Econ7115: Structural Models and Numerical Methods in Economics
Assignment W10

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1. Consider the following dynamic discrete choice model:

   - Each agent, with discounting factor $\beta \in (0,1)$, owns a non-divisible asset, valued $x_t > 0$ at period t
   - The agent chooses whether to sell the asset: $a_t = 0$, not sell; $a_t = 1$, sell
   - Utility from the asset: $u(x_t, a_t) + \epsilon_t(a_t)$ where $\epsilon_t(.) \sim G_\epsilon$ is an idiosyncratic utility shock that is unobserved by the econometrician

   $$u\left(x_t, a_t\right) = \begin{cases} 0, & \text{if } a_t = 0 \\ \frac{x_t^{1-\gamma}}{1-\gamma} & \text{if } a_t = 1 \end{cases} \tag{1}$$

   - If $a_t = 1$, then $x_s = 0$ for all $s > t$
   - Let $\Delta > 0$ be the growing size and $\bar{x} = x_0 + 50\Delta$ be the upper bound of asset value. If $a_t = 0$, the Markov transition probability of x_t is defined as

   $$x_{t+1} = \begin{cases} x_t + \Delta, & \text{with prob. } q \in (0,1), \text{ if } x_t < \bar{x} \\ x_t, & \text{with prob. } 1-q, \text{ if } x_t < \bar{x} \\ x_t, & \text{with prob. } 1, \text{ if } x_t = \bar{x} \end{cases} \tag{2}$$

   - $\epsilon_t(a_t)$ is drawn IID from the Type I Extreme Value (TIEV) Distribution

   1. Let $V\left(x, \epsilon\right)$ be the value function. Please write down the Bellman equation in terms of $V\left(x, \epsilon\right)$.
   2. Let $\bar{V}\left(x\right) \equiv \int V\left(x, \epsilon\right) dG_\epsilon$. Please write the Bellman equation in terms of $\bar{V}\left(x\right)$.
   3. Let $x_0 = 1$, $\gamma = 0.5$, $\Delta = 0.1$, $q = 0.2$, and $\beta = 0.9$. Please compute the likelihood $Pr\left[a = 1 | x\right]$.
   4. Utilizing the numerical results above, please discuss the implications of (i) γ, (ii) Δ, and (iii) q, for the likelihood $Pr\left[a = 1 | x\right]$.