Econ7115: Structural Models and Numerical Methods in Economics
Assignment W04 (Comprehensive)

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1. Consider N countries in the world. We index countries by $n = 1, 2, \ldots, N$. Country n includes S_n internal regions. We denote the set of regions in country n by $\mathcal{L}_n$. We denote countries by i and n and regions by ℓ and m. Country n is endowed with $\bar{L}_n$ labors. Labors are immobile across countries but mobile within country.

   Each region produces a distinctive variety of goods. In region $\ell \in \mathcal{L}_n$, the representative consumer has a CES preference over varieties from all regions:

   $$U_\ell \equiv B_\ell \left[\sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} C_{m\ell}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1, \tag{1}$$

   where $C_{m\ell}$ is the quantity of variety m consumed by the representative consumer in region ℓ and B_ℓ is the amenity in region ℓ that can be expressed as

   $$B_\ell \equiv \bar{B}_\ell L_\ell^{-\mu_n}, \quad \mu_n \geq 0, \tag{2}$$

   where L_ℓ is the labor in region ℓ.

   Variety ℓ is produced by labor under perfect competition. The unit cost of variety ℓ is

   $$c_\ell = \frac{w_\ell}{A_\ell}, \quad A_\ell \equiv \bar{A}_\ell L_\ell^{\alpha} \tag{3}$$

   where w_ℓ is the wage in region ℓ, $\bar{A}_\ell$ is the exogenous productivity, and $\alpha \geq 0$ represents local agglomeration externality.

   Trade from region ℓ to region m incurs an iceberg trade cost $\tau_{\ell m} \geq 1$ with $\tau_{\ell\ell} = 1$. Notice that $\tau_{\ell m}$ represents both domestic and international trade costs. Moreover, trade from region $\ell \in \mathcal{L}_i$ to $m \in \mathcal{L}_n$ also incurs import tariffs $t_{in} \geq 0$ with $t_{ii} = 0$.

   1. Denote $X_{\ell m}$ as the trade value from region $\ell \in \mathcal{L}_i$ to $m \in \mathcal{L}_n$. Denote X_m as the total expenditure in region m. Please derive the expressions for $\lambda_{\ell m} \equiv \frac{X_{\ell m}}{X_m}$.

2. Please derive the expressions for the aggregate price index in region m, P_m.
3. Please derive the expressions for equilibrium labor allocation within each country i, $(L_\ell)_{\ell \in \mathcal{L}_i}$.
4. Suppose that tariff revenues are distributed evenly to all workers in the importing country. Please derive the equilibrium system.
5. Please define the problem for country 1 to choose its import tariffs in order to maximize its national welfare.
6. Please derive the equilibrium in relative changes. Which parameters are required to conduct this "exact-hat" algebra?
7. Please define the optimal import tariffs in country 1 utilizing the "exact-hat" algebra above.

2. Consider the following special case of the model above: there are two countries, $N = 2$. Each country has two regions. Denote $\mathcal{L}_1 = \{1, 2\}$ and $\mathcal{L}_2 = \{3, 4\}$. Domestic trade costs are assumed to be $\tau_{12} = \tau_{21} = \tau_{34} = \tau_{43} = 1.5$. We assume that region 1 and 3 are coastal regions and region 2 and 4 are inland regions. We assume that coastal regions are directly connected via international trade, whereas inland regions are engaged into international trade through coastal regions. In particular, $\tau_{14} = \tau_{41} = \tau_{13}\tau_{34}$, $\tau_{23} = \tau_{32} = \tau_{31}\tau_{12}$ and $\tau_{24} = \tau_{42} = \tau_{21}\tau_{13}\tau_{34}$. We set $\tau_{13} = \tau_{31} = 1.2$. Moreover, there is no tariff initially, *i.e.* $t_{\ell m} = 0$ for all (ℓ, m).

   Countries and regions are symmetric in size, technology and amenity: $\bar{L}_1 = \bar{L}_2 = 1$, $\bar{A}_\ell = \bar{B}_\ell = 1$ for any region ℓ. Moreover, we assume that $\theta \equiv \sigma - 1 = 4$ and $\alpha = 0.1$. In our baseline case, we set $\mu_1 = \mu_2 = 0.5$. We take the following numeraire: $\sum_{i=1}^{N} \sum_{\ell \in \mathcal{L}_i} w_\ell = 1$.

   1. Please compute the equilibrium outcomes under zero tariffs.
   2. Please derive the unilaterally optimal import tariffs for country 1, assuming country 2 imposes zero import tariffs.
   3. Please derive the Nash tariffs for country 1 and 2.
   4. Reduce μ_1 from 0.5 to 0.4 and recompute the above three exercises. Discuss the computational results.