Quantitative Spatial Models

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Spring 2025

Quantitative Spatial Models

- Comparing with quantitative trade models, quantitative spatial models are drawing increasing attention
  - Policy relevance:
    - Within country
    - Classical externalities
  - Identification: quasi-experimental shocks
- Road map:
  - Basic elements in quantitative spatial models: Allen and Arkolakis (2014)
  - Identifying externalities and spillovers over space: Ahlfeldt, Redding, Sturm, and Wolf (2015)
  - Transportation networks: Allen and Arkolakis (2022)

Productivities and Amenities over Space: Allen and Arkolakis (2014)

- The stylized nature of geography (e.g. line or circle) in large set of economic geography papers makes it difficult to take them directly to the data.

- Allen and Arkolakis (2014) develop a quantitative framework to determine the spatial distribution of economic activity with a rich geography.

- Derive sufficient conditions for existence and uniqueness of equilibrium in spatial geography models (with a continuum of locations)

- Illustrative quantitative exercise: Estimate the underlying geography of the United States and calculate the welfare effect of the interstate highway system.

AA (2014): Model – Geography

- Continuum of locations, $i \in S$ (S is a closed and bounded set of a finite dimensional Euclidean space)

- Location $i \in S$:
  - Endowed with differentiated variety (Armington assumption)
  - Productivity: $A(i) = \bar{A}(i) L(i)^{\alpha}$ (exogenous part: $\bar{A}(i)$)
  - Amenity: $u(i) = \bar{u}(i) L(i)^{\beta}$ (exogenous part: $\bar{u}(i)$)

- $\alpha \geq 0$, $\beta \leq 0$. Note that these spillovers are assumed to be local in nature (i.e. do not affect productivity or amenities in nearby regions)

- For all $i, j \in S$, symmetric iceberg bilateral trade cost $T(i, j)$

AA (2014): Model – Workers

- CES preferences over differentiated varieties with elasticity of substitution $\sigma > 1$

- Can choose to live/work in any location (free mobility)

- Receive wage $w(i)$ for their inelastically supplied unit of labor

- Welfare in location i is:

$$W(i) = \left(\int_{s \in S} q(s,i)^{\frac{\sigma-1}{\sigma}} ds \right)^{\frac{\sigma}{\sigma-1}} u(i)$$

  where $q(s,i)$ is the per capita quantity consumed in location i of the good produced in location s and $u(i)$ is the local amenity.

- Due to CES preference:

$$W(i) = \bar{u}(i)\, L(i)^{\beta} \frac{w(i)}{P(i)}. \tag{1}$$

AA (2014): Model – Workers

- Equivalent setting: Roy model that a worker can draw idiosyncratic amenity shock ξ in each location from a Frechet distribution:

$$G(\xi) = \exp\{-\bar{U}(i)\xi^{-\mu}\}. \tag{2}$$

- Then the labor distribution is

$$\frac{L(i)}{\bar{L}} = \frac{\bar{U}(i)(w(i)/P(i))^{\mu}}{\sum_k \bar{U}(k)(w(k)/P(k))^{\mu}}. \tag{3}$$

- Notice that

$$E(\xi(i)w(i)/P(i)|\text{work in i}) = \Gamma(1 - \frac{1}{\mu})[\sum_k (w(k)/p(k))^{\mu}]^{\frac{1}{\mu}}. \tag{4}$$

- Omitting the constant:

$$W(i) = L(i)^{-\frac{1}{\mu}} \frac{w(i)}{P(i)}. \tag{5}$$

AA (2014): Model – Production

- Labor is the only factor of production, $L(i)$ is the density of workers.
- Productivity of worker in location i is $A(i)$
- Price of good from i is $\frac{w(i)}{A(i)} T(i,j)$ in location j
- Trade flows: $X(i,j) = \left(\frac{T(i,j)w(i)}{A(i)P(j)}\right)^{1-\sigma} w(j)L(j)$
- Price index: $P(j)^{1-\sigma} = \int_S T(s,j)^{1-\sigma} A(s)^{\sigma-1} w(s)^{1-\sigma} ds$

AA (2014): Model – Equilibrium definitions

- A spatial equilibrium is a distribution of economic activity such that:
  - Markets clear, i.e. $w(i)\,L(i) = \int_S X(i,s)\,ds$,
  - Welfare is equalized, i.e. $W \in \mathbb{R}_{++}$ such that for all $i \in S$, $W(i) \leq W$, with equality if $L(i) > 0$,
  - The aggregate labor market clears, i.e. $\int_S L(s)\,ds = \bar{L}$.
- A spatial equilibrium is regular if L and w are continuous and strictly positive (i.e. every location is inhabited).
- A spatial equilibrium is point-wise locally stable if $\frac{dW(i)}{dL(i)} < 0$ for all $i \in S$ (i.e. no small number of workers can increase welfare by moving to another location).

AA (2014): Model – With spillovers

- **Theorem**: Consider any regular geography with endogenous productivity and amenities with T symmetric. Define $\gamma_1 = 1 - \alpha(\sigma - 1) - \beta\sigma$, and $\gamma_2 = 1 + \alpha\sigma + (\sigma - 1)\beta$. If $\gamma_1 \neq 0$, then:
  1. There exists a regular equilibrium.
  2. If $\gamma_1 < 0$, no regular equilibria are point-wise locally stable.
  3. If $\gamma_1 > 0$, all equilibria are regular and point-wise locally stable.
  4. If $\frac{\gamma_2}{\gamma_1} \in [-1, 1]$, the equilibrium is unique.
- Sufficient conditions for uniqueness satisfied only if no net spillovers, i.e. $\alpha + \beta \leq 0$

AA (2014): Model – With spillovers

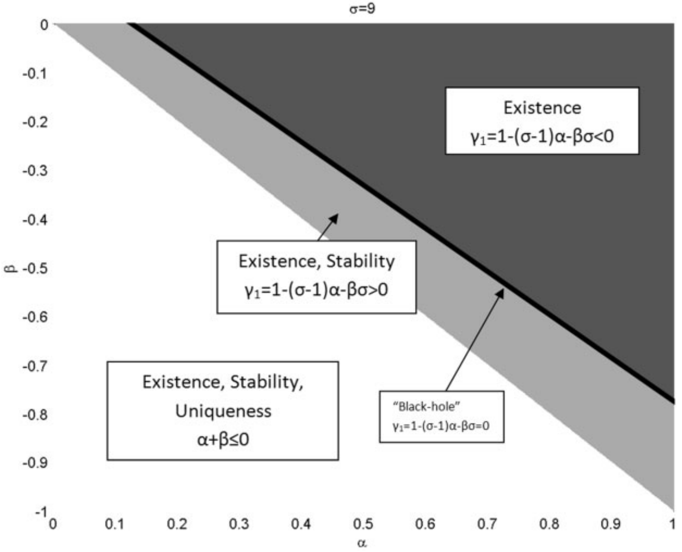


FIGURE I

AA (2014): Estimating trade costs

- For any $i, j \in S$, traders t choosing mode $m \in \{1, ..., M\}$ of transit where cost is $\exp\left(\tau_m d_m\left(i,j\right) + f_m + \nu_{tm}\right)$ and ν_{tm} is i.i.d. from a Gumbel dist. $Pr\left[\nu_{tm} \leq \nu\right] = \exp\left[-\exp\left(-\theta\nu\right)\right]$

- Then mode-specific bilateral trade shares are:

$$\pi_m\left(i,j\right) = \frac{\exp\left(-a_m d_m\left(i,j\right) - b_m\right)}{\sum_k \left(\exp\left(-a_k d_k\left(i,j\right) - b_k\right)\right)}, \tag{6}$$

where $a_m = \theta\tau_m$ and $b_m = \theta f_m$.

- Combined with model, yields gravity equation:

$$\ln X(i,j) = \frac{\sigma - 1}{\theta} \ln \sum_m \left(\exp\left(-a_m d_m(i,j) - b_m\right)\right) + \left(1 - \sigma\right)\beta' \ln \mathbf{C}(i,j) + \delta(i) + \delta(j) \tag{7}$$

- Estimate a_m and b_m using bilateral trade share eq (6), θ using gravity eq (7). Assume $\sigma = 9$.

- Given σ and the symmetry of $T(i,j)$, there is a simpler way to recover $T(i,j)$ from $X(i,j)$

AA (2014): Trade cost estimates

Table II: Estimated mode-specific relative cost of travel

	All CFS Areas				Only MSAs			
Geographic trade costs	Road	Rail	Water	Air	Road	Rail	Water	Air
Variable cost	0.5636***	0.1434***	0.0779***	0.0026	0.4542***	0.1156***	0.0628***	0.0021
	(0.0120)	(0.0063)	(0.0199)	(0.0085)	(0.0233)	(0.0210)	(0.0265)	(0.0176)
Fixed cost	0	0.4219***	0.5407***	0.5734***	0	0.34***	0.4358***	0.4621***
	N/A	(0.0097)	(0.0236)	(0.0129)	N/A	(0.0235)	(0.0375)	(0.0264)
Estimated shape parameter (θ)	14.225***				17.6509***			
	(0.3375)				(1.4194)			
Non-geographic trade costs								
Similar ethnicity	-0.0888***				-0.0803***			
	(0.0153)				(0.0275)			
Similar language	0.063***				0.0286			
	(0.0223)				(0.0359)			
Similar migrants	-0.0191				-0.0135			
	(0.0119)				(0.0203)			
Same state	-0.2984***				-0.3104***			
	(0.0101)				(0.0176)			
R-squared (within)	0.4487				0.4113			
R-squared (overall)	0.6456				0.5995			
Observations with positive bilateral flows	9601	9601	9601	9601	3266	3266	3266	3266
Observations with positive mode-specific bilateral flows	9311	1499	78	1016	3144	340	26	471

AA (2014): Recovering Productivities and Amenities

- Notice that

$$
\begin{aligned}
A(i)^{\sigma-1} &= \left[\frac{W^{1-\sigma}}{w(i)^{\sigma} L(i)} \int_S T(i,s)^{1-\sigma} u(s)^{\sigma-1} w(s)^{\sigma} L(s)\, ds\right]^{-1} \\
u(i)^{\sigma-1} &= \left[\frac{W^{1-\sigma}}{w(i)^{1-\sigma}} \int_S T(s,i)^{1-\sigma} A(s)^{\sigma-1} w(s)^{1-\sigma}\, ds\right]^{-1}
\end{aligned}
\tag{8}
$$

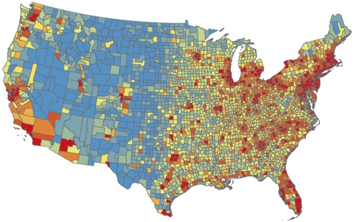
- $T(i,s)$ have been recovered using the gravity equation

- Knowing σ and $(w(i), L(i))_{i\in S}$, $\left(A(i)^{\sigma-1}, u(i)^{\sigma-1}\right)_{i\in S}$ can be recovered by solving a recursive system

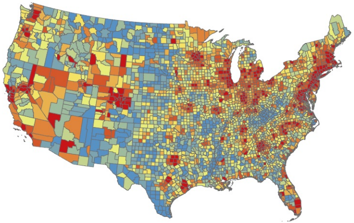
- We need (α,β) to recover $\left(\bar{A}(i), \bar{u}(i)\right)$ from $\left(A(i)^{\sigma-1}, u(i)^{\sigma-1}\right)$: IVs are required for identifying (α,β)

AA (2014): Population and wages – data

Figure 12: United States population density and wages in 2000



Population density

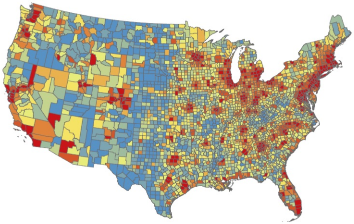


Wages

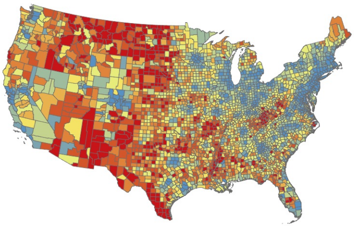
Notes: This figure shows the relative population density (top) and wages (bottom) within the United States in the year 2000 by decile. The data are reported at the county level; red (blue) indicate higher (lower) deciles. (Source: MPC (2011a)).

AA (2014): Estimated composite productivity and amenity

Figure 13: Estimated composite productivity and amenity



Composite productivity

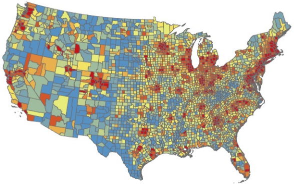


Composite amenity

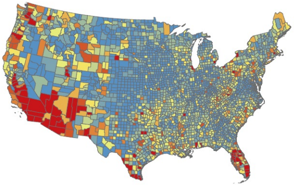
Notes: This figure shows the estimated composite productivity (top) and amenity (bottom) by decile. The data are reported at the county level; red (blue) indicate higher (lower) deciles.

AA (2014): Estimated exogenous productivity and amenity

Figure 14: Estimated exogenous productivity and amenity



Exogenous productivity

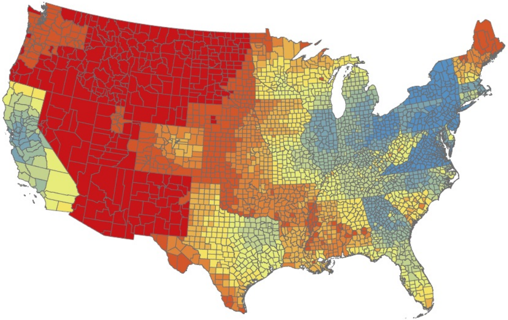


Exogenous amenity

Notes: This figure shows the estimated exogenous productivity $\bar{A}$ (top) and amenity $\bar{u}$ (bottom) by decile assuming $\alpha = 0.1$ and $\beta = -0.3$. The data are reported at the county level; red (blue) indicate higher (lower) deciles.

AA (2014): Estimated price index

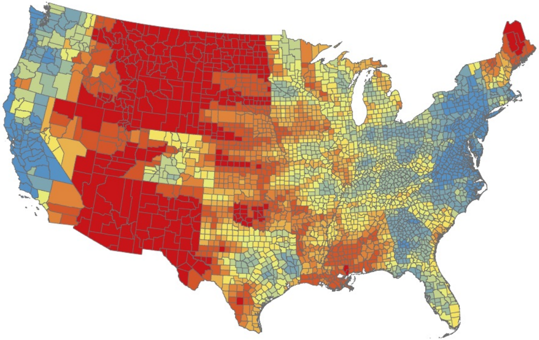
Figure 15: Estimated price index



Notes: This figure shows the estimated price index by decile. The data are reported at the county level; red (blue) indicate higher (lower) deciles.

AA (2014): Estimated increase in price index from removing IHS

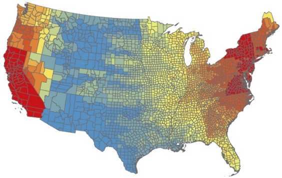
Figure 17: Estimated increase in the price index from removing the Interstate Highway System



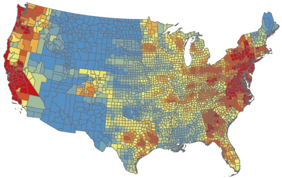
Notes: This figure depicts the estimated increase in the price index (by decile) across space from removing the Interstate Highway System (IHS), holding wages and productivities constant at the 2000 U.S. levels. Red (blue) indicate higher (lower) deciles (e.g. the removal of the IHS disproportionately increased the economic remoteness in red regions).

AA (2014): Estimated change in population from removing IHS

Figure 18: Estimated change in the population from removing the Interstate Highway System



$\alpha = 0$, $\beta = 0$



$\alpha = 0.1$, $\beta = -0.3$

Notes: This figure shows the estimated change in population (in deciles) from the removal of the Interstate Highway System (IHS). The top map reports the estimated population changes when there are no spillovers (i.e. $\alpha = \beta = 0$), while the bottom map reports the estimated population changes when spillovers are chosen to approximately match those from the literature (i.e. $\alpha = 0.1$ and $\beta = -0.3$). Red (blue) indicates higher (lower) deciles (e.g. the removal of the IHS increased the relative population in red areas).

Ahlfeldt, Redding, Sturm, and Wolf (2015)

- Economic activity is highly unevenly distributed across space.
- Economic geography literature is interested to separate locational fundamentals, from agglomeration (e.g increasing returns) and dispersion forces (e.g land scarcity, commuting costs)
- Suppose one regresses productivity, wages or employment on the density of economic activity. Empirical challenge:
  - Third variables can affect both productivity and wages and density
  - Difficult to find instruments that only affect productivity or wages through density

Ahlfeldt, Redding, Sturm, and Wolf (2015)

- Develop a quantitative model of city structure to determine agglomeration and dispersion forces
- Combine the model with data for thousands of city blocks in Berlin in 1936, 1986 and 2006 on:
  - Land prices
  - Workplace employment
  - Residence employment
- Use the division of Berlin in the aftermath of the Second World War and its reunification in 1989 as a source of exogenous variation in the surrounding concentration of economic activity

Historical Background

- A protocol signed during the Second World War organized Germany into American, British, French and Soviet occupation zones
- Although 200km within the Soviet zone, Berlin was to be jointly occupied and organized into four occupation red:
  - Boundaries followed pre-war district boundaries, with the same East-West orientation as the occupation zones, and created sectors of roughly equal pre-war population (prior to French sector which was created from part of the British sector)
  - Protocol envisioned a joint city administration ("Kommandatura")
- Following the onset of the Cold War
  - East and West Germany founded as separate states and separate city governments created in East and West Berlin in 1949
  - The adoption of Soviet-style policies of command and control in East Berlin limited economic interactions with West Berlin
  - To stop civilians leaving for West Germany, the East German authorities constructed the Berlin Wall in 1961

Historical Background: Berlin

ARSW (2015): Model

- Reservation level of utility ($\bar{U}$) for living outside the city.
- The city consists of a set of discrete blocks indexed by i
- Floor space can be used for residential or commercial use
- Workers choose a block of residence, a block of employment, and consumption of the final good
- Firms choose a block of production and inputs of labor and floor space
- Single final good (which is also the numeraire)

ARSW (2015): Model – Workers

- Utility for worker ω residing in block i and working in block j:

$$U_{ij\omega} = \frac{B_i z_{ij\omega}}{d_{ij}} \left(\frac{c_{ij}}{\beta}\right)^{\beta} \left(\frac{\ell_{ij}}{1-\beta}\right)^{1-\beta}, \qquad 0 < \beta < 1,$$

  - Consumption of the final good (c_{ij}), numeraire ($p_i = 1$)
  - Residential floor space (ℓ_{ij}), price Q_i
  - Residential amenity B_i
  - Commuting costs d_{ij}
  - Idiosyncratic shock $z_{ij\omega}$
  - Wage w_j

- Indirect utility

$$U_{ij\omega} = \frac{z_{ij\omega} B_i w_j Q_i^{\beta-1}}{d_{ij}},$$

- The idiosyncratic shock to worker productivity is drawn from a Fréchet distribution:

$$F(z_{ij\omega}) = e^{-T_i E_j z_{ij\omega}^{-\epsilon}}, \qquad T_i, E_j > 0, \ \epsilon > 1,$$

ARSW (2015): Model – Commuting Decisions

- Probability worker chooses to live in block i and work in block j is:

$$\pi_{ij} = \frac{T_i E_j \left(d_{ij} Q_i^{1-\beta}\right)^{-\epsilon} \left(B_i w_j\right)^{\epsilon}}{\sum_{r=1}^{S} \sum_{s=1}^{S} T_r E_s \left(d_{rs} Q_r^{1-\beta}\right)^{-\epsilon} \left(B_r w_s\right)^{\epsilon}} \equiv \frac{\Phi_{ij}}{\Phi}. \tag{9}$$

- Residential and workplace choice probabilities

$$\pi_{Ri} = \sum_{j=1}^{S} \pi_{ij} = \frac{\sum_{j=1}^{S} \Phi_{ij}}{\Phi}, \qquad \pi_{Mj} = \sum_{i=1}^{S} \pi_{ij} = \frac{\sum_{i=1}^{S} \Phi_{ij}}{\Phi}. \tag{10}$$

- Conditional on living in block i, the probability that a worker commutes to block j follows a gravity equation:

$$\pi_{ij|i} = \frac{E_j \left(w_j / d_{ij}\right)^{\epsilon}}{\sum_{s=1}^{S} E_s \left(w_s / d_{is}\right)^{\epsilon}},$$

ARSW (2015): Model – Commuting

- Employment in block j equals the sum across all blocks i of people living in residence times the probability of commuting from i to j:

$$H_{Mj} = \sum_{i=1}^{S} \frac{E_j \left(w_j / d_{ij} \right)^{\epsilon}}{\sum_{s=1}^{S} E_s \left(w_s / d_{is} \right)^{\epsilon}} H_{Ri}, \qquad d_{ij} = e^{\kappa \tau_{ij}}. \tag{11}$$

- This equation be useful to determine equilibrium wages.

ARSW (2015): Model – Consumers

- Consumers decide before idiosyncratic shocks $z_{ij\omega}$ are realized whether to move to the city or not.

- Population mobility implies that expected utility equals reservation utility level:

$$\mathbb{E}[U] = \gamma \left[\sum_{r=1}^{S} \sum_{s=1}^{S} T_r E_s \left(d_{rs} Q_r^{1-\beta} \right)^{-\epsilon} (B_r w_s)^{\epsilon} \right]^{1/\epsilon} = \bar{U}, \tag{12}$$

- Residential amenities are influenced by both fundamentals (b_i) and spillovers (Ω_i)

$$B_i = b_i \Omega_i^{\eta}, \qquad \Omega_i \equiv \left[\sum_{s=1}^{S} e^{-\rho \tau_{is}} \left(\frac{H_{Rs}}{K_s} \right) \right].$$

ARSW (2015): Model – Production

- A single final good (numeraire) is produced under conditions of perfect competition, constant returns to scale and zero trade costs with a larger economy:

$$y_j = A_j \left(H_{Mj}\right)^{\alpha} \left(L_{Mj}\right)^{1-\alpha}, \qquad 0 < \alpha < 1,$$

- H_{Mj} is workplace employment

- L_{Mj} is measure of floor space used commercially

- Productivity (A_j) depends on fundamentals (a_j) and spillovers (Υ_j):

$$A_j = a_j \Upsilon_j^{\lambda}, \qquad \Upsilon_j \equiv \left[\sum_{s=1}^{S} e^{-\delta \tau_{is}} \left(\frac{H_{Ms}}{K_s}\right)\right],$$

- δ is the rate of decay of spillovers

- λ captures the relative importance of spillovers

ARSW (2015): Land prices

- The share of floor space used commercially:

$$\begin{aligned} \theta_i = 1 \quad &\text{if} \qquad q_i > \xi_i Q_i, \\ \theta_i \in [0,1] \quad &\text{if} \qquad q_i = \xi_i Q_i, \\ \theta_i = 0 \quad &\text{if} \qquad q_i < \xi_i Q_i. \end{aligned} \tag{13}$$

- $\xi_i \geq 1$ represents 1 plus the tax equivalent of land use regulations

- Assume observed land price is maximum of commercial and residential price: $\mathbb{Q}_i = \max\{q_i, Q_i\}$

ARSW (2015): Model – Production

- Firms choose a block of production, effective employment and commercial land use to maximize profits taking as given goods and factor prices, productivity and the locations of other firms/workers

- Zero profits imply for the price of commercial land q_j:

$$q_j = (1-\alpha)\left(\frac{\alpha}{w_j}\right)^{\frac{\alpha}{1-\alpha}} A_j^{\frac{1}{1-\alpha}}. \tag{14}$$

ARSW (2015): Model – Land Market Clearing

- Utility max implies demand for residential floor space:

$$(1-\theta_i)L_i = \frac{(1-\beta)\mathbb{E}(w \mid i)}{Q_i} H_{Ri}. \tag{15}$$

- Profit max implies demand for commercial floor space:

$$\theta_i L_i = H_{Mi}\left(\frac{(1-\alpha)A_i}{q_i}\right)^{\frac{1}{\alpha}}. \tag{16}$$

- Floor space L supplied by a competitive construction sector using geographic land K and capital M as inputs

$$L_i = \varphi_i K_i^{1-\mu}, \qquad \varphi_i = M_i^{\mu},$$

- Density of development (φ_i) from land market clearing:

$$\varphi_i = \frac{L_i}{K_i^{1-\mu}} = \frac{(1-\theta_i)L_i + \theta_i L_i}{K_i^{1-\mu}}$$

ARSW (2015): Equilibrium with exogenous loc. charact.

- **Proposition 1:** Given the model's parameters $[\alpha, \beta, \mu, \epsilon, \kappa]$, the reservation utility $\bar{U}$, and vectors of exogenous location characteristics $[T, E, A, B, \phi, K, \xi, \tau]$, there exists a unique general equilibrium vector $[\pi_M, \pi_R, H, Q, q, w, \theta]$, where H denotes total city population.

- These seven components are determined by the system of seven equations: Commercial land market clearing (16), Residential land market clearing (15), zero profits (14), no arbitrage between alternative uses of land (13), the residential choice probability (π_{Ri} in (10)), the workplace choice probability (π_{Mi} in (10)), and the population mobility (12).

ARSW (2015): Overview of remainder of paper

- Next the authors present a reduced-form empirical exercise on land price and population changes
- Then the authors estimate simple version of the model without agglomeration effects. They show that when doing counterfactuals (i.e. establishing / removing a wall, with it's effects on where you can live, work) the counterfactual predictions are different from the data
- They then do a structural estimation with agglomeration effects. We care here about two things: (a) the measurement of the agglomeration effects per se, (b) the counterfactual predictions from that model are much closer to the data (note: multiple equilbria are here an issue for conducting the counterfactual)

ARSW (2015): Data

- Data on land prices, workplace employment, residence employment and bilateral travel times

- Data for Greater Berlin in 1936 and 2006

- Data for West Berlin in 1986

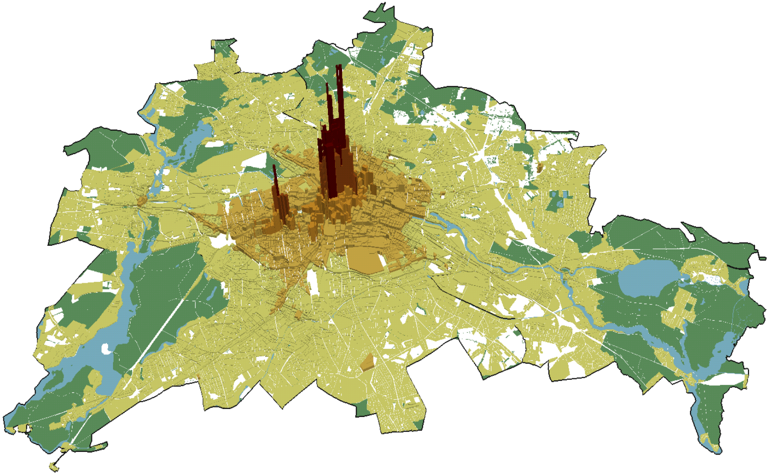
- Data at the following levels of spatial aggregation:
  - Pre-war districts ("Bezirke"), 20 in Greater Berlin, 12 in West Berlin
  - Statistical areas ("Gebiete"), around 90 in West Berlin
  - Statistical blocks, around 9,000 in West Berlin

- Land prices: official assessed land value of a representative undeveloped property or the fair market value of a developed property if it were not developed

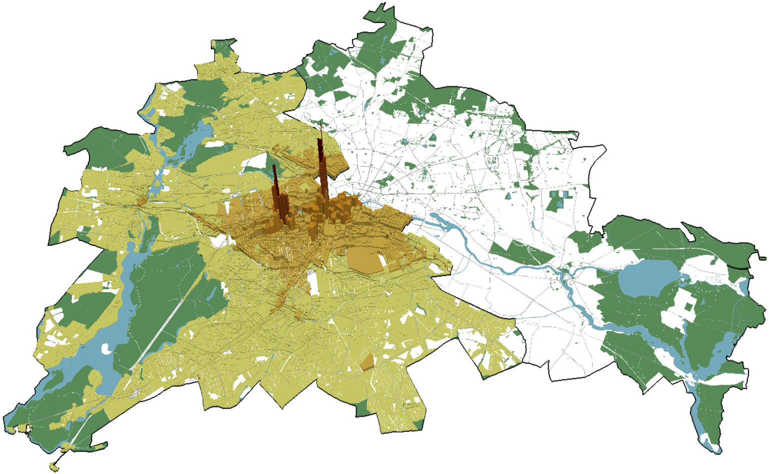
- Geographical Information Systems (GIS) data on:
  - land area, land use, building density, proximity to U-Bahn (underground) and S-Bahn (suburban) stations, schools, parks, lakes, canals and rivers, Second World War destruction, location of government buildings and urban regeneration programs

ARSW (2015): Land prices Berlin 1936

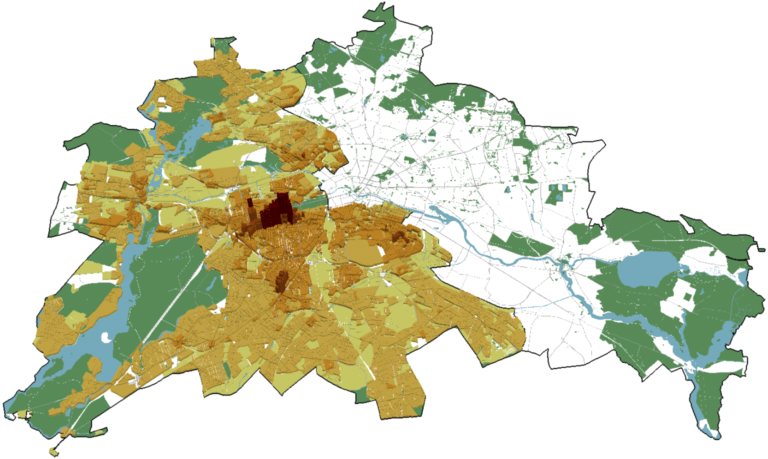
Land prices are normalized to have a mean of 1 in each year.



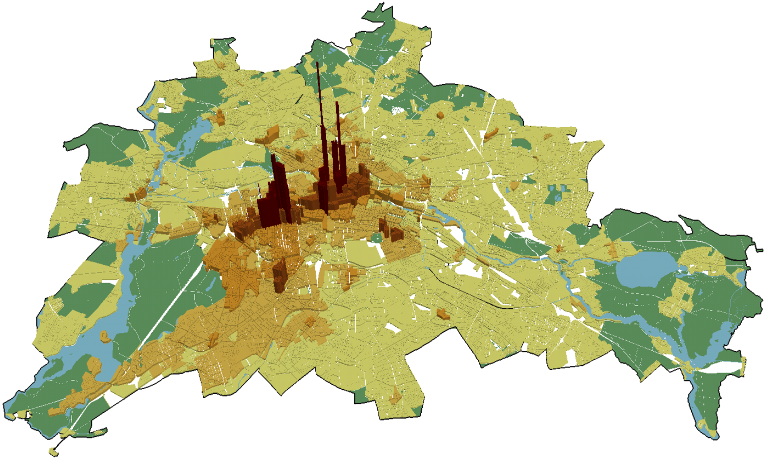
ARSW (2015): Land prices West Berlin 1936



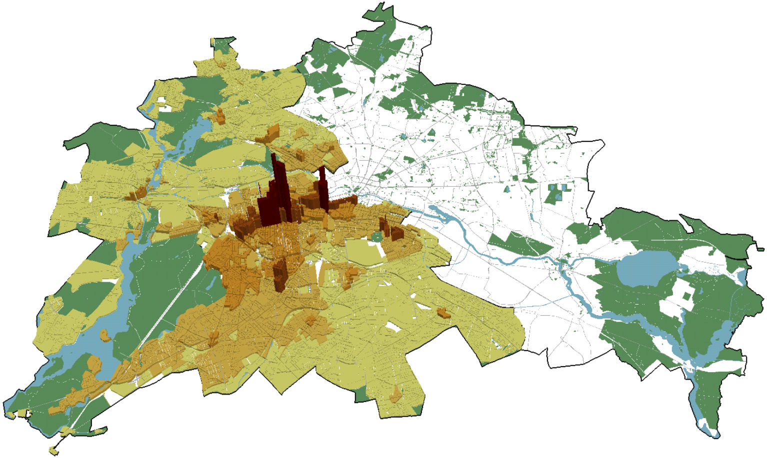
ARSW (2015): Land prices West Berlin 1986



ARSW (2015): Land prices Berlin 2006



ARSW (2015): Land prices West Berlin 2006



ARSW (2015): Difference-in-Differences Specification

- Estimate difference in difference specification for division and reunification separately (for areas in West Berlin):

$$\triangle \ln Q_i = \psi + \sum_{k=1}^{K} I_{ik}\beta_k + \ln X_i\zeta + \chi_i, \tag{17}$$

- I_{ik} is a $(0,1)$ dummy which equals one if block i lies within distance grid cell k from the pre-war CBD and zero otherwise

- Observable block characteristics (X_i): Land area, land use, distance to nearest U-Bahn station, S-Bahn station, school, lake, river or canal, and park, war destruction, government buildings and urban regeneration programs

Division and Pre-War CBD

ARSW (2015): D.i.D. West Berlin 1936-86

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Δ ln Q	Δ ln Q	Δ ln Q	Δ ln Q	Δ ln Q	Δ ln EmpR	Δ ln EmpR	Δ ln EmpW	Δ ln EmpW
CBD 1	-0.800***	-0.567***	-0.524***	-0.503***	-0.565***	-1.332***	-0.975***	-0.691*	-0.639*
	(0.071)	(0.071)	(0.071)	(0.071)	(0.077)	(0.383)	(0.311)	(0.408)	(0.338)
CBD 2	-0.655***	-0.422***	-0.392***	-0.360***	-0.400***	-0.715**	-0.361	-1.253***	-1.367***
	(0.042)	(0.047)	(0.046)	(0.043)	(0.050)	(0.299)	(0.280)	(0.293)	(0.243)
CBD 3	-0.543***	-0.306***	-0.294***	-0.258***	-0.247***	-0.911***	-0.460**	-0.341	-0.471**
	(0.034)	(0.039)	(0.037)	(0.032)	(0.034)	(0.239)	(0.206)	(0.241)	(0.190)
CBD 4	-0.436***	-0.207***	-0.193***	-0.166***	-0.176***	-0.356**	-0.259	-0.512***	-0.521***
	(0.022)	(0.033)	(0.033)	(0.030)	(0.026)	(0.145)	(0.159)	(0.199)	(0.169)
CBD 5	-0.353***	-0.139***	-0.123***	-0.098***	-0.100***	-0.301***	-0.143	-0.436***	-0.340***
	(0.016)	(0.024)	(0.024)	(0.023)	(0.020)	(0.110)	(0.113)	(0.151)	(0.124)
CBD 6	-0.291***	-0.125***	-0.094***	-0.077***	-0.090***	-0.360***	-0.135	-0.280**	-0.142
	(0.018)	(0.019)	(0.017)	(0.016)	(0.016)	(0.100)	(0.089)	(0.130)	(0.116)
Inner Boundary 1-6			Yes	Yes	Yes		Yes		Yes
Outer Boundary 1-6			Yes	Yes	Yes		Yes		Yes
Kudamm 1-6				Yes	Yes		Yes		Yes
Block Characteristics					Yes		Yes		Yes
District Fixed Effects		Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	6260	6260	6260	6260	6260	5978	5978	2844	2844
R-squared	0.26	0.51	0.63	0.65	0.71	0.19	0.43	0.12	0.33

Note: Q denotes the price of floor space. EmpR denotes employment by residence. EmpW denotes employment by workplace. CBD1-CBD6 are six 500m distance grid cells for distance from the pre-war CBD. Inner Boundary 1-6 are six 500m grid cells for distance to the Inner Boundary between East and West Berlin. Outer Boundary 1-6 are six 500m grid cells for distance to the outer boundary between West Berlin and East Germany. Kudamm 1-6 are six 500m grid cells for distance to Breitscheid Platz on the Kurfürstendamm. The coefficients on the other distance grid cells are reported in Table A2 of the web appendix. Block characteristics include the logarithm of distance to schools, parks and water, the land area of the block, the share of the block's built-up area destroyed during the Second World War, indicators for residential, commercial and industrial land use, and indicators for whether a block includes a government building and urban regeneration policies post-reunification. Heteroscedasticity and Autocorrelation Consistent (HAC) standard errors in parentheses (Conley 1999). * significant at 10%; ** significant at 5%; *** significant at 1%.

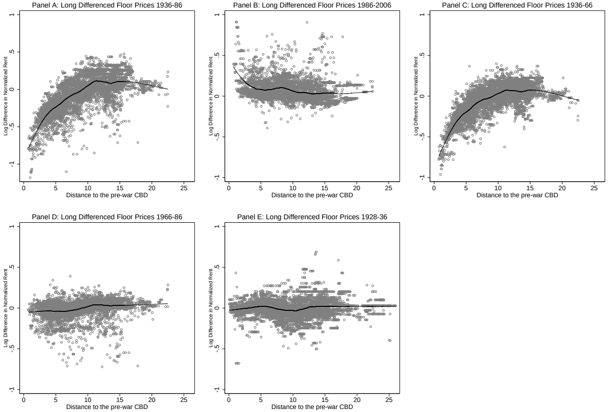
ARSW (2015): D.i.D. West Berlin 1986-2006

	(1) $\Delta \ln Q$	(2) $\Delta \ln Q$	(3) $\Delta \ln Q$	(4) $\Delta \ln Q$	(5) $\Delta \ln Q$	(6) $\Delta \ln EmpR$	(7) $\Delta \ln EmpR$	(8) $\Delta \ln EmpW$	(9) $\Delta \ln EmpW$
CBD 1	0.398***	0.408***	0.368***	0.369***	0.281***	1.079***	1.025***	1.574***	1.249**
	(0.105)	(0.090)	(0.083)	(0.081)	(0.088)	(0.307)	(0.297)	(0.479)	(0.517)
CBD 2	0.290***	0.289***	0.257***	0.258***	0.191**	0.589*	0.538*	0.684**	0.457
	(0.111)	(0.096)	(0.090)	(0.088)	(0.087)	(0.315)	(0.299)	(0.326)	(0.334)
CBD 3	0.122***	0.120***	0.110***	0.115***	0.063**	0.340*	0.305*	0.326	0.158
	(0.037)	(0.033)	(0.032)	(0.032)	(0.028)	(0.180)	(0.158)	(0.216)	(0.239)
CBD 4	0.033***	0.031	0.030	0.034	0.017	0.110	0.034	0.336**	0.261
	(0.013)	(0.023)	(0.022)	(0.021)	(0.020)	(0.068)	(0.066)	(0.161)	(0.185)
CBD 5	0.025***	0.018	0.020	0.020	0.015	-0.012	-0.056	0.114	0.066
	(0.010)	(0.015)	(0.014)	(0.014)	(0.013)	(0.056)	(0.057)	(0.118)	(0.131)
CBD 6	0.019**	-0.000	-0.000	-0.003	0.005	0.060	0.053	0.049	0.110
	(0.009)	(0.009)	(0.012)	(0.012)	(0.011)	(0.039)	(0.041)	(0.095)	(0.098)
Inner Boundary 1-6			Yes	Yes	Yes		Yes		Yes
Outer Boundary 1-6			Yes	Yes	Yes		Yes		Yes
Kudamm 1-6				Yes	Yes		Yes		Yes
Block Characteristics					Yes		Yes		Yes
District Fixed Effects		Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	7050	7050	7050	7050	7050	6718	6718	5602	5602
R-squared	0.08	0.32	0.34	0.35	0.43	0.04	0.07	0.03	0.06

Note: Q denotes the price of floor space. EmpR denotes employment by residence. EmpW denotes employment by workplace. CBD1-CBD6 are six 500m distance grid cells for distance from the pre-war CBD. Inner Boundary 1-6 are six 500m grid cells for distance to the Inner Boundary between East and West Berlin. Outer Boundary 1-6 are six 500m grid cells for distance to the outer boundary between West Berlin and East Germany. Kudamm 1-6 are six 500m grid cells for distance to Breitscheid Platz on the Kurfürstendamm. The coefficients on the other distance grid cells are reported in Table A4 of the web appendix. Block characteristics include the logarithm of distance to schools, parks and water, the land area of the block, the share of the block's built-up area destroyed during the Second World War, indicators for residential, commercial and industrial land use, and indicators for whether a block includes a government building and urban regeneration policies post-reunification. Heteroscedasticity and Autocorrelation Consistent (HAC) standard errors in parentheses (Conley 1999).* significant at 10%; ** significant at 5%; *** significant at 1%.

ARSW (2015): Land prices over time

Placebo test: Land price change from 1928 to 1936. Of the change from 1936 - 1986, most of the effect is already in place by 1966.



Note: Log floor prices are normalized to have a mean of zero in each year before taking the long difference. Solid lines are fitted values from locally-weighted linear least squares regressions.

ARSW (2015): Gravity

- From (9) – gravity equation for commuting from residence i to workplace j:

$$\ln \pi_{ij} = -\nu\tau_{ij} + \vartheta_i + \varsigma_j + e_{ij},$$

- where τ_{ij} is travel time in minutes and $\nu = \epsilon\kappa$
- ϑ_i are residence fixed effects
- ς_j are workplace fixed effects

ARSW (2015): Gravity Results

	(1) ln Bilateral Commuting Probability 2008	(2) ln Bilateral Commuting Probability 2008	(3) ln Bilateral Commuting Probability 2008	(4) ln Bilateral Commuting Probability 2008
Travel Time ($-\kappa\varepsilon$)	-0.0697***	-0.0702***	-0.0771***	-0.0706***
	(0.0056)	(0.0034)	(0.0025)	(0.0026)
Estimation	OLS	OLS	Poisson PML	Gamma PML
More than 10 Commuters		Yes	Yes	Yes
Fixed Effects	Yes	Yes	Yes	Yes
Observations	144	122	122	122
R-squared	0.8261	0.9059	-	-

Note: Gravity equation estimates based on representative micro survey data on commuting for Greater Berlin for 2008. Observations are bilateral pairs of 12 workplace and residence districts (post 2001 Bezirke boundaries). Travel time is measured in minutes. Fixed effects are workplace district fixed effects and residence district fixed effects. The specifications labelled more than 10 commuters restrict attention to bilateral pairs with 10 or more commuters. Poisson PML is Poisson Pseudo Maximum Likelihood estimator. Gamma PML is Gamma Pseudo Maximum Likelihood Estimator. Standard errors in parentheses are heteroscedasticity robust. * significant at 10%; ** significant at 5%; *** significant at 1%.

ARSW (2015): Backing out amenities and productivities

- Using estimated ν, and data on residence and workplace employment, one can solve for transformed wages $\omega_j = w_j^{\epsilon}$ from equation (11)

- Recover overall productivity A_j from equation (14) (zero profits):

$$\ln\left(\frac{A_{it}}{\bar{A}_t}\right) = (1-\alpha)\ln\left(\frac{\mathbb{Q}_{it}}{\bar{\mathbb{Q}}_t}\right) + \frac{\alpha}{\epsilon}\ln\left(\frac{\omega_{it}}{\bar{\omega}_t}\right)$$

  where $\bar{A}_t = \exp(1/S\sum_{s=1}^{S}\ln A_{st})$ (geometric mean)

- High floor prices and wages require high final good productivity for zero profits to be satisfied

ARSW (2015): Backing out amenities and productivities

- Recover amenities B_i from equation (10) (residential choice probabilities):

$$\ln\left(\frac{B_{it}}{\bar{B}_t}\right) = \frac{1}{\epsilon}\ln\left(\frac{H_{Rit}}{\bar{H}_{Rt}}\right) + (1-\beta)\ln\left(\frac{\mathbb{Q}_{it}}{\bar{\mathbb{Q}}_t}\right) + \frac{1}{\epsilon}\ln\left(\frac{W_{it}}{\bar{W}_t}\right)$$

  where $W_{it} = \sum_{s=1}^{S} \omega_{st}/e^{\nu\tau_{ist}}$ and variables with an upper bar denote that variable's geometric mean.

- High floor prices and high residence employment must be explained either by high wage commuting access or high amenities.

- (So far not making assumptions about the relative importance of production and residential externalities versus fundamentals)

ARSW (2015): Backing out amenities and productivities

- Fix other parameters (α, ...) to values from the literature.
- Estimate ϵ from dispersion of log adjusted wages backed out in the model and log wages in the data for a selected year.
- Next table shows in columns 1-4 how the amenities and productivities changed over time.
- Also, simulate the impact of the division on West Berlin, holding productivity, amenities constant at their 1936 values. Column 5 shows that the models prediction are off from the diff and diff results (-.4 versus -.8 log points) Similarly for column 6 (re-unification; 0 versus -.4 log points).

ARSW (2015): Changes in Amenities and Productivity

TABLE IV

PRODUCTIVITY, AMENITIES, AND COUNTERFACTUAL FLOOR PRICES[a]

	(1) $\Delta \ln A$ 1936–1986	(2) $\Delta \ln B$ 1936–1986	(3) $\Delta \ln A$ 1986–2006	(4) $\Delta \ln B$ 1986–2006	(5) $\Delta \ln QC$ 1936–1986	(6) $\Delta \ln QC$ 1986–2006
CBD 1	-0.207^{***}	-0.347^{***}	0.261^{***}	0.203^{***}	-0.408^{***}	-0.010
	(0.049)	(0.070)	(0.073)	(0.054)	(0.038)	(0.020)
CBD 2	-0.260^{***}	-0.242^{***}	0.144^{**}	0.109^{*}	-0.348^{***}	0.079^{**}
	(0.032)	(0.053)	(0.056)	(0.058)	(0.017)	(0.036)
CBD 3	-0.138^{***}	-0.262^{***}	0.077^{***}	0.059^{**}	-0.353^{***}	0.036
	(0.021)	(0.037)	(0.024)	(0.026)	(0.022)	(0.031)
CBD 4	-0.131^{***}	-0.154^{***}	0.057^{***}	0.010	-0.378^{***}	0.093^{***}
	(0.016)	(0.023)	(0.015)	(0.008)	(0.021)	(0.026)
CBD 5	-0.095^{***}	-0.126^{***}	0.028^{**}	-0.014^{*}	-0.380^{***}	0.115^{***}
	(0.014)	(0.013)	(0.013)	(0.007)	(0.022)	(0.033)
CBD 6	-0.061^{***}	-0.117^{***}	0.023^{**}	0.001	-0.354^{***}	0.066^{***}
	(0.015)	(0.015)	(0.010)	(0.005)	(0.018)	(0.023)
Counterfactuals					Yes	Yes
Agglomeration Effects					No	No
Observations	2,844	5,978	5,602	6,718	6,260	7,050
R^2	0.09	0.06	0.02	0.03	0.07	0.03

ARSW (2015): Structural Estimation

- Next, the authors use the exogenous variation from Berlin's division and reunification to structurally estimate the model's parameters including the agglomeration forces.
- From the previous equations and the definition of productivities / amenities it follows:

$$\Delta \ln \left(\frac{a_{it}}{\bar{a}_t}\right) = (1-\alpha)\Delta \ln \left(\frac{\mathbb{Q}_{it}}{\bar{\mathbb{Q}}_t}\right) + \frac{\alpha}{\epsilon}\Delta \ln \left(\frac{\omega_{it}}{\bar{\omega}_t}\right) - \lambda \Delta \ln \left(\frac{\Upsilon_{it}}{\bar{\Upsilon}_t}\right)$$

$$\Delta \ln \left(\frac{b_{it}}{\bar{b}_t}\right) = \frac{1}{\epsilon}\Delta \ln \left(\frac{H_{Rit}}{\bar{H}_{Rt}}\right) + (1-\beta)\Delta \ln \left(\frac{\mathbb{Q}_{it}}{\bar{\mathbb{Q}}_t}\right) + \frac{1}{\epsilon}\Delta \ln \left(\frac{W_{it}}{\bar{W}_t}\right) - \eta \Delta \ln \left(\frac{\Omega_{it}}{\bar{\Omega}_t}\right)$$

- Production externalities Υ_{it} depend on travel-time weighted sum of observed workplace employment densities
- Residential externalities Ω_{it} depend on travel-time weighted sum of observed residence employment densities
- Adjusted fundamentals relative to geometric mean are structural residuals

ARSW (2015): Parameters

Assumed Parameter		Source	Value
Residential land	$1-\beta$	Morris-Davis (2008)	0.25
Commercial land	$1-\alpha$	Valentinyi-Herrendorf (2008)	0.20
Fréchet Scale	T	(normalization)	1
Expected Utility	$\bar{u}$	(normalization)	1000

Estimated Parameter	
Production externalities elasticity	λ
Production externalities decay	δ
Residential externalities elasticity	η
Residential externalities decay	ρ
Commuting semi-elasticity	$\nu = \epsilon\kappa$
Commuting heterogeneity	ϵ

ARSW (2015): Moment Conditions

- Changes in adjusted fundamentals uncorrelated with exogenous change in surrounding economic activity from division/reunification

$$\mathbb{E}\left[\mathbb{I}_k \times \triangle \ln\left(a_{it}/\bar{a}_t\right)\right] = 0, \qquad k \in \{1, \ldots, K_{\mathbb{I}}\},$$

$$\mathbb{E}\left[\mathbb{I}_k \times \triangle \ln\left(b_{it}/\bar{b}_t\right)\right] = 0, \qquad k \in \{1, \ldots, K_{\mathbb{I}}\}.$$

where $\mathbb{I}_k$ are indicators for distance grid cells from pre-war CBD

- Other moments are fraction of workers that commute less than 30 minutes and wage dispersion

$$\mathbb{E}\left[\vartheta H_{Mj} - \sum_{i \in \aleph_j}^{S} \frac{\omega_j / e^{\nu \tau_{ij}}}{\sum_{s=1}^{S} \omega_s / e^{\nu \tau_{is}}} H_{Ri}\right] = 0,$$

$$\mathbb{E}\left[(1/\epsilon)^2 \ln\left(\omega_j\right)^2 - \sigma^2_{\ln w_i}\right] = 0,$$

ARSW (2015): Estimated Parameters

TABLE V

GENERALIZED METHOD OF MOMENTS (GMM) ESTIMATION RESULTS[a]

	(1) Division Efficient GMM	(2) Reunification Efficient GMM	(3) Division and Reunification Efficient GMM
Commuting Travel Time Elasticity ($\kappa\varepsilon$)	0.0951***	0.1011***	0.0987***
	(0.0016)	(0.0016)	(0.0016)
Commuting Heterogeneity (ε)	6.6190***	6.7620***	6.6941***
	(0.0939)	(0.1005)	(0.0934)
Productivity Elasticity (λ)	0.0793***	0.0496***	0.0710***
	(0.0064)	(0.0079)	(0.0054)
Productivity Decay (δ)	0.3585***	0.9246***	0.3617***
	(0.1030)	(0.3525)	(0.0782)
Residential Elasticity (η)	0.1548***	0.0757**	0.1553***
	(0.0092)	(0.0313)	(0.0083)
Residential Decay (ρ)	0.9094***	0.5531	0.7595***
	(0.2968)	(0.3979)	(0.1741)

ARSW (2015): Localized Externalities

TABLE VI

EXTERNALITIES AND COMMUTING COSTS[a]

	(1) Production Externalities $(1 \times e^{-\delta\tau})$	(2) Residential Externalities $(1 \times e^{-\rho\tau})$	(3) Utility After Commuting $(1 \times e^{-\kappa\tau})$
0 minutes	1.000	1.000	1.000
1 minute	0.696	0.468	0.985
2 minutes	0.485	0.219	0.971
3 minutes	0.338	0.102	0.957
5 minutes	0.164	0.022	0.929
7 minutes	0.079	0.005	0.902
10 minutes	0.027	0.001	0.863
15 minutes	0.004	0.000	0.802
20 minutes	0.001	0.000	0.745
30 minutes	0.000	0.000	0.642

Counterfactuals

TABLE VII

COUNTERFACTUALS[a]

	(1) $\Delta \ln QC$ 1936–1986	(2) $\Delta \ln QC$ 1936–1986	(3) $\Delta \ln QC$ 1936–1986	(4) $\Delta \ln QC$ 1936–1986	(5) $\Delta \ln QC$ 1986–2006	(6) $\Delta \ln QC$ 1986–2006	(7) $\Delta \ln QC$ 1986–2006
CBD 1	−0.836***	−0.613***	−0.467***	−0.821***	0.363***	1.160***	0.392***
	(0.052)	(0.032)	(0.060)	(0.051)	(0.041)	(0.052)	(0.043)
CBD 2	−0.560***	−0.397***	−0.364***	−0.624***	0.239***	0.779***	0.244***
	(0.034)	(0.025)	(0.019)	(0.029)	(0.028)	(0.044)	(0.027)
CBD 3	−0.455***	−0.312***	−0.336***	−0.530***	0.163***	0.594***	0.179***
	(0.036)	(0.030)	(0.030)	(0.036)	(0.031)	(0.045)	(0.031)
CBD 4	−0.423***	−0.284***	−0.340***	−0.517***	0.140***	0.445***	0.143***
	(0.026)	(0.019)	(0.022)	(0.031)	(0.021)	(0.042)	(0.021)
CBD 5	−0.418***	−0.265***	−0.351***	−0.512***	0.177***	0.403***	0.180***
	(0.032)	(0.022)	(0.027)	(0.039)	(0.032)	(0.038)	(0.032)
CBD 6	−0.349***	−0.222***	−0.304***	−0.430***	0.100***	0.334***	0.103***
	(0.025)	(0.016)	(0.022)	(0.029)	(0.024)	(0.034)	(0.023)
Counterfactuals	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Agglomeration Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	6,260	6,260	6,260	6,260	7,050	6,260	7,050
R^2	0.11	0.13	0.07	0.13	0.12	0.24	0.13

Allen and Arkolakis (2022)

- "The Welfare Effects of Transportation Infrastructure Improvements"
- A framework to characterize the impact of infrastructure investment on welfare.
- Traders optimally travel across the complete transportation network. So infrastructure investment affects:
  - The cost of shipping goods between directly connected locations.
  - The total bilateral cost between any two locations.
- Two characterizations:
  - How infrastructure investment between any two connected locations decreases the total trade costs between all pairs of locations.
  - How the cost reduction between any two locations affects welfare.

Model Setup

- $i \in \{1, \ldots, N\}$ locations.
- Infrastructure matrix $T = [t_{ij} \geq 1]$ where t_{ij} is the iceberg cost incurred from moving directly from i to j.
- A path p between i and j: a sequence of locations beginning with location i and ending with location j, $\{i = p_0, p_1, \ldots, p_K = j\}$.
- The aggregate trade cost from i to j on a path p:

$$\tilde{\tau}_{ij}(p) = \prod_{k=1}^{K} t_{p_{k-1}, p_k}. \tag{18}$$

- Trade between i and j is undertaken by a continuum of heterogeneous traders $\nu \in [0, 1]$ who travel along endogenously chosen paths to get from i to j.

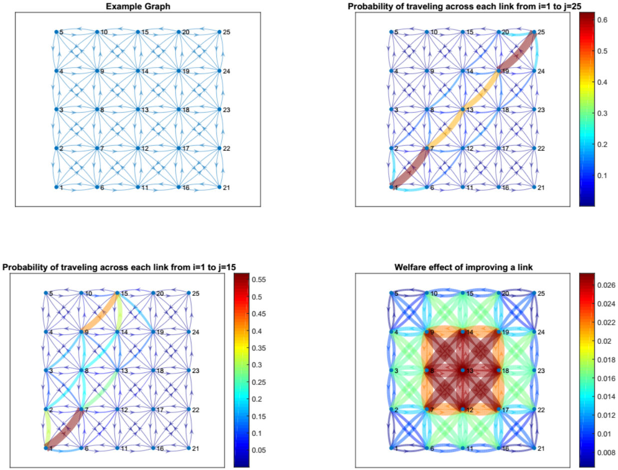
Traders

- Each trader incurs a path-specific idiosyncratic trade cost shock $\varepsilon_{ij}(p,\nu)$.
- $\varepsilon_{ij}(p,\nu)$ is Frechet distributed with shape parameter $\theta > 0$.
- Trader ν solves:

$$\tau_{ij}(\nu) = \min_{p \in P_{K,ij}, K \geq 0} \tilde{\tau}_{ij}(p)\varepsilon_{ij}(p,\nu). \tag{19}$$

- Discussions:
  - Allow traders to choose any possible path.
  - The extent to which traders differ in which path they choose is determined by θ.
  - When $\theta \to \infty$, all traders choose the route with the minimum aggregate trade cost.
  - Donaldson (2012): Djikstra method. Allen and Arkolakis (2014): the fast marching method.

Figure 1: An Example Geography



Optimal Routes

- The expected trade cost from i to j across all traders:

$$\tau_{ij} = E_\nu[\tau_{ij}(\nu)] = c \left(\sum_{k=0}^{\infty} \sum_{p \in P_{ij,K}} \tilde{\tau}_{ij}(p)^{-\theta} \right)^{-\frac{1}{\theta}}. \tag{20}$$

- Then

$$\tau_{ij}^{-\theta} = c^{-\theta} \sum_{K=0}^{\infty} \sum_{p \in P_{ij,K}} \prod_{k=1}^{K} t_{p_{k-1},p_k}^{-\theta}. \tag{21}$$

- Let $a_{ij} := t_{ij}^{-\theta}$ and $A = [a_{ij}]$. Then

$$\tau_{ij}^{-\theta} = c^{-\theta} \sum_{K=0}^{\infty} \sum_{p \in P_{ij,K}} \prod_{k=1}^{K} a_{p_{k-1},p_k}. \tag{22}$$

  - $a_{ij} \in [0,1]$.
  - $a_{ij} = 0$ if $t_{ij} = \infty$.

Optimal Routes

- Then

$$\tau_{ij}^{-\theta} = c^{-\theta} \sum_{K=0}^{\infty} \left(\sum_{k_1=1}^{N} \sum_{k_2=1}^{N} \sum_{k_{K-1}=1}^{N} \left(a_{i,k_1} \times a_{k_1,k_2} \times \ldots \times a_{k_{K-2},k_{K-1}} \times a_{k_{K-1},j} \right) \right). \tag{23}$$

- Matrix operation:

$$\tau_{ij}^{-\theta} = c^{-\theta} \sum_{K=0}^{\infty} A_{ij}^{K}, \tag{24}$$

where A_{ij}^K is the (i,j) element of the matrix A^K.

- As long as the spectral radius of A is less than one,

$$B = [b_{ij}] = (I - A)^{-1} = \sum_{K=0}^{\infty} A^K. \tag{25}$$

- Therefore,

$$\tau_{ij} = c b_{ij}^{-\frac{1}{\theta}}. \tag{26}$$

Optimal Routes

- A sufficient condition for the spectral radius being less than 1: $\sum_j t_{ij}^{-\theta} < 1$ for all i.
    - Trade costs between connected locations are sufficiently large.
    - A is sufficiently sparse.
    - The heterogeneity across traders is sufficiently small.

Properties of the endogenous trade costs

- The probability of using link $t_{k\ell}$ when traveling from i to j:

$$\pi_{ij}^{kl} = \frac{b_{ik} a_{kl} b_{lj}}{b_{ij}} = \left(\frac{1}{c} \frac{\tau_{ij}}{\tau_{ik} t_{kl} \tau_{lj}} \right)^{\theta}. \tag{27}$$

Properties of the endogenous trade costs

- How a change in the infrastructure matrix changes the endogenous trade costs:

$$\frac{\partial \log \tau_{ij}}{\partial \log t_{kl}} = \frac{b_{ik} a_{kl} b_{lj}}{b_{ij}} = \left(\frac{1}{c} \frac{\tau_{ij}}{\tau_{ik} t_{kl} \tau_{lj}} \right)^{\theta}. \tag{28}$$

Properties of the endogenous trade costs

- The distance between i and j on path p:

$$\tilde{d}_{ij}(p) = \sum_{k=1}^{K} d_{p_{k-1},p_k}. \tag{29}$$

- The expected distance between i and j:

$$E\left(\tilde{d}_{ij}(p)\right) = \sum_{k=1}^{N}\sum_{l=1}^{N} d_{kl} \times \left(\frac{1}{c}\frac{\tau_{ij}}{\tau_{ik}t_{kl}\tau_{lj}}\right)^{\theta}. \tag{30}$$

Trade

- Armington model: CES preference for location-specific varieties with elasticity of substitution $\sigma > 1$.
- Gravity equation:
$$X_{ij} = \tau_{ij}^{1-\sigma} A_i^{\sigma-1} w_i^{1-\sigma} P_j^{\sigma-1} E_j. \tag{31}$$
- Total income:
$$Y_i = \sum_{j=1}^{N} X_{ij}. \tag{32}$$
- Total expenditure:
$$E_i = \sum_{j=1}^{N} X_{ji}. \tag{33}$$
- Market clear:
$$Y_i = E_i = w_i L_i. \tag{34}$$

The Spatial Economy

- Welfare:

$$W_i := \frac{w_i}{P_i} u_i. \tag{35}$$

- Replace P_i by W_i and insert welfare equalization:

$$w_i L_i = W^{1-\sigma} \sum_j \left(\frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} w_j^{\sigma} L_j. \tag{36}$$

$$w_i L_i = W^{1-\sigma} \sum_j \left(\frac{\tau_{ji}}{A_j u_i} \right)^{1-\sigma} w_j^{1-\sigma} w_i^{\sigma} L_j. \tag{37}$$

Social Planner's Problem

- Social planner:

$$\max_{\{\{w_i\},\{L_i\},W\}} \log W,$$
$$\sum_{i=1}^{N} w_i L_i = \sum_{i=1}^{N}\sum_{j=1}^{N} \left(\frac{\tau_{ij}}{A_i u_j}\right)^{1-\sigma} w_i^{1-\sigma} W^{1-\sigma} w_j^{\sigma} L_j. \tag{38}$$

- It can be shown that the solution to this social planner's problem is identical to decentralized equilibrium.
- Envelope theorem:

$$\frac{\partial \log W}{\partial \log \tau_{ij}} = -\lambda \left(\frac{\tau_{ij}}{A_i u_j}\right)^{1-\sigma} w_i^{1-\sigma} W^{1-\sigma} w_j^{\sigma} L_j = -\frac{X_{ij}}{Y^W}, \tag{39}$$

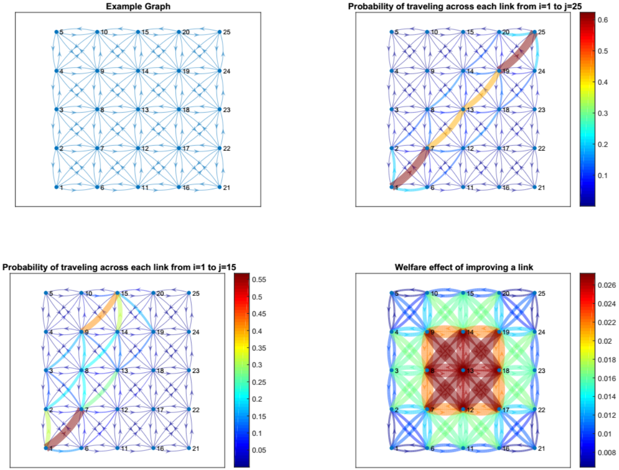
where λ is the Lagrange multiplier and Y^W is the world income.

The Welfare Effects of Infrastructure

- $$-\frac{d\log W}{d\log t_{ij}} = \sum_{k=1}^{N}\sum_{l=1}^{N}\frac{X_{kl}}{Y^W}\frac{b_{ki}a_{ij}b_{jl}}{b_{kl}} = c^{\theta}\sum_{k=1}^{N}\sum_{l=1}^{N}\frac{X_{kl}}{Y^W}\left(\frac{\tau_{kl}}{\tau_{ki}t_{ij}\tau_{jl}}\right)^{\theta}. \tag{40}$$
- Sufficient statistics:

$$-\frac{d\log W}{d\log t_{ij}} = \left(\frac{t_{ij}\tau_{ij}}{c}\right)^{-\theta} X_{ji}^{\frac{\theta}{1-\sigma}}\sum_{k=1}^{N}\sum_{l=1}^{N}\frac{X_{kl}}{Y^W}\left(\frac{X_{ki}X_{jl}}{X_{kl}}\right)^{\frac{\theta}{1-\sigma}}. \tag{41}$$

Figure 1: An Example Geography



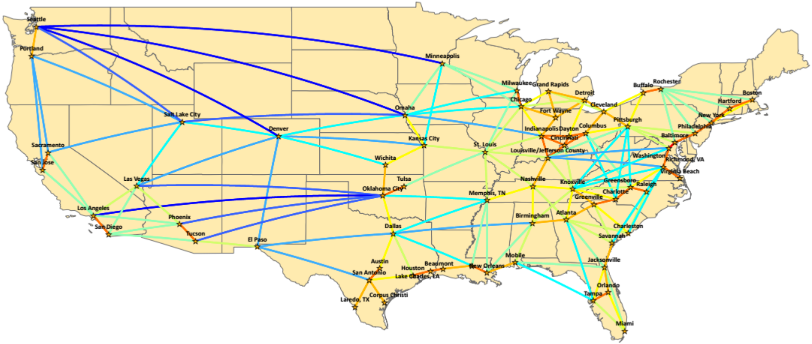
The Welfare Effects of Improving the U.S. Interstate Highway System

Figure 2: The U.S. Interstate Highway Network

Panel A: *The Observed U.S. Interstate Highway Network*

The Welfare Effects of Improving the U.S. Interstate Highway System

Panel B: *A Graphical Representation of the U.S. Interstate Highway Network*



Data

- Shipment-level data between 67 U.S. cities from 2012 Commodity Flow Survey (CFS).
- For all origin-destination city pairs:
  - Aggregate value of bilateral trade flows.
  - Mean distance traveled between the pair.
  - The standard deviation of distance traveled between the pair.

Estimation

- Let $t_{ij} = \exp\{\kappa \text{time}_{ij}\}$.
- Methods of moments:
  1. The mean distance.
  2. The observed value.
  3. An average trade cost of 20%. ad valorem tariff equivalent.
- Results:
  - $\kappa = 0.0108$.
  - $\sigma = 7.92$.
  - $\theta = 136.13$.

Figure 3: The Variability of Route Distances Across Shipments

Panel A: *Variation in Distance Traveled between Major City Pairs*

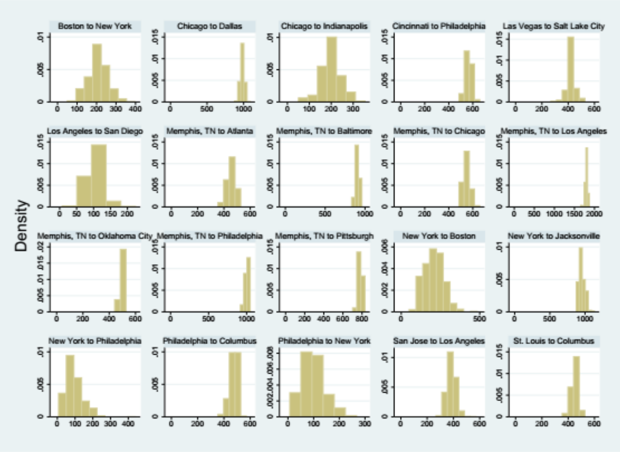
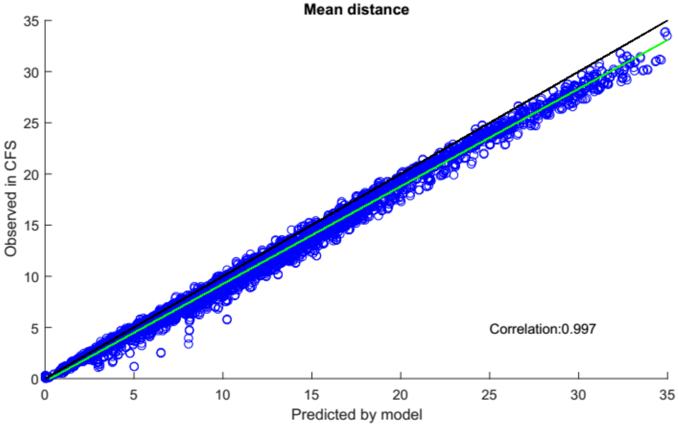


Figure 4: Model Fit



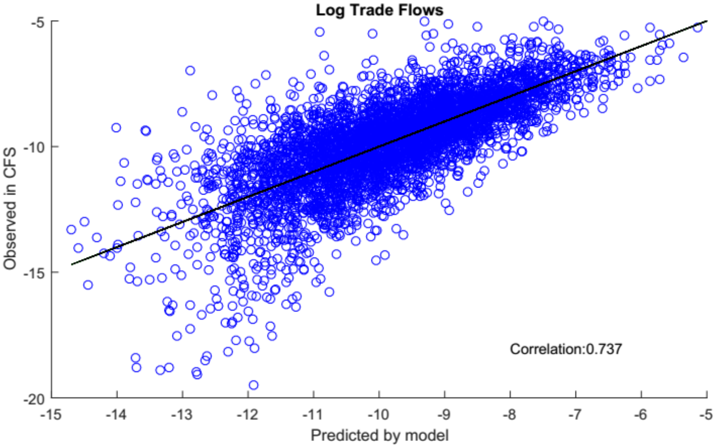

Log Trade Flows
Observed in CFS
Predicted by model
Correlation:0.737
-5
-10
-15
-20
-15
-14
-13
-12
-11
-10
-9
-8
-7
-6
-5

Figure 6: Probability of Using Different Highways from LA to NY

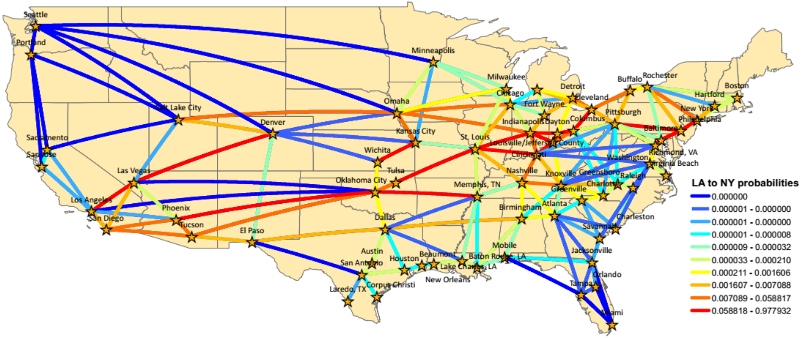


Figure 7: The Welfare Effects of Transportation Infrastructure Improvements

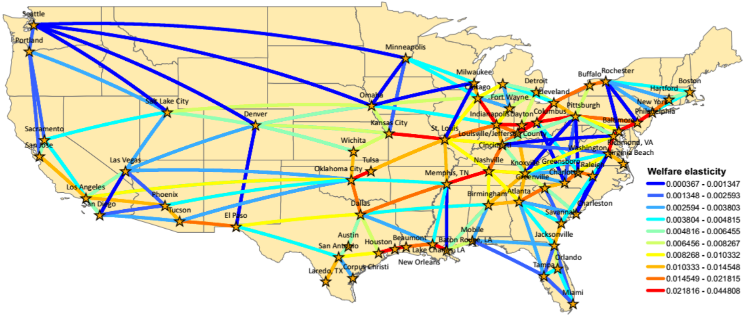


Table 2: Top 20 Highways

	Origin City	Destination City	Interstate	Welfare Elasticity
1	New York	Philadelphia	95 (South)	0.04481
2	Baltimore	Washington	95 (South)	0.04477
3	Columbus	Dayton	70 (West)	0.0438
4	Dayton	Columbus	70 (East)	0.04094
5	Dayton	Cincinnati	75 (South)	0.04007
6	Philadelphia	Baltimore	95 (South)	0.03961
7	Cincinnati	Dayton	75 (North)	0.03691
8	Philadelphia	New York	95 (South)	0.03292
9	Washington	Baltimore	95 (North)	0.03069
10	Washington	Richmond, VA	95 (South)	0.03025
11	Beaumont	Houston	10 (West)	0.02874
12	Tulsa	Oklahoma City	44 (West)	0.02874
13	Pittsburgh	Columbus	70(West)	0.02868
14	Lake Charles, LA	Beaumont	10 (West)	0.02858
15	Baltimore	Philadelphia	95 (North)	0.02793
16	Columbus	Cincinnati	71 (South)	0.02742
17	Cincinnati	Louisville/Jefferson County	71 (South)	0.02662
18	Louisville/Jefferson County	Nashville	65 (South)	0.02659
19	Columbus	Cleveland	71 (North)	0.02629
20	Nashville	Memphis, TN	40 (West)	0.02617

Notes: This table reports the twenty (one-way) interstate links that have the greatest improvement on U.S. welfare for a given percentage reduction in trade costs on that connection.

Useful Reviews

- Allen and Arkolakis (2025) “Quantitative Regional Economics”
- Fajgelbaum and Gaubert (2025) “Optimal Spatial Policies”
- Redding, Stephen (2025) “Quantitative Urban Economics”