UM-SJTU Joint Institute
Physicis Laboratory II
(VP241)

Laboratory Report

Exercise 2

The Hall Probe: Characteristics and Applications

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1 Introduction

1.1 Hall Effect

Consider a conducting sheet (made of a metal or a semiconductor) placed in a magnetic field so that the plane of the sheet is perpendicular to the direction of the magnetic field B (Figure 1). When the electric current I passes through the sheet in the direction shown in Figure 1, an electric potential difference between the sides a and b of the sheet is generated. The corresponding electric field is perpendicular to both the direction of the current and the direction of the magnetic field. This effect is known as the Hall effect, and the electric potential difference is called the Hall voltage U_H.

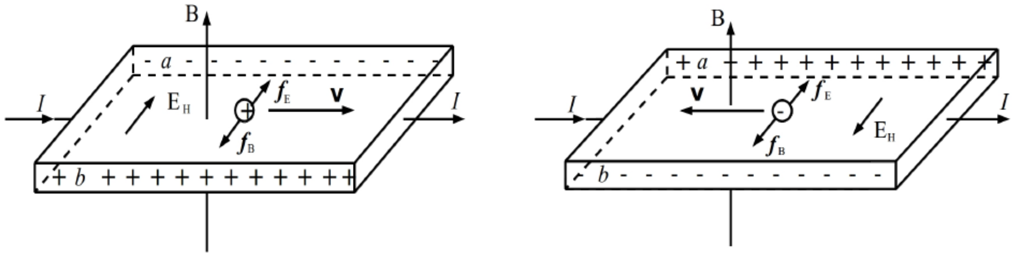

Figure 1: The Principle of Hall Effect

The origin of the Hall Voltage comes from the Lorentz force $\mathbf{F}_B$. It is perpendicular to the magnetic field and the moving direction of the charge carring particle. So the particles' moving direction is deviated, and consequently an electric field is generated. The electric field also imposes a force upon the particles. In the end, the electric force balances the Lorentz force and a stablized Hall voltage is generated.

When the external magnetic field is not very large, the Hall Voltage is proportional to both the current and the magnitude of the magnetic field, and inversely proportional to the thickness of the sheet d:

$$U_H = R_H \frac{IB}{d} = KIB, \tag{1}$$

where R_H is the so-called Hall coefficient and $K = R_H/d = K_H/I$, where K_H is the so-called sensitivity of the Hall element.

1.2 Integrated Hall Probe

The magnitude of the magnetic field can be found by measuring the Hall voltage with a Hall probe when the sensitivity K_H and the current I are fixed. Since the Hall Voltage is usually very small, it should be amplified after it is measured. Silicon can be used to design both the Hall probe and the integrated circuits, so it is convenient to arrange the Hall probe and the electric circuits into a single device. Such a device is called an integrated Hall probe.

The integrated Hall probe SS495A is made up of a Hall sensor, an amplifier and a voltage compensator (Figure 2). The relation between the output voltage U and the magnitude of the magnetic field is

$$B = \frac{U - U_0}{K_H} \tag{2}$$

1.3 Magnetic Field Distribution Inside a Solenoid

The magnetic field distribution on the axis of a single layer solenoid can be calculated from the following formula

$$B(x) = \mu_0 \frac{N}{L} I_M \left\{ \frac{L+2x}{2\sqrt{D^2 + (L+2x)^2}} + \frac{L-2x}{2\sqrt{D^2 + (L-2x)^2}} \right\} = C(x) I_M, \tag{3}$$

where N is the number of turns of the solenoid, L is its length, I_M is the current through the solenoid wire, and D is the solenoid's diameter. The magnetic permeability of vacuum is $\mu_0 = 4\pi \times 10^{-7}$ H/m.

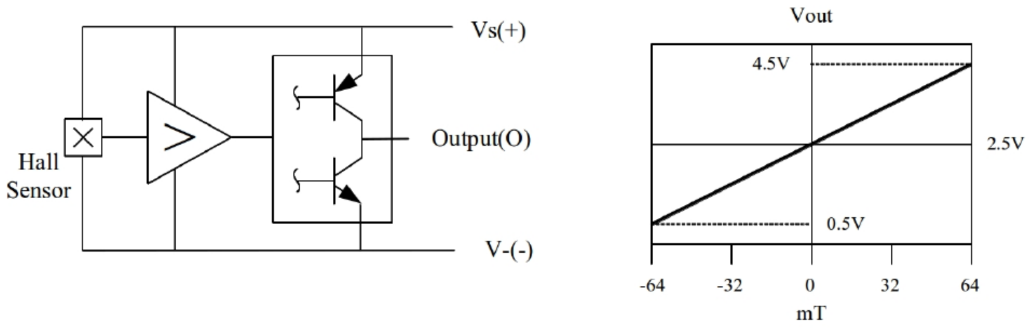

Figure 2: The Circuit Inside the Integrated Hall Probe SS495A(Left). The Relation between the Output Voltage U and the Magnitude of the Magnetic Field B(Right).

Based on the information given in the manual, the theoretical magnitude of the magnetic field at the center of the solenoid when $I_M = 0.1\text{A}$ is $B = 1.4366[\text{mT}]$(The uncertainty of this data is zero). Table 1 lists the theoretical value of the magnitude of the magnetic field when $I = 0.1\text{A}$.

x[cm]	B[mT]	x[cm]	B[mT]
± 0.0	1.4366	± 8.0	1.4057
± 1.0	1.4363	± 9.0	1.3856
± 2.0	1.4356	± 10.0	1.3478
± 3.0	1.4343	± 11.0	1.2685
± 4.0	1.4323	± 11.5	1.1963
± 5.0	1.4292	± 12.0	1.0863
± 6.0	1.4245	± 12.5	0.9261
± 7.0	1.4173	± 13.0	0.7233

Table 1: Theoretical Value of the Magnetic Field inside the Solenoid

2 Apparatus & Measurement Procedure

2.1 Apparatus

The experimental setup shown in Figure 3 consists of an integrated Hall probe SS495A (see Figure 4) with $K_H = 31.25 \pm 1.25$ [V/T] (at the working voltage 5 V), a solenoid, a power supply, a voltmeter, a DC voltage divider, and a set of connecting wires.

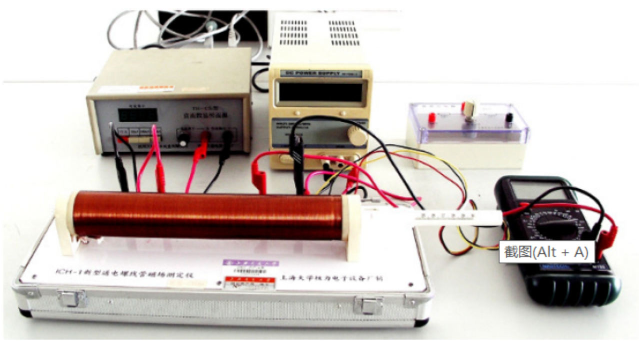

Figure 3: Measurement Setup

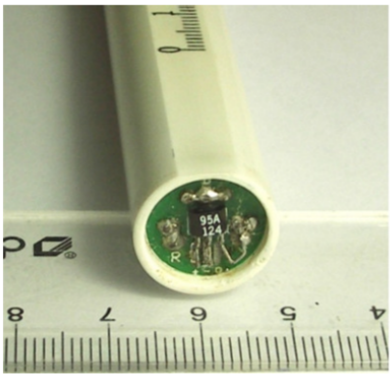

Figure 4: Integrated Hall probe SS495A.

Physical Quantity	Uncertainty
Voltage source	0.5%[V]
Distance	0.05[cm]
Current source	2%[mA]
Voltmeter	$0.05\% + 6 \times 10^{-3}$ or 6×10^{-4}[V]

Table 2: The Uncertainty of the Equipment Used in the Lab

2.2 Measurement Procedure

2.2.1 Relation Between Sensitivity K_H and Working Voltage U_S

1. I first placed the integrated hall probe at the center of the solenoid(let the reading on the solenoid be 13[cm]). Then I set the working voltage to be $U_S = 5$[V] and measured the output voltage $U_0(I_M = 0)$ and $U(I_M = 250$[mA]). Then I took the theoretical value of $B(x = 0)$ from Section 1.3 and calculated the sensitivity of the probe K_H^* using Equation 2.
2. Then I measured K_H for different values of U_S (from 2.8 V to 10 V) and calculated K_H/U_S and plotted the curve K_H/U_S vs. U_S.

2.2.2 Relation Between Output Voltage U and Magnetic Field B

1. I first let the input current to be zero so that $B = 0$ and the working voltage $U_S = 5$[V], while the probe remains to be at the center of the solenoid. Then I connected the 2.4-2.6V output terminal of the DC voltage divider and the negative port of the voltmeter. Then I adjust the resistance of the divider until the output voltage is 0.
2. Then I measured the output voltage U for different values of I_M ranging from 0 to 500mA, with intervals of 50mA.

2.2.3 Magnetic Field Distribution Inside the Solenoid

1. I set the input current $I_M = 250$[mA], and then I recorded the output voltage U and the corresponding position x.

3 Result

3.1 Relation Between Sensitivity K_H and Working Voltage U_S

Table 3 and 4 are two original data tables for this section. We first need to calculate the magnetic field in the center of the solenoid when $I_M = 250$[mA]:

$$B = B' \frac{I_M}{I_0} = 1.4366 \times 10^{-3} \times \frac{250}{100} = 0.00359[\mathrm{T}] \pm 0.00007[\mathrm{T}].$$

U_S[V]	Uncertainty[V]	$U_0(I_M = 0)$[V]	Uncertainty[V]	$U(I_M = 250\text{mA})$[V]	Uncertainty[V]
5.00	0.003	2.472	0.007	2.599	0.007

Table 3: The Original Data for K_H^*

U_S[V]	Uncertainty[V]	U_0[V]	Uncertainty[V]	U[V]	Uncertainty[V]
2.840	0.014	1.4004	0.0013	1.4689	0.0013
3.200	0.016	1.5777	0.0014	1.6552	0.0014
3.660	0.018	1.8076	0.0015	1.8963	0.0015
4.04	0.02	1.9969	0.0016	2.0946	0.0016
4.38	0.02	2.161	0.007	2.266	0.007
4.80	0.02	2.373	0.007	2.489	0.007
5.23	0.03	2.584	0.007	2.711	0.007
5.63	0.03	2.785	0.007	2.919	0.007
6.01	0.03	2.971	0.007	3.107	0.008
6.43	0.03	3.175	0.008	3.321	0.008
6.80	0.03	3.362	0.008	3.515	0.008
7.26	0.04	3.583	0.008	3.736	0.008
7.65	0.04	3.778	0.008	3.940	0.008
8.07	0.04	3.984	0.008	4.152	0.008
8.39	0.04	4.140	0.008	4.310	0.008
8.95	0.04	4.418	0.008	4.592	0.008
9.42	0.05	4.647	0.008	4.826	0.008
9.92	0.05	4.889	0.008	5.075	0.009

Table 4: The Original Data

Then we calculate the value of K_H^* using data in Table 3.

$$K_H^* = \frac{U - U_0}{B} = \frac{2.599 - 2.472}{0.00359} = 35[\text{V/T}] \pm 3[\text{V/T}].$$

Then we calculate the value of K_H using data in Table 4 and get the data in Table 5. For example, when $U_0 = 1.4004$[V], $U = 1.4689$[V], $U_S = 2.840$[V],

$$K_H = \frac{U - U_0}{B} = \frac{1.4689 - 1.4004}{0.00359} = 19.1[\text{V/T}] \pm 0.6[\text{V/T}]$$
$$\frac{K_H}{U_S} = \frac{19.1}{2.84} = 6.7[\text{T}^{-1}] \pm 0.2[\text{T}^{-1}]$$

K_H[V/T]	Uncertainty[V/T]	K_H/U_S[T^{-1}]	Uncertainty[T^{-1}]	U_S[V]	Uncertainty[V]
19.1	0.6	6.7	0.2	2.840	0.014
21.6	0.7	6.8	0.2	3.200	0.016
24.7	0.8	6.7	0.2	3.660	0.018
27.2	0.8	6.7	0.2	4.04	0.02
29	3	6.6	0.7	4.38	0.02
32	3	6.7	0.6	4.80	0.02
35	3	6.7	0.6	5.23	0.03
37	3	6.6	0.5	5.63	0.03
38	3	6.3	0.5	6.01	0.03
41	3	6.4	0.5	6.43	0.03
43	3	6.3	0.4	6.80	0.03
43	3	5.9	0.4	7.26	0.04
45	3	5.9	0.4	7.65	0.04
47	3	5.8	0.4	8.07	0.04
47	3	5.6	0.4	8.39	0.04
48	3	5.4	0.3	8.95	0.04
50	3	5.3	0.3	9.42	0.05
52	4	5.2	0.4	9.92	0.05

Table 5: The Data for K_H vs. U_S.

Using the last four columns of Table 5, I obtained Figure 5.

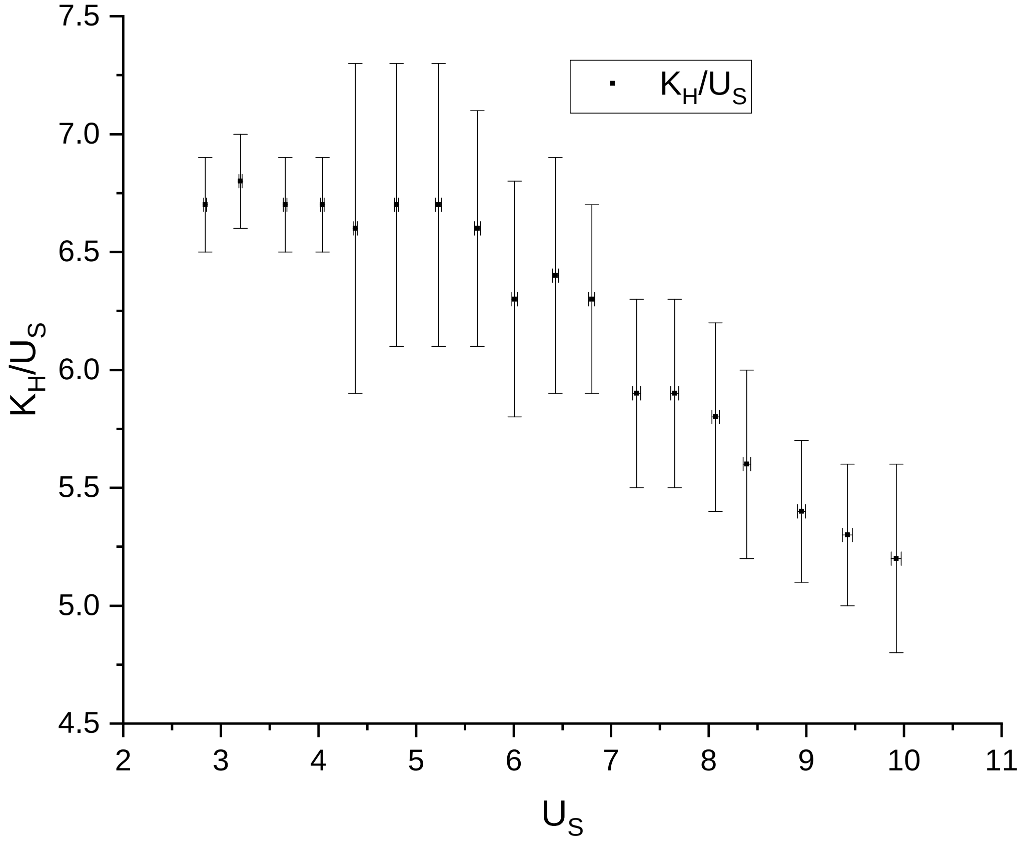

Figure 5: The Relation of K_H/U_S vs. U_S

3.2 Relation Between Output Voltage U and Magnetic Field B

Table 6 is the original data of the output voltage under different input current. Use the relation

$$B = B' I_M / I_0,$$

we can calculate the magnitude of the magnetic field. For example, when $I_M = 0.200$[A],

$$B = 1.4366 \times 10^{-3} \times 0.200/0.1 = 0.00287[\mathrm{T}] \pm 0.00006[\mathrm{T}].$$

I_M[A]	Uncertainty[A]	U[V]	Uncertainty[V]
0	0	0.0000	0.0006
0.0500	0.0010	0.0263	0.0006
0.100	0.002	0.0530	0.0006
0.150	0.003	0.0720	0.0006
0.200	0.004	0.0996	0.0006
0.250	0.005	0.1222	0.0007
0.300	0.006	0.1413	0.0007
0.350	0.007	0.1679	0.0007
0.400	0.008	0.1901	0.0007
0.450	0.009	0.2116	0.0007
0.500	0.010	0.2373	0.0007

Table 6: The Original Data

Then I used the data in Table 7 to apply linear fit and got Figure 6. From it, we can read the slope, whose physical meaning is K_H, as

$$K_H = 32.6[\mathrm{V/T}] \pm 0.7[\mathrm{V/T}].$$

B[T]	Uncertainty[T]	U[V]	Uncertainty[V]
0	0	0.0000	0.0006
0.000718	0.000014	0.0263	0.0006
0.00144	0.00003	0.0530	0.0006
0.00215	0.00004	0.0720	0.0006
0.00287	0.00006	0.0996	0.0006
0.00359	0.00007	0.1222	0.0007
0.00431	0.00009	0.1413	0.0007
0.00503	0.00010	0.1679	0.0007
0.00575	0.00011	0.1901	0.0007
0.00646	0.00013	0.2116	0.0007
0.00718	0.00014	0.2373	0.0007

Table 7: The Data for the Linear Fit

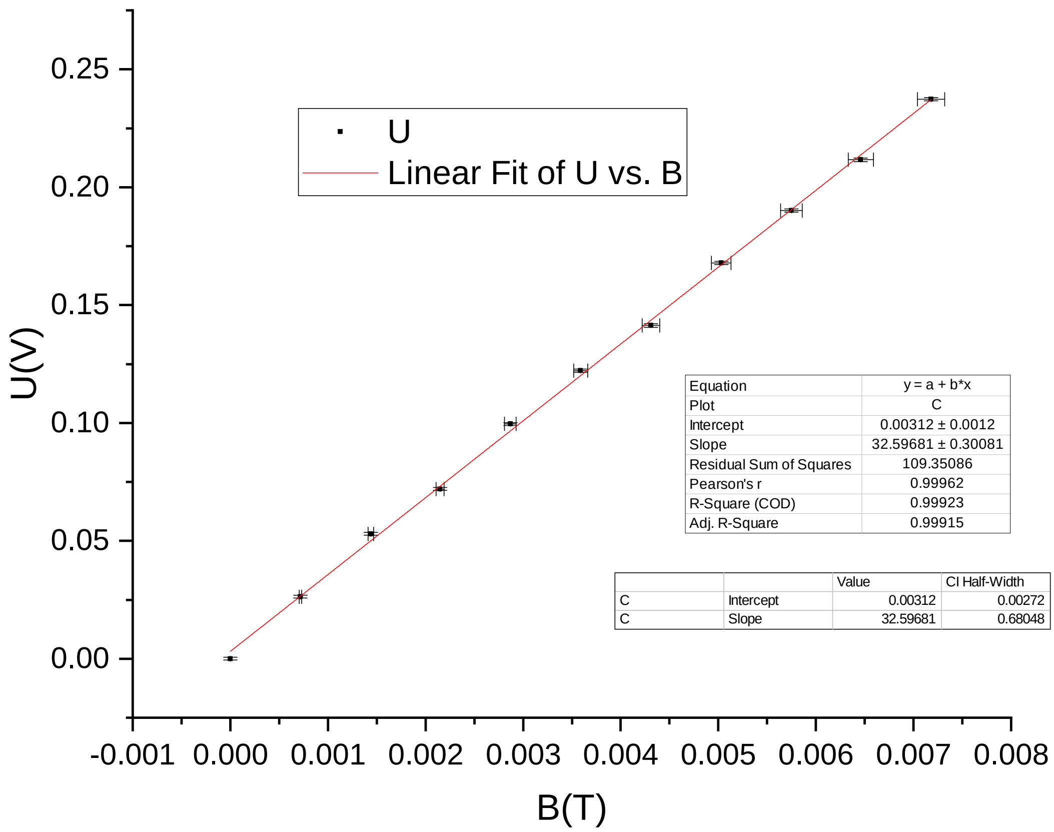

Figure 6: The Linear Fit of U vs. B

3.3 Magnetic Field Distribution Inside the Solenoid

x[cm]	Uncertainty[cm]	U[V]	Uncertainty[V]	x[cm]	Uncertainty[cm]	U[V]	Uncertainty[V]
0.00	0.05	0.0097	0.0006	16.00	0.05	0.1219	0.0007
0.50	0.05	0.0131	0.0006	17.00	0.05	0.1222	0.0007
1.00	0.05	0.0178	0.0006	18.00	0.05	0.1223	0.0007
1.50	0.05	0.0253	0.0006	19.00	0.05	0.1223	0.0007
2.00	0.05	0.0367	0.0006	19.50	0.05	0.1220	0.0007
2.50	0.05	0.0510	0.0006	20.00	0.05	0.1219	0.0007
3.00	0.05	0.0689	0.0006	20.50	0.05	0.1218	0.0007
3.50	0.05	0.0852	0.0006	21.00	0.05	0.1216	0.0007
4.00	0.05	0.0969	0.0006	21.50	0.05	0.1214	0.0007
4.50	0.05	0.1043	0.0007	22.00	0.05	0.1210	0.0007
5.00	0.05	0.1093	0.0007	22.50	0.05	0.1207	0.0007
5.50	0.05	0.1129	0.0007	23.00	0.05	0.1204	0.0007
6.00	0.05	0.1152	0.0007	23.50	0.05	0.1198	0.0007
6.50	0.05	0.1170	0.0007	24.00	0.05	0.1190	0.0007
7.00	0.05	0.1183	0.0007	24.50	0.05	0.1180	0.0007
7.50	0.05	0.1191	0.0007	25.00	0.05	0.1168	0.0007
8.00	0.05	0.1198	0.0007	25.50	0.05	0.1151	0.0007
8.50	0.05	0.1202	0.0007	26.00	0.05	0.1130	0.0007
9.00	0.05	0.1208	0.0007	26.50	0.05	0.1096	0.0007
9.50	0.05	0.1211	0.0007	27.00	0.05	0.1047	0.0007
10.00	0.05	0.1214	0.0007	27.50	0.05	0.0975	0.0006
11.00	0.05	0.1218	0.0007	28.00	0.05	0.0866	0.0006
12.00	0.05	0.1219	0.0007	28.50	0.05	0.0704	0.0006
13.00	0.05	0.1220	0.0007	29.00	0.05	0.0524	0.0006
14.00	0.05	0.1218	0.0007	29.50	0.05	0.0354	0.0006
15.00	0.05	0.1216	0.0007	30.00	0.05	0.0258	0.0006

Table 8: The Original Data for Section 3.3

Table 8 shows the original data of the output voltage for different places inside a solenoid. Since I need to plot a graph with the origin representing the center of the solenoid, and I consider $x = 13$[cm] as the center of the solenoid, in Table 9, all the value of x are subtracted by 13. For example, if $x = 1.00$[cm] in Table 8, $x = 1 - 13 = -12.00$[cm] in Table 1. As for the calculation of the experimental value of B,

$$B = \frac{U}{K_H},$$

where $K_H = 32.6[\mathrm{V/T}] \pm 0.7[\mathrm{V/T}]$ known from Section 3.2. So, for example, when $U = 0.0131$[V],

$$B = \frac{0.0131}{32.6} = 0.40[\mathrm{mT}] \pm 0.02[\mathrm{mT}].$$

x[cm]	u_x[cm]	B[mT]	u_B[mT]	x[cm]	u_x[cm]	B[mT]	u_B[mT]
-13.00	0.05	0.298	0.019	3.00	0.05	3.74	0.08
-12.50	0.05	0.40	0.02	4.00	0.05	3.75	0.08
-12.00	0.05	0.55	0.02	5.00	0.05	3.75	0.08
-11.50	0.05	0.78	0.02	6.00	0.05	3.75	0.08
-11.00	0.05	1.13	0.03	6.50	0.05	3.74	0.08
-10.50	0.05	1.56	0.04	7.00	0.05	3.74	0.08
-10.00	0.05	2.11	0.05	7.50	0.05	3.74	0.08
-9.50	0.05	2.61	0.06	8.00	0.05	3.73	0.08
-9.00	0.05	2.97	0.07	8.50	0.05	3.72	0.08
-8.50	0.05	3.20	0.07	9.00	0.05	3.71	0.08
-8.00	0.05	3.35	0.08	9.50	0.05	3.70	0.08
-7.50	0.05	3.46	0.08	10.00	0.05	3.69	0.08
-7.00	0.05	3.53	0.08	10.50	0.05	3.67	0.08
-6.50	0.05	3.59	0.08	11.00	0.05	3.65	0.08
-6.00	0.05	3.63	0.08	11.50	0.05	3.62	0.08
-5.50	0.05	3.65	0.08	12.00	0.05	3.58	0.08
-5.00	0.05	3.67	0.08	12.50	0.05	3.53	0.08
-4.50	0.05	3.69	0.08	13.00	0.05	3.47	0.08
-4.00	0.05	3.71	0.08	13.50	0.05	3.36	0.08
-3.50	0.05	3.71	0.08	14.00	0.05	3.21	0.07
-3.00	0.05	3.72	0.08	14.50	0.05	2.99	0.07
-2.00	0.05	3.74	0.08	15.00	0.05	2.66	0.06
-1.00	0.05	3.74	0.08	15.50	0.05	2.16	0.05
0.00	0.05	3.74	0.08	16.00	0.05	1.61	0.04
1.00	0.05	3.74	0.08	16.50	0.05	1.09	0.03
2.00	0.05	3.73	0.08	17.00	0.05	0.79	0.03

Table 9: The Experimental Result of B vs. x

Also, we need to calculate the theoretical value of B when $I_M = 0.25$[A]. $B = B' I_M / I_0$, where B' is the value in Table 1 and $I_0 = 0.1$[A]. For example, when $B' = 1.4366$[mT],

$$B = \frac{1.4366 \times 0.25}{0.1} = 3.59[\text{mT}] \pm 0.07[\text{mT}].$$

Table 10 shows the theoretical value of B vs. x when $I_M = 250$[mA] using data in Table 1. Then I used the data in Table 9 and 10 to plot Figure 7, which is the magnetic field inside a solenoid.

x[cm]	B[mT]	Uncertainty[mT]
$\pm$0.0	3.59	0.07
$\pm$1.0	3.59	0.07
$\pm$2.0	3.59	0.07
$\pm$3.0	3.59	0.07
$\pm$4.0	3.58	0.07
$\pm$5.0	3.57	0.07
$\pm$6.0	3.56	0.07
$\pm$7.0	3.54	0.07
$\pm$8.0	3.51	0.07
$\pm$9.0	3.46	0.07
$\pm$10.0	3.37	0.07
$\pm$11.0	3.17	0.06
$\pm$11.5	2.99	0.06
$\pm$12.0	2.72	0.05
$\pm$12.5	2.32	0.05
$\pm$13.0	1.81	0.04

Table 10: The Theoretical Result of B vs. x

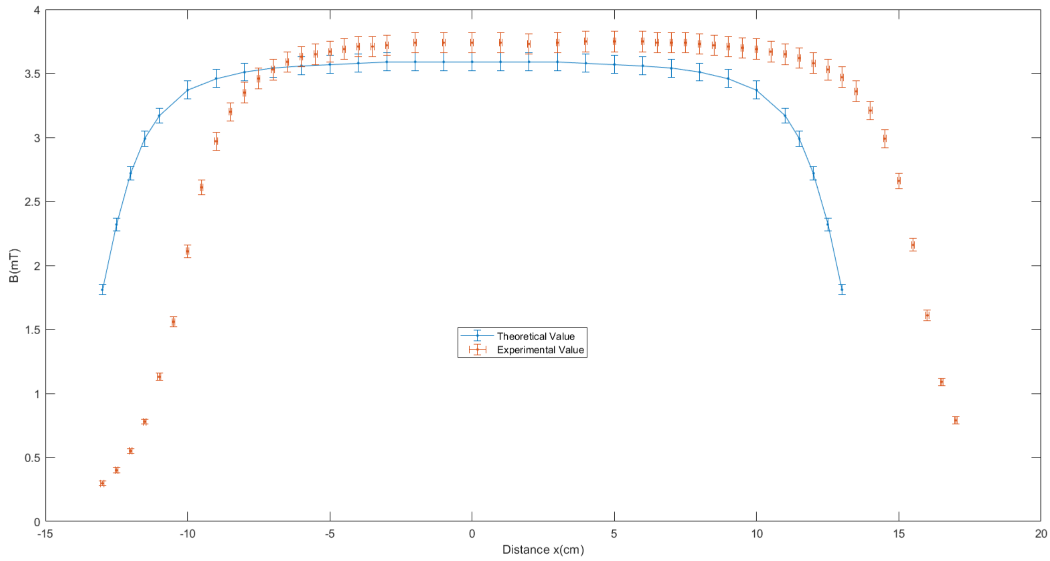

Figure 7: The Theoretical and Experimental Magnetic Field inside a Solenoid

4 Discussion

In the first part of this lab, we measure the Hall sensitivity K_H under working voltage $U_S = 5[\text{V}]$. We get

$$K_H^* = 35[\text{V/T}] \pm 3[\text{V/T}]$$
$$K_{H,t} = 31.25[\text{V/T}] \pm 1.25[\text{V/T}]$$

We can see that although the value of K_H^* and K_H(theoretical) is within the uncertainty of each other, the uncertainty of K_H^* is rather huge. Also, we have obtained

$$K_H = 32.6[\text{V/T}] \pm 0.7[\text{V/T}]$$

from the second part of this lab through linear fit. All of these three values lie in the range of one another's uncertainty, but K_H clearly has a smaller uncertainty than K_H^*. And that is why I used K_H instead of K_H^* in the third part to calculate the experimental value of B.

In the first part, we have measured K_H under different working voltage U_S and obtained Figure 5. From this figure, we can not see any pattern between K_H and U_S. But I can conclude that under different working voltage, K_H tends to be different. Also, from Table 5, the trend is that the higher the working voltage, the large the K_H.

In the third part of the experiment, I obtained both theoretical and experimental values of B in different positions inside a solenoid and plotted Figure 7. If we view the two sets of data separately (either experimental or theoretical), we can conclude that there is a range near the center of the solenoid where the magnetic field is uniform, while it decreases as we approach either end of the solenoid. However, the starting and ending point of the uniform magnetic field is different for the theoretical and experimental value. The reason might be that the center of the solenoid should be 15[cm] instead of 13[cm]. Also, the magnitude of the uniform magnetic field is different. The reason is probably that the K_H I used to calculate the experimental values is larger than the actual K_H.

5 Conclusion

In this lab, we have measured the Hall sensitivity of the Hall probe under different working voltages. Also, we have obtained two experimental value of K_H using two methods. One is direct measurement and the other is linear fit. In addition, we have measured the magnetic field inside a solenoid. All these three experiments are very successful as no results deviates from the theoretical values too much. However, it is unsatisfactory that some of the values (such as K_H^*) have very large uncertainty. In order to optimize the result and have more accurate data, here are some suggestions:

1. Change for better equipment, especially the voltage source. During the experiment, the output voltage kept increasing bit by bit after I stopped touch the knob, which made the reading on the multimeter keep changing, which causes much trouble for reading.

2. We can measure the output voltage several time when measuring K_H^*, which can minimize the effect of Type-A uncertainty.

6 Reference

1. Qin Tian, Wang Zhiyu, Lin Yiqiao, Mateusz Krzyzosiak, "Exercise 2- lab manual [rev. 3.8]".

A Uncertainty Analysis

A.1 The Uncertainty of the Voltages in Section 3.1

We first calculate the uncertainty in table 4. For the uncertainty of U_S, $u = U_S \times 0.5\%$. For example, when $U_S = 2.84$[V],

$$u = U_S \times 0.5\% = 2.84 \times 0.5\% = 0.014[\mathrm{V}].$$

Then the uncertainty of U_0 and U. $u = U \times 0.05\% + 6 \times 10^{-3}$ or 6×10^{-4}. For instance, when $U_0 = 1.4004$[V],

$$u = 1.4004 \times 0.05\% + 0.0006 = 0.0013[\mathrm{V}].$$

When $U = 2.266$[V],

$$u = 2.266 \times 0.05\% + 0.006 = 0.007[\mathrm{V}].$$

A.2 The Uncertainty of the Magnetic Field in Section 3.1

As we know,

$$B = B'\frac{I}{I_0}.$$

where $B' = 1.4366$[mT], $I_0 = 0.1$[A], $I = 0.25$[A], $u_I = I \times 2\% = 0.005$[A], and the uncertainty of both B' and I_0 is zero because these data are theoretical values. Then the uncertainty of the calculated magnetic field is

$$u_B = \left|\frac{\partial B}{\partial I}u_I\right| = \frac{B'}{I_0}u_I = \frac{1.4366 \times 10^{-3}}{0.1} \times 0.005 = 0.00007[\mathrm{T}].$$

A.3 The Uncertainty of K_H and K_H/U_S in Section 3.1

Since $K_H = \frac{U-U_0}{B}$, the uncertainty can be calculated as

$$\begin{aligned}
u &= \sqrt{\left(\frac{\partial K_H}{\partial U}u_U\right)^2 + \left(\frac{\partial K_H}{\partial B}u_B\right)^2 + \left(\frac{\partial K_H}{\partial U_0}u_{U_0}\right)^2} \\
&= \sqrt{\frac{u_U^2}{B^2} + \frac{u_{U_0}^2}{B^2} + \frac{(U-U_0)^2 u_B^2}{B^4}} \\
&= \frac{1}{B^2}\sqrt{(u_U^2 + u_{U_0}^2)B^2 + (U-U_0)^2 u_B^2}.
\end{aligned}$$

For example, when $B = 0.00359$[T], $U = 1.4689$[V], $U_0 = 1.4004$[V], $u_B = 0.00007$[T], $u_U = 0.0013$[V], $u_{U_0} = 0.0013$[V], then the uncertainty of K_H is

$$u = \frac{1}{0.00359^2}\sqrt{(2 \times 0.0013^2) \times 0.00359^2 + (1.4689 - 1.4004)^2 \times 0.00007^2} = 0.6[\mathrm{V/T}].$$

The same calculation can be applied to the uncertainty of K_H^*.

As for the uncertainty of K_H/U_S,

$$\begin{aligned}
u &= \sqrt{\left(\frac{\partial}{\partial K_H}\left(\frac{K_H}{U_S}\right)u_{K_H}\right)^2 + \left(\frac{\partial}{\partial U_S}\left(\frac{K_H}{U_S}\right)u_{U_S}\right)^2} \\
&= \sqrt{\frac{u_{K_H}^2}{U_S^2} + \frac{K_H^2 u_{U_S}^2}{U_S^4}} \\
&= \frac{1}{U_S^2}\sqrt{u_{K_H}^2 U_S^2 + K_H^2 u_{U_S}^2}.
\end{aligned}$$

So, when $K_H = 19.1$[V/T], $U_S = 2.84$[V], $u_{K_H} = 0.6$[V/T], $u_{U_S} = 0.014$[V], we have

$$u = \frac{1}{2.84^2} \times \sqrt{0.6^2 \times 2.84^2 + 19.1^2 \times 0.014^2} = 0.2[\mathrm{T}^{-1}].$$

A.4 The Uncertainty of I_M, U and B in Section 3.2

The uncertainty for I_M is $u = I_M \times 2\%[\mathrm{A}]$. So, for example, when $I_M = 0.1[\mathrm{A}]$,

$$u = 0.1 \times 2\% = 0.002[\mathrm{A}].$$

The uncertainty for U is $u = U \times 0.05\% + 0.0006[\mathrm{V}]$. So, when $U = 52.97[\mathrm{mV}]$,

$$u = 52.97/1000 \times 0.05\% + 0.0006 = 0.0006[\mathrm{V}].$$

Since $B = B'I_M/I_0$, where $I_0 = 0.1\mathrm{A}$ and $B' = 1.4366 \times 10^{-3}\mathrm{T}$, the uncertainty of the magnetic field is

$$u = \left|\frac{\partial B}{\partial I_M} u_{I_M}\right| = \frac{B' u_{I_M}}{I_0}.$$

For example, when $I_M = 0.1[\mathrm{A}]$, $u_{I_M} = 0.002[\mathrm{A}]$,

$$u = \frac{1.4366 \times 10^{-3} \times 0.002}{0.1} = 0.00003[\mathrm{T}].$$

A.5 The Uncertainty of the experimental and theoretical magnetic field B in Section 3.3

The uncertainty of the voltages is $u = U \times 0.05\% + 0.0006[\mathrm{V}]$. For example, when $U = 0.0131[\mathrm{V}]$,

$$u = 0.0131 \times 0.05\% + 0.0006 = 0.0006[\mathrm{V}]$$

We then calculate the uncertainty of the experimental value of B. Since here $B = U/K_H$, we have

$$\begin{aligned} u &= \sqrt{\left(\frac{\partial B}{\partial U} u_U\right)^2 + \left(\frac{\partial B}{\partial K_H} u_{K_H}\right)^2} \\ &= \sqrt{\frac{u_U^2}{K_H^2} + \frac{U^2 u_{K_H}^2}{K_H^4}} \\ &= \frac{1}{K_H^2}\sqrt{u_U^2 K_H^2 + U^2 u_{K_H}^2}. \end{aligned}$$

Here we use the experimental value of K_H, which was obtained in Section 3.2: $K_H = 32.6[\mathrm{V/T}]$, $u_{K_H} = 0.7[\mathrm{V/T}]$. For example, when $U = 0.0131[\mathrm{V}]$, $u_U = 0.0006[\mathrm{V}]$,

$$u = \frac{1}{32.6^2} \times \sqrt{0.0006^2 \times 32.6^2 + 0.0131^2 \times 0.7^2} = 0.00002[\mathrm{T}] = 0.02[\mathrm{mT}].$$

As for the theoretical value of the magnetic field, $B_t = B'I_M/I_0$, where $I_0 = 0.1\mathrm{A}$, and B' is the value listed in Table 1, so the uncertainty is

$$u = \left|\frac{\partial B_t}{\partial I_M} u_{I_M}\right| = \frac{B' u_{I_M}}{I_0}.$$

Here, $u_{I_M} = 0.25 \times 2\% = 0.005[\mathrm{A}]$. For example, when $B' = 1.4366[mT]$,

$$u = \frac{1.4366 \times 0.005}{0.1} = 0.07[\mathrm{mT}].$$

B Data Sheet

The data sheets are attached to this report.