Econ7115: Structural Models and Numerical Methods in Economics
Assignment W12

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1. Consider the following dynamic programming problem for a firm:

$$\begin{aligned} V(k,z) &= \max_{i\geq 0}\left\{d+\beta E\left[V\left(k',z'\right)|k,z\right]\right\}, \quad k,z>0 \\ \text{s.t. } d &= zk^{\alpha}-i-\frac{c}{2}\frac{i^2}{k}, \quad k'=i+(1-\delta)k \end{aligned} \tag{1}$$

   Notice that d is the current cash flow, depending on (i) output zk^{α} which combines exogenous productivity z and capital stock k, (ii) investment i, and (iii) the quadratic investment cost $\frac{c}{2}\frac{i^2}{k}$. The next period capital stock, k', depends on investment and the non-depreciated capital. We assume that $z \in \{z_L, z_H\}$ and the transition probabilities are $\pi_{jj'} = prob\left[z_{j'}|z_j\right]$ where $j, j \in \{L, H\}$. We assume that $z_L = 1-\bar{z}$ whereas $z_H = 1+\bar{z}$.

   Consider the following parameterization:

   - Predetermined parameters: $\alpha = 1/3$, $\delta = 0.06$, $\beta = 0.9$
   - Parameters to be estimated: $\boldsymbol{\theta} \equiv (c, \pi_{HH}, \pi_{LL}, \bar{z})$

   Please answer the following questions:

   1. Suppose that we have observed capital stocks and cash flows for S firms over T periods, $(k_{st}, d_{st})_{s=1,...,S;t=1,...,T}$. Please propose estimators of $\boldsymbol{\theta}$ based on indirect inference.
   2. Please find the simulated observations for $(k_{st}, d_{st})_{s=1,...,S;t=1,...,T}$ in the attachment. "k_mat_obs.csv" contains $(k_{st})_{s=1,...,S;t=1,...,T}$ and "d_mat_obs.csv" contains $(d_{st})_{s=1,...,S;t=1,...,T}$, with each row being a firm and each column being a period. Please estimate $\boldsymbol{\theta}$ from these simulated observations based on your proposed estimators. (Hint: when you simulate the model, please drop the first 100 periods of simulations to avoid the impacts of initial conditions.)