Demand Estimation

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Demand Estimation

- Demand parameters for a class of oligopolistic differentiated products markets
  - Own- and cross-price elasticities
  - Demands with respect to product characteristics
- Policy evaluations based on the industry model: *e.g.*
  - Trade policies: imported prices $\Rightarrow$ market shares of domestic firms
  - Merger policies: M&A $\Rightarrow$ market shares of other firms
  - Environmental policies: regulations on products with certain characteristics

Demand Estimation: Berry et al. (1995, BLP)

- The utility of consumer i for product $j = 0, 1, \ldots, J$ at time t:

$$u_{ijt} = \underbrace{x'_{jt}}_{\text{characteristics}} \beta_i + \underbrace{\xi_{jt}}_{\text{market-level shock}} + \underbrace{\varepsilon_{ijt}}_{\text{idio. shock} \sim F_\varepsilon} \tag{1}$$

- Random taste parameters:

$$\beta_i = \bar{\beta} + \eta_i, \quad \eta_i \sim F_\eta \left(.|\Sigma\right) \tag{2}$$

- Therefore,

$$u_{ijt} = \underbrace{x'_{jt}\bar{\beta} + \xi_{jt}}_{\delta_{jt} \equiv \delta_{jt}\left(\bar{\beta}, \xi_{jt}\right)} + x'_{jt}\eta_i + \varepsilon_{ijt} \tag{3}$$

BLP (1995): Share Inversion

- Suppose that ε is drawn i.i.d. from a Type-I Extreme Value distribution. Then the market share for good j at time t:

$$\begin{aligned}\sigma_{jt} &= \int \mathbf{1}\left\{u_{ijt} \geq u_{ikt}, \forall k \neq j\right\} dF_{\eta}\left(\eta_i | \Sigma\right) dF_{\varepsilon}\left(\varepsilon_{it}\right) \\ &\int \frac{\exp\left\{\delta_{jt}\left(\bar{\beta}, \xi_{jt}\right) + x_{jt}'\eta_i\right\}}{1 + \sum_{k=1}^{J} \exp\left\{\delta_{kt}\left(\bar{\beta}, \xi_{kt}\right) + x_{kt}'\eta_i\right\}} dF_{\eta}\left(\eta_i | \Sigma\right)\end{aligned} \tag{4}$$

- Given the observed market share s_{jt}, we want to recover δ_{jt}:

$$s_{jt} = \sigma_{jt}\left(\delta_{1t}, \ldots, \delta_{Jt}, \Sigma\right) \Rightarrow \delta_{jt} = \sigma_{jt}^{-1}\left(s_{1t}, \ldots, s_{Jt}, \Sigma\right) \tag{5}$$

BLP (1995): Contraction Mapping

- Given Σ, initial guess $\left(\sigma_{jt}^{(0)}\right)$:

$$\delta_{jt}^{(h+1)} = \delta_{jt}^{(h)} + \log\left(s_{jt}\right) - \log\left(\int \frac{\exp\left\{\delta_{jt}\left(\bar{\beta}, \xi_{jt}\right) + x_{jt}'\eta_i\right\}}{1 + \sum_{k=1}^{J} \exp\left\{\delta_{kt}\left(\bar{\beta}, \xi_{kt}\right) + x_{kt}'\eta_i\right\}} dF_{\eta}\left(\eta_i | \Sigma\right)\right) \tag{6}$$

- Monte Carlo integration:
  - Draw a set of $\eta_i \sim F_{\eta}\left(\eta_i | \Sigma\right)$ prior to the contraction mapping
  - For each iteration, plug η_i into the share expression and take an average

BLP (1995): Estimation

- Suppose that there is an instrumental variable z_{jt} such that

$$E\left[\xi_{jt} | z_{jt}, x_{jt}\right] = 0 \tag{7}$$

  - BLP instrument: product characteristics of other firms in the same market
  - affect price due to competition but are uncorrelated with unobserved demand shocks

- Sample moments:

$$E\left[\zeta\left(\Sigma\right)\right] = 0, \quad \zeta\left(\Sigma\right) = \frac{1}{TJ}\sum_{t=1}^{T}\sum_{j=1}^{J}\xi_{jt}\left(\Sigma\right) z_{jt} \tag{8}$$

- GMM estimator:

$$\min_{\Sigma} Q\left(\Sigma\right) = \zeta\left(\Sigma\right)' W \zeta\left(\Sigma\right) \tag{9}$$

BLP (1995): Estimation

- Outer loop: search over Σ (*e.g.* quasi-Newton)
- Inner loop:
  1. Use the contraction mapping to solve for δ_{jt}
  2. Use 2SLS to estimate linear coefficient $\bar{\beta}$: $\delta_{jt} = x_{jt}'\bar{\beta} + \xi_{jt}$
  3. Obtain the residual $\hat{\xi}_{jt} = \delta_{jt} - x_{jt}'\hat{\bar{\beta}}$ and construct the sample moments as $\hat{\zeta}(\Sigma) = \frac{1}{TJ}\sum_{t=1}^{T}\sum_{j=1}^{J}\hat{\xi}_{jt}(\Sigma)\, z_{jt}$
  4. Evaluate the GMM objective function: $Q(\Sigma) = \hat{\zeta}(\Sigma)' W \hat{\zeta}(\Sigma)$

Comments

- BLP (1995) utilize industry-level data to estimate the demand model: macro BLP
- Utilizing firm-level or consumer-level data could facilitate model identification: *e.g.* BLP (2004) micro BLP