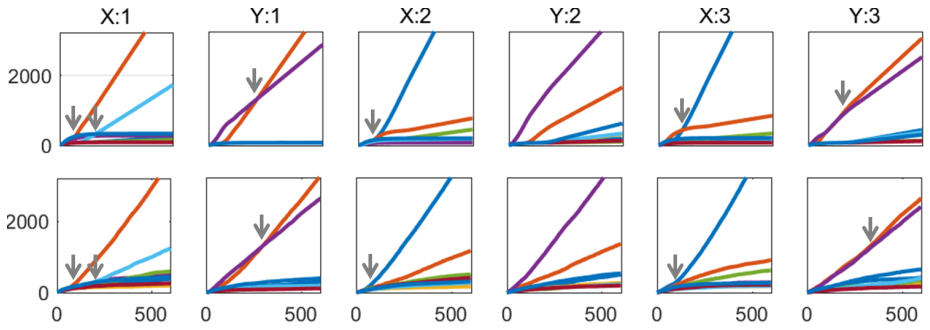

Figure 2: **Results:** Accumulated strategy used of player (X, Y) in Game (1, 2, 3) of theory (top) and experiment (bottom) of the two-players games (1-600 rounds). The crossover points labelled by arrows.

in long-run, and (3) the order of the distribution of Nash strategies, which related to Nash equilibrium.

Using existed measurements, result from the data confirms that the observed Nash distribution and cycles meet theoretical expectations well. Main finding in this work is counter-intuitive — the pulse signal, which dominates the game at the beginning, could come from a dominated strategies before its extinction during the strategy space collapse. We show that, all the observations can be explained by evolutionary game dynamics **simultaneously**, which has better performance here than the rational analysis tech in classical game theory. Our experiment suggests that, in the panorama of the Nash Equilibrium finding, the strategy space collapses is an important area.

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