Quantitative Trade Models

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Numerical Methods

- Non-linear solvers
- Non-linear optimizers (unconstrained/constrained)
- Differentiation and integration
- Dynamic programming—solving for the (functional) fixed point

Quantitative Trade Models

- Essence: Transmissions of shocks or policies via networks
- Shocks/policies $\rightarrow_{\text{Trade or other networks}}$ Local economies $\rightarrow_{\text{Local conditions}}$ Local changes $\rightarrow_{\text{Spillovers}}$ Other regions $\rightarrow_{\text{Feedback loops}}$ Aggregate implications
- Example:
  - Labor market: Lee, Eunhee (2020) JIE
  - Innovation: Sampson, Thomas (2023) AER
  - Environment: Shapiro, Joseph (2021) QJE
  - Transportation: Wong, Woan Foong (2022) AEJ: Applied
  - Quality: Fieler, Cecilia and Jonathan Eaton (2025) Econometrica

Road Map

- Workhorse model of trade and industrial policies
  - Elements: comparative advantage, trade frictions, input-output linkages, increasing returns to scale
  - Policies: import/export tariffs, industrial subsidies
- Using micro data to estimate GE trade model: Eaton, Kortum, and Kramarz (2011)
  - Firm heterogeneity in trade models a la Melitz (2003)
  - Micro-fit? Why can't we bring Melitz model to data?
  - Macro-tractability? How to keep elegant aggregation in Melitz (2003)

Probabilistic Approach: Eaton and Kortum (2002)

- N countries.

- Country i is endowed with labor L_i.

- The representative consumer in country i consumes a final good consisting of a continuum of varieties indexed by $\omega \in [0,1]$. These varieties are aggregated by a CES function:

$$U_i = Q_i = \left[\int_0^1 Q_i(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1, \tag{1}$$

  where $Q_i(\omega)$ is the quantity of variety ω consumed by the representative consumer in country i.

- Each good ω is produced by a firm using labor and final goods under perfect competition. Let w_i be the wage and P_i be the price index of final good derived later. The unit cost for producing variety ω in country i is assumed to be

$$c_i(\omega) = \frac{w_i}{z_i(\omega)}, \tag{2}$$

  where $z_i(\omega)$ is the productivity for producing variety ω in country i.

Probabilistic Approach: Eaton and Kortum (2002)

- Perfect competition in the market for each ω.
- Trade from country i to country n incurs an iceberg trade cost $\tau_{in} \geq 1$.
- For each country i and variety ω, the productivity $z_i(\omega)$ is drawn independently from a Fréchet distribution:

$$Prob\{z_i(\omega) \leq z\} = F_i(z) \equiv \exp\{-T_i z^{-\theta}\}, \quad z > 0, \quad T_i > 0, \quad \theta > \max\{1, \sigma - 1\}. \tag{3}$$

  - Scale parameter T_i: the average productivity of country i.
  - Shape parameter θ: dispersion of productivities across varieties.

Probabilistic Approach: Eaton and Kortum (2002)

- From which country will the representative consumer in country n purchase variety ω?
  - The consumers will only buy a particular variety from the lowest-cost country.

- The price of variety ω produced in country i served to country n can be given by

$$p_{in}(\omega) = \left[\frac{w_i}{z_i(\omega)}\right]\tau_{in}. \tag{4}$$

- The price actually paid by consumers in country n is

$$p_n(\omega) = \min_i \left\{p_{in}(\omega)\right\}. \tag{5}$$

Probabilistic Approach: Eaton and Kortum (2002)

Lemma

The CDF of the price of variety ω actually paid by consumers in country n is

$$Pr(p_n(\omega) \leq p) = 1 - \exp\{-\Phi_n p^{\theta}\}, \tag{6}$$

where $\Phi_n = \sum_{i=1}^{N} T_i (w_i \tau_{in})^{-\theta}$.

Probabilistic Approach: Eaton and Kortum (2002)

Proposition

The probability that country i provides a good at the lowest price in country n is

$$\pi_{in} = \frac{T_i (w_i \tau_{in})^{-\theta}}{\Phi_n}. \tag{7}$$

Probabilistic Approach: Eaton and Kortum (2002)

Proposition

The price of variety ω that country n actually buys from any country i satisfies:

$$prob\left(p_{in}(\omega) \leq p | p_{in}(\omega) \leq \min\left\{p_{kn}(\omega); k \neq i\right\}\right) = 1 - \exp\{-\Phi_n p^{\theta}\}. \qquad (8)$$

- The expenditure share of country n on goods from country i is *equal to* the probability that country i provides the lowest cost to country n:
  - When τ_{in} decreases, country i will provide more varieties to country n.
  - For each variety ω it serves country n, the relative price stays unchanged.

Probabilistic Approach: Eaton and Kortum (2002)

- The price index of the final good in country n can be derived from the ideal price index under CES preference:

$$P_n = E\left[p_n(\omega)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = \gamma \Phi_n^{-\frac{1}{\theta}}, \tag{9}$$

where the constant $\gamma = \left[\Gamma\left(\frac{\theta-(\sigma-1)}{\theta}\right)\right]^{\frac{1}{1-\sigma}}$ and $\Gamma(y) = \int_0^\infty x^{y-1}e^{-x}dx$ is the gamma function.

- The expression in Equation (9) comes from the moment generating function of the Weibull distribution.

Probabilistic Approach: Eaton and Kortum (2002)

Definition

Given the environment $(L_i, T_i, \tau_{in}, \theta)$, the equilibrium consists of wage $\{w_i\}$ such that

- Labor markets clear

$$w_i L_i = \sum_{n=1}^{N} \frac{T_i \left(w_i \tau_{in}\right)^{-\theta}}{\sum_{k=1}^{N} T_k \left(w_k \tau_{kn}\right)^{-\theta}} w_n L_n. \tag{10}$$

- Existence and uniqueness of the equilibrium
  - Please show that the excess labor demand function $\zeta_i\left(\mathbf{w}\right) := \frac{1}{w_i} \sum_{n=1}^{N} \frac{T_i (w_i \tau_{in})^{-\theta}}{\sum_{k=1}^{N} T_k (w_k \tau_{kn})^{-\theta}} w_n L_n - L_i$ satisfies the *gross substitutes property*.
  - General discussions of the equilibrium existence and uniqueness: Allen, Arkolakis, and Li (2024)

Probabilistic Approach: Eaton and Kortum (2002)

- Eaton and Kortum (2002) delivers the exactly identical equilibrium system with the Armington model
- More reasonable micro-foundations: Ricardian comparative advantage
- Varieties in the EK model $\neq$ Disaggregated goods in data
- Probabilistic approach: tractable aggregation of heterogeneous agents' behaviors $\rightarrow$ few parameters/statistics

A Workhorse Model of Trade and Industrial Policies

- N countries with $\{L_i\}_{i=1}^{N}$.

- J sectors:

$$U(C_n) = \prod_{j=1}^{J} \left(C_n^j\right)^{\alpha_n^j}, \quad \sum_{j=1}^{J} \alpha_n^j = 1. \tag{11}$$

- A continuum of varieties $\omega^j \in [0,1]$ in each sector j, aggregated via a CES function with elasticity of substitution σ^j and produced under perfect competition.

- Unit cost of production factors:

$$c_i^j\left(\omega^j\right) = \frac{c_i^j}{z_i^j\left(\omega^j\right)\left(L_i^j\right)^{\psi^j}}, \quad c_i^j \equiv w_i^{\gamma_i^j} \prod_{k=1}^{J} \left(P_i^k\right)^{\gamma_i^{k,j}}, \tag{12}$$

where L_i^j is the total labor in sector j of country i and $\psi_j \geq 0$ reflects the external economies of scale.

A Workhorse Model of Trade and Industrial Policies

- $z_n^j(\omega^j)$ draws from a Frechet distribution:

$$Prob\{z_i^j\left(\omega^j\right) \leq z\} = F_i(z) \equiv \exp\{-T_i^j z^{-\theta^j}\}, \quad z > 0, \quad T_i^j \geq 0, \quad \theta^j > \max\{1, \sigma^j - 1\}. \quad (13)$$

- Trade costs:
  - Iceberg trade costs: $\tau_{in}^j \geq 1$ with $\tau_{ii}^j = 1$
  - Export tariff: $e_{in}^j \geq 0$ with $e_{ii}^j = 0$
  - Import tariff: $t_{in}^j \geq 0$ with $t_{ii}^j = 0$
- Ad-valorem ndustrial subsidy: $s_i^j \geq 0$
  - Define $\kappa_{in}^j \equiv \tau_{in}^j \left(1 - s_i^j\right)\left(1 + e_{in}^j\right)\left(1 + t_{in}^j\right)$

Equilibrium

- Trade share and price index:

$$\lambda_{in}^j = \frac{T_i^j \left(L_i^j\right)^{\theta^j \psi^j} \left(c_i^j \kappa_{in}^j\right)^{-\theta^j}}{\sum_{h=1}^N T_h^j \left(L_h^j\right)^{\theta^j \psi^j} \left(c_h^j \kappa_{hn}^j\right)^{-\theta^j}}, \quad P_n^j = \left[\sum_{i=1}^N T_i^j \left(L_i^j\right)^{\theta^j \psi^j} \left(c_i^j \kappa_{in}^j\right)^{-\theta^j}\right]^{-\frac{1}{\theta^j}}. \tag{14}$$

- Labor market clearing:

$$L_i^j = \frac{\gamma_i^j}{w_i} \sum_{n=1}^N \frac{\lambda_{in}^j X_n^j}{\left(1-s_i^j\right)\left(1+e_{in}^j\right)\left(1+t_{in}^j\right)}, \quad \sum_{j=1}^J L_i^j = L_i. \tag{15}$$

- Total expenditure:

$$X_i^j = \alpha_i^j Y_i + \sum_{k=1}^J \gamma_i^{j,k} \sum_{n=1}^N \frac{\lambda_{in}^k X_n^k}{\left(1-s_i^k\right)\left(1+e_{in}^k\right)\left(1+t_{in}^k\right)}, \tag{16}$$

where

$$Y_i = w_i L_i + \sum_{j=1}^J \sum_{n=1}^N \left[-\frac{s_i^j}{1-s_i^j} + \frac{e_{in}^j}{\left(1-s_i^j\right)\left(1+e_{in}^j\right)}\right] \lambda_{in}^j X_n^j + \sum_{j=1}^J \sum_{n=1}^N \frac{t_{ni}^j}{\left(1-s_n^j\right)\left(1+e_{ni}^j\right)\left(1+t_{ni}^j\right)} \lambda_{ni}^j X_i^j. \tag{17}$$

Exact-Hat Algebra

- Policy changes: $\left(\widehat{1-s_i^j}, \widehat{1+e_{in}^j}, \widehat{1+t_{in}^j}\right)$
- Equilibrium changes: $\left(\hat{w}_i, \hat{X}_i^j, \hat{P}_i^j, \hat{L}_i^j\right)$
- Parameters: $\left(\alpha_i^j, \gamma_i^j, \gamma_i^{k,j}, \psi^j, \theta^j\right)$
- Data: $\left(X_{in}^j, s_i^j, e_{in}^j, t_{in}^j\right)$

Estimating θ^j

- Fixed-effect gravity:
  -
  $$\log \lambda_{in}^j = -\theta^j \log\left(1 + t_{in}^j\right) + D_{in}\delta^j + fe_i^j + fe_n^j + \epsilon_{in}^j \tag{18}$$
  -
  $$\Delta \log \lambda_{in}^j = -\theta^j \Delta \log\left(1 + t_{in}^j\right) + fe_i^j + fe_n^j + \Delta\epsilon_{in}^j \tag{19}$$
  - What are the structural interpretations of fixed effects and errors in two equations above? What are the assumptions of identifying θ^j?
- Caliendo and Parro (2015)

$$\log\left(\frac{X_{in}^j X_{nh}^j X_{hi}^j}{X_{ni}^j X_{ih}^j X_{hn}^j}\right) = -\theta^j \log\left(\frac{\left(1 + t_{in}^j\right)\left(1 + t_{nh}^j\right)\left(1 + t_{hi}^j\right)}{\left(1 + t_{ni}^j\right)\left(1 + t_{ih}^j\right)\left(1 + t_{hn}^j\right)}\right) + \epsilon_{inh}^j \tag{20}$$

What are the structural interpretations of the error? What are the assumptions of identifying θ^j?

Estimating ψ^j

- fe_i^j in the gravity equation (in levels):

$$fe_i^j = \left(\theta^j \psi^j\right) \log L_i^j + \underbrace{\log\left(T_i^j\right) - \theta^j \log\left(c_i^j\left(1 - s_i^j\right)\right)}_{\nu_i^j} \tag{21}$$

- Regress fe_i^j on $\log\left(L_i^j\right)$?

- Ideas for IV: Bartelme et al. (2025)
  - Demand shifters on labor in sector j of country i
  - A measure of demand-predicted sector size: Bartik instrument

Firm Heterogeneity in Trade: A Sketch of Melitz (2003)

- Firm with productivity φ in country i is self-selected into exporting to country j:
  - Profit function: $\pi_{ij}(\varphi) = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{w_i\tau_{ij}}{\varphi}\right)^{1-\sigma} Y_j P_j^{\sigma-1}$
  - Fixed cost of exporting $f_{ij} \Rightarrow$ Export iff $\pi_{ij}(\varphi) \geq f_{ij}$
  - Productivity cutoff: $\varphi \geq \varphi_{ij}^* \equiv \left(\frac{\sigma f_{ij}\left(\frac{\sigma}{\sigma-1}w_i\tau_{ij}\right)^{\sigma-1}}{Y_j P_j^{\sigma-1}}\right)^{\frac{1}{\sigma-1}}$
  - φ is drawn from a Pareto distribution with CDF:
    $G_i(\varphi) = 1 - T_i\varphi^{-\theta_i}, \quad \varphi \geq T_i^{\frac{1}{\theta_i}}, \quad \theta_i > \sigma - 1$
  - Closed-form aggregation of a truncated distribution
- Mechanism: Most productive firms are selected into exporting $\Rightarrow$ Labor relocated to these firms $\Rightarrow$ Least productive firms exit $\Rightarrow$ Aggregate productivity $\uparrow$

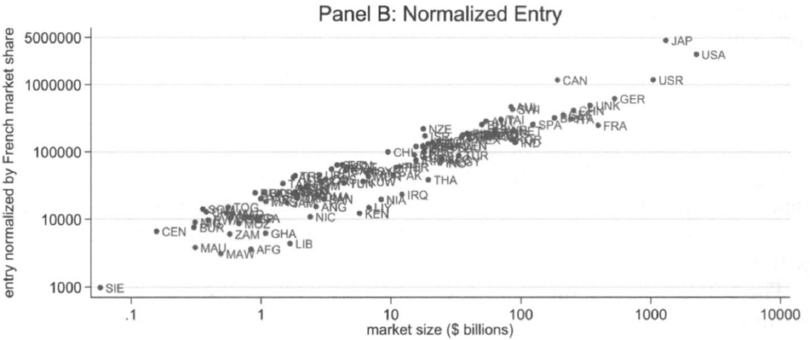
Eaton, Kortum, and Kramarz (2011)

- Linking to micro to macro:
    - How relevant is firm heterogeneity to exports in micro data?
    - Trade liberalization $\Rightarrow$ Firm sales and Welfare.
- We need a structural model that can
    - capture firm heterogeneity from the micro data, and
    - account the impacts of firm heterogeneity on aggregates.
- Melitz (2003) cannot match firm data directly:
    - Prediction: more productive firms are more likely to enter into Every market.
    - Not the case in micro data.
- Eaton, Kortum, and Kramarz (2011): An anatomy of international trade: evidence from French firms.
    - Carefully match their model into the firm-level data.
    - Elegant aggregation techniques.

A Very Standard Quantitative Trade Paper

0. Motivational facts.
1. Theoretical framework: $y = g(x)$.
2. Practical specification: $y = g(x, \xi, \Theta)$.
3. Solving the model.
4. Understanding how the model works:
   - Connecting model predictions to motivational facts.
   - Preparing the model to fit the data.
5. Estimation.
6. Validation:
   - Goodness-of-fit/Out-of-sample predictions.
   - Interpretation.
   - Structurally decomposing the magnitudes of factors in interest.
7. Counterfactual experiments:
   - Existing/hypothetical policies and fundamental shocks.

Targeted Facts



(Note: normalized entry is equal to $\frac{N_{nF}}{X_{nF}/X_n}$ where N_{nF} is the number of French firms selling to a market, X_{nF} is total exports of French firms to market n, and X_n is the market size.)

Targeted Facts

TABLE II

FRENCH FIRMS SELLING TO STRINGS OF TOP-SEVEN COUNTRIES

	Number of French Exporters		
Export String[a]	Data	Under Independence	Model
BE[a]	3988	1700	4417
BE–DE	863	1274	912
BE–DE–CH	579	909	402
BE–DE–CH–IT	330	414	275
BE–DE–CH–IT–UK	313	166	297
BE–DE–CH–IT–UK–NL	781	54	505
BE–DE–CH–IT–UK–NL–US	2406	15	2840
Total	9260	4532	9648

[a]The string BE means selling to Belgium but no other among the top 7; BE–DE means selling to Belgium and Germany but no other, and so forth.

Targeted Facts

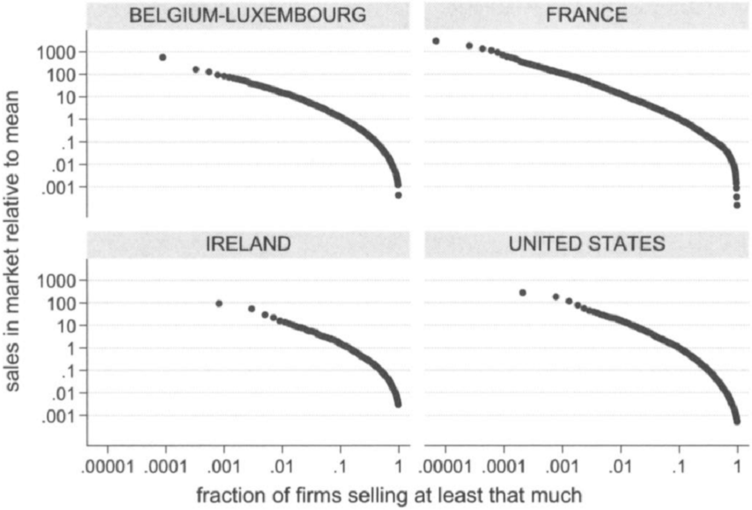


FIGURE 2.—Sales distributions of French firm: Graphs by country.

Targeted Facts

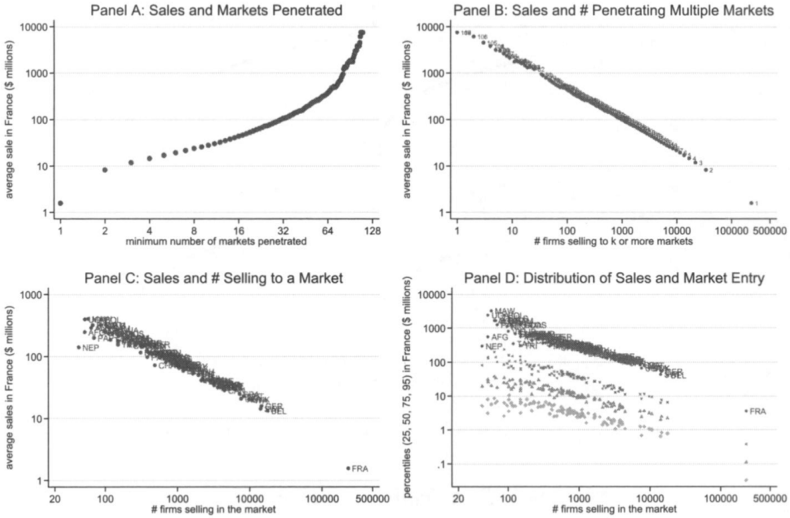


FIGURE 3.—Sales in France and market entry.

Model: Firm Heterogeneity

- Cost of serving market d:

$$c_{ni}(j) = \frac{w_i d_{ni}}{z_i(j)}. \tag{22}$$

- The measure of potential producers in country i with efficiency at least z:

$$\mu_i^z(z) = T_i z^{-\theta}, \quad z > 0. \tag{23}$$

- The measure of goods with cost below c:

$$\mu_{ni}(c) = \Phi_{ni} c^{\theta}, \quad \Phi_{ni} = T_i (w_i d_{ni})^{-\theta}. \tag{24}$$

Model: Entry and Demand Shocks

- A market n contains a unit measure of potential buyers. To sell to a fraction f of them, a producer in country i must incur a fixed cost:

$$E_{ni}(j) = \varepsilon_n(j) E_{ni} M(f), \tag{25}$$

where

$$M(f) = \frac{1 - (1-f)^{1-\frac{1}{\lambda}}}{1 - \frac{1}{\lambda}}, \quad \lambda > 0. \tag{26}$$

- Melitz (2003): $\lambda = \infty$.

- Monopolistic competition:

$$X_n(j) = \alpha_n(j) f p^{1-\sigma} X_n P_n^{\sigma-1}. \tag{27}$$

Model: the Firm's Problem

- Profit in n:

$$\Pi_{ni}(p,f) = \left(1 - \frac{c_n(j)}{p}\right)\alpha_n(j) f p^{1-\sigma} X_n P_n^{\sigma-1} - \varepsilon_n(j) E_{ni} M(f). \tag{28}$$

- Optimal pricing:

$$p_n(j) = \bar{m} c_n(j), \quad \bar{m} = \frac{\sigma}{\sigma - 1}. \tag{29}$$

- Optimal fraction:

$$f_{ni}(j) = \max\left\{1 - \left[\eta_n(j)\frac{X_n}{\sigma E_{ni}}\left(\frac{\bar{m} c_n(j)}{P_n}\right)^{-(\sigma-1)}\right]^{-\lambda}, 0\right\}, \tag{30}$$

where $\eta_n(j) = \alpha_n(j)/\varepsilon_n(j)$.

Model: Export Entry and Sales

- A firm enters market n if and only if

$$c \leq \bar{c}_{ni}(\eta) := \left(\eta \frac{X_n}{\sigma E_{ni}}\right)^{\frac{1}{\sigma-1}} \frac{P_n}{\bar{m}}. \tag{31}$$

- The optimal fraction, sales, and fixed cost can be expressed by c and $\bar{c}_{ni}(\eta)$:
  - $f_{ni}(\eta, c) = 1 - (\frac{c}{\bar{c}_{ni}(\eta)})^{\lambda(\sigma-1)}$.
  - $X_{ni}(\alpha, \eta, c) = \frac{\alpha}{\eta}\left[1 - \left(\frac{c}{\bar{c}_{ni}(\eta)}\right)^{\lambda(\sigma-1)}\right]\left(\frac{c}{\bar{c}_{ni}(\eta)}\right)^{-(\sigma-1)} \sigma E_{ni}$.
  - $E_{ni}(\alpha, \eta, c) = \frac{\alpha}{\eta} E_{ni} \frac{1-(c/\bar{c}_{ni}(\eta))^{(\lambda-1)(\sigma-1)}}{1-1/\lambda}$.

Aggregation: Price

- The price index in country n is

$$P_n^{1-\sigma} = \bar{m}\left[\int\int\left(\sum_{i=1}^{N}\int_0^{\bar{c}_{ni}(\eta)}\alpha f_{ni}(\eta,c)c^{1-\sigma}d\mu_{ni}(c)\right)g(\alpha,\eta)d\alpha d\eta\right]. \tag{32}$$

- Integrating over c, we have

$$P_n^{-\theta} = \kappa_1\Psi_n X_n^{\frac{\theta-(\sigma-1)}{\sigma-1}}, \tag{33}$$

where $\Psi_n = \sum_{i=1}^{N}\Phi_{ni}E_{ni}^{-\frac{\theta-(\sigma-1)}{\sigma-1}}$.

Aggregation: Entry, Sales, Fixed Costs

- Entry condition: $\bar{c}_{ni}(\eta) = \eta^{1/(\sigma-1)} \left(\frac{X_n}{\kappa_1 \Psi_n}\right)^{\frac{1}{\theta}} E_{ni}^{-\frac{1}{\sigma-1}}$.
- Trade share: $\pi_{ni} = \frac{X_{ni}}{X_n} = \frac{\Phi_{ni} E_{ni}^{-\frac{\theta-(\sigma-1)}{\sigma-1}}}{\Psi_n}$.
- Total measure of firm in i serving n: $J_{ni} = \frac{\kappa_2}{\kappa_1} \frac{\pi_{ni} X_n}{E_{ni}}$.
- Total fixed costs: $\bar{E}_{ni} = \frac{\theta-(\sigma-1)}{\theta\sigma} \pi_{ni} X_n$.

A Streamlined Representation

- Prepare the model to match the data.
- Let $u(j) = T_F z_F(j)^{-\theta}$ be standardized unit cost of firm j in France. The measure of firms with $u(j) \leq u$:

$$\mu_F^z((T_F/u)^{1/\theta}) = u. \tag{34}$$

- Firm j enters market n if its $u(j)$ and $\eta_n(j)$ satisfy

$$u(j) \leq \bar{u}_{nF}(\eta_n(j)) = \left(\frac{\pi_{nF} X_n}{\kappa_1 \sigma E_{nF}}\right) \eta_n(j)^{\tilde{\theta}}, \quad \tilde{\theta} = \frac{\theta}{\sigma - 1}. \tag{35}$$

- Conditional on entry firm j's sales in market n is

$$X_{nF}(j) = \varepsilon_n(j) \left[1 - \left(\frac{u(j)}{\bar{u}_{nF}(\eta_n(j))}\right)^{\lambda/\tilde{\theta}}\right] \left(\frac{u(j)}{\bar{u}_{nF}(\eta_n(j))}\right)^{-1/\tilde{\theta}} \sigma E_{nF}. \tag{36}$$

Connecting the Model to the Empirical Regularities

- Entry:
  - A relationship between the number of French firms selling to market n and the size of market n:
$$\frac{N_{nF}}{\pi_{nF}} = \frac{\kappa_2}{\kappa_1} \frac{X_n}{\sigma E_{nF}}. \tag{37}$$
  - Calculate the fixed export costs directly from
$$\sigma E_{nF} = \frac{\kappa_2}{\kappa_1} \frac{\pi_{nF} X_n}{N_{nF}} = \frac{\kappa_2}{\kappa_1} \bar{X}_{nF}. \tag{38}$$
  - Entry condition:
$$u(j) \leq \bar{u}_{nF}(\eta_n(j)) = \frac{N_{nF}}{\kappa_2} \eta_n(j)^{\tilde{\theta}}. \tag{39}$$

Connecting the Model to the Empirical Regularities

- Sales in a market:
  - Conditional on a firm's entry into market n, the term

$$\nu_{nF}(j) = \frac{u(j)}{\bar{u}_{nF}(\eta_n(j))} \tag{40}$$

    is distributed uniformly on $[0, 1]$.
  - Then the sales

$$X_{nF}(j) = \varepsilon_n(j)[1 - \nu_{nF}(j)^{\lambda/\tilde{\theta}}]\nu_{nF}(j)^{-1/\tilde{\theta}} \frac{\kappa_2}{\kappa_1} \bar{X}_{nF}. \tag{41}$$

Connecting the Model to the Empirical Regularities

- Sales in France conditional on entry in a foreign market:

$$X_{FF}(j)|n = \frac{\alpha_F(j)}{\eta_n(j)} \left[1 - \nu_{nF}(j)^{\lambda/\tilde{\theta}} \left(\frac{N_{nF}}{N_{FF}} \right)^{\lambda/\tilde{\theta}} \left(\frac{\eta_n(j)}{\eta_F(j)} \right)^{\lambda} \right] \times \nu_{nF}(j)^{-1/\tilde{\theta}} \left(\frac{N_{nF}}{N_{FF}} \right)^{-1/\tilde{\theta}} \frac{\kappa_2}{\kappa_1} \bar{X}_{FF}. \tag{42}$$

- Normalized export intensity:

$$X_{nF}(j)/X_{FF}(j)\bar{X}_{nF}/\bar{X}_{FF} = \frac{\alpha_n(j)}{\alpha_F(j)} \left[\frac{1 - \nu_{nF}(j)^{\lambda/\tilde{\theta}}}{1 - \nu_{nF}(j)^{\lambda/\tilde{\theta}} \left(\frac{N_{nF}}{N_{FF}} \right)^{\lambda/\tilde{\theta}} \left(\frac{\eta_n(j)}{\eta_F(j)} \right)^{\lambda}} \right] \left(\frac{N_{nF}}{N_{FF}} \right)^{1/\tilde{\theta}}. \tag{43}$$

Estimation

- Assume that $\log(\alpha)$ and $\log(\eta)$ are normally distributed with zero mean and variance σ_α^2 and σ_η^2, and correlation ρ. Then
  - $\kappa_1 = \left[\frac{\tilde{\theta}}{\tilde{\theta}-1} - \frac{\tilde{\theta}}{\tilde{\theta}+\lambda-1}\right] \exp\{\frac{\sigma_\alpha+2\rho\sigma_\alpha\sigma_\eta(\tilde{\theta}-1)+\sigma_\eta(\tilde{\theta}-1)^2}{2}\}$.
  - $\kappa_2 = \exp\left\{\frac{(\tilde{\theta}\sigma_\eta)^2}{2}\right\}$.

- Only five parameters to estimate: $\Theta = \{\tilde{\theta}, \lambda, \sigma_\alpha, \sigma_\eta, \rho\}$.

- Simulated Methods of Moments:
  - Given Θ, simulate artificial samples.
  - Compute the moments from simulated samples.
  - Compare the simulated moments to the data moments.
  - Change Θ to minimize the distance between simulated moments and the data moments.

Simulated Methods of Moments

- Artificial French exporter s with the number S. 113 destinations.
- Draw S realizations of $\nu(s)$ independently from $U[0,1]$.
- Draw $S \times 113$ realizations of $a_n(s)$ and $h_n(s)$ independently from $N(0,1)$.
- A given simulation of the model requires Θ, X_{nF}, and N_{nF}.

Simulated Methods of Moments

1. Calculate κ_1 and κ_2.
2. Calculate σE_{nF} for each destination n.
3. Construct $S \times 113$ realizations for each of $\log \alpha_n(s)$ and $\log \eta_n(s)$.
4. Construct $S \times 113$ entry hurdles $\bar{u}(s) = \frac{N_{nF}}{\kappa_2} \eta_n(s)^{\tilde{\theta}}$.
5. Calculate $\bar{u}^X(s) = \max_{n \neq F}\{\bar{u}_n(s)\}$ and $\bar{u}(s) = \min\{\bar{u}_F(s), \bar{u}^X(s)\}$.
6. $u(s)$ is a realization from $U[0, \bar{u}(s)]$. So $u(s) = \nu(s)\bar{u}(s)$.
7. Artificial French exporter s gets an importance weight $\bar{u}(s)$.
8. Market entry index $\delta_{nF}(s) = 1$ if $u(s) \leq \bar{u}_n(s)$ and 0 otherwise. With $\delta_{nF}(s) = 1$,

$$X_{nF}(s) = \frac{\alpha_n(s)}{\eta_n(s)} \left[1 - \left(\frac{u(s)}{\bar{u}_n(s)}\right)^{\lambda/\tilde{\theta}}\right] \left(\frac{u(s)}{\bar{u}_n(s)}\right)^{-1/\tilde{\theta}} \sigma E_{nF}. \tag{44}$$

Moments

- Moments in this exercise: the number of firms that fall into sets of exhaustive and mutually exclusive bins.
- N^k: the number of firms achieving some outcome k in the actual data.
- $\hat{N}^k$: corresponding number in simulated data, as $\hat{N}^k = \frac{1}{S}\sum_{s=1}^{S} \bar{u}(s)\delta^k(s)$ where $\delta^k(s)$ is an indicator for achieving outcome k.

Four Sets of Moments

1. $\hat{m}^k(1;\Theta)$: the proportion of simulated exporters selling to each possible combination k of the seven most popular export destinations.
2. $\hat{m}^k(2;\Theta)$: the proportion of simulated exporters in each market falling into $(50, 75, 95)$ quantiles of exporting sales.
3. $\hat{m}^k(3;\Theta)$: the proportion of simulated exporters in each market falling into the $(50, 75, 95)$ quantiles of French sales.
4. $\hat{m}^k(4;\Theta)$: the proportion of simulated exporters in each market falling into the $(50, 75)$ quantiles of export intensities.

Estimation

- $y(\Theta) = m - \hat{m}(\Theta)$.
- We get the SMM estimator by:

$$\hat{\Theta} = \arg\min_{\Theta}\{y(\Theta)' W y(\Theta)\}. \tag{45}$$

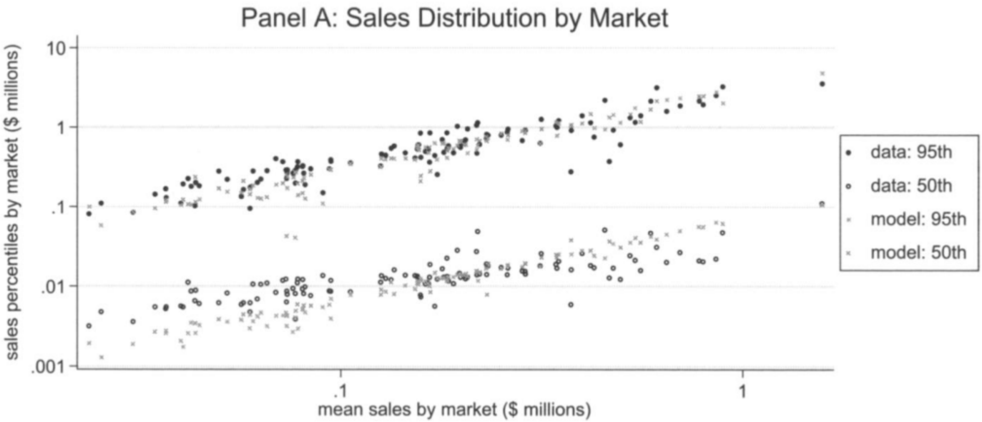
- Search for $\hat{\Theta}$ using the simulated annealing algorithm.
- Calculate standard errors using a bootstrap technique.

$\tilde{\theta}$	λ	σ_α	σ_η	ρ
2.46	0.91	1.69	0.34	−0.65
(0.10)	(0.12)	(0.03)	(0.01)	(0.03)

Implications

- $\tilde{\theta} = 2.46$: fixed costs dissipate 59% of gross profit in any destination.
- Enormous idiosyncratic variation in a firm's sales across destinations but less variation in the entry shock.
- Consistent with the entry of firms into markets where they sell very little: (i) $\lambda = 1$, (ii) negative covariance between the sales and entry shocks.

Model-fit



Panel A: Sales Distribution by Market
sales percentiles by market ($ millions)
10
1
.1
.01
.001
.1
1
mean sales by market ($ millions)
data: 95th
data: 50th
model: 95th
model: 50th

Sources of Variation

- The fraction of the variance of entry in each market that can be explained by the cost draw u alone:

$$R_n^E = 1 - \frac{E[V_n^C(u)]}{V_n^U}, \tag{46}$$

where V_n^U is the variance of entry decisions in market n conditional only on u and V_n^U is the unconditional variance.

- Result: on average 57% of the variation in entry in a market can be attributed to the core efficiency.

Productivity

- The average value-added per worker of exporters is 1.22 times the average for all firms.
- Let $I(j)$ be the intermediate expenditure and β be the value-added share:

$$I_i(j) = (1-\beta)\bar{m}^{-1}Y_i(j) + E_i(j). \tag{47}$$

- Then $q_i(j) = \frac{V_i(j)}{\beta\bar{m}^{-1}Y_i(j)} = \frac{\bar{m}-(1-\beta)}{\beta} - \frac{\bar{m}}{\beta}\frac{E_i(j)}{Y_i(j)}$.
- Estimates: $\sigma = 2.98$ and $\beta = 0.34$.
- $\theta = 4.87$.

General Equilibrium

- Factor bundles: $w_i = W_i^{\beta} P_i^{1-\beta}$.
- Manufactures have a share γ in final consumption. Non-manufactures are produced by labor.
- $E_{ni} = W_n F_{ni}$.
- Each countries manufacturing deficit D_i and total deficit D_i^A are held at their 1986 values.
- General equilibrium definition and counterfactual computation are the same as EK(2002) and DEK (2008).

Counterfactuals

TABLE IV

COUNTERFACTUALS: FIRM TOTALS[a]

		Counterfactual	
	Baseline	Change From Baseline	Percentage Change
Number			
All firms	231,402	–26,589	–11.5
Exporting	32,969	10,716	32.5
Values ($ millions)			
Total sales	436,144	16,442	3.8
Domestic sales	362,386	–18,093	–5.0
Exports	73,758	34,534	46.8

Counterfactuals

TABLE V

COUNTERFACTUALS: FIRM ENTRY AND EXIT BY INITIAL SIZE

	All Firms			Exporters		
		Counterfactual			Counterfactual	
Initial Size Interval (percentile)	Baseline No. of Firms	Change From Baseline	Change (%)	Baseline No. of Firms	Change From Baseline	Change (%)
Not active	0	1118	—	0	1118	—
0 to 10	23,140	−11,551	−49.9	767	15	2.0
10 to 20	23,140	−5702	−24.6	141	78	55.1
20 to 30	23,140	−3759	−16.2	181	192	106.1
30 to 40	23,140	−2486	−10.7	357	357	100.0
40 to 50	23,140	−1704	−7.4	742	614	82.8
50 to 60	23,138	−1141	−4.9	1392	904	65.0
60 to 70	23,142	−726	−3.1	2450	1343	54.8
70 to 80	23,140	−405	−1.8	4286	1829	42.7
80 to 90	23,140	−195	−0.8	7677	2290	29.8
90 to 99	20,826	−38	−0.2	12,807	1915	15.0
99 to 100	2314	0	0.0	2169	62	2.8

Summary of EKK (2011)

- A GE model that can match the micro data carefully and quantify aggregate effects.
- Very rigorous empirical implementation.
- Firm heterogeneity in productivity accounts for 57% of export entry.
- Trade liberalization promotes aggregate productivities by reallocating labors to most productive firms.

Trends in Quantitative Trade Literature

- Policy-driven
  - Uncertainties: Alessandria et al. (2025)
  - Geopolitics: Alekseev and Lin (2025)
- Interacting with other fields
  - Agriculture: Farrokhi and Pellegrina (2023)
  - Market power: Graziano, Alejandro (2024)
- Dynamics
  - Downward nominal wage rigidities: Rodriguez-Clare et al. (2024)