Instrument Variables in Structural Modeling

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Model Estimation

- Structural model:

$$G\left(Y_t, X_t, \epsilon_t; \theta\right) = 0 \tag{1}$$

  - Data on policies X_t and outcomes Y_t + Assumptions on shocks $\epsilon_t \Rightarrow$ estimates of θ
  - Threats to identification: confounding factors $\epsilon_t \not\perp X_t$; clearer under linear approximation

$$\Delta Y_t = -\left[\nabla_Y G\right]^{-1}\left[\nabla_X G\right]\Delta X_t - \left[\nabla_Y G\right]^{-1}\left[\nabla_\epsilon G\right]\Delta\epsilon_t \tag{2}$$

- Advantage of structural modeling in identification
  - Simultaneously characterize many interdependent Y's $\Rightarrow$ Better controls:
  - e.g. returns to education: Keane and Wolpin (1997) model occupation and education choices simultaneously
- Limitations of structural modeling in identification
  - Still unobserved/unexplained shocks ϵ_t
  - Mis-specified $G(.)$

Davis et al. (2014): Model

- An infinitely lived representative household; a unit mass of homogeneous members; inelastically supply a unit of labor each period
- Member's utility: local provided housing services, h; freely traded consumption good
- Each period, the household freely allocates across cities its workers, business capital, and residential structures
- The household also chooses for the next period these two stocks and infrastructure in each city
- Each city's exogenous productivity shock z: stationary discrete Markov
- The household does not take into account the effects of its actions on the density of production in each location

Davis et al. (2014): Competitive Equilibrium

- Let $\mu_t(z^t)$ denote the time t distribution of cities across productivity histories and $x(z^t) \equiv y(z^t)/l_b(z^t)$ be the output density. The planning problem is

$$\max_{\substack{\{C_t, K_{bt+1}, K_{ht+1}, y(z^t), l_b(z^t), l_h(z^t), n(z^t), \\ k_b(z^t), k_h(z^t), k_{ft+1}(z^t), h(z^t)\}_{t=0}^{\infty}}} \sum_{t=0}^{\infty} \beta^t \ln C_t$$

$$+ \psi \sum_{t=0}^{\infty} \beta^t \sum_{z^t} \mu_t(z^t) n(z^t) \ln \frac{h(z^t)}{n(z^t)}$$

subject to

$$C_t + \Gamma_{bt}[K_{bt+1} - (1-\kappa_b)K_{bt}] + \Gamma_{ht}[K_{ht+1} - (1-\kappa_h)K_{ht}] + \Gamma_{ft} \sum_{z^t} \mu_t(z^t)[k_{ft+1}(z^t) - (1-\kappa_f)k_{ft}(z^{t-1})] \leq \left[\sum_{z^t} \mu_t(z^t) y(z^t)^{\eta}\right]^{1/\eta}, \tag{3}$$

$$y(z^t) \leq [Y_t z_t]^{(1-\alpha)\phi} x(z^t)^{(\lambda-1)/\lambda} l_b(z^t)^{1-\phi} k_b(z^t)^{\alpha\phi} n(z^t)^{(1-\alpha)\phi}, \quad \forall z^t, \tag{4}$$

$$h(z^t) \leq l_h(z^t)^{1-\omega} k_h(z^t)^{\omega}, \quad \forall z^t, \tag{5}$$

$$l_h(z^t) + l_b(z^t) \leq k_{ft}(z^{t-1})^{\zeta}, \quad \forall z^t, \tag{6}$$

$$\sum_{z^t} \mu_t(z^t) k_b(z^t) \leq K_{bt}, \tag{7}$$

$$\sum_{z^t} \mu_t(z^t) k_h(z^t) \leq K_{ht}, \tag{8}$$

$$\sum_{z^t} \mu_t(z^t) n(z^t) \leq N_t, \tag{9}$$

Davis et al. (2014): Balanced Growth Path

- Along BGP, the growth rate of consumption satisfies

$$
\begin{aligned}
g_c =& \gamma_a^{(1-\alpha)\delta/(1-\delta\alpha+(\sigma-1)\zeta)} \gamma_b^{\alpha\delta/(1-\delta\alpha+(\delta-1)\zeta)} \\
& \times \gamma_f^{\zeta(1-\delta)/(1-\delta\alpha+(\delta-1)\zeta)} \gamma_n^{(1-\zeta)(\delta-1)/(1-\delta\alpha+(\delta-1)\zeta)},
\end{aligned}
\tag{3}
$$

  where

  - $\gamma_j > 0$ is the growth rate of the technology for producing the j-type investment good relative to that for consumption, for all $j = b, f, h$
  - $\gamma_a > 0$ is the TFP growth
  - $\delta \equiv \phi\lambda$

Davis et al. (2014): Empirical Strategy

- Let $\tilde{x}$ be the point estimate of any variable x and g_{p_j} denotes the growth rate of the price of investment good j. Then the estimate of γ_a:

$$\tilde{\gamma}_a = \tilde{g}_c^{(1-\alpha)\delta+(\zeta-1)(\delta-1)/((1-\alpha)\delta)} \tilde{\gamma}_n^{(1-\zeta)(1-\delta)/((1-\alpha)\delta)} \tilde{g}_{p_b}^{\alpha/(1-\alpha)} \tilde{g}_{p_f}^{\zeta(1-\delta)/((1-\alpha)\delta)}. \tag{4}$$

- Let g_c^* be the counterfactual growth rate of consumption without agglomeration, $\delta = \phi$. Then

$$g_c^* = \tilde{g}_c^{\frac{1-\alpha\delta-\zeta(1-\delta)}{\lambda[1-\alpha\phi-\zeta(1-\phi)]}} \tilde{\gamma}_n^{\frac{(1-\lambda)(1-\zeta)}{\lambda[1-\alpha\phi-\zeta(1-\phi)]}} \tilde{g}_{p_f}^{\frac{\zeta(1-\lambda)}{\lambda[1-\alpha\phi-\zeta(1-\phi)]}} \tag{5}$$

- The increase in per capita consumption growth due to agglomeration:

$$\Lambda = \frac{\tilde{g}_c - g_c^*}{g_c^* - 1} \tag{6}$$

  - growth rates: g_c, g_{p_b}, g_{p_f}, γ_n
  - parameters: λ, α, ϕ, ζ

Davis et al. (2014): Empirical Strategy

- Data:
  - Identify λ usiing panel data for 22 cities in the US over 1978-2009
  - Remaining parameters and growth rates: aggregate time series on prices and quantities
- Let $\hat{x}_{it} \equiv \log x_{it} - \frac{1}{N}\sum_{j=1}^{N} \log x_{jt}$. Then the TFP satisfies

$$\hat{a}_{it} = \frac{\lambda - 1}{\lambda} \hat{r}_{it} + (1 - \alpha)\,\phi \hat{z}_{it}, \tag{7}$$

  where $\hat{r}_{it}$ is the local-good-denominated land rents: $\hat{r}_{it} = \frac{1}{\omega}\hat{r}_{hit} - \hat{p}_{yit}$ where r_h denotes the rental price of housing and p_y denotes the output price
- Endogeneity: the idiosyncratic technology term, $\hat{z}_{it}$, is correlated with local-output-denominated land rents

Davis et al. (2014): Estimation

- Zipf's law $\Rightarrow \Delta \hat{z}_{it} = \varepsilon_{it}$ where ε_{it} is independently distributed across time and space. Then

$$\Delta \hat{a}_{it} = \frac{\lambda - 1}{\lambda} \Delta \hat{r}_{it} + (1 - \alpha)\, \phi \varepsilon_{it} \tag{8}$$

- By construction, any variable dated $t - 1$ and earlier is orthogonal to ε_{it}

$$E \left\{ \left[\Delta \hat{a}_{it} - \frac{\lambda - 1}{\lambda} \Delta \hat{r}_{it} \right] \times \hat{\nu}_{it-j} \right\} = 0, \quad j \geq 1. \tag{9}$$

- Together with the moment condition above, there are 13 moment conditions that involve aggregate variables to identify 13 remaining parameters (including growth rates)
    1. Estimate 13 parameters by 13 macro moments
    2. Inserting the estimated parameters to construct micro moments, estimate λ
    3. Obtain Λ

- To allow for IID measurement error, use variable dated $t - 2$ and earlier, because first-difference measurement error may be correlated with instruments dated at $t - 1$
    - Instruments: $\hat{r}_{hit-2}$, $\hat{w}_{it-2}$, $\hat{p}_{yit-2}$, log house prices, and log per capita income

Davis et al. (2014): Empirical Results

TABLE I

ESTIMATES OF λ[a]

Methodology	Estimate	Std. Error
GMM with estimated TFP & land price	1.069	0.030
OLS with TFP & land price as data		
Panel data	1.054	0.016
Mean of 22 time series	1.074	0.019
Mean of 29 cross sections	1.088	0.032
2SLS with TFP & land price as data		
Panel data	1.099	0.029
Mean of 22 time series	1.084	0.036
Mean of 29 cross sections	1.065	0.045

[a] GMM estimates and standard errors are from estimating equations (23) and (34) with $R = 1.05$. Standard errors for GMM include the sampling uncertainty that arises from estimating the parameters in the TFP and land price measurement equations (26). The OLS and 2SLS estimates are from estimating (22) taking TFP and land prices as data.

Davis et al. (2014): Empirical Results

TABLE II

BASELINE PARAMETER ESTIMATES[a]

Parameter	Description	Estimate	Std. Error
Λ	Effect of agglomeration on growth	0.102	0.050
δ	Net effect of density on productivity	1.041	0.018
ζ	Infrastructure share in finished land	0.545	0.053
ϕ	Non-land income share	0.974	0.002
α	Capital's share of non-land income	0.299	0.002
ω	Structure's share of housing	0.668	0.024
ξ	Skilled–unskilled labor substitutability	0.545	0.035
γ_n	Population growth	1.012	0.000
g_{p_f}	Growth of infrastructure prices	1.006	0.002
g_{p_b}	Growth of business capital prices	0.995	0.001
g_{p_s}	Growth of residential structures price	1.008	0.001
g_{p_l}	Growth of finished land prices	1.010	0.003
g_c	Per capita consumption growth	1.017	0.001
κ_f	Infrastructure depreciation rate	0.021	0.000
κ_b	Business capital depreciation rate	0.107	0.001
κ_s	Structures depreciation rate	0.016	0.000

[a]Estimates and standard errors are from estimating equations (18), (23), and (34). Estimates are based on $R = 1.05$.

Davis et al. (2014): Empirical Results

TABLE IV

EFFECTS OF MODEL ASSUMPTIONS ON BASELINE PARAMETER ESTIMATES[a]

Model	Λ	δ	ζ	ϕ	α
Baseline	0.102	1.041	0.545	0.974	0.299
	(0.050)	(0.018)	(0.053)	(0.002)	(0.002)
$\eta = 1$	0.128	1.057	0.545	0.974	0.299
	(0.069)	(0.020)	(0.053)	(0.002)	(0.002)
$g_{p_f} = g_{p_l} = g_c \gamma_n$	0.133	1.032	0.571	0.983	0.306
	(0.052)	(0.010)	(0.016)	(0.001)	(0.002)

[a]Standard errors are given in parentheses. In both perturbations to the baseline, estimation is based on the same set of instruments as in the baseline.

Comments: Two Styles of Structural Estimation

- Pooling all moments $\Rightarrow$ Recovering all parameters simultaneously
  - Pros: fully utilize the model structure (interdependence among parameters); more efficient
  - Cons: estimates of key parameters are sensitive to the moments identifying other parameters; difficult to interpret and adjust; more exposed to model misspecification
- Separately identify key parameters with others: usually partial equilibrium reduced-form equations to identify key parameters
  - Pros: transparent for interpretation; easy to adjust; less affected by the moments identifying other parameters; more robust under model misspecification
  - Cons: not always feasible (unjustifiable assumptions necessary to separation); incompatible estimates due to the ignorance of interdependence; less efficient
  - Example: Deng et al. (2024) "Local Corporate Taxes and the Geography of Foreign Multinationals"

Adao, Costinot, and Donaldson (2024): Model Validation

- Quantitative model:

$$y_t = f\left(\tau_t, \epsilon_t\right), \tag{10}$$

  $y_t \equiv \{y_{n,t}\}$: vector of endogenous variables; $\tau_t \equiv \{\tau_{k,t}\}$: vector of policies in interest; ϵ_t: vector of all other time-varying shocks

- True data-generating process

$$y_t = f^*\left(\tau_t, \epsilon_t^*\right) \tag{11}$$

- Question: How close is $W\left(\Delta x\right)$ is to $W\left(\Delta x^*\right)$, where
  - $\Delta x \equiv f\left(\tau_{t+1}, \epsilon_{t+1}\right) - f\left(\tau_t, \epsilon_{t+1}\right)$
  - $\Delta x^* \equiv f^*\left(\tau_{t+1}, \epsilon_{t+1}^*\right) - f^*\left(\tau_t, \epsilon_{t+1}^*\right)$
  - $W(.)$: some statistic of interest

Adao, Costinot, and Donaldson (2024): Model Validation

- Philosophy:
  - we are ultimately not interested in assessing whether f is close to f^*–in which case one may use any shock and any variable
  - but instead we are interested in assessing whether the causal impact of the policy change predicted by the model on a statistic of interest $W(\Delta x)$ is close to $W(\Delta x^*)$
- Main inputs for model validation: for all relevant variables $n \in \mathcal{N}_W$
  - A model's predicted causal impact, Δx_n
  - The change observed in the data, $\Delta y_n \equiv y_{n,t+1} - y_{n,t}$
- Practical problem with using the raw differences: $\{\Delta y_n - \Delta x_n\}$:
  - Even around well-known episodes of policy change, there may be other non-policy shocks occurring whose magnitudes may be large and whose distribution may be unknown

Adao, Costinot, and Donaldson (2024): Model Validation

- The change observed in the data can be structurally decomposed into

$$\begin{aligned}\Delta y &= f^*\left(\tau_{t+1}, \epsilon^*_{t+1}\right) - f^*\left(\tau_t, \epsilon^*_t\right) \\ &= \underbrace{\left[f^*\left(\tau_{t+1}, \epsilon^*_{t+1}\right) - f^*\left(\tau_t, \epsilon^*_{t+1}\right)\right]}_{\Delta x^*} + \underbrace{\left[f^*\left(\tau_t, \epsilon^*_{t+1}\right) - f^*\left(\tau_t, \epsilon^*_t\right)\right]}_{\Delta \eta^*} \end{aligned} \tag{12}$$

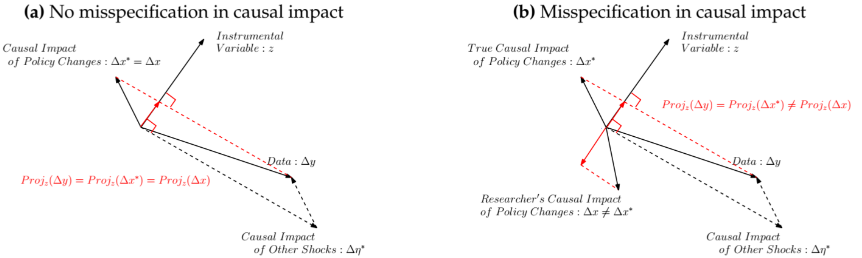
- Therefore, the difference

$$\Delta y - \Delta x = \left(\Delta x^* - \Delta x\right) + \Delta \eta^* \tag{13}$$

- Instrument variable:
  - We may not have strong priors about the magnitude and distribution of $\Delta\eta^*$, but we may be confident that they are orthogonal to some exogenous shifters of policy changes, $\Delta\tau_{IV}$
  - Compare the projections of Δx and Δx^* on $z \equiv \tilde{z}\left(\Delta\tau_{IV}\right)$
    - The later coincides with the projection of Δy on z under the assumption that z and $\Delta\eta^*$ are orthogonal
    - If $\Delta x_n = \Delta x^*_n$ for all $n \in \mathcal{N}_W$, then these two projections will be identical

Adao, Costinot, and Donaldson (2024): Model Validation

Figure 1: An IV-Based Measure of Goodness of fit



Notes: Notes: The figure shows how one can compare the true causal impact of policy changes Δx^* to the causal impact of policy changes in the researcher's model Δx by comparing the projection of Δx on the IV z to the projection of Δy on the same IV z. Panel (a) focuses on the situation where the two causal impacts coincide, $\Delta x^* = \Delta x$, so that $Proj_z(\Delta y) = Proj_z(\Delta x^*) = Proj_z(\Delta x)$. Panel (b) focuses on the situation where the two causal impacts differ, $\Delta x^* \neq \Delta x$, and $Proj_z(\Delta y) = Proj_z(\Delta x^*) \neq Proj_z(\Delta x)$.

Adao, Costinot, and Donaldson (2024): Model Validation

- For any causal prediction Δx and statistic of interest W, the goodness of fit of the researcher's prediction when using the IV z is

$$\hat{\beta}_z \equiv \frac{1}{N_W} \sum_{n \in \mathcal{N}_W} z_n \left(\Delta y_n - \Delta x_n \right), \tag{14}$$

where $N_W \equiv |\mathcal{N}_W|$ denotes the number of observations entering the statistic of interest W

- Assumptions:
  - **A1.** Conditional on the realization of period t's policy τ_t and other shocks ϵ_t^*, policy shifters are mean zero and independent of other shocks in period $t+1$: $\Delta\tau_{IV} \perp \epsilon_{t+1}^* | \epsilon_t^*, \tau_t$
  - **A2.** For any $n \in \mathcal{N}_W$, the instrument variable takes the form $z_n = \sum_k s_{nk} \Delta\tau_{IV,k}$, where the vector $\{s_{nk}\}$ may be a function of, and only of, the realization of period t's policy τ_t and other shocks ϵ_t^*
  - **A3.** For any $n \in \mathcal{N}_W$, $\Delta x_n^* = \Delta x_n$

Adao, Costinot, and Donaldson (2024): Model Validation

Proposition (Moment Restriction)

Take any IV z that satisfies A1 and A2. If A3 holds, then $E_t\left[\hat{\beta}_z\right] = 0$

Proposition (Asymptotic Distribution)

Take any IV z that satisfies A1 and A2. If A3 holds and (i) $\tau_{IV,k}$ *are i.i.d. across* $k = 1, \ldots, K$*, (ii)* $\frac{1}{N_W^2}\sum_k (S_k)^2 \to 0$ *with* $S_k \equiv \sum_n |s_{nk}|$*, and (iii)* $Var_t\left[\Delta\tau_{IV,k}\right]$ *and* $\Delta\eta_n^*$ *are uniformly bounded, then* $\hat{\beta}_z \to_p 0$*. Furthermore, if (iv)* $\frac{\max_k (S_k)^2}{\sum_k S_k^2} \to 0$*, (v)* $E_t\left[(\Delta\tau_{IV,k})^4\right]$ *is uniformly bounded, and (vi)* $\frac{1}{\sum_k S_k^2} Var_t\left[\sum_{n\in\mathcal{N}_W} z_n \Delta\eta_n^* | \epsilon_{t+1}^*\right] \to_p \bar{V}_\beta$ *and non-random, then* $r_\beta \hat{\beta}_z \to_d \mathcal{N}\left(0, \bar{V}_\beta\right)$ *where* $r_\beta \equiv \frac{N_W}{\sqrt{\sum_k S_k^2}}$*. The asymptotic variance of* $\hat{\beta}_z$ *can be estimated by*

$$\hat{V}\left[\hat{\beta}_z\right] = \sum_k (\Delta\tau_{IV,k})^2 \left[\sum_{n\in\mathcal{N}_W} s_{nk}\left(\Delta y_n - \Delta x_n\right)/N_W\right]^2 \tag{15}$$

Adao, Costinot, and Donaldson (2024): Model Validation

- A general procedure of model validation:
  - Use a subset of moments for estimation
  - Use another subset of untargeted moments to validate model's predictions
- This IV-based test can be regarded as the second step, addressing
  - can we reject the null that $\Delta x_n^* = \Delta x_n$ for all $n \in \mathcal{N}_W$?
  - rather than the broader: "can we reject the null that $f = f^*$?", relying on which may lead to
    - reject a model even though its causal answer of interest is correct–because other shocks in the model are misspecified
    - fail to reject a model even though its causal answer is incorrect–due to focusing on moments with little relevance to the causal mechanism of interest

Adao, Costinot, and Donaldson (2024): Model Validation

- Correlation between Δy_n and Δx_n:
    - $\text{Corr}(\Delta y_n, \Delta x_n)$; R-squared of the OLS regression of Δy_n on Δx_n; mean squared error
    - Depend on the magnitude of $\text{Var}(\Delta \eta_n^*)$
- Other untargeted moments:
    - unsure whether the untargeted moments reflects the causal effects in interest
- Integrate $\hat{\beta}_z = 0$ in estimation?
    - In many cases, estimating moments are partial equilibrium but $\hat{\beta}_z = 0$ is general equilibrium in nature

Adao, Costinot, and Donaldson (2024): Application

- Fajgelbaum et al. (FGKK, 2020):
  - US-China trade war $\Rightarrow$ US welfare
  - multiple countries $i \in \mathcal{I}$; US: $i = H$; regions in the US: $r \in \mathcal{R}$; sector $s \in \mathcal{S}$
  - time t; labor: primary factor of production; $L_{rs,t}$: exogenous labor supply; $w_{rs,t}$: wage
- Preferences: one non-tradable sector $s = NT$; tradable sectors $s \in \mathcal{S}_T$, product set $\mathcal{P}_s$

$$
\begin{aligned}
U_t &= \left(C_{NT,t}\right)^{\beta_{NT,t}} \left(C_{T,t}\right)^{1-\beta_{NT,t}}, \quad C_{T,t} = \prod_{s \in \mathcal{S}_T} \left(C_{Ts,t}\right)^{\beta_{s,t}} \\
C_{Ts,t} &= \left[\sum_{j=H,F} \left(A_{js,t}\right)^{\frac{1}{\kappa}} \left(C_{js,t}\right)^{\frac{\kappa-1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}}, \forall s \in \mathcal{S}_T \\
C_{js,t} &= \left[\sum_{\nu \in \mathcal{P}_s} \left(a_{j\nu,t}\right)^{\frac{1}{\eta}} \left(c_{j\nu,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, j = H, F, s \in \mathcal{S}_T \\
c_{F\nu,t} &= \left[\sum_{i \neq H} \left(a_{i\nu,t}\right)^{\frac{1}{\sigma}} \left(c_{j\nu,t}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \forall n \in \mathcal{P}_s, s \in \mathcal{S}_T
\end{aligned}
\tag{16}
$$

Adao, Costinot, and Donaldson (2024): Application

- Technology:
  - Non-tradable: $Q_{rNT,t} = Z_{rNT,t} L_{rNT,t}, \quad \forall r \in \mathcal{R}$
  - Tradables: $Q_{rs,t} = Z_{rs,t} \left(M_{rs,t}\right)^{\alpha_{ls,t}} \left(L_{rs,t}\right)^{\alpha_{Ls,t}}$, where $M_{rs,t} = \prod_{k \in \mathcal{S}_T} \left(M_{krs,t}\right)^{\alpha_{ks,t}}$ for all $r \in \mathcal{R}$, $s \in \mathcal{S}_T$
- Prices:
  - US import price: $p^H_{i\nu,t} = \left(1 + \tau^H_{i\nu,t}\right) \bar{p}^F_{i\nu,t}$, for all $i \neq H$, $\nu \in \mathcal{P}_s$, and $s \in \mathcal{S}_T$
  - US export price: $p^F_{i\nu,t} = \left(1 + \tau^F_{i\nu,t}\right) \bar{p}^H_{i\nu,t}$, for all $i \neq H$, $\nu \in \mathcal{P}_s$, and $s \in \mathcal{S}_T$
- US government budget constraint:

$$T_t = \sum_{s \in \mathcal{S}_T, \nu \in \mathcal{P}_s, i \in \mathcal{I}} \tau^H_{i\nu,t} \bar{p}^F_{i\nu,t} \left(c_{i\nu,t} + \sum_{r \in \mathcal{R}, s \in \mathcal{S}_T} q_{i\nu rs,t} \right) + F_t \tag{17}$$

Adao, Costinot, and Donaldson (2024): Application

- Foreign import demand: $c^F_{i\nu,t} = a^F_{i\nu,t}\left(p^F_{i\nu,t}\right)^{-\sigma_F}$

- Foreign export supply: $q^F_{i\nu,t} = \left(z^F_{i\nu,t}\right)^{\frac{1}{\omega_F}}\left(\bar{p}^F_{i\nu,t}\right)^{\frac{1}{\omega_F}}$

- Market clearing:

$$
\begin{aligned}
q_{i\nu,t} &= c^F_{i\nu,t}, \quad \forall i \neq H, \nu \in \mathcal{P}_s, s \in \mathcal{S}_T \\
q_{H\nu,t} &= c_{H\nu,t} + \sum_{r\in\mathcal{R}, s\in\mathcal{S}_T} q_{H\nu rs,t}, \quad \forall \nu \in \mathcal{P}_s, s \in \mathcal{S}_T \\
q^F_{i\nu,t} &= c_{i\nu,t} + \sum_{r\in\mathcal{R}, s\in\mathcal{S}_T} q_{i\nu rs,t}, \quad \forall i \neq H, \nu \in \mathcal{P}_s, s \in \mathcal{S}_T \\
Q_{rNT,t} &= C_{rNT,t}, \quad \forall r \in \mathcal{R}
\end{aligned}
\tag{18}
$$

- Competitive equilibrium: given tariffs $\tau_t \equiv \left\{\tau^H_t, \tau^F_t\right\}$, a competitive equilibrium corresponds to prices $p_t \equiv \left\{p^H_t, p^F_t, \bar{p}^H_t, \bar{p}^F_t, w_{rs}, p_{rNT,t}\right\}$, quantities $q_t \equiv \left\{c_{i\nu,t}, c^F_{i\nu,t}, q_{i\nu rs,t}, q_{i\nu,t}, q^F_{i\nu,t}, Q_{rNT,t}\right\}$, and a lump-sum transfer T_t such that the US household maximizes its utility; US firms maximize their profits; import and export prices are non-arbitrage; US government's budget constraint holds; good markets clear.

Adao, Costinot, and Donaldson (2024): Application

- Parameters
  - Time-invariant: $\theta \equiv \{\sigma, \omega_F, \sigma_F, \eta, \kappa\}$
  - Time-varying:
    $\epsilon_t = \left\{\beta_{s,t}, A_{Hs,t}, A_{Fs}, a_{H\nu}, a_{F\nu}, a_{i\nu}, Z_{rs,t}, \alpha_{ls,t}, \alpha_{Ls,t}, \alpha_{ks,t}, z_{i\nu,t}, a^F_{i\nu,t}, z^F_{i\nu}, F_t, L_{rs,t}\right\}$
- FGKK estimates of elasticities: US tariffs and foreign retaliatory tariffs $\Rightarrow$ Trade floes
  - domestic vs. foreign: $\kappa = 1.19$
  - different products within domestic or foreign: $\eta = 1.53$
  - across different foreign origins within foreign: $\sigma = 2.53$
  - foreign imported demand: $\sigma_F = 1.04$
  - foreign inverse supply: $\omega_F = 0$

Adao, Costinot, and Donaldson (2024): Application

- Calibrate (ϵ_t):
  - Given estimated θ and the initial period tariff τ_t
  - FGKK's model exactly matches trade and production date from the US in 2016
    1. variety-level quantities and values for both exports and imports (71 foreign countries i; 10228 tradable products ν; 88 tradable sectors s)
    2. sector-level revenues and expenditures on both labor and intermediates for each tradable s
    3. region-sector employment, with each region r a US county

Adao, Costinot, and Donaldson (2024): Application

- The first-order impact of a change in tariffs on US welfare (envelope theorem):

$$dW = \sum_{i \neq H, \nu} q_{i\nu,t} d\bar{p}^{H}_{i\nu,t} - \sum_{i \neq H, \nu} q^{F}_{i\nu,t} d\bar{p}^{F}_{i\nu,t} + \sum_{i \neq H, \nu} \tau^{H}_{i\nu,t} \bar{p}^{F}_{i\nu,t} dq^{F}_{i\nu,t} \tag{19}$$

- Re-expression:

$$W(\Delta x) = \sum_{i,\nu} \left[\underbrace{\omega^{X}_{i\nu} \left(\Delta x^{X}_{i\nu} \right)}_{\text{exports}} + \underbrace{\omega^{M}_{i\nu} \left(-\Delta x^{M}_{i\nu} \right)}_{\text{imports}} + \underbrace{\omega^{R}_{i\nu} \left(\Delta x^{R}_{i\nu} \right)}_{\text{tariff revenues}} \right] \tag{20}$$

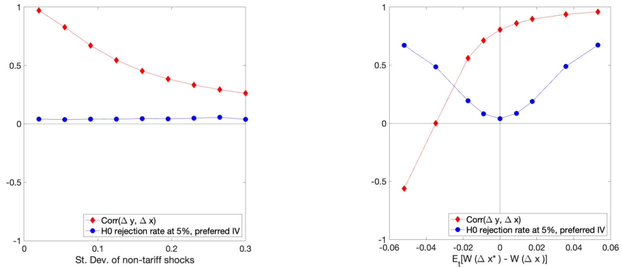
Adao, Costinot, and Donaldson (2024): Application

- Monte Carlo simulations: $b = 1, \ldots, 1500$ counterfactual economies
  - counterfactual tariff changes, $(\Delta\tau)^b$: IID normal with mean $\mu_{\Delta\tau} = 0.02$ and s.d. $\sigma_{\Delta\tau} = 0.06$
  - counterfactual other changes, $(\Delta\epsilon)^b$: IID normal with mean $\mu_{\Delta\epsilon} = 0$ and s.d. $\sigma_{\Delta\epsilon} = 0.06$
  - assume either (i) FGKK's model is true; or (ii) it's subject to some misspecification
  - compute the counterfactual changes in all equilibrium variables, $(\Delta y)^b$
  - compute the causal impact of changes in tariffs in the true model and the researcher's model, $(\Delta x^*)^b$ and $(\Delta x)^b$, respectively

Adao, Costinot, and Donaldson (2024): Application

Figure 2: Correlation- versus IV-based Tests

(a) No misspecification **(b)** Uniform misspecification



Notes: This figure reports the rejection rate of an IV-based test at a 5% significance level (blue circles) and the correlation between $(\Delta y_n)^b$ and $(\Delta x_n)^b$ (red diamonds) across $b = 1, ..., 1500$ simulated economies. The shifters and shares entering the IV $(z_n^{\text{pref}})^b$ are described in equations (9) and (10). Figure 2a focuses on the case where the researcher's model is the true model and varies the standard deviation of other shocks σ_ϵ used in the simulations. Figure 2b instead assumes that the researcher's model misspecifies the pass-through rate of all statutory tariffs into applied tariffs and varies the true pass-through rate used in the simulations, which results in the corresponding amount of average welfare misspecification $E_t[W(\Delta x^*) - W(\Delta x)]$ shown on the x-axis.

Adao, Costinot, and Donaldson (2024): Application

- US-China trade war: Trumpian tariffs + foreign retaliatory tariffs
- FGKK's conclusion: the aggregate real income loss was \$7.2 billion, or 0.04% of GDP
- Shocks fed into the researcher's model:
  - Tariff shocks: data
  - Non-tariff shocks: FGKK's model matches the changes in outcomes Δy_n observed over the period from January 2018 to April 2019

Adao, Costinot, and Donaldson (2024): Application

Table 1: Testing Predictions about the Welfare Impact of the US-China Trade War

Goodness-of-fit measure:	Correlation	IV-Based Test	
		Naive IV	Preferred IV
	$Corr\left(\Delta y_n, \Delta x_n(\hat{\theta})\right)$	$\hat{\beta}_{z^{\text{naive}}}(\hat{\theta})$	$\hat{\beta}_{z^{\text{pref}}}(\hat{\theta})$
	(1)	(2)	(3)
Point estimate	0.04	0.59	0.15
Inference ignoring estimation uncertainty			
Std. error		0.66	0.21
p-value of H0: $\hat{\beta} = 0$		0.36	0.48
Inference accounting for estimation uncertainty			
Std. error		0.67	0.23
p-value of H0: $\hat{\beta} = 0$		0.37	0.52

Notes: All statistics are based on the pooled sample of changes in 24,193 welfare-relevant outcomes, and FGKK's estimates $\hat{\theta}$, as described in Section 4.1. Column (1) reports the correlation between actual changes and the predicted impact of the US-China trade war across all outcomes. Columns (2) and (3) implement our IV-based test using the naive IV z^{naive}, as defined by equations (11) and (13), and our preferred IV z^{pref}, as defined by equations (10) and (13). Inference ignoring estimation uncertainty is as described in Section 2.3. Inference accounting for estimation uncertainty is as described in Appendix C.1.

Comments

- FGKK's model: partial equilibrium in other countries + first-order approximation
  - Simple sufficient statistics for welfare changes
  - May not be ideal for analyzing big policy shocks such as the US-China trade wars
- Assignment W11:
  - validate predictions of a multi-country GE model on the US-China trade wars, utilizing the exact-hat algebra
  - IV-based test proposed by ACD is still applicable