Demand under the CES preference

Consider a consumer with nominal income X. She consumes N types of goods, with the following CES utility function:

$$U = \left[\sum_{i=1}^{N} C_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where C_i is the consumed quantity of good i.

The consumer faces price p_i for good i. The consumer's problem is then

$$\max_{(C_i)_{i=1}^N} U = \left[\sum_{i=1}^{N} C_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad s.t. \sum_{i=1}^{N} p_i C_i = X$$

Please show the following:

$$X_i \equiv p_i C_i = p_i^{1-\sigma} P^{\sigma-1} X$$

where $P \equiv \left[\sum_{i=1}^{N} p_i^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$.