

Demand Estimation

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Demand Estimation

- Demand parameters for a class of oligopolistic differentiated products markets
 - Own- and cross-price elasticities
 - Demands with respect to product characteristics
- Policy evaluations based on the industry model: e.g.
 - Trade policies: imported prices \Rightarrow market shares of domestic firms
 - Merger policies: M&A \Rightarrow market shares of other firms
 - Environmental policies: regulations on products with certain characteristics

Demand Estimation: Berry et al. (1995, BLP)

- The utility of consumer i for product $j = 0, 1, \dots, J$ at time t :

$$u_{ijt} = \underbrace{x'_{jt}}_{\text{characteristics}} \beta_i + \underbrace{\xi_{jt}}_{\text{market-level shock}} + \underbrace{\varepsilon_{ijt}}_{\text{idio. shock} \sim F_\varepsilon} \quad (1)$$

- Random taste parameters:

$$\beta_i = \bar{\beta} + \eta_i, \quad \eta_i \sim F_\eta(\cdot | \Sigma) \quad (2)$$

- Therefore,

$$u_{ijt} = \underbrace{x'_{jt}\bar{\beta} + \xi_{jt}}_{\delta_{jt} \equiv \delta_{jt}(\bar{\beta}, \xi_{jt})} + x'_{jt}\eta_i + \varepsilon_{ijt} \quad (3)$$

BLP (1995): Share Inversion

- Suppose that ε is drawn i.i.d. from a Type-I Extreme Value distribution. Then the market share for good j at time t :

$$\begin{aligned}\sigma_{jt} &= \int \mathbf{1} \{ u_{ijt} \geq u_{ikt}, \forall k \neq j \} dF_\eta(\eta_i | \Sigma) dF_\varepsilon(\varepsilon_{it}) \\ &\quad \int \frac{\exp \{ \delta_{jt} (\bar{\beta}, \xi_{jt}) + x'_{jt} \eta_i \}}{1 + \sum_{k=1}^J \exp \{ \delta_{kt} (\bar{\beta}, \xi_{kt}) + x'_{kt} \eta_i \}} dF_\eta(\eta_i | \Sigma)\end{aligned}\tag{4}$$

- Given the observed market share s_{jt} , we want to recover δ_{jt} :

$$s_{jt} = \sigma_{jt}(\delta_{1t}, \dots, \delta_{Jt}, \Sigma) \Rightarrow \delta_{jt} = \sigma_{jt}^{-1}(s_{1t}, \dots, s_{Jt}, \Sigma)\tag{5}$$

BLP (1995): Contraction Mapping

- Given Σ , initial guess $(\sigma_{jt}^{(0)})$:

$$\delta_{jt}^{(h+1)} = \delta_{jt}^{(h)} + \log(s_{jt}) - \log \left(\int \frac{\exp \left\{ \delta_{jt} (\bar{\beta}, \xi_{jt}) + x'_{jt} \eta_i \right\}}{1 + \sum_{k=1}^J \exp \left\{ \delta_{kt} (\bar{\beta}, \xi_{kt}) + x'_{kt} \eta_i \right\}} dF_\eta(\eta_i | \Sigma) \right) \quad (6)$$

- Monte Carlo integration:
 - Draw a set of $\eta_i \sim F_\eta(\eta_i | \Sigma)$ prior to the contraction mapping
 - For each iteration, plug η_i into the share expression and take an average

BLP (1995): Estimation

- Suppose that there is an instrumental variable z_{jt} such that

$$E [\xi_{jt} | z_{jt}, x_{jt}] = 0 \quad (7)$$

- BLP instrument: product characteristics of other firms in the same market
- affect price due to competition but are uncorrelated with unobserved demand shocks
- Sample moments:

$$E [\zeta (\Sigma)] = 0, \quad \zeta (\Sigma) = \frac{1}{TJ} \sum_{t=1}^T \sum_{j=1}^J \xi_{jt} (\Sigma) z_{jt} \quad (8)$$

- GMM estimator:

$$\min_{\Sigma} Q (\Sigma) = \zeta (\Sigma)' W \zeta (\Sigma) \quad (9)$$

BLP (1995): Estimation

- Outer loop: search over Σ (e.g. quasi-Newton)
- Inner loop:
 1. Use the contraction mapping to solve for δ_{jt}
 2. Use 2SLS to estimate linear coefficient $\bar{\beta}$: $\delta_{jt} = x'_{jt}\bar{\beta} + \xi_{jt}$
 3. Obtain the residual $\hat{\xi}_{jt} = \delta_{jt} - x'_{jt}\hat{\beta}$ and construct the sample moments as
$$\hat{\zeta}(\Sigma) = \frac{1}{TJ} \sum_{t=1}^T \sum_{j=1}^J \hat{\xi}_{jt}(\Sigma) z_{jt}$$
 4. Evaluate the GMM objective function: $Q(\Sigma) = \hat{\zeta}(\Sigma)' W \hat{\zeta}(\Sigma)$

Comments

- BLP (1995) utilize industry-level data to estimate the demand model: macro BLP
- Utilizing firm-level or consumer-level data could facilitate model identification:
e.g. BLP (2004) micro BLP