# Quantitative Trade Models

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# Quantitative Trade Models

- Essence: Transmissions of shocks or policies via networks

- Shocks/policies $\rightarrow_{\text{Trade or other networks}}$ Local economies $\rightarrow_{\text{Local conditions}}$ Local changes $\rightarrow_{\text{Spillovers}}$ Other regions $\rightarrow_{\text{Feedback loops}}$ Aggregate implications

- Example:
  - Labor market: Lee, Eunhee (2020) JIE

  - Innovation: Sampson, Thomas (2023) AER

  - Environment: Shapiro, Joseph (2021) QJE

  - Transportation: Wong, Woan Foong (2022) AEJ: Applied

  - Quality: Fieler, Cecilia and Jonathan Eaton (2025) Econometrica

# Road Map

- Probabilistic Approach: Eaton and Kortum (2002)

- A workhorse model of quantitative trade: Trade+IO linkages+Scale economies in production

- Estimating the heterogeneous-firm-trade model using firm-level data: Eaton, Kortum, and Kramarz (2011)

# Probabilistic Approach: Eaton and Kortum (2002)

- $N$ countries.

- Country $i$ is endowed with labor $L_i$.

- The representative consumer in country $i$ consumes a final good consisting of a continuum of varieties indexed by $\omega \in [0, 1]$. These varieties are aggregated by a CES function:

$$U_i = Q_i = \left[ \int_0^1 Q_i(\omega)^{\frac{\sigma-1}{\sigma}} \, d\omega \right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1, \tag{1}$$

where $Q_i(\omega)$ is the quantity of variety $\omega$ consumed by the representative consumer in country $i$.

- Each good $\omega$ is produced by a firm using labor and final goods under perfect competition. Let $w_i$ be the wage and $P_i$ be the price index of final good derived later. The unit cost for producing variety $\omega$ in country $i$ is assumed to be

$$c_i(\omega) = \frac{w_i}{z_i(\omega)}, \tag{2}$$

where $z_i(\omega)$ is the productivity for producing variety $\omega$ in country $i$.

# Probabilistic Approach: Eaton and Kortum (2002)

- Perfect competition in the market for each $\omega$.

- Trade from country $i$ to country $n$ incurs an iceberg trade cost $\tau_{in} \geq 1$.

- For each country $i$ and variety $\omega$, the productivity $z_i(\omega)$ is drawn independently from a Fréchet distribution:

$$Prob\{z_i(\omega) \leq z\} = F_i(z) \equiv \exp\{-T_i z^{-\theta}\}, \quad z > 0, \quad T_i > 0, \quad \theta > \max\{1, \sigma-1\}. \tag{3}$$

  - Scale parameter $T_i$: the average productivity of country $i$.

  - Shape parameter $\theta$: dispersion of productivities across varieties.

# Probabilistic Approach: Eaton and Kortum (2002)

- From which country will the representative consumer in country $n$ purchase variety $\omega$?
  - The consumers will only buy a particular variety from the lowest-cost country.

- The price of variety $\omega$ produced in country $i$ served to country $n$ can be given by

$$p_{in}(\omega) = \left[\frac{w_i}{z_i(\omega)}\right] \tau_{in}. \tag{4}$$

- The price actually paid by consumers in country $n$ is

$$p_n(\omega) = \min_i \{p_{in}(\omega)\} . \tag{5}$$

# Probabilistic Approach: Eaton and Kortum (2002)

**Lemma**

*The CDF of the price of variety $\omega$ actually paid by consumers in country $n$ is*

$$Pr(p_n(\omega) \leq p) = 1 - \exp\{-\Phi_n p^{\theta}\}, \tag{6}$$

*where $\Phi_n = \sum_{i=1}^{N} T_i(w_i \tau_{in})^{-\theta}$.*

# Probabilistic Approach: Eaton and Kortum (2002)

**Proposition**

*The probability that country i provides a good at the lowest price in country n is*

$$\pi_{in} = \frac{T_i(w_i \tau_{in})^{-\theta}}{\Phi_n}. \tag{7}$$

# Probabilistic Approach: Eaton and Kortum (2002)

### Proposition

*The price of variety $\omega$ that country $n$ actually buys from any country $i$ satisfies:*

$$prob\left(p_{in}(\omega) \le p \,|\, p_{in}(\omega) \le \min\{p_{kn}(\omega); k \ne i\}\right) = 1 - \exp\{-\Phi_n p^{\theta}\}. \qquad (8)$$

- The expenditure share of country $n$ on goods from country $i$ is *equal to* the probability that country $i$ provides the lowest cost to country $n$:
    - When $\tau_{in}$ decreases, country $i$ will provide more varieties to country $n$.

    - For each variety $\omega$ it serves country $n$, the relative price stays unchanged.

# Probabilistic Approach: Eaton and Kortum (2002)

- The price index of the final good in country $n$ can be derived from the ideal price index under CES preference:

$$P_n = E\left[p_n(\omega)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = \gamma \Phi_n^{-\frac{1}{\theta}}, \tag{9}$$

where the constant $\gamma = \left[\Gamma\left(\frac{\theta-(\sigma-1)}{\theta}\right)\right]^{\frac{1}{1-\sigma}}$ and $\Gamma(y) = \int_0^\infty x^{y-1}e^{-x}dx$ is the gamma function.

- The expression in Equation (9) comes from the moment generating function of the Weibull distribution.

# Probabilistic Approach: Eaton and Kortum (2002)

## Definition
Given the environment $(L_i, T_i, \tau_{in}, \theta)$, the equilibrium consists of wage $\{w_i\}$ such that

- Labor markets clear

$$w_i L_i = \sum_{n=1}^{N} \frac{T_i \left(w_i \tau_{in}\right)^{-\theta}}{\sum_{k=1}^{N} T_k \left(w_k \tau_{kn}\right)^{-\theta}} w_n L_n. \qquad (10)$$

- Existence and uniqueness of the equilibrium
  - Please show that the excess labor demand function
    $\zeta_i(\mathbf{w}) := \frac{1}{w_i} \sum_{n=1}^{N} \frac{T_i(w_i \tau_{in})^{-\theta}}{\sum_{k=1}^{N} T_k(w_k \tau_{kn})^{-\theta}} w_n L_n - L_i$ satisfies the *gross substitutes property*.

  - General discussions of the equilibrium existence and uniqueness: Allen, Arkolakis, and Li (2024)

# Probabilistic Approach: Eaton and Kortum (2002)

- Eaton and Kortum (2002) delivers the exactly identical equilibrium system with the Armington model

- More reasonable micro-foundations: Ricardian comparative advantage

- Varieties in the EK model $\neq$ Disaggregated goods in data

- Probabilistic approach: tractable aggregation of heterogeneous agents' behaviors $\rightarrow$ few parameters/statistics

# Last Time

- Dynamic programming:
    - Dynamic decisions made by individuals

- Probabilistic approach: Eaton and Kortum (2002)
    - Given the realizations of idiosyncratic shocks, solve for individual decisions

    - Aggregate individual decisions based on the distributions of idiosyncratic shocks

    - Distributions: parsimonious (macro-tractable) vs. realistic (micro-fit)

# Today

- Workhorse model of trade and industrial policies
    - Elements: comparative advantage, trade frictions, input-output linkages, increasing returns to scale

    - Policies: import/export tariffs, industrial subsidies

- Using micro data to estimate GE trade model: Eaton, Kortum, and Kramarz (2011)
    - Firm heterogeneity in trade models a la Melitz (2003)

    - Micro-fit? Why can't we bring Melitz model to data?

    - Macro-tractability? How to keep elegant aggregation in Melitz (2003)

# A Workhorse Model of Trade and Industrial Policies

- $N$ countries with $\{L_i\}_{i=1}^N$.

- $J$ sectors:

$$U(C_n) = \prod_{j=1}^J \left(C_n^j\right)^{\alpha_n^j}, \quad \sum_{j=1}^J \alpha_n^j = 1. \tag{11}$$

- A continuum of varieties $\omega^j \in [0,1]$ in each sector $j$, aggregated via a CES function with elasticity of substitution $\sigma^j$ and produced under perfect competition.

- Unit cost of production factors:

$$c_i^j\left(\omega^j\right) = \frac{c_i^j}{z_i^j\left(\omega^j\right)\left(L_i^j\right)^{\psi^j}}, \quad c_i^j \equiv w_i^{\gamma_i^j} \prod_{k=1}^J \left(P_i^k\right)^{\gamma_i^{k,j}}, \tag{12}$$

where $L_i^j$ is the total labor in sector $j$ of country $i$ and $\psi_j \geq 0$ reflects the external economies of scale.

# A Workhorse Model of Trade and Industrial Policies

- $z_n^j(\omega^j)$ draws from a Frechet distribution:

$$Prob\{z_i^j\left(\omega^j\right) \leq z\} = F_i(z) \equiv \exp\{-T_i^j z^{-\theta^j}\}, \quad z > 0, \quad T_i^j \geq 0, \quad \theta^j > \max\{1, \sigma^j - 1\}. \quad (13)$$

- Trade costs:
    - Iceberg trade costs: $\tau_{in}^j \geq 1$ with $\tau_{ii}^j = 1$
    - Export tariff: $e_{in}^j \geq 0$ with $e_{ii}^j = 0$
    - Import tariff: $t_{in}^j \geq 0$ with $t_{ii}^j = 0$

- Ad-valorem ndustrial subsidy: $s_i^j \geq 0$
    - Define $\kappa_{in}^j \equiv \tau_{in}^j \left(1 - s_i^j\right)\left(1 + e_{in}^j\right)\left(1 + t_{in}^j\right)$

# Equilibrium

- Trade share and price index:

$$\lambda_{in}^j = \frac{T_i^j \left(L_i^j\right)^{\theta^j \psi^j} \left(c_i^j \kappa_{in}^j\right)^{-\theta^j}}{\sum_{h=1}^N T_h^j \left(L_h^j\right)^{\theta^j \psi^j} \left(c_h^j \kappa_{hn}^j\right)^{-\theta^j}}, \quad P_n^j = \left[\sum_{i=1}^N T_i^j \left(L_i^j\right)^{\theta^j \psi^j} \left(c_i^j \kappa_{in}^j\right)^{-\theta^j}\right]^{-\frac{1}{\theta^j}}. \tag{14}$$

- Labor market clearing:

$$L_i^j = \frac{\gamma_i^j}{w_i} \sum_{n=1}^N \frac{\lambda_{in}^j X_n^j}{\left(1 - s_i^j\right)\left(1 + e_{in}^j\right)\left(1 + t_{in}^j\right)}, \quad \sum_{j=1}^J L_i^j = L_i. \tag{15}$$

- Total expenditure:

$$X_i^j = \alpha_i^j Y_i + \sum_{k=1}^J \gamma_i^{j,k} \sum_{n=1}^N \frac{\lambda_{in}^k X_n^k}{\left(1 - s_i^k\right)\left(1 + e_{in}^k\right)\left(1 + t_{in}^k\right)}, \tag{16}$$

where

$$Y_i = w_i L_i + \sum_{j=1}^J \sum_{n=1}^N \left[-\frac{s_i^j}{1 - s_i^j} + \frac{e_{in}^j}{\left(1 - s_i^j\right)\left(1 + e_{in}^j\right)}\right] \lambda_{in}^j X_n^j + \sum_{j=1}^J \sum_{n=1}^N \frac{t_{ni}^j}{\left(1 - s_n^j\right)\left(1 + e_{ni}^j\right)\left(1 + t_{ni}^j\right)} \lambda_{ni}^j X_i^j. \tag{17}$$

# Exact-Hat Algebra

- Policy changes: $\left( \widehat{1 - s_i^j}, \widehat{1 + e_{in}^j}, \widehat{1 + t_{in}^j} \right)$

- Equilibrium changes: $\left( \hat{w}_i, \hat{X}_i^j, \hat{P}_i^j, \hat{L}_i^j \right)$

- Parameters: $\left( \alpha_i^j, \gamma_i^j, \gamma_i^{k,j}, \psi^j, \theta^j \right)$

- Data: $\left( X_{in}^j, s_i^j, e_{in}^j, t_{in}^j \right)$

# Estimating $\theta^j$

- Fixed-effect gravity:

  -

$$\log \lambda_{in}^j = -\theta^j \log \left(1 + t_{in}^j\right) + D_{in}\delta^j + fe_i^j + fe_n^j + \epsilon_{in}^j \tag{18}$$

  -

$$\Delta \log \lambda_{in}^j = -\theta^j \Delta \log \left(1 + t_{in}^j\right) + fe_i^j + fe_n^j + \Delta \epsilon_{in}^j \tag{19}$$

  - What are the structural interpretations of fixed effects and errors in two equations above? What are the assumptions of identifying $\theta^j$?

- Caliendo and Parro (2015)

$$\log \left(\frac{X_{in}^j X_{nh}^j X_{hi}^j}{X_{ni}^j X_{ih}^j X_{hn}^j}\right) = -\theta^j \log \left(\frac{\left(1 + t_{in}^j\right)\left(1 + t_{nh}^j\right)\left(1 + t_{hi}^j\right)}{\left(1 + t_{ni}^j\right)\left(1 + t_{ih}^j\right)\left(1 + t_{hn}^j\right)}\right) + \epsilon_{inh}^j \tag{20}$$

What are the structural interpretations of the error? What are the assumptions of identifying $\theta^j$?

# Estimating $\psi^j$

- $fe_i^j$ in the gravity equation (in levels):

$$fe_i^j = \left(\theta^j \psi^j\right) \log L_i^j + \underbrace{\log\left(T_i^j\right) - \theta^j \log\left(c_i^j \left(1 - s_i^j\right)\right)}_{\nu_i^j} \qquad (21)$$

- Regress $fe_i^j$ on $\log\left(L_i^j\right)$?

- Ideas for IV: Bartelme et al. (2025)
    - Demand shifters on labor in sector $j$ of country $i$

    - A measure of demand-predicted sector size: Bartik instrument

# Eaton, Kortum, and Kramarz (2011)

- Linking to micro to macro:
    - How relevant is firm heterogeneity to exports in micro data?
    - Trade liberalization $\Rightarrow$ Firm sales and Welfare.

- We need a structural model that can
    - capture firm heterogeneity from the micro data, and
    - account the impacts of firm heterogeneity on aggregates.

- Melitz (2003) cannot match firm data directly:
    - Prediction: more productive firms are more likely to enter into Every market.
    - Not the case in micro data.

- Eaton, Kortum, and Kramarz (2011): An anatomy of international trade: evidence from French firms.
    - Carefully match their model into the firm-level data.
    - Elegant aggregation techniques.

# A Very Standard Quantitative Trade Paper

0. Motivational facts.
1. Theoretical framework: $y = g(x)$.
2. Practical specification: $y = g(x, \xi, \Theta)$.
3. Solving the model.
4. Understanding how the model works:
    - Connecting model predictions to motivational facts.
    - Preparing the model to fit the data.
5. Estimation.
6. Validation:
    - Goodness-of-fit/Out-of-sample predictions.
    - Interpretation.
    - Structurally decomposing the magnitudes of factors in interest.
7. Counterfactual experiments:
    - Existing/hypothetical policies and fundamental shocks.

# Targeted Facts



Panel B: Normalized Entry
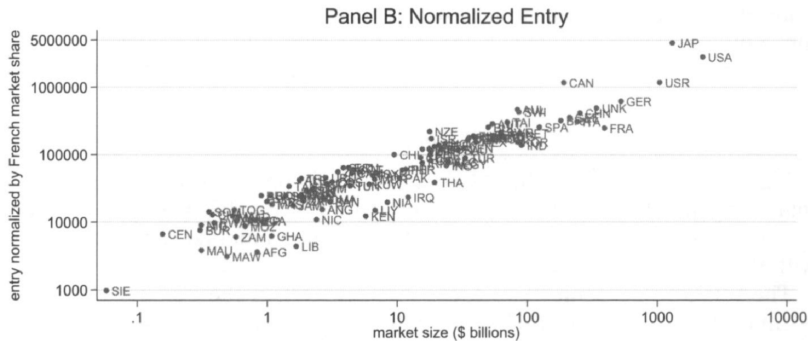
(Note: normalized entry is equal to $\frac{N_{nF}}{X_{nF}/X_n}$ where $N_{nF}$ is the number of French firms selling to a market, $X_{nF}$ is total exports of French firms to market $n$, and $X_n$ is the market size.)

# Targeted Facts

## TABLE II

### FRENCH FIRMS SELLING TO STRINGS OF TOP-SEVEN COUNTRIES

| Export String[a] | Number of French Exporters | | |
|---|---|---|---|
| | Data | Under Independence | Model |
| BE[a] | 3988 | 1700 | 4417 |
| BE–DE | 863 | 1274 | 912 |
| BE–DE–CH | 579 | 909 | 402 |
| BE–DE–CH–IT | 330 | 414 | 275 |
| BE–DE–CH–IT–UK | 313 | 166 | 297 |
| BE–DE–CH–IT–UK–NL | 781 | 54 | 505 |
| BE–DE–CH–IT–UK–NL–US | 2406 | 15 | 2840 |
| Total | 9260 | 4532 | 9648 |

[a] The string BE means selling to Belgium but no other among the top 7; BE–DE means selling to Belgium and Germany but no other, and so forth.
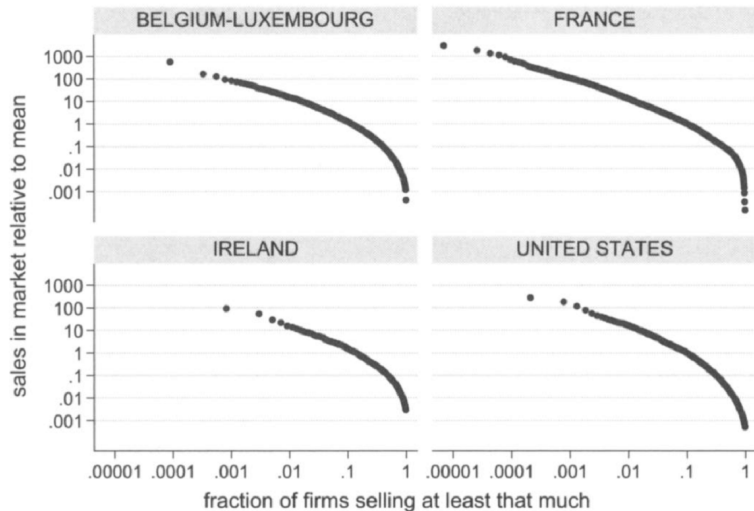
# Targeted Facts



FIGURE 2.—Sales distributions of French firm: Graphs by country.
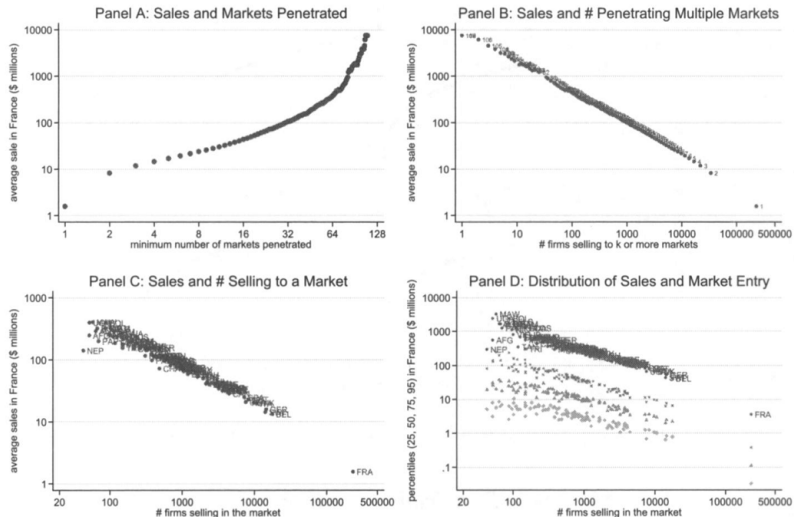
# Targeted Facts



FIGURE 3.—Sales in France and market entry.

# Model: Firm Heterogeneity

- Cost of serving market $d$:

$$c_{ni}(j) = \frac{w_i d_{ni}}{z_i(j)}. \tag{22}$$

- The measure of potential producers in country $i$ with efficiency at least $z$:

$$\mu_i^z(z) = T_i z^{-\theta}, \quad z > 0. \tag{23}$$

- The measure of goods with cost below $c$:

$$\mu_{ni}(c) = \Phi_{ni} c^{\theta}, \quad \Phi_{ni} = T_i (w_i d_{ni})^{-\theta}. \tag{24}$$

# Model: Entry and Demand Shocks

- A market $n$ contains a unit measure of potential buyers. To sell to a fraction $f$ of them, a producer in country $i$ must incur a fixed cost:

$$E_{ni}(j) = \varepsilon_n(j) E_{ni} M(f), \tag{25}$$

where

$$M(f) = \frac{1 - (1-f)^{1-\frac{1}{\lambda}}}{1 - \frac{1}{\lambda}}, \quad \lambda > 0. \tag{26}$$

- Melitz (2003): $\lambda = \infty$.

- Monopolistic competition:

$$X_n(j) = \alpha_n(j) f p^{1-\sigma} X_n P_n^{\sigma-1}. \tag{27}$$

# Model: the Firm's Problem

- Profit in $n$:

$$\Pi_{ni}(p, f) = \left(1 - \frac{c_n(j)}{p}\right) \alpha_n(j) f p^{1-\sigma} X_n P_n^{\sigma-1} - \varepsilon_n(j) E_{ni} M(f). \qquad (28)$$

- Optimal pricing:

$$p_n(j) = \bar{m} c_n(j), \quad \bar{m} = \frac{\sigma}{\sigma - 1}. \qquad (29)$$

- Optimal fraction:

$$f_{ni}(j) = \max\left\{1 - \left[\eta_n(j)\frac{X_n}{\sigma E_{ni}}\left(\frac{\bar{m} c_n(j)}{P_n}\right)^{-(\sigma-1)}\right]^{-\lambda}, 0\right\}, \qquad (30)$$

where $\eta_n(j) = \alpha_n(j)/\varepsilon_n(j)$.

## Model: Export Entry and Sales

- A firm enters market $n$ if and only if

$$c \leq \bar{c}_{ni}(\eta) := \left( \eta \frac{X_n}{\sigma E_{ni}} \right)^{\frac{1}{\sigma-1}} \frac{P_n}{\bar{m}}. \tag{31}$$

- The optimal fraction, sales, and fixed cost can be expressed by $c$ and $\bar{c}_{ni}(\eta)$:
  - $f_{ni}(\eta, c) = 1 - \left( \frac{c}{\bar{c}_{ni}(\eta)} \right)^{\lambda(\sigma-1)}$.
  - $X_{ni}(\alpha, \eta, c) = \frac{\alpha}{\eta} \left[ 1 - \left( \frac{c}{\bar{c}_{ni}(\eta)} \right)^{\lambda(\sigma-1)} \right] \left( \frac{c}{\bar{c}_{ni}(\eta)} \right)^{-(\sigma-1)} \sigma E_{ni}$.
  - $E_{ni}(\alpha, \eta, c) = \frac{\alpha}{\eta} E_{ni} \frac{1 - (c/\bar{c}_{ni}(\eta))^{(\lambda-1)(\sigma-1)}}{1 - 1/\lambda}$.

## Aggregation: Price

- The price index in country $n$ is

$$P_n^{1-\sigma} = \bar{m} \left[ \int \int \left( \sum_{i=1}^{N} \int_0^{\bar{c}_{ni}(\eta)} \alpha f_{ni}(\eta, c) c^{1-\sigma} d\mu_{ni}(c) \right) g(\alpha, \eta) d\alpha d\eta \right]. \qquad (32)$$

- Integrating over $c$, we have

$$P_n^{-\theta} = \kappa_1 \Psi_n X_n^{\frac{\theta - (\sigma-1)}{\sigma-1}}, \qquad (33)$$

where $\Psi_n = \sum_{i=1}^{N} \Phi_{ni} E_{ni}^{-\frac{\theta - (\sigma-1)}{\sigma-1}}$.

## Aggregation: Entry, Sales, Fixed Costs

- Entry condition: $\bar{c}_{ni}(\eta) = \eta^{1/(\sigma-1)} \left( \frac{X_n}{\kappa_1 \Psi_n} \right)^{\frac{1}{\theta}} E_{ni}^{-\frac{1}{\sigma-1}}$.

- Trade share: $\pi_{ni} = \frac{X_{ni}}{X_n} = \frac{\Phi_{ni} E_{ni}^{-\frac{\theta-(\sigma-1)}{\sigma-1}}}{\Psi_n}$.

- Total measure of firm in $i$ serving $n$: $J_{ni} = \frac{\kappa_2}{\kappa_1} \frac{\pi_{ni} X_n}{E_{ni}}$.

- Total fixed costs: $\bar{E}_{ni} = \frac{\theta-(\sigma-1)}{\theta\sigma} \pi_{ni} X_n$.

# A Streamlined Representation

- Prepare the model to match the data.
- Let $u(j) = T_F z_F(j)^{-\theta}$ be standardized unit cost of firm $j$ in France. The measure of firms with $u(j) \leq u$:

$$\mu_F^z((T_F/u)^{1/\theta}) = u. \tag{34}$$

- Firm $j$ enters market $n$ if its $u(j)$ and $\eta_n(j)$ satisfy

$$u(j) \leq \bar{u}_{nF}(\eta_n(j)) = \left(\frac{\pi_{nF} X_n}{\kappa_1 \sigma E_{nF}}\right) \eta_n(j)^{\tilde{\theta}}, \quad \tilde{\theta} = \frac{\theta}{\sigma - 1}. \tag{35}$$

- Conditional on entry firm $j$'s sales in market $n$ is

$$X_{nF}(j) = \varepsilon_n(j) \left[1 - \left(\frac{u(j)}{\bar{u}_{nF}(\eta_n(j))}\right)^{\lambda/\tilde{\theta}}\right] \left(\frac{u(j)}{\bar{u}_{nF}(\eta_n(j))}\right)^{-1/\tilde{\theta}} \sigma E_{nF}. \tag{36}$$

# Connecting the Model to the Empirical Regularities

- Entry:
  - A relationship between the number of French firms selling to market $n$ and the size of market $n$:

  $$\frac{N_{nF}}{\pi_{nF}} = \frac{\kappa_2}{\kappa_1} \frac{X_n}{\sigma E_{nF}}. \tag{37}$$

  - Calculate the fixed export costs directly from

  $$\sigma E_{nF} = \frac{\kappa_2}{\kappa_1} \frac{\pi_{nF} X_n}{N_{nF}} = \frac{\kappa_2}{\kappa_1} \bar{X}_{nF}. \tag{38}$$

  - Entry condition:

  $$u(j) \leq \bar{u}_{nF}(\eta_n(j)) = \frac{N_{nF}}{\kappa_2} \eta_n(j)^{\tilde{\theta}}. \tag{39}$$

# Connecting the Model to the Empirical Regularities

- Sales in a market:
  - Conditional on a firm's entry into market $n$, the term

  $$\nu_{nF}(j) = \frac{u(j)}{\bar{u}_{nF}(\eta_n(j))} \qquad (40)$$

  is distributed uniformly on $[0, 1]$.
  - Then the sales

  $$X_{nF}(j) = \varepsilon_n(j)[1 - \nu_{nF}(j)^{\lambda/\tilde{\theta}}]\nu_{nF}(j)^{-1/\tilde{\theta}}\frac{\kappa_2}{\kappa_1}\bar{X}_{nF}. \qquad (41)$$

# Connecting the Model to the Empirical Regularities

- Sales in France conditional on entry in a foreign market:

$$X_{FF}(j)|n = \frac{\alpha_F(j)}{\eta_n(j)} \left[ 1 - \nu_{nF}(j)^{\lambda/\tilde{\theta}} \left( \frac{N_{nF}}{N_{FF}} \right)^{\lambda/\tilde{\theta}} \left( \frac{\eta_n(j)}{\eta_F(j)} \right)^{\lambda} \right]$$

$$\times \nu_{nF}(j)^{-1/\tilde{\theta}} \left( \frac{N_{nF}}{N_{FF}} \right)^{-1/\tilde{\theta}} \frac{\kappa_2}{\kappa_1} \bar{X}_{FF}.$$

(42)

- Normalized export intensity:

$$X_{nF}(j)/X_{FF}(j)\bar{X}_{nF}/\bar{X}_{FF} =$$

$$\frac{\alpha_n(j)}{\alpha_F(j)} \left[ \frac{1 - \nu_{nF}(j)^{\lambda/\tilde{\theta}}}{1 - \nu_{nF}(j)^{\lambda/\tilde{\theta}} \left( \frac{N_{nF}}{N_{FF}} \right)^{\lambda/\tilde{\theta}} \left( \frac{\eta_n(j)}{\eta_F(j)} \right)^{\lambda}} \right] \left( \frac{N_{nF}}{N_{FF}} \right)^{1/\tilde{\theta}}.$$

(43)

# Estimation

- Assume that $\log(\alpha)$ and $\log(\eta)$ are normally distributed with zero mean and variance $\sigma_\alpha^2$ and $\sigma_\eta^2$, and correlation $\rho$. Then
  - $\kappa_1 = \left[ \frac{\tilde{\theta}}{\tilde{\theta}-1} - \frac{\tilde{\theta}}{\tilde{\theta}+\lambda-1} \right] \exp\{ \frac{\sigma_\alpha + 2\rho\sigma_\alpha\sigma_\eta(\tilde{\theta}-1) + \sigma_\eta(\tilde{\theta}-1)^2}{2} \}$.
  - $\kappa_2 = \exp\left\{ \frac{(\tilde{\theta}\sigma_\eta)^2}{2} \right\}$.

- Only five parameters to estimate: $\Theta = \{\tilde{\theta}, \lambda, \sigma_\alpha, \sigma_\eta, \rho\}$.

- Simulated Methods of Moments:
  - Given $\Theta$, simulate artificial samples.
  - Compute the moments from simulated samples.
  - Compare the simulated moments to the data moments.
  - Change $\Theta$ to minimize the distance between simulated moments and the data moments.

# Simulated Methods of Moments

- Artificial French exporter $s$ with the number $S$. 113 destinations.

- Draw $S$ realizations of $\nu(s)$ independently from $U[0, 1]$.

- Draw $S \times 113$ realizations of $a_n(s)$ and $h_n(s)$ independently from $N(0, 1)$.

- A given simulation of the model requires $\Theta$, $X_{nF}$, and $N_{nF}$.

# Simulated Methods of Moments

1. Calculate $\kappa_1$ and $\kappa_2$.
2. Calculate $\sigma E_{nF}$ for each destination $n$.
3. Construct $S \times 113$ realizations for each of $\log \alpha_n(s)$ and $\log \eta_n(s)$.
4. Construct $S \times 113$ entry hurdles $\bar{u}(s) = \frac{N_{nF}}{\kappa_2} \eta_n(s)^{\bar{\theta}}$.
5. Calculate $\bar{u}^X(s) = \max_{n \neq F}\{\bar{u}_n(s)\}$ and $\bar{u}(s) = \min\{\bar{u}_F(s), \bar{u}^X(s)\}$.
6. $u(s)$ is a realization from $U[0, \bar{u}(s)]$. So $u(s) = \nu(s)\bar{u}(s)$.
7. Artificial French exporter $s$ gets an importance weight $\bar{u}(s)$.
8. Market entry index $\delta_{nF}(s) = 1$ if $u(s) \leq \bar{u}_n(s)$ and 0 otherwise. With $\delta_{nF}(s) = 1$,

$$X_{nF}(s) = \frac{\alpha_n(s)}{\eta_n(s)}\left[1 - \left(\frac{u(s)}{\bar{u}_n(s)}\right)^{\lambda/\bar{\theta}}\right]\left(\frac{u(s)}{\bar{u}_n(s)}\right)^{-1/\bar{\theta}}\sigma E_{nF}. \tag{44}$$

# Moments

- Moments in this exercise: the number of firms that fall into sets of exhaustive and mutually exclusive bins.

- $N^k$: the number of firms achieving some outcome $k$ in the actual data.

- $\hat{N}^k$: corresponding number in simulated data, as $\hat{N}^k = \frac{1}{S} \sum_{s=1}^{S} \bar{u}(s) \delta^k(s)$ where $\delta^k(s)$ is an indicator for achieving outcome $k$.

# Four Sets of Moments

1. $\hat{m}^k(1; \Theta)$: the proportion of simulated exporters selling to each possible combination $k$ of the seven most popular export destinations.
2. $\hat{m}^k(2; \Theta)$: the proportion of simulated exporters in each market falling into $(50, 75, 95)$ quantiles of exporting sales.
3. $\hat{m}^k(3; \Theta)$: the proportion of simulated exporters in each market falling into the $(50, 75, 95)$ quantiles of French sales.
4. $\hat{m}^k(4; \Theta)$: the proportion of simulated exporters in each market falling into the $(50, 75)$ quantiles of export intensities.

# Estimation

- $y(\Theta) = m - \hat{m}(\Theta)$.
- We get the SMM estimator by:

$$\hat{\Theta} = \arg\min_{\Theta}\{y(\Theta)' W y(\Theta)\}. \tag{45}$$

- Search for $\hat{\Theta}$ using the simulated annealing algorithm.
- Calculate standard errors using a bootstrap technique.

| $\tilde{\theta}$ | $\lambda$ | $\sigma_\alpha$ | $\sigma_\eta$ | $\rho$ |
|---|---|---|---|---|
| 2.46 | 0.91 | 1.69 | 0.34 | $-0.65$ |
| (0.10) | (0.12) | (0.03) | (0.01) | (0.03) |

# Implications

- $\tilde{\theta} = 2.46$: fixed costs dissipate 59% of gross profit in any destination.

- Enormous idiosyncratic variation in a firm's sales across destinations but less variation in the entry shock.

- Consistent with the entry of firms into markets where they sell very little: (i) $\lambda = 1$, (ii) negative covariance between the sales and entry shocks.

# Model-fit



Panel A: Sales Distribution by Market
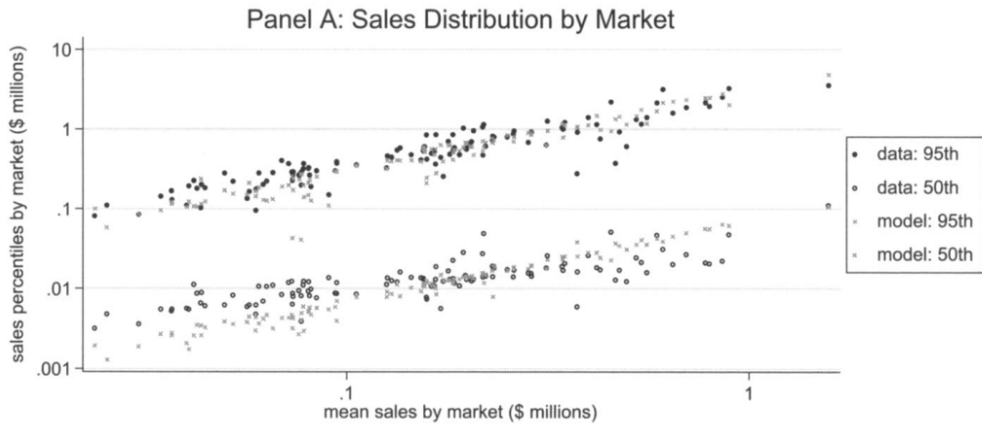
# Sources of Variation

- The fraction of the variance of entry in each market that can be explained by the cost draw $u$ alone:

$$R_n^E = 1 - \frac{E[V_n^C(u)]}{V_n^U},\qquad (46)$$

where $V_n^U$ is the variance of entry decisions in market $n$ conditional only on $u$ and $V_n^U$ is the unconditional variance.

- Result: on average 57% of the variation in entry in a market can be attributed to the core efficiency.

# Productivity

- The average value-added per worker of exporters is 1.22 times the average for all firms.

- Let $I(j)$ be the intermediate expenditure and $\beta$ be the value-added share:

$$I_i(j) = (1 - \beta)\bar{m}^{-1} Y_i(j) + E_i(j). \tag{47}$$

- Then $q_i(j) = \frac{V_i(j)}{\beta \bar{m}^{-1} Y_i(j)} = \frac{\bar{m} - (1-\beta)}{\beta} - \frac{\bar{m}}{\beta} \frac{E_i(j)}{Y_i(j)}$.

- Estimates: $\sigma = 2.98$ and $\beta = 0.34$.

- $\theta = 4.87$.

# General Equilibrium

- Factor bundles: $w_i = W_i^\beta P_i^{1-\beta}$.

- Manufactures have a share $\gamma$ in final consumption. Non-manufactures are produced by labor.

- $E_{ni} = W_n F_{ni}$.

- Each countries manufacturing deficit $D_i$ and total deficit $D_i^A$ are held at their 1986 values.

- General equilibrium definition and counterfactual computation are the same as EK(2002) and DEK (2008).

# Counterfactuals

## TABLE IV
### COUNTERFACTUALS: FIRM TOTALS[a]

| | Baseline | Counterfactual | |
| | | Change From Baseline | Percentage Change |
|---|---|---|---|
| Number | | | |
| All firms | 231,402 | −26,589 | −11.5 |
| Exporting | 32,969 | 10,716 | 32.5 |
| Values ($ millions) | | | |
| Total sales | 436,144 | 16,442 | 3.8 |
| Domestic sales | 362,386 | −18,093 | −5.0 |
| Exports | 73,758 | 34,534 | 46.8 |

# Counterfactuals

## TABLE V
### COUNTERFACTUALS: FIRM ENTRY AND EXIT BY INITIAL SIZE

| Initial Size Interval (percentile) | Baseline No. of Firms | All Firms | | Baseline No. of Firms | Exporters | |
|---|---|---|---|---|---|---|
| | | Counterfactual | | | Counterfactual | |
| | | Change From Baseline | Change (%) | | Change From Baseline | Change (%) |
| Not active | 0 | 1118 | — | 0 | 1118 | — |
| 0 to 10 | 23,140 | −11,551 | −49.9 | 767 | 15 | 2.0 |
| 10 to 20 | 23,140 | −5702 | −24.6 | 141 | 78 | 55.1 |
| 20 to 30 | 23,140 | −3759 | −16.2 | 181 | 192 | 106.1 |
| 30 to 40 | 23,140 | −2486 | −10.7 | 357 | 357 | 100.0 |
| 40 to 50 | 23,140 | −1704 | −7.4 | 742 | 614 | 82.8 |
| 50 to 60 | 23,138 | −1141 | −4.9 | 1392 | 904 | 65.0 |
| 60 to 70 | 23,142 | −726 | −3.1 | 2450 | 1343 | 54.8 |
| 70 to 80 | 23,140 | −405 | −1.8 | 4286 | 1829 | 42.7 |
| 80 to 90 | 23,140 | −195 | −0.8 | 7677 | 2290 | 29.8 |
| 90 to 99 | 20,826 | −38 | −0.2 | 12,807 | 1915 | 15.0 |
| 99 to 100 | 2314 | 0 | 0.0 | 2169 | 62 | 2.8 |

# Summary of EKK (2011)

- A GE model that can match the micro data carefully and quantify aggregate effects.

- Very rigorous empirical implementation.

- Firm heterogeneity in productivity accounts for 57% of export entry.

- Trade liberalization promotes aggregate productivities by reallocating labors to most productive firms.

# Trends in Quantitative Trade Literature

- Policy-driven
    - Uncertainties: Alessandria et al. (2025)

    - Geopolitics: Alekseev and Lin (2025)

- Interacting with other fields
    - Agriculture: Farrokhi and Pellegrina (2023)

    - Market power: Graziano, Alejandro (2024)

- Dynamics
    - Downward nominal wage rigidities: Rodriguez-Clare et al. (2024)