Indirect Inference

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Indirect Inference

- Structural model:
$$G\left(Y, X, \epsilon; \theta\right) = 0, \tag{1}$$
where ϵ is IID with CDF $F(.)$ and $G(.)$ is complex and difficult to invert.

- Reduced-form relationship between Y and X:
$$H(Y, X, u; \beta) = 0, \tag{2}$$
where $H(.)$ is simple (e.g. linear) and β can be easily estimated using data on Y and $X \Rightarrow \hat{\beta}$

- Indirect inference:
    - Simulate ϵ based on $F(.)$
    - Given initial guess θ and X, simulate $Y\left(X, \epsilon; \theta\right)$ that satisfy $G\left(Y, X, \epsilon; \theta\right) = 0$
    - Utilize the simulated $\left(Y\left(X, \epsilon; \theta\right), X\right)$ to obtain $\hat{\beta}\left(X, \epsilon; \theta\right)$
    - Recover θ from $\hat{\beta}\left(X, \epsilon; \theta\right) = \hat{\beta}$

Catherine et al. (2022): Production

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- Firm i's demand curve:
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- Firm i's demand curve:
$$q_{it} = Q p_{it}^{-\phi}, \quad \phi > 1. \tag{5}$$
- Labor is a static input. So the total profits of the firm is given by
$$\pi\left(z_{it}; k_{it}\right) = \max_{l_{it}} \left\{ p_{it} q_{it} - w l_{it} \right\} = b Q^{1-\theta} w^{-\frac{1-\alpha}{\alpha}\theta} e^{\frac{\theta}{\alpha} z_{it}} k_{it}^{\theta} \tag{6}$$
where b is a constant and $\theta \equiv \frac{\alpha(\phi-1)}{1+\alpha(\phi-1)} < 1$

Catherine et al. (2022): Investment

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$$k_{it+1} = k_{it} + i_{it} - \delta k_{it} \tag{7}$$

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- Firm's net debt: d_{it}; $d_{it} < 0$ means cash holding
  - One-period maturity: the firm commit to repay $(1 + r)\, d_{it+1}$ at $t + 1$
  - If $d_{it} < 0$, the firm receives a positive cash inflow of $-(1 + (1 - m)\, r)\, d_{it+1}$ with $m \in (0, 1)$

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- Firm profits net of interest payments and capital depreciation are taxed at rate τ (tax credit when profits are negative)
- Financing frictions:
  1. Equity issuance cost: if preissuance cash flows are x, cash flows net of issuance costs: $G(x) = x\,(1 + e\mathbf{1}_{x<0})$ where $e > 0$ parameterizes the cost of equity issuance
  2. Collateral constraint (limited enforcement): $(1+r)\, d_{it+1} \leq s\,[(1-\delta)\, k_{it+1} + E\,[p_{t+1}|p_t] \times h]$ where h is the quantity of real estate and p_t is the associated price, with $\log p_t$ follows a discretized AR(1) process

Catherine et al. (2022): Firm's Dynamic Decisions

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Catherine et al. (2022): Firm's Dynamic Decisions

- Firm is subject to a death shock with probability D
- Firm optimally chooses capital and debt to maximize a discounted sum of per-period cash flows, subject to the financing constraint. Let $S_{it} = \{k_{it}, d_{it}\}$ and $X_{it} = \{z_{it}, p_t\}$. The firm solves

$$
\begin{aligned}
& V\left(S_{it}; X_{it}\right) = \max_{S_{it+1}} \left\{ CF + \frac{1-D}{1+r} E\left[V\left(S_{it+1}; X_{it+1}\right)|X_{it}\right] + \frac{D}{1+r}\left(k_{it+1} - \left(1+\tilde{r}_{it}\right) d_{it+1}\right) \right\} \\
& \text{s.t. } \left(1+r\right) d_{it+1} \leq s\left[\left(1-\delta\right) k_{it+1} + E\left[p_{t+1}|p_t\right] \times h\right] \\
& \text{with: } CF = G\left(\pi\left(z_{it}; k_{it}\right) - i_{it} - \frac{c}{2}\frac{i_{it}^2}{k_{it}} + d_{it+1} - \left(1+\tilde{r}_{it}\right) d_{it} - \tau\left[\pi\left(z_{it}; k_{it}\right) - \tilde{r}_{it} d_{it} - \delta k_{it}\right]\right) \\
& i_{it} = k_{it+1} - \left(1-\delta\right) k_{it} \\
& \tilde{r}_{it} = r \text{ if } d_{it} > 0 \text{ and } (1-m)r \text{ if } d_{it} \leq 0,
\end{aligned}
\tag{8}
$$

where $\frac{D}{1+r}\left(k_{it+1} - \left(1+\tilde{r}_{it}\right) d_{it+1}\right)$ corresponds to the shareholder's payoff in the event of firm death

Catherine et al. (2022): Indirect Inference

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- Target: search for the set of parameters Ω such that model-generated moments $\hat{m}(\Omega)$ on simulated data fit a predetermined set of data moments m
- No analytical solution to the model $\Rightarrow$ cannot invert the system of equations $\Rightarrow$ Indirect inference
  - For a given set of parameters, solve the Bellman equation and obtain policy function $S_{it+1} = (d_{it+1}, k_{it+1})$ as a function of $S_{it} = (d_{it}, k_{it})$ and exogenous variables $X_{it} = (z_{it}, p_t)$
  - Minimize the distance from simulated to data moments

$$\hat{\Omega} = \arg\min_{\Omega} (m - \hat{m}(\Omega))' W (m - \hat{m}(\Omega)). \tag{9}$$

Catherine et al. (2022): Predefined Parameters

- capital share: $\alpha = 1/3$ from Bartelsman, Haltiwanger, and Scarpetta (2013)
- demand elasticity: $\phi = 6.7$ from Broda and Weinstein (2006)
- $\log p_t$: AR(1) process, with persistence of 0.62 and innovation volatility of 0.06
- capital depreciation rate: $\delta = 6\%$ from Midrigan and Xu (2014)
- risk-free borrowing rate $r = 3\%$; the lending rate $(1 - m)\,r = 2\%$
- death rate $D = 8\%$; corporate tax rate $\tau = 33\%$
- h is set to match the average ratio of real estate to capital
- normalize $w = 0.03$ and $Q = 1$

Catherine et al. (2022): Estimated Parameters

- $\log z_t$: persistence ρ and innovation volatility σ
- collateral parameter s
- adjustment cost c
- the cost of equity issuance e

Catherine et al. (2022): Moments

- short- and long-term volatility of output $\Rightarrow$ persistence and volatility of log sales
  - volatility of $(\log sales_{it} - \log sales_{it-1})$ is 0.327; that of $(\log sales_{it} - \log sales_{it-5})$ is 0.912
  - the fact that five-year growth is less than five times more volatile than one-year growth contributes to the identification of persistence
  - targeting these two moments instead of directly matching the persistence coefficient of log sales makes the estimation less sensitive to short-panel bias

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- autocorrelation of investment $\Rightarrow$ quadratic adjustment costs
  - for each firm, compute i_{it}/k_{it-1}; then regress this ratio on firm fixed effects and extract the residuals; then compute the autocorrelation of these residuals (result 0.165)
  - adjustment costs compel the firm to smooth its investment policy in response to a productivity shock

Catherine et al. (2022): Moments

- a direct measure of financing constraints $\Rightarrow$ collateral constraint parameter s, the sensitivity of investment to real estate value
  - Regression: $\frac{i_{it}}{k_{it-1}} = \alpha + \beta \frac{REValue_{it}}{k_{it-1}} + Offprice_{it} + controls_{it} + \nu_{it}$
  - $\hat{\beta} = 0.06$ and t-stat$= 6.1$: this coefficient would be statistically insignificant absent financing frictions
  - Identify the level of financing frictions through indirect inference

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  - $\hat{\beta} = 0.06$ and t-stat$= 6.1$: this coefficient would be statistically insignificant absent financing frictions
  - Identify the level of financing frictions through indirect inference

- average ratio of net positive equity issuance to value-added $\Rightarrow$ the cost of equity issuance
  - the average of this ratio across all firms: 0.026

Catherine et al. (2022): Identification

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- large number of simulated observations: $1,000,000$ firms over 10 years

Catherine et al. (2022): Identification

Table I

Elasticity of Moments with Respect to Parameters

This table reports the elasticity of simulated moments with respect to the estimated structural parameters. First, we start with the SMM estimate $\widehat{\Omega}$ of the parameters Ω. For each $k = 1, \dots, 4$, we set $\omega_l = \hat{\omega}_l$ for all $l \neq k$, and vary the parameter ω_k around the estimated $\hat{\omega}_k$ to compute the elasticity of moments to parameters in the vicinity of the SMM estimate. For each moment m_n, we compute

$$\epsilon_{n,k} = \frac{\log m_n^+ - \log m_n^-}{\log \omega_k^+ - \log \omega_k^-} \approx \frac{\partial \log(\hat{m}_n)}{\partial \log(\hat{\omega}_k)},$$

where $\hat{m}_n$ is the n^{th} data moment, m_n^+ is the moment based on data simulated with parameter $\hat{\omega}_k^+$. m_n^- is the average of moments based on data simulated with parameter ω_k^-. For each parameter, we consider a 10-grid point scale as in Figures IA.1 to IA.5. Parameters ω_k^+ and ω_k^- are values just above and below the SMM estimate $\hat{\omega}_k$. For example, around the SMM estimate, a 1% increase in s is associated with a 1.2% decrease in the sensitivity of investment to real estate and a 1.1% increase in leverage.

	s.d. Δ log sales	s.d. Δ_5 log sales	Net Debt / Assets	β(*Inv*, *RE*)	Autocorr. Invest.	Equity Issues / Value-Added
Pledgeability s	0.066	0.071	1.1	1.2	−0.64	−0.23
Adj. cost c	−0.02	−0.013	0.029	−0.0058	0.31	−.071
Volatility σ	1.0	1.1	−0.7	0.7	0.26	3.8
Persistence ρ	0.81	2.1	−0.76	−2.5	5.5	13.0
Issuance cost e	−0.057	−0.13	−0.2	0.21	−0.72	−2.1

Catherine et al. (2022): Identification

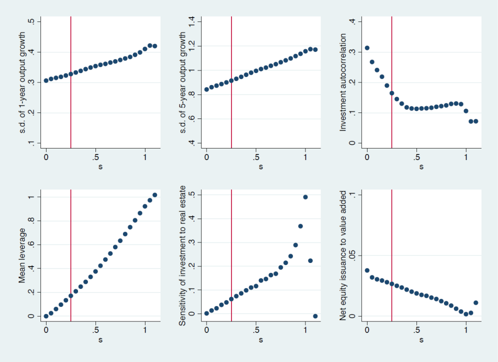

Figure IA1. Sensitivity of moments to pledgeability s. In this figure, we set all of the estimated parameters (s, c, ρ, σ, H and e) at their SMM estimate in our preferred specification – as per column (3), Panel A in Table II of the main article. We fix w and Q at their reference levels: $w = 0.03$ and $Q = 1$. We then vary s from zero to one. For each value of s that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of s.

Catherine et al. (2022): Identification

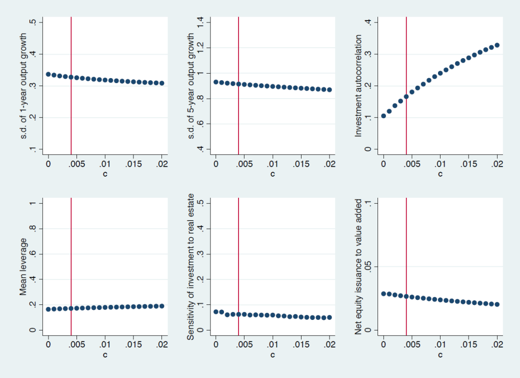

Figure IA2. Sensitivity of moments to pledgeability c**.** In this figure, we set all of the estimated parameters (s, c, ρ, σ, H and e) at their SMM estimate in our preferred specification – as per column (3), Panel A in Table II of the main article. We fix w and Q at their reference levels: $w = 0.03$ and $Q = 1$. We then vary c from zero to 0.02. For each value of c that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of c.

Catherine et al. (2022): Identification

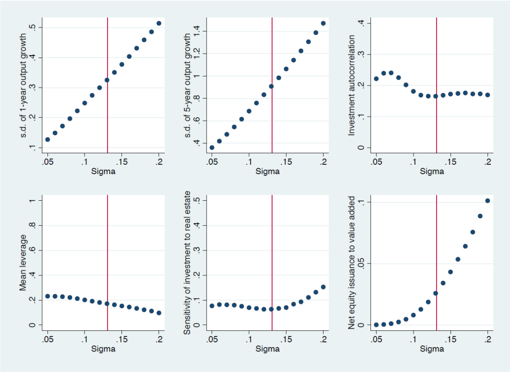

Figure IA3. Sensitivity of moments to pledgeability σ. In this figure, we set all of the estimated parameters (s, c, ρ, σ, H and e) at their SMM estimate in our preferred specification – as per column (3), Panel A in Table II of the main article. We fix w and Q at their reference levels: $w = 0.03$ and $Q = 1$. We then vary σ from zero to 0.2. For each value of σ that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of σ.

Catherine et al. (2022): Identification

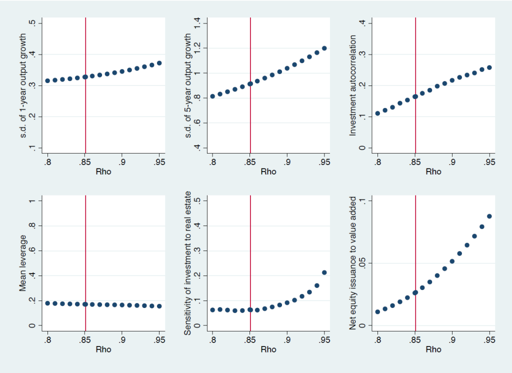

Figure IA4. Sensitivity of moments to pledgeability ρ. In this figure, we set all of the estimated parameters (s, c, ρ, σ, H and e) at their SMM estimate in our preferred specification – as per column (3), Panel A in Table II of the main article. We fix w and Q at their reference levels: $w = 0.03$ and $Q = 1$. We then vary ρ from 0.8 to 0.95. For each value of ρ that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of ρ.

Catherine et al. (2022): Identification

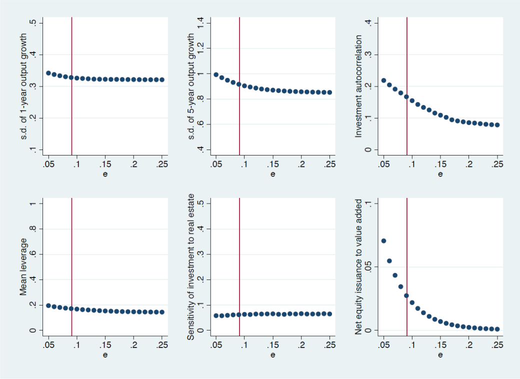

Figure IA5. Sensitivity of moments to pledgeability e**.** In this figure, we set all of the estimated parameters (s, c, ρ, σ, H and e) at their SMM estimate in our preferred specification – as per column (3), Panel A in Table II of the main article. We fix w and Q at their reference levels: $w = 0.03$ and $Q = 1$. We then vary e from 0.05 to 0.25. For each value of e that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of e.

Catherine et al. (2022): Identification

Table II

Parameter Estimates (SMM)

This table reports the results of our SMM estimations. The estimation procedure is described in the text and in Section II of the Internet Appendix. Columns (1) to (3) correspond to SMMs using different models. Column (1) assumes no adjustment cost and infinite cost of equity issuance ($c = 0, e = +\infty$). Column (2) introduces adjustment costs but maintains $e = +\infty$. Column (3) further allows for a finite cost of equity issuance. For each of these estimations, Panel A shows the estimated parameters, along with standard errors (obtained via bootstrapping) in parentheses. Panel B shows the value of a set of moments, measured on simulated data (with 1,000,000 observations). Moments with a superscript "+" are those that are targeted in the estimation. The other moments are not targeted. The last column (labeled "data") reports the empirical moments.

Specification:	Model 1: $c = 0, e = +\infty$ (1)	Model 2: $c > 0, e = +\infty$ (2)	Model 3: $c > 0, e > 0$ (3)	Data (4)
Panel A: Estimated Parameters				
ρ	0.922	0.893	0.851	
	(0.013)	(0.020)	(0.017)	
σ	0.124	0.134	0.131	
	(0.007)	(0.004)	(0.003)	
s	0.196	0.216	0.250	
	(0.078)	(0.080)	(0.048)	
c	0	0.008	0.004	
		(0.003)	(0.002)	
e	$+\infty$	$+\infty$	0.091	
			(0.012)	
Panel B: Moments (Targeted Indicated with "+")				
SD one-year sales growth	0.327^{+}	0.327^{+}	0.327^{+}	0.327
SD five-year sales growth	0.913^{+}	0.912^{+}	0.912^{+}	0.912
Real-estate to assets	0.140^{+}	0.140^{+}	0.141^{+}	0.140
$\beta(Inv, RE)$	0.060^{+}	0.060^{+}	0.060^{+}	0.060
Autocorrelation of Inv.	0.041	0.165^{+}	0.165^{+}	0.165
Net equity issuance to value-added	0	0	0.026^{+}	0.026
$\beta(D, RE)$	0.059	0.053	0.070	0.060
Net debt to assets	0.030	0.090	0.171	0.098

Catherine et al. (2022): Determinants of Financing Constraints

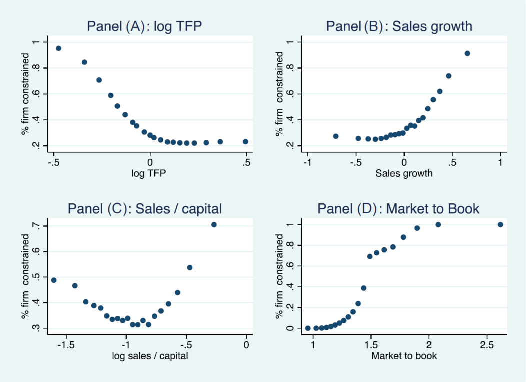

Figure 1. Financing constraints as a function of firm characteristics. This figure shows how the extent of financing constraints covaries with firm characteristics in the cross section of simulated firms. We simulate a data set of 1,000,000 firms over 215 years using parameters from our preferred specification (Table II, Panel A, column (3)). We remove the first 200 years to ensure firms are in steady state. For each characteristic x, we then sort firms into 20 equal-sized bins of x, and, for each bin, compute the average share of constrained firms. We label a firm-year as "constrained" if its market value is less than 95% of its unconstrained market value. Unconstrained market value is computed using the same set of state variables (z, k, b) at the beginning of the period and a model for which the cost of equity issuance e is set to zero. We use the following conditioning variables x: z (Panel A), $\log pq_t - \log pq_{t-1}$ (Panel B), $\log \frac{pq}{k}$ (Panel C), and $\frac{V}{k}$ (Panel D). (Color figure can be viewed at wileyonlinelibrary.com)

Catherine et al. (2022): General Equilibrium Analysis

- Endogenize aggregate demand Q and the real wage w

Catherine et al. (2022): General Equilibrium Analysis

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- Firms:
  - CES aggregation: $Q_t = \left(\sum_{i=1}^{N} q_{it}^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}}$
  - Price index: $P_t = \left(\sum_i p_{it}^{1-\phi} \right)^{\frac{1}{1-\phi}}$; normalize $P_t = 1$
  - Demand: $q_{it} = Q_t \left(\frac{p_{it}}{P_t} \right)^{-\phi}$

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- Consumption:
  - Final goods: $Q_t = C_t + I_t + Adj.Cost_t$, where $Adj.Cost_t = \sum_i \frac{c}{2} i_{it}^2 / k_{it}$ and $I_t = \sum_i i_{it}$
  - Consumer's behavior: $U_s = \sum_{t \geq s} \beta^{t-s} u_t, \quad u_t = C_t - \bar{L}^{-\frac{1}{\epsilon}} \frac{L_t^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}}$
  - Labor supply: $L_t^s = \bar{L} w_t^{\epsilon}$

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    - Labor supply: $L_t^s = \bar{L} w_t^{\epsilon}$

- Steady-state:
    - $r_t = \frac{1}{\beta} - 1$; no aggregate demand and productivity shocks: (Q_t, w_t) are constant over time
    - Market clearing: $\bar{L} w^{\epsilon} = \sum_{i=1}^{N} l^d\left((Q, w); z_{it}, k_{it}(Q, w)\right)$ and $PQ = \sum_{i=1}^{N} p_{it} q\left((Q, w); z_{it}, k_{it}(Q, w)\right)$

Catherine et al. (2022): General Equilibrium Analysis

Table III

Aggregate Effects of Collateral Constraints

This table reports results of the counterfactual analysis for different SMM parameter estimates. The general equilibrium analysis is described in Section IV and reported in Panel A. Columns (1) to (3) correspond to the three different models described in columns (1) to (3) of Table II: Column (1) assumes no adjustment cost ($c = 0$) and infinite cost of equity issuance ($e = +\infty$). Column (2) allows for adjustment cost but still assumes infinite cost of equity issuance. Column (3) also allows for finite cost of equity issues. Panel B implements the same methodology, but it holds the aggregate demand shifter Q constant while the wage w clears the labor market. Panel C holds both the aggregate demand shifter Q and wage w constant. Results in both panels are shown as log deviations from the constrained estimated model to the unconstrained benchmark. The unconstrained benchmark corresponds to an equilibrium in which firms face the same set of parameters as in the SMM estimate—reported in the same column, Table II, Panel A—but do not face a constraint on equity issuance ($e = 0$). In this unconstrained benchmark, investment reaches first best, but firms still benefit from the debt tax shield. For example, column (1) (no adjustment cost, no equity issuance) shows that the aggregate TFP loss compared to a benchmark without financing constraints is 3.1%.

Specification:	Model 1 $c = 0, e = +\infty$ (1)	Model 2 $c > 0, e = +\infty$ (2)	Model 3 $c > 0, e > 0$ (3)
Panel A: General Equilibrium Results			
Δ log(TFP)	0.031	0.027	0.014
Δ log(Output)	0.151	0.120	0.071
Δ log(wage)	0.101	0.080	0.048
Δ log(L)	0.051	0.040	0.024
Δ log(K)	0.282	0.215	0.137
Panel B: Partial Equilibrium Results, Holding Q Fixed Only			
Δ log(TFP)	0.012	0.012	0.005
Δ log(Output)	0.110	0.088	0.052
Δ log(wage)	0.073	0.059	0.035
Δ log(L)	0.037	0.029	0.017
Δ log(K)	0.240	0.185	0.117
Panel C: Partial Equilibrium Results, Holding (Q, w) Fixed			
Δ log(TFP)	−0.040	−0.029	−0.020
Δ log(Output)	0.400	0.320	0.189
Δ log(wage)	-	-	-
Δ log(L)	0.400	0.320	0.189
Δ log(K)	0.531	0.417	0.254

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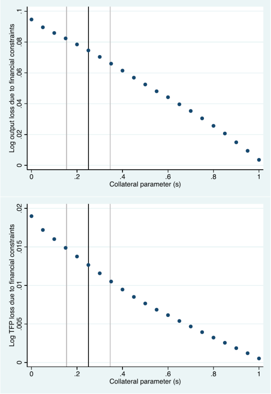

Figure 2. General equilibrium effect of pledgeability *s*. This figure reports the general equilibrium effect of changing the collateral parameter s from zero (the capital stock cannot be pledged as collateral) to one (100% of the capital stock can be pledged to lenders). For each value of s, we first compute aggregate output and TFP in a general equilibrium economy in which firms face a collateral parameter s and all other parameters are set to their estimate in Table II, Panel A, column (3). We then compute aggregate output and TFP in another general equilibrium economy where firms face the same collateral parameter s and all other parameters are set to their estimate in Table II, Panel A, column (3), except for the equity issuance parameter, which is now set to zero. This other economy corresponds to the unconstrained benchmark: in the absence of equity issuance costs, firms' investment will be first best. For each value of s, we then compute the log difference of output and TFP between these two economies. The vertical black line corresponds to the SMM estimate of s (0.25) and gray lines denote the limits of the 90% confidence interval. For example, when s increases from 0.1 to 0.6, the output loss relative to the unconstrained benchmark goes from 10% to 5%. (Color figure can be viewed at wileyonlinelibrary.com)

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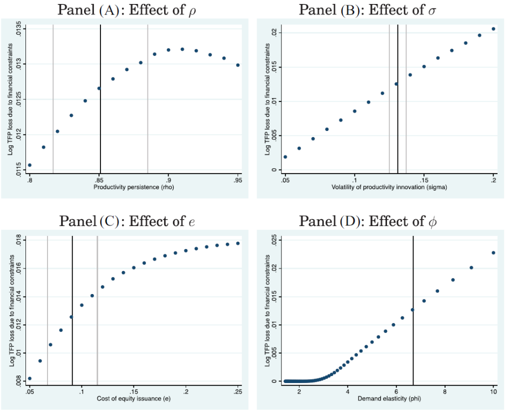

Figure 3. Total factor productivity: Additional comparative statics. This figure reports the effect of changing various parameters on the TFP loss of financing constraints. We vary productivity persistence in Panel A (ρ), productivity innovation volatility in Panel B (σ), equity issuance costs in Panel C (e), and price elasticity ϕ in Panel D. In Panels A, B, and C, the vertical black line correspond to the SMM estimates and the gray lines the borders of the 90% confidence interval. In Panel D, we do not report confidence intervals because ϕ is calibrated, not estimated. The values we span correspond to the range of values in Broda and Weinstein (2006).
(Color figure can be viewed at wileyonlinelibrary.com)

Comments

- Indirect inference: matching the reduced-form estimates in the observed and simulated data
  - Why not directly matching the moments in the observed and simulated data?
  - Moments: higher dimensional information but greater noises
  - Reduced-form estimates: shrunk but cleaner

Comments

- Indirect inference: matching the reduced-form estimates in the observed and simulated data
  - Why not directly matching the moments in the observed and simulated data?
  - Moments: higher dimensional information but greater noises
  - Reduced-form estimates: shrunk but cleaner
- Utilizing IV in estimating the reduced-form model: $H(Y, X, u; \beta) = 0$
  - Do we need IV in estimating the reduced-form model using the simulated data?
  - No. We could simulate the structural errors so that they are exogenous