# Dynamic Trade and Spatial Models

Zi Wang
HKBU

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# Dynamic Structural Models

- Some problems are dynamic in nature: *e.g.*
  - Expectation/uncertainties
  - Migration
  - Innovation
- Dynamic settings have different implications with static ones: *e.g.*
  - Regulating carbon emissions $\Rightarrow$ Production costs increase in a static world $\Rightarrow$ Green innovation $\Rightarrow$ Production costs in a dynamic world

# Alessandria et al. (2025) "Trade-Policy Dynamics: Evidence from 60 Years of U.S.-China Trade"

- Trade policies $\Rightarrow$ Trade volumes
  - Present policies: static trade models
  - Past policies: trade responds gradually to past policy changes
  - Future policies: uncertainty about future policy can affect trade in the present
  - Interaction: The effects of past and future policy are intertwined
- Application:
  - How expectations of U.S. trade policy on China evolved
  - How these expectations, together with changes in tariffs, shaped Chinese export growth

# Empirical Evidence

- Two-pronged approach:
  - Short- and long-run elasticities of trade to the 1980 tariff reduction and the speed of adjustment
  - The elasticity of trade to the risk of reversing the tariff reduction
- Data:
  - Annual data on U.S. imports from 1974-2008: 5-digit SITC industries (good $g$ and exporting country $j$)— log of FOB import value $v_{gjt}$
  - Trade policy: applied and statutory tariff rates
    - Non-Normal Trade Relations (NNTR): high tariffs
    - Normal Trade Relations (NTR): low tariffs

# Trade and Policy Dynamics

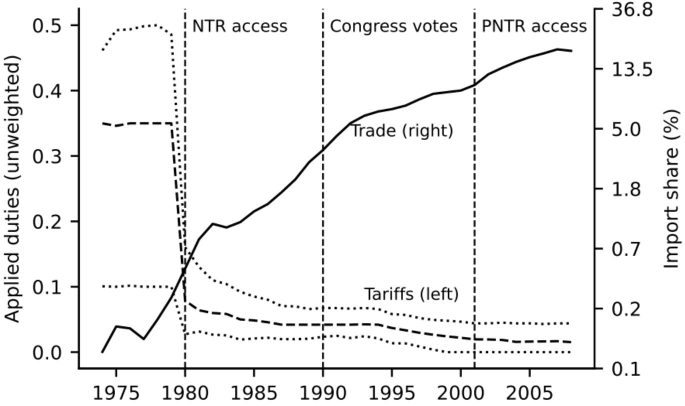


Fig. 1 – Growth of aggregate imports from China. Solid line: China's log share of total U.S. imports. Dashed line: median tariff across goods. Dotted lines: 25th and 75th percentiles.

# Slow adjustment to the 1980 granting of NTR status

- An error correction model (ECM):

$$\Delta v_{jgt} = \delta_{jt} + \delta_{jg} + \delta_{gt} + \left[\sigma^{SR}_{China}\Delta\tau_{jgt} + \gamma_{China}\left(v_{jg,t-1} - \sigma^{LR}_{China}\tau_{jg,t-1}\right)\right]\mathbb{1}_{\{j=China\}}$$
$$+ \left[\sigma^{SR}_{Others}\Delta\tau_{jgt} + \gamma_{Others}\left(v_{jg,t-1} - \sigma^{LR}_{Others}\tau_{jg,t-1}\right)\right]\mathbb{1}_{\{j=Others\}} + u_{jgt}. \quad (1)$$

- Local projection (LP):

$$\Delta_h v_{jg,1979} = \sigma^{h}_{China}\mathbb{1}_{\{j=China\}}\Delta_h\tau_{jg,1979} + \sigma^{h}_{Others}\mathbb{1}_{\{j\neq China\}}\Delta_h\tau_{jg,1979} + \delta_{jh} + \delta_{gh} + u_{jg}, \quad (2)$$

# Slow adjustment to the 1980 granting of NTR status

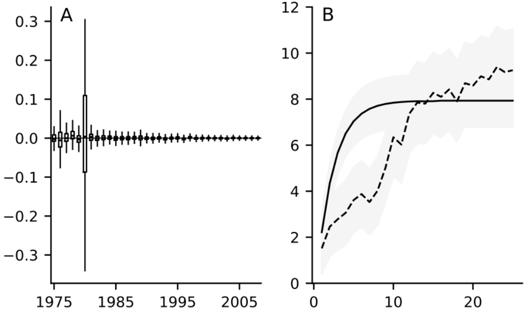


Fig. 2 – A: box-and-whisker plot of distribution of residuals from regressing $\Delta_1 \tau_{jgt}$ on $\delta_{jt}, \delta_{jg}$, and $\delta_{gt}$. B: elasticities of U.S. imports from China to tariff changes. Solid line shows ECM estimates from (1). Dashed line shows local-projections estimates from (2). Shaded areas: 95-percent confidence intervals.

- Short-run trade elasticity: $-2.22$ vs. Long-run trade elasticity: $-7.93$

# Effects of the risk of losing NTR status

- Time-invariant NTR gap: $GAP_g = \log\left(1 + \tau_g^{NNTR} - \tau_{g,2001}^{NTR}\right)$

- Estimating equation:

$$v_{jgt} = \sum_{t'=1974}^{2007} \beta_t \mathbf{1}_{\{t=t' \wedge j=China\}} GAP_g + \delta_{jt} + \delta_{jg} + \delta_{gt} + u_{jgt}. \tag{1}$$

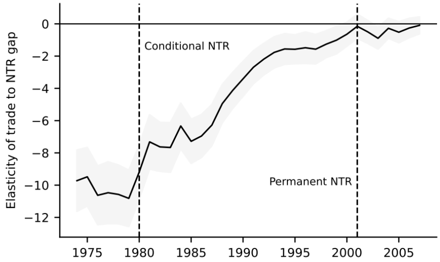


Fig. 3 – Elasticity of U.S. imports from China to the NTR gap. Solid line: estimates of $\hat{\beta}_t$ from (3). Shaded area: 95-percent confidence interval.

# Model of Trade-Policy Dynamics

- Purpose: to isolate the roles of gradual adjustment and policy uncertainty on the growth of U.S. imports from China

- $G$ goods: 5-digit SITC goods

- Within each good $g$, a continuum of heterogeneous Chinese firms produce differentiated varieties
    - Productivity $z$
    - Variable trade cost $\xi$
    - Exogenous death rate: $1 - \delta(z)$, with $\delta'(z) > 0$
    - Fixed mass of firms in each $g$: when a firm dies, a new firm replace it.
    - To export, a firm pays a fixed cost that depends on whether it exported in the previous period

# Model of Trade-Policy Dynamics

- Tariffs $\tau_{gt}(s)$
  - Trade-policy regime: NTR $s = 1$ or NNTR $s = 2$
  - Markov process with transition probabilities: $\omega_t(s, s')$
  - Firms know the entire path of regime-switching probabilities, $\{\omega_t(s, s')\}_{t=0}^{\infty}$
- Production and demand
  - Firms use labor to produce $y = z\ell$
  - Productivity $z$ is independent across firms and transitioned as $h_g(z, z')$
  - U.S. demand for a firm's good: $d_{gt}(p, s) = (p\tau_{gt}(s))^{-\theta_g} D_{gt}$, where $D_{gt}$ is an aggregate demand shifter

# Model of Trade-Policy Dynamics

- Variable trade cost $\xi$:
  - $\xi \in \{\xi_{gL}, \xi_{gH}, \infty\}$
  - $\xi = \infty$: nonexporter
  - When a nonexporter chooses to export, it begins with $\xi_{gH}$ in the next period
  - Exporters with $\xi < \infty$ retain their current variable trade cost with prob. $\rho_\xi$ and switch with prob. $1 - \rho_\xi$
- Fixed trade cost $f$: the firm pays to export in the next period
  - A nonexporter pays $f_{g0}$ to start exporting in the next period
  - An exporter pays $f_{g1}$ to continue exporting

# Model of Trade-Policy Dynamics

- Firm's static problem

$$\pi_{gt}(z,\xi,s) = \max_p p d_{gt}\left(p,\tau_{gt}(s)\right) - w\frac{d_{gt}\left(p,\tau_{gt}(s)\right)\xi}{z}. \tag{2}$$

- The values of exporting and not exporting at $t+1$:

$$\begin{aligned} V^1_{gt}(z,\xi,s) &= -f_g(s) + \frac{\delta(z)}{1+r}\sum_{s'}\omega_t\left(s,s'\right)\underset{z',\xi'}{E_t} V_{g,t+1}\left(z',\xi',s'\right) \\ V^0_{gt}(z,\xi,s) &= \frac{\delta(z)}{1+r}\sum_{s'}\omega_t\left(s,s'\right)\underset{z'}{E_t} V_{g,t+1}\left(z',\infty,s'\right) \end{aligned} \tag{3}$$

- The value of a firm:

$$V_{gt}\left(z,\xi,s\right) = \pi_{gt}\left(z,\xi,s\right) + \max\left\{V^1_{gt}(z,\xi,s), V^0_{gt}(z,\xi,s)\right\} \tag{4}$$

- Break-even exporter has productivity $\bar{z}_{gt}(\xi,s)$ such that $V^1_{gt}(\bar{z}_{gt}(\xi,s),\xi,s) = V^0_{gt}(\bar{z}_{gt}(\xi,s),\xi,s)$

# Aggregation

- $z \in \mathcal{Z}$:
  - $h_g(\mathcal{Z}, z)$ is the prob. of surviving and drawing a new productivity in $\mathcal{Z}$ conditional on today's $z$
  - $\bar{h}_g(\mathcal{Z})$ is the prob. of dying and being replaced by a new firm with productivity in $\mathcal{Z}$
- $\varphi_{gt}(z, \xi)$ is the distribution of $z$ and $\xi$; $Q_{gt}(\mathcal{Z}, z, \xi) \equiv h_g(\mathcal{Z}, z)\varphi_{gt}(z, \xi)$
- $\varphi_{g,t+1}(\mathcal{Z}, \infty) = \sum_{\xi} \left[ \int_0^{\bar{z}_{gt}(\xi, s_t)} Q_{gt}(\mathcal{Z}, z, \xi)\, dz + \int_{\bar{z}_{gt}(\xi, s_t)}^{\infty} \bar{h}_g(\mathcal{Z})\, \varphi_{gt}(z, \xi) dz \right]$
- $\varphi_{g,t+1}(\mathcal{Z}, \xi_{gH}) =$
  $\int_{\bar{z}_{gt}(\infty, s_t)}^{\infty} Q_{gt}(\mathcal{Z}, z, \infty)\, dz + \rho_{\xi} \int_{\bar{z}_{gt}(\xi_{gH}, s_t)}^{\infty} Q_{gt}(\mathcal{Z}, z, \xi_{gH})\, dz + (1 - \rho_{\xi}) \int_{\bar{z}_{gt}(\xi_{gL}, s_t)}^{\infty} Q_{gt}(\mathcal{Z}, z, \xi_{gL})\, dz$
- $\varphi_{g,t+1}(\mathcal{Z}, \xi_{gL}) = \rho_{\xi} \int_{\bar{z}_{gt}(\xi_{gL}, s_t)}^{\infty} Q_{gt}(\mathcal{Z}, z, \xi_{gL})\, dz + (1 - \rho_{\xi}) \int_{\bar{z}_{gt}(\xi_{gH}, s_t)}^{\infty} Q_{gt}(\mathcal{Z}, z, \xi_{gH})\, dz$

# Aggregation

- Although the decision rule $\bar{z}_{gt}(\xi, s)$ respond immediately to trade-policy changes, the stock of exporters across trade costs adjusts gradually

- Aggregate trade volumes respond slowly to policy changes:

$$EX_{gt}(s) = \sum_{\xi \in \left\{\xi_{gL}, \xi_{gH}\right\}} \int_{z} p\left(z, \xi, \tau_{gt}(s)\right) y(z, \xi, \tau_{gt}(s)) \varphi_{gt}(z, \xi) dz. \tag{5}$$

# Calibration: Step 1–Moments from Micro Data

- Sector-level statistics about dynamics of Chinese firms that export to the U.S. (15 sectors over 2004-2007)
  - The dispersion in export sales
  - The fraction of firms that export
  - The fraction of exporters who stop exporting each period
  - The average exports of incumbent exporters divided by the average exports of new exporters

# Calibration: Step 2–Assigned Parameters

- $w_t = 1$ and interest rate is 4%
- $\tau_{gt}(1)$ and $\tau_{gt}(2)$ directly from the data
- $z = \frac{1}{\theta_g - 1} \log a$, $\log a' = \rho_z \log a + \varepsilon$, $\varepsilon \sim N\left(0, \sigma_{gz}\right)$
- $1 - \delta(a) = \max\left\{0, \min\left[e^{-\delta_0 a} + \delta_1, 1\right]\right\}$: $\rho_z, \delta_0, \delta_1$ from the literature
- $\theta_g$ from the literature: HS4 level to 5-digit SITC and aggregate into 15 sectors by taking the average

# Calibration: Step 3–Steady State

- Calibrate $\sigma_{gz}$, $f_{g0}$, $f_{g1}$, $\xi_{gH}$, $\xi_{gL}$ so that the model's steady state matches four sets of Moments derived in Step 1

Table 1: Chinese exporter dynamics statistics, 2004–2007

| | Sector | Export part. | Exit rate | Incumbent size prem. | CV log exports |
|---|---|---|---|---|---|
| 1 | Food, beverage, tobacco | 19 | 16 | 2.71 | 0.91 |
| 2 | Textile, clothing, footwear | 45 | 10 | 1.99 | 1.06 |
| 3 | Wood and straw products | 24 | 13 | 2.05 | 1.09 |
| 4 | Paper, printing products | 12 | 17 | 3.10 | 1.30 |
| 5 | Energy products, chemicals | 19 | 15 | 3.23 | 1.48 |
| 6 | Rubber, plastic products | 29 | 10 | 2.69 | 1.08 |
| 7 | Non-metallic mineral products | 16 | 18 | 2.26 | 0.85 |
| 8 | Base metal manuf. | 12 | 21 | 3.96 | 1.15 |
| 9 | Calendered metal manuf | 29 | 10 | 2.48 | 1.24 |
| 10 | Other machinery, equipment | 23 | 13 | 3.33 | 1.54 |
| 11 | Computer, electrical, optical | 48 | 7 | 4.82 | 1.94 |
| 12 | Electrical equipment manuf. | 32 | 10 | 3.35 | 1.55 |
| 13 | Vehicle manuf. | 23 | 12 | 4.07 | 1.31 |
| 14 | Furniture, other manuf. | 59 | 7 | 1.76 | 0.95 |
| 15 | Non-manufacturing | 28 | 13 | 2.99 | 1.25 |

*Notes:* The data are described in the appendix. Reported moments are averages for 2004–2007.

# Calibration: Step 3–Steady State

- Calibrate $\sigma_{gz}$, $f_{g0}$, $f_{g1}$, $\xi_{gH}$, $\xi_{gL}$ so that the model's steady state matches four sets of Moments derived in Step 1

Table 2: Sector-level model parameters

| | Sector | $\theta_g$ | $f_{g0}$ | $f_{g1}$ | $\xi_{gH}$ | $\sigma_{gz}$ |
|---|---|---|---|---|---|---|
| 1 | Food, beverage, tobacco | 3.09 | 0.14 | 0.12 | 3.34 | 0.91 |
| 2 | Textile, clothing, footwear | 3.17 | 0.20 | 0.13 | 2.57 | 1.02 |
| 3 | Wood and straw products | 2.79 | 0.26 | 0.17 | 3.71 | 1.03 |
| 4 | Paper, printing products | 3.43 | 0.19 | 0.16 | 3.46 | 1.08 |
| 5 | Energy products, chemicals | 2.99 | 0.27 | 0.20 | 4.56 | 1.17 |
| 6 | Rubber, plastic products | 3.16 | 0.18 | 0.12 | 3.40 | 0.99 |
| 7 | Non-metallic mineral products | 2.85 | 0.16 | 0.14 | 3.56 | 0.90 |
| 8 | Base metal manuf. | 3.04 | 0.13 | 0.15 | 4.60 | 0.99 |
| 9 | Calendered metal manuf. | 2.73 | 0.29 | 0.17 | 4.62 | 1.07 |
| 10 | Other machinery, equipment | 3.74 | 0.23 | 0.15 | 3.03 | 1.20 |
| 11 | Computer, electronic, optical | 3.18 | 0.46 | 0.20 | 4.81 | 1.35 |
| 12 | Electrical equipment manuf. | 3.27 | 0.30 | 0.16 | 3.86 | 1.20 |
| 13 | Vehicle manuf. | 3.06 | 0.20 | 0.15 | 4.81 | 1.07 |
| 14 | Furniture, other manuf. | 3.26 | 0.20 | 0.11 | 2.25 | 0.98 |
| 15 | Non-manufacturing | 2.97 | 0.22 | 0.16 | 4.04 | 1.07 |

# Calibration: Step 4–Transition

- Calibrate $\rho_\xi$ and $\omega_t\left(s, s'\right)$ to match
  - Long-run trade elasticity of $-7.93$
  - Annual NTR-gap elasticities
- Simulations:
  - Starting from all non-exporters and feed in the realized sequence of trade-policy regimes
  - Update the distribution of $\varphi_{gt}$ and compute aggregate exports
  - Estimate ECM and NTR-gap equations on the simulated data
- $\rho_\xi = 0.91$: from long-run aggregate trade elasticity
- $\omega_t(2, 1)$ is from the NTR-gap elasticities during the 1970s, when China was in the NNTR, whereas $\omega_t(1, 2)$ is from the NTR-gap elasticity from 1981 onward

# Estimates of Trade-Policy Expectations

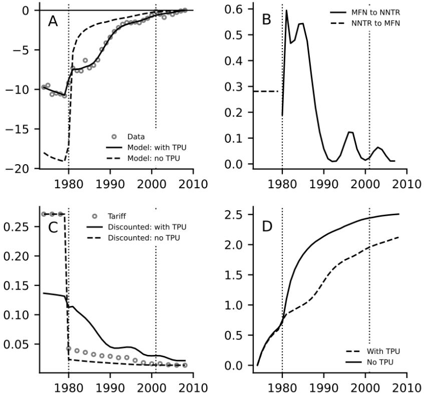


Fig. 5 – Model results. A: NTR-gap elasticities. B: policy transition probabilities. C: discounted expected value of tariffs. D: aggregate trade.

# The Effects of Policy Uncertainty on Trade

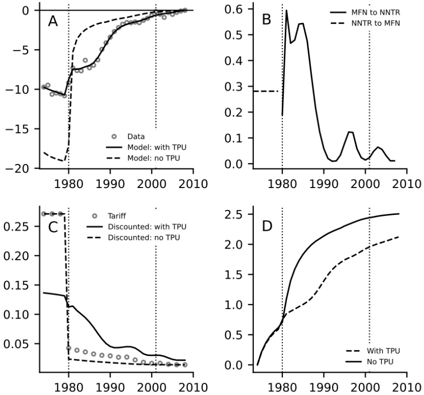


Fig. 5 – Model results. A: NTR-gap elasticities. B: policy transition probabilities. C: discounted expected value of tariffs. D: aggregate trade.

# The Role of Slow Adjustment

- Pre-PNTR elasticity of trade to the NTR gap:

$$v_{gjt} = \beta \mathbf{1}_{\{t<2000 \wedge j=China\}} GAP_g + (\theta_g + 1)\, \tau_{gjt} + \delta_{jt} + \delta_{jg} + \delta_{gt} + u_{jgt} \quad (6)$$

- Estimate using the observed data: $\beta = -0.67$
- Estimate using simulated data from the no-TPU counterfactual: $\beta = -0.3$
- Even if China's PNTR had never been in doubt, exports of high-gap goods still would have grown faster than imports of low-gap goods after China gained PNTR in 2001
- A gradual adjustment to the earlier reform accounts for about 40% of the overall effect of PNTR on trade

# The Role of Time-Varying Policy Uncertainty

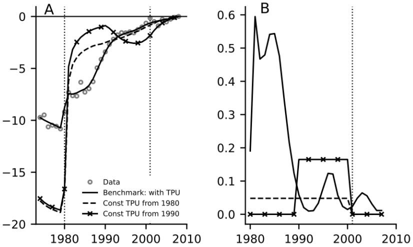


Fig. 6 – Benchmark model vs. constant-TPU models. A: NTR-gap elasticities. B: policy transition probabilities.

- The rising credibility of U.S. trade policy toward China during the mid to late 1980s was an important factor in explaining the growth of U.S. imports from China over the next two decades, and ignoring this trend overstates the degree to which uncertainty fell after China gained PNTR status in 2001

# Summary of Alessandria et al. (2025)

- A partial equilibrium model: fix the wage and the final demand
- Closely connected with the empirical results: the long-term elasticities of US imports from China to the NTR gap
- Only with the model we can separate:
    - Slow adjustment to the 1980 granting of NTR status
    - Effects of the risk of losing NTR status
- Separately identify static and dynamic parameters: steady state vs. transition
    - One-step estimation could be more efficient
    - Two-step: clear, transparent, intuitive about which moments identify which parameters

# Caliendo et al. (2019) “Trade and Labor Market Dynamics: General Equilibrium Analysis of the China Trade Shock”

- Dynamic decisions in migration: option value of the migration destination

- Trade shocks $\Rightarrow$ Local labor markets
  - Migration friction is the first-order issue in the long run
  - Higher exposure to import competition + Lower mobility = Greater losses from trade shocks

- Challenge:
  - High dimensionality in migration decision: $N$ regions
  - Computation of the option values: value function iteration
  - Aggregation across heterogeneous households: numerical integration

# CDP (2019): Model

- $N$ locations: $n$, $i$; $J$ sectors: $j$, $k$
- Each region-sector combination:
  - A competitive labor market
  - A continuum of perfectly competitive firms producing intermediates
- Time is discrete and goes to infinity: $t = 0, 1, 2, \ldots$

# CDP (2019): Household

- $L_0^{nj}$ of households in $(n, j)$ at $t = 0$
  - Employed: wage $w_t^{nj}$
  - Unemployed: home production $b^n = C_t^{n0}$–sector zero
- Consumption:

$$C_t^{nj} = \prod_{k=1}^{J} \left( c_t^{nj,k} \right)^{\alpha^k}. \tag{7}$$

- Forward looking: discounting factor $\beta \geq 0$
  - Labor allocation cost from $(n, j)$ to $(i, k)$: $\tau^{nj,ik} \geq 0$
  - Additive idiosyncratic shock for each choice: $\epsilon_t^{ik}$

# CDP (2019): Migration

- Bellman's equation for migrating from $(n, j)$ to $(i, k)$

$$v_t^{nj} = U\left(C_t^{nj}\right) + \max_{\{i,k\}_{i=1,k=0}^{N,J}} \left\{\beta E\left[v_{t+1}^{ik}\right] - \tau^{nj,ik} - \nu\epsilon_t^{ik}\right\}, \tag{8}$$

where $C_t^{nj} = b^n$ if $j = 0$, and $\frac{w_t^{nj}}{P_t^n}$ otherwise

- $\epsilon$ is i.i.d. distributed Type-I Extreme Value with zero mean:

$$V_t^{nj} \equiv E\left[v_t^{nj}\right] = U\left(C_t^{nj}\right) + \nu \log\left(\sum_{i=1}^{N}\sum_{k=0}^{J} \exp\left(\beta V_{t+1}^{ik} - \tau^{nj,ik}\right)^{\frac{1}{\nu}}\right). \tag{9}$$

- Aggregation: The fraction of households that relocate from $(n, j)$ to $(i, k)$

$$\mu_t^{nj,ik} = \frac{\exp\left(\beta V_{t+1}^{ik} - \tau^{nj,ik}\right)^{\frac{1}{\nu}}}{\sum_{m=1}^{N}\sum_{h=0}^{J} \exp\left(\beta V_{t+1}^{mh} - \tau^{nj,mh}\right)^{\frac{1}{\nu}}}. \tag{10}$$

- Labor dynamics: Labor supply at $t$ is fully determined at $t-1$

$$L_{t+1}^{nj} = \sum_{i=1}^{N}\sum_{k=0}^{J} \mu_t^{ik,nj} L_t^{ik}. \tag{11}$$

# CDP (2019): Production

- Each variety is produced by labor (wage $w_t^{nj}$), structure (price $r_t^{nj}$), and intermediates

$$x_t^{nj} = \left[ \left( r_t^{nj} \right)^{\xi^n} \left( w_t^{nj} \right)^{1-\xi^n} \right]^{\gamma^{nj}} \prod_{k=1}^{J} \left( P_t^{nk} \right)^{\gamma^{nj,nk}} . \tag{12}$$

- EK setting of productivities: The import share in market $(n, j)$ on goods $j$ from market $i$

$$\pi_t^{nj,ij} = \frac{\left( x_t^{ij} \kappa_t^{nj,ij} \right)^{-\theta^j} \left( A_t^{ij} \right)^{\theta^j \gamma^{ij}}}{\sum_{m=1}^{N} \left( x_t^{mj} \kappa_t^{nj,mj} \right)^{-\theta^j} \left( A_t^{mj} \right)^{\theta^j \gamma^{mj}}}, \tag{13}$$

where $\kappa_t^{nj,ij}$ is the iceberg trade cost of good $j$ from $i$ to $n$

- Price index:

$$P_t^{nj} = \left( \sum_{m=1}^{N} \left( x_t^{mj} \kappa_t^{nj,mj} \right)^{-\theta^j} \left( A_t^{mj} \right)^{\theta^j \gamma^{mj}} \right)^{-\frac{1}{\theta^j}} . \tag{14}$$

# CDP (2019): Market Clearing

- A unit mass of immobile rentier in each region:
    - Receive a constant share $\iota^n$ from the global portfolio of structure revenue: $\chi_t = \sum_{i=1}^{N} \sum_{k=1}^{J} r_t^{ik} H^{ik}$
- Goods market clearing:

$$X_t^{nj} = \alpha^j \left( \sum_{k=1}^{J} w_t^{nk} L_t^{nk} + \iota^n \chi_t \right) + \sum_{k=1}^{J} \gamma^{nk,nj} \sum_{i=1}^{N} \pi_t^{ik,nk} X_t^{ik}. \tag{15}$$

- Labor market clearing:

$$w_t^{nj} L_t^{nj} = \gamma^{nj} (1 - \xi^n) \sum_{i=1}^{N} \pi_t^{ij,nj} X_t^{ij}. \tag{16}$$

- Structure market clearing: note that $r_t^{nj}$ can be expressed by $w_t^{nj}$, $L_t^{nj}$, and $H^{nj}$

$$r_t^{nj} H^{nj} = \gamma^{nj} \xi^n \sum_{i=1}^{N} \pi_t^{ij,nj} X_t^{ij}. \tag{17}$$

# CDP (2019): Equilibrium

- Temporary equilibrium: given $L_t^{nj}$, $w_t^{nj}$
- Sequential competitive equilibrium: $\left(L_t^{nj}, \mu_t^{nj,ik}, V_t^{nj}, w_t^{nj}\right)_{t=0}^{\infty}$
- Stationary equilibrium: $\left(L_t^{nj}, \mu_t^{nj,ik}, V_t^{nj}, w_t^{nj}\right)_{t=0}^{\infty}$ are constant for all $t$

# CDP (2019): Static Hat Algebra

- Given the allocation of temporary equilibrium at $t$, $\{L_t, \pi_t, X_t\}$, the solution to the temporary equilibrium at $t+1$ for a given change in $\left(\dot{L}^{nj}_{t+1} \equiv L^{nj}_{t+1}/L^{nj}_{t}\right)$ and $\dot{\Theta}_{t+1}$, where $\Theta \equiv (A_t, \kappa_t)$ denotes time-varying parameters, can be solved by
  - $\dot{x}^{nj}_{t+1} = \left(\dot{L}^{nj}_{t+1}\right)^{\gamma^{nj}\xi^{n}} \left(\dot{w}^{nj}_{t+1}\right)^{\gamma^{nj}} \prod_{k=1}^{J} \left(\dot{P}^{nk}_{t+1}\right)^{\gamma^{nj,nk}}$
  - $\dot{P}^{nj}_{t+1} = \left(\sum_{i=1}^{N} \pi^{nj,ij}_{t} \left(\dot{x}^{ij}_{t+1}\dot{\kappa}^{nj,ij}_{t+1}\right)^{-\theta^{j}} \left(\dot{A}^{ij}_{t+1}\right)^{\theta^{j}\gamma^{ij}}\right)^{-\frac{1}{\theta^{j}}}$
  - $\pi^{nj,ij}_{t+1} = \pi^{nj,ij}_{t} \left(\frac{\dot{x}^{ij}_{t+1}\dot{\kappa}^{nj,ij}_{t+1}}{\dot{P}^{nj}_{t+1}}\right)^{-\theta_{j}} \left(\dot{A}^{ij}_{t+1}\right)^{\theta^{j}\gamma^{ij}}$
  - $X^{nj}_{t} = \alpha^{j}\left(\sum_{k=1}^{J} \dot{w}^{nk}_{t}\dot{L}^{nk}_{t} + \iota^{n}\chi_{t}\right) + \sum_{k=1}^{J} \gamma^{nk,nj} \sum_{i=1}^{N} \pi^{ik,nk}_{t} X^{ik}_{t}$ where $\chi_{t+1} = \sum_{i,k} \frac{\xi^{i}}{1-\xi^{i}} \dot{w}^{ik}_{t+1}\dot{L}^{ik}_{t+1} w^{ik}_{t} L^{ik}_{t}$
  - $\dot{w}^{nj}_{t+1}\dot{L}^{nj}_{t+1} w^{nj}_{t} L^{nj}_{t} = \gamma^{nj}\left(1-\xi^{n}\right) \sum_{i=1}^{N} \pi^{ij,nj}_{t+1} X^{ij}_{t+1}$

# CDP (2019): Dynamic Hat Algebra

- Suppose that $U\left(C_t^{nj}\right) \equiv \log\left(C_t^{nj}\right)$. Given $(L_0, \pi_0, X_0, \mu_{-1})$ and an anticipated convergent $(\Theta_t)_{t=1}^{\infty}$ where $\lim_{t\to\infty}\dot{\Theta}_t = 1$, sequential equilibrium can be expressed as
  - $\mu_{t+1}^{nj,ik} = \frac{\mu_t^{nj,ik}\left(\dot{u}_{t+2}^{ik}\right)^{\frac{\beta}{\nu}}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\mu_t^{nj,mh}\left(\dot{u}_{t+2}^{mh}\right)^{\frac{\beta}{\nu}}}$, where $u_t^{nj} \equiv \exp\left(V_t^{nj}\right)$
  - $\dot{u}_{t+1}^{nj} = \dot{\omega}^{nj}\left(\dot{L}_{t+1}, \dot{\Theta}_{t+1}\right)\left(\sum_{i=1}^{N}\sum_{k=0}^{J}\mu_t^{nj,ik}\left(\dot{u}_{t+2}^{ik}\right)^{\frac{\beta}{\nu}}\right)^{\nu}$ where $\omega_t^{nj} \equiv \frac{w_t^{nj}}{P_t^n}$
  - $L_{t+1}^{nj} = \sum_{i=1}^{N}\sum_{k=0}^{J}\mu_t^{ik,nj}L_t^{ik}$.

# CDP (2019): Counterfactuals

- Given a baseline economy $\{L_t, \mu_{t-1}, \pi_t, X_t\}_{t=0}^{\infty}$ and a counterfactual convergent sequence of changes in fundamentals, $\left\{\hat{\Theta}_t\right\}_{t=1}^{\infty}$, the counterfactual sequential equilibrium $\{L'_t, \mu'_{t-1}, \pi'_t, X'_t\}_{t=0}^{\infty}$ can be solved by
  - $\mu_t^{'nj,ik} = \frac{\mu_{t-1}^{'nj,ik} \dot{\mu}_t^{nj,ik} \left(\hat{u}_{t+1}^{ik}\right)^{\frac{\beta}{\nu}}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu_{t-1}^{'nj,mh} \dot{\mu}_t^{nj,mh} \left(\hat{u}_{t+1}^{mh}\right)^{\frac{\beta}{\nu}}}$
  - $\hat{u}_t^{nj} = \hat{\omega}^{nj}\left(\hat{L}_t, \hat{\Theta}_t\right)\left(\sum_{i=1}^{N} \sum_{k=0}^{J} \mu_{t-1}^{'nj,ik} \dot{\mu}_t^{nj,ik} \left(\hat{u}_{t+1}^{ik}\right)^{\frac{\beta}{\nu}}\right)^{\nu}$
  - $L_{t+1}^{'nj} = \sum_{i=1}^{N} \sum_{k=0}^{J} \mu_t^{'ik,nj} L_t^{'ik}$

# CDP (2019): Parameters

- Bilateral trade flows $\pi_t^{nj,ij}$ and value added $w_t^{nj} L_t^{nj} + r_t^{nj} H_t^{nj}$
- Employment distribution $L_t$ and migration flows $\mu_t^{nj,ik}$
- Value-added share $\gamma^{nj}$, the IO share $\gamma^{nj,nk}$, the structure share $\xi^n$, the final consumption share $\alpha^j$, and the global portfolio share $\iota^n$
- Trade elasticities $\theta^j$, the migration elasticity $\frac{1}{\nu}$, and the discount factor $\beta$

# CDP (2019): Estimating $\nu$ and China Shock

- The cross-sectional migration flows contain information on expected values that depend on future wages and the option value of migration across markets, and future migration flows are sufficient statistics for these option values:

$$\log\left(\frac{\mu_t^{nj,nk}}{\mu_t^{nj,nj}}\right) = \tilde{C} + \frac{\beta}{\nu}\log\left(\frac{w_{t+1}^{nk}}{w_{t+1}^{nj}}\right) + \beta\log\left(\frac{\mu_{t+1}^{nj,nk}}{\mu_{t+1}^{nk,nk}}\right) + \tilde{\omega}_{t+1} \tag{18}$$

- The China trade shock on the U.S.: isolate supply shocks from China

$$\Delta M_{\text{USA},j} = a_1 + a_2 \Delta M_{\text{other},j} + u_j \tag{19}$$

# CDP (2019): Impacts of China Shock

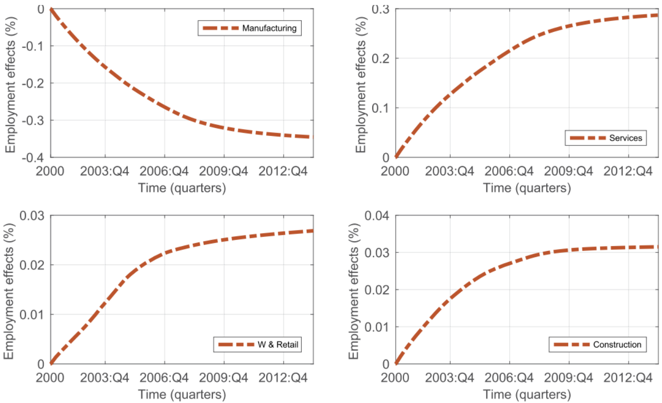


FIGURE 1.—The effect of the China shock on employment shares. Note: The figure presents the effects of the China shock measured as the change in employment shares by sector (manufacturing, services, wholesale and retail, and construction) over total employment between the economy with all fundamentals changing as in the data and the economy with all fundamentals changing except for the estimated sectoral changes in productivities in China (the economy without the China shock).

# CDP (2019): Impacts of China Shock

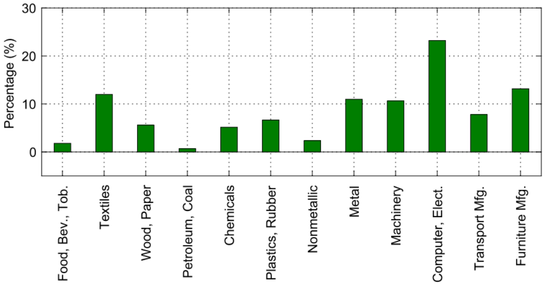


FIGURE 2.—Manufacturing employment declines due to the China trade shock (percent of total). Note: The figure presents the contribution of each manufacturing industry to the total reduction in the manufacturing employment due to the China shock.

# CDP (2019): Impacts of China Shock

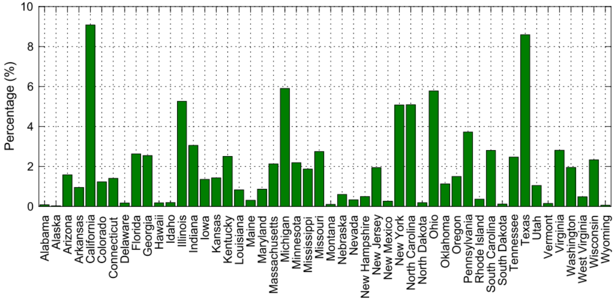


FIGURE 3.—Regional contribution to U.S. aggregate manufacturing employment decline (percent). Note: The figure presents the contribution of each state to the total reduction in manufacturing sector employment due to the China shock.

# CDP (2019): Impacts of China Shock

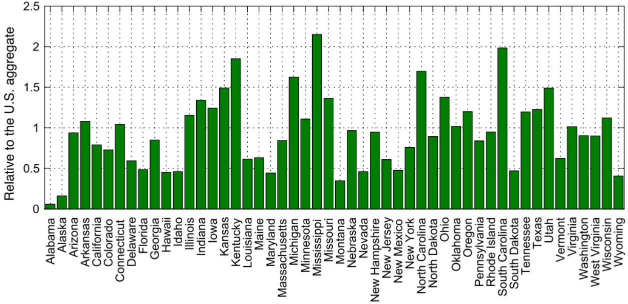


FIGURE 4.—Regional contribution to U.S. aggregate manufacturing employment decline, normalized by regional employment share. Note: The figure presents the contribution of each state to the U.S. aggregate reduction in manufacturing sector employment due to the China shock, normalized by the employment of each state relative to the U.S. aggregate employment.

# CDP (2019): Impacts of China Shock

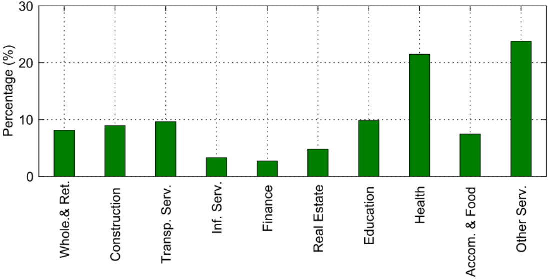


FIGURE 5.—Non-manufacturing employment increases due to the China trade shock (percent of total). Note: The figure presents the contribution of each non-manufacturing sector to the total increase in non-manufacturing employment due to the China shock.

# CDP (2019): Impacts of China Shock

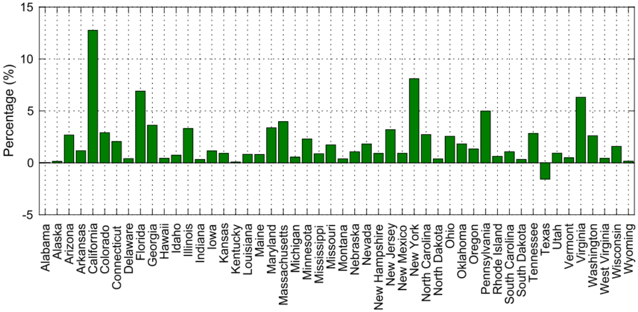


FIGURE 6.—Regional contribution to U.S. aggregate non-manufacturing employment increase (percent). Note: The figure presents the contribution of each state to the total rise in non-manufacturing employment due to the China shock.

# CDP (2019): Impacts of China Shock

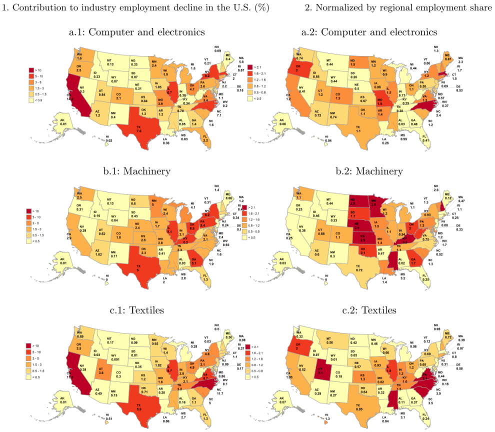


FIGURE 7.—Regional employment declines in manufacturing industries. Note: This figure presents the reduction in local employment in manufacturing industries. Column 1 presents the contribution of each state to the U.S. aggregate reduction in industry employment due to the China shock. Column 2 presents the contribution of each state to the U.S. aggregate reduction in industry employment normalized by the employment size of each state relative to U.S. aggregate employment. Panels a present the results for the computer and electronics industry. Panels b present the results for the machinery industry. Panels c present the results for the textiles industry.

# CDP (2019): Impacts of China Shock

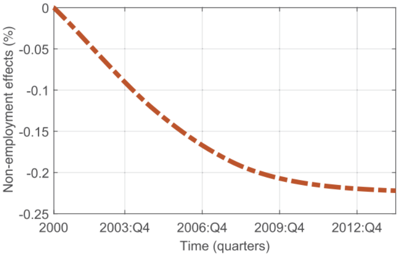


FIGURE 9.—The effect of the China shock on non-employment shares. Note: The figure presents the effects of the China shock, measured as the difference in the non-employment shares between the economy with all fundamentals changing as in the data and the economy with all fundamentals changing except for the estimated sectoral changes in productivities in China (the economy without the China shock).

# CDP (2019): Impacts of China Shock

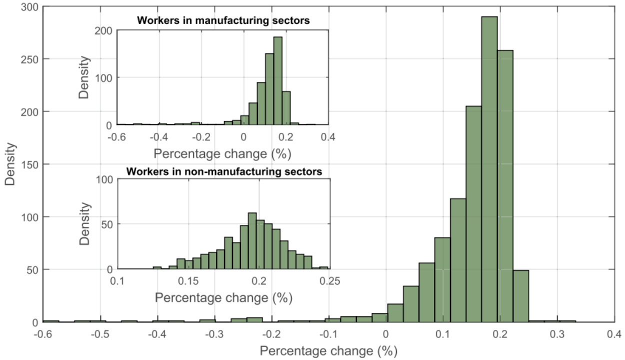


FIGURE 10.—Welfare effects of the China shock across U.S. labor markets. Note: The figure presents the change in welfare across all labor markets (central figure), for workers in manufacturing sectors (top-left panel), and for workers in non-manufacturing sectors (bottom-left panel) as a consequence of the China shock. The largest and smallest 1 percentile are excluded in each figure. The percentage change in welfare is measured in terms of consumption equivalent variation.

# Summary of CDP (2019)

- CDP (2019) is essentially a finite-period model:
  - Convergence to the stationary equilibrium
  - No persistent aggregate uncertainties
- Key tricks: additive migration frictions + Type I Extreme Value distribution
- Migration is the only dynamic decision. Extensions with endogenous capital accumulation:
  - Kleinman et al. (2023); Cai et al. (2025): capital accumulation is determined by immobile land owners
  - Giannone et al. (2023): finite-period model