UM-SJTU Joint Institute
Physicis Laboratory II
(VP241)

Laboratory Report

Exercise 4

Polarization of Light

Name: **Wang Dawei** ID: 518370910063 Group: 19
Name: Hu Jie ID: 518370910057 Group: 19
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1 Introduction & Theoretical Background

Light is a kind of electro-magnetic wave formed by time-varying perpendicular magnetic and electric field. General we consider the electric field vector with the maximum magnitude as the light vector, which is always perpendicular to the direction along which light propagates. Thus, light is a kind of transverse wave. In natural light, the direction of the oscillation is in all possible direction (on a plane perpendicular to the direction along which light propagates), and the amplitude of these oscillations are all equal. In other words, the electric field vector's distribution is uniform. Therefore, natural light is also called unpolarized light. If, on the other hand, at any given particular time point, the light oscillation is not uniform, this kind of light is called polarized light.

1.1 Polarization of Light

There are many kinds of polarized light. If the light vector's direction is always the same, it will be a linearly polarized light (Figure 1). The direction of the light vector is referred to as the polarization axis. If the end point of the light

Figure 1: Linearly polarized light whose polarization axis is parallel to the page plane

vector is rotating about the propagation direction of light and its trajectory on a plane is a circle, the light is circularly polarized. If its trajectory is an ellipse, it is called elliptically polarized(Figure 2). If the light is a combination of a

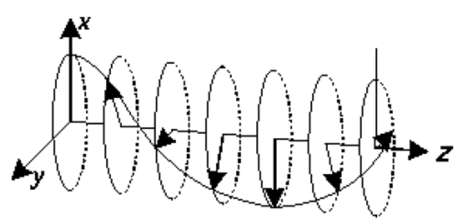

Figure 2: Elliptically polarized light

polarized light and natural light, it is called partially polarized light.

1.2 Polarizer

A polaroid(also known as polarizer) is a device commonly used in labs to generate polarized light. It polarizes the light using the principle of dichroism. It allows the light polarized in a certain direction (direction of the crystal alignment) to pass through the material, while the light polarized in all other directions is absorbed. This turns the incident natural light into linearly polarized. The polaroid can also be used as an analyzer(See the next section).

1.3 Malus's Law

Malus's Law is a quantitative law describing the change of the brightness of a linearly polarized light passing through an analyzer. Suppose that we have two polarizers arranged so that their planes are parallel − the left one plays the role of a polarizer, the other one is an analyzer (Figure 3). Let the angle between their transmission directions (polarization axes) be θ. The light is incident normally on the polarizer and then continues to the analyzer. The intensity of the linearly polarized light leaving the analyzer is

$$I_{light} = I_{light,0} \cos^2 \theta$$

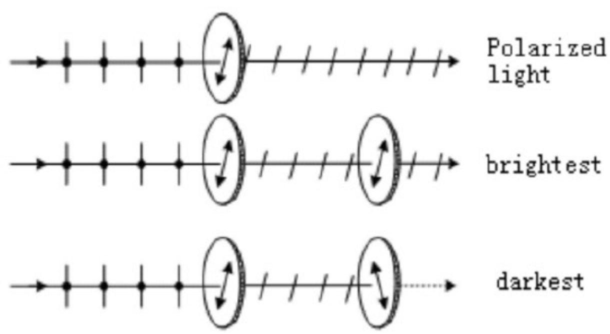

Figure 3: Change in the brightness of the light depends on the mutual orientation of the polarizer and the analyzer.

We can use the analyzer to determine the incident light's polarization. If it is natural light or circularly polarized light, when we rotate the analyzer, the light intensity will remain a constant. If it is partially polarized or elliptically polarized, the minimum intensity will not be zero. If it is linearly polarized, there will be a certain angle where the light intensity becomes zero.

1.4 The Generation of Elliptically and Circularly Polarized Light. Half-wave and Quarter-wave Plates

When a linearly polarized light is incident normally on a crystal plate whose surface is parallel to its optical axis, the light is resolved into two waves: an *e*-wave with the oscillation direction parallel to the optical axis of the plate (extraordinary axis) and an o-wave whose oscillation direction is perpendicular to the optical axis (ordinary axis)(Figure 4). Suppose the angle between the polarizing axis and the optical axis of the plate is α. The two waves propagate in the same direction, but with different speeds. The resulting optical path difference over the thickness d of the plate is

$$\Delta = (n_e - n_o)d,$$

where n_e and n_o are the refraction index of the e-wave and o-wave in the crystal. And the phase difference will be

$$\delta = \frac{2\pi(n_e - n_o)d}{\lambda},$$

where λ is the wave length of the light.

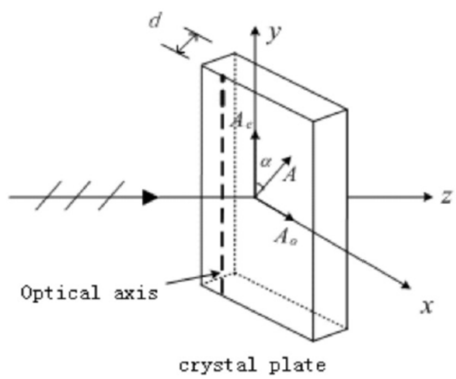

Figure 4: Linearly polarized light passing through a waveplate.

As shown in Figure 4, when the light propagates through the crystal plate, the two components of the light vector are

$$\begin{aligned} E_x &= A_o \cos \omega t \\ E_y &= A_e \cos(\omega t + \delta), \end{aligned}$$

where $A_o = A \sin \alpha$, $A_e = A \cos \alpha$. Eliminating time from the above equations one obtains

$$\frac{E_x^2}{A_o^2} + \frac{E_y^2}{A_e^2} - 2\frac{E_x E_y}{A_o A_e} \cos \delta = \sin^2 \delta. \tag{1}$$

When the thickness of the plate changes, the optical path difference changes as well. Some cases of particular interest, are discussed below:

- If $\Delta = k\lambda (k = 0, 1, 2, \dots)$, the phase difference δ will be 0, and Equation 1 becomes

$$E_y = \frac{A_e}{A_o} E_x,$$

which is a linear equation. Hence the transmitted light is linearly polarized with the oscillation direction remaining unchanged. A waveplate that satisfies this condition is called a full-wave plate. The light goes through a full-wave plate without changing its polarization state.

- If $\Delta = (2k+1)\lambda/2 (k = 0, 1, 2, \dots)$, the phase difference δ will be π, and Equation 1 is
$$E_y = -\frac{A_e}{A_o} E_x.$$
The transmitted light is also linearly polarized with the polarization axis rotated by the angle of 2α. A waveplate that satisfies the condition is called 1/2-wave plate or half-wave plate. When a polarized light passes through a half-wave plate, its polarization axis gets rotated by an angle 2α. If $\alpha = \pi/4$, then the polarization axis of the transmitted light is perpendicular to that of the incident light.

- If $\Delta = (2k+1)\lambda/4 (k = 0, 1, 2, \dots)$, the phase difference δ will be $\pm\pi/2$, and Equation 1 becomes
$$\frac{E_x^2}{A_o^2} + \frac{E_y^2}{A_e^2} = 1.$$
The transmitted light is elliptically polarized. A waveplate that satisfies the above condition is called a 1/4-wave plate or a quarter-waveplate and is an important optical element in many polarization experiments.

If $A_e = A_o = A$, then $E_x^2 + E_y^2 = A^2$, and the transmitted light is circularly polarized. Since the amplitudes of the *o*-wave and the *e*-wave are both functions of α, the polarization state after passing through a 1/4-wave plate will vary, depending on the angle:

- if $\alpha = 0$, the transmitted light is linearly polarized with the polarization axis parallel to the optical axis of the 1/4-wave plate;
- if $\alpha = \pi/2$, the transmitted light is linearly polarized with the polarization axis perpendicular to the optical axis of the 1/4-wave plate;
- if $\alpha = \pi/4$, the transmitted light is circularly polarized;
- otherwise, the transmitted light is elliptically polarized.

2 Apparatus & Measurement Procedure

2.1 Apparatus

The measurement setup consists of: a semiconductor laser, a silicon photo-cell, a UT51 digital universal meter, as well as two polarizers, 1/2-wave and 1/4-wave plates. The elements are placed on an optical bench. The uncertainty of the apparatus is listed in Table 1.

Apparatus	Uncertainty
Digital Universal Meter	0.001[μA]
Two polarizers, 1/2-wave and 1/4-wave plates	2[°]

Table 1: The uncertainty of the apparatus used in this lab section

2.2 Measurement Procedure

2.2.1 Demostration of Malus's Law

1. Take the polarizers and the plates off the optical bench. Turn on the universal meter and the laser. Make sure that laser is incident into the transmitter with the mark "Φ6". After we have got some readings on the universal meter, do not adjust the laser's direction for the rest of the lab.

2. Put one of the polarizers on the optical bench. Adjust the direction so that the reflection light is not too deviated from the source. Make sure that the reading on the universal meter is around 0.8 − 1 [μA].

3. Put the other polarizer on the optical bench. Rotate it so that the reading becomes zero on the universal meter. Let this angle be 90°.

4. Rotate the analyzer for 5°(the direction does not matter), and record the reading. Repeat this until the rotation angle becomes 0°.

2.2.2 Linearly Polarized Light and the Half-wave Plate

1. Put the half wave plate on the optical bench between the polarizer and the analyzer.

2. Rotate the analyzer until the reading becomes zero on the universal meter. Record the angle on the analyzer.

3. Rotate the half-wave plate for 10°, and then rotate the analyzer until the reading on the universal meter becomes zero again. Read the angle on the analyzer and record the difference of it between the angle in Step 2.

4. Repeat Step 3 until the half-wave plate has rotated 90°.

2.2.3 Circularly and Elliptically Polarized Light and the 1/4-wave Plate

1. Take the half wave plate off, and then rotate the analyzer until the reading becomes zero.

2. Put on the quarter-wave plate, and the rotate the plate until the reading becomes zero again. Record the angle both on the analyzer and the plate.

3. Take this reading as the data for $\theta = 90°$. Rotate the analyzer for 10°, record it as the next data.

4. Repeat Step 3 until the analyzer is rotated for a full round.

5. Choose the maximum value of current in the 36 results and make it the "Maximum Electric Current I_0".

6. Rotate the analyzer to its angle in Step 2, and then rotate the quarter plate for 20°. Repeat Step 3 to 5.

7. Rotate the quarter plate for another 25°, and repeat step 3 to 5.

3 Result

3.1 Demostration of Malus's Law

Maximum Electric Current I_0		$0.780 \pm 0.001[\mu\text{A}]$	
$\theta[°] \pm 2[°]$	$I[\mu\text{A}] \pm 0.001[\mu\text{A}]$	$\theta[°] \pm 2[°]$	$I[\mu\text{A}] \pm 0.001[\mu\text{A}]$
0	0.780	50	0.334
5	0.779	55	0.259
10	0.766	60	0.207
15	0.742	65	0.148
20	0.702	70	0.094
25	0.659	75	0.056
30	0.607	80	0.028
35	0.538	85	0.008
40	0.467	90	0.000
45	0.404		

Table 2: Measurement data of Malus's law demostration

Table 2 is the result of the verification of Malus's law. I performed a linear fit of the I_0/I versus $\cos^2\theta$ using the data in Table 3. When $I = 0.780[\mu\text{A}]$ and $\theta = 0°$, the sample calculations are as follows:

$$\frac{I}{I_0} = \frac{0.780}{0.780} = 1.0000 \pm 0.0018$$
$$\cos^2\theta = \cos^2(0 \times \pi/180) = 1 \pm 0.$$

Figure 5 is the graph of the linear fit. From it we can directly read that

$$\frac{I}{I_0\cos^2\theta} = 1.01 \pm 0.01.$$

As the expected result($I/I_0\cos^2\theta = 1$) is within the uncertainty, we can conclude that Malus's Law has been verified.

I/I_0	Uncertainty	$\cos^2\theta$	Uncertainty
1.0000	0.0018	1	0
0.9987	0.0018	0.992	0.006
0.9821	0.0018	0.970	0.012
0.9513	0.0018	0.93	0.02
0.9000	0.0017	0.88	0.03
0.8449	0.0017	0.82	0.03
0.7782	0.0016	0.75	0.03
0.6897	0.0016	0.67	0.03
0.5987	0.0015	0.59	0.03
0.5179	0.0014	0.50	0.03
0.4282	0.0014	0.41	0.03
0.3321	0.0014	0.33	0.03
0.2654	0.0013	0.25	0.03
0.1897	0.0013	0.18	0.03
0.1205	0.0013	0.12	0.02
0.0718	0.0013	0.067	0.018
0.0359	0.0013	0.030	0.012
0.0103	0.0013	0.007	0.006
0.0000	0.0013	0	0

Table 3: Data for linear fit of Malus's law demostration

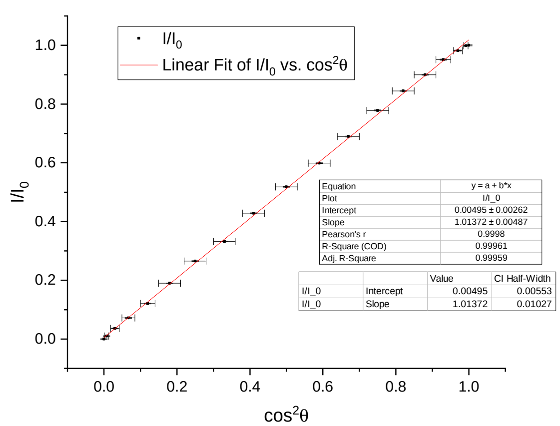

Figure 5: The linear fit of Malus's law demostration

3.2 Linearly Polarized Light and the Half-wave Plate

Table 4 is the result of the linear polarized light passing through a half wave plate. The expected result is that $\Delta\theta = 2\theta$, where $\Delta\theta$ is the rotation angle

Uncertainty of the angle: $\pm 2[°]$

Rotation angle of the 1/2-wave plate[°]	Rotation angle of the analyzer[°]
initial	0
10	17
20	32
30	58
40	78
50	98
60	118
70	139
80	158
90	178

Table 4: Measurement data for the 1/2-wave plate

$\theta[°]$	Uncertainty[°]	$\Delta\theta[°]$	Uncertainty
0	2	0	2
10	2	17	2
20	2	32	2
30	2	58	2
40	2	78	2
50	2	98	2
60	2	118	2
70	2	139	2
80	2	158	2
90	2	178	2

Table 5: The data for linear fit of $\Delta\theta$ vs. θ

of the analyzer, and θ is the rotation angle of the half-wave plate. The data used in the linear fit is listed in Table 5. From Figure 6, we can read that

$$\frac{\Delta\theta}{\theta} = 2.01 \pm 0.06.$$

3.3 The light intensity when the quarter wave plate's angle is 0°

Table 6 is the original data for the light intensity of the transmitted light passing through a quarter wave plate. I drew the relation between $\sqrt{I/I_0}$ vs. θ under polar coordinate using the modified data in Table 7 and get Figure 7.

3.4 The light intensity when the angle is 20°

Table 8 is the original data in this section. I tried to plot the relation of $\sqrt{I/I_0}$ vs. θ in polar coordinate with data in Table 9, and got Figure 8. Note

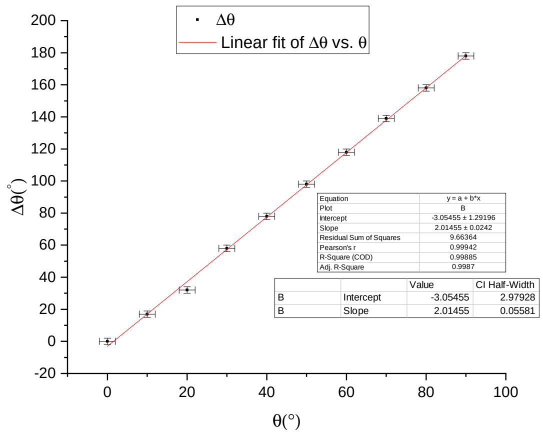

Figure 6: The linear fit of $\Delta\theta$ vs. θ

Rotation angle of 1/4-wave plate: $0[°]$			
Maximum Electric Current I_0		$0.498 \pm 0.001[\mu\text{A}]$	
$\theta[°] \pm 2[°]$	$I[\mu\text{A}] \pm 0.001[\mu\text{A}]$	$\theta[°] \pm 2[°]$	$I[\mu\text{A}] \pm 0.001[\mu\text{A}]$
0	0.487	180	0.498
10	0.472	190	0.481
20	0.440	200	0.440
30	0.379	210	0.379
40	0.298	220	0.294
50	0.208	230	0.203
60	0.126	240	0.116
70	0.056	250	0.052
80	0.013	260	0.011
90	0.000	270	0.001
100	0.016	280	0.020
110	0.061	290	0.067
120	0.133	300	0.139
130	0.221	310	0.227
140	0.308	320	0.315
150	0.387	330	0.392
160	0.452	340	0.452
170	0.492	350	0.498

Table 6: Measurement data for the 1/4-wave plate(rotation angle 0°).

that there is a point outside the trajectory. That point is the point when the rotation angle is 70°.

$\sqrt{\frac{I}{I_0}}$	Uncertainty	$\theta[^\circ]$	Uncertainty$[^\circ]$
0.9889	0.0014	0	2
0.9736	0.0014	10	2
0.9400	0.0014	20	2
0.8723	0.0015	30	2
0.7736	0.0015	40	2
0.6463	0.0017	50	2
0.503	0.002	60	2
0.335	0.002	70	2
0.162	0.006	80	2
0	$0^\star$	90	2
0.179	0.006	100	2
0.350	0.003	110	2
0.517	0.002	120	2
0.6662	0.0017	130	2
0.7864	0.0015	140	2
0.8815	0.0014	150	2
0.9527	0.0014	160	2
0.9940	0.0014	170	2
1.0000	0.0014	180	2
0.9828	0.0014	190	2
0.9400	0.0014	200	2
0.8724	0.0015	210	2
0.7684	0.0015	220	2
0.6385	0.0017	230	2
0.483	0.002	240	2
0.323	0.003	250	2
0.149	0.007	260	2
0.04	0.02	270	2
0.200	0.005	280	2
0.367	0.003	290	2
0.528	0.002	300	2
0.6752	0.0016	310	2
0.7953	0.0015	320	2
0.8872	0.0014	330	2
0.9527	0.0014	340	2
1.0000	0.0014	350	2

Table 7: The data for the polar coordinate plot when angle is 0°
*Here the uncertainty according to the formula is infinity. This is due to the fact that there is zero on the denominator. I consider it zero because this data is the base of the experiment. I delibrately set it to be zero.

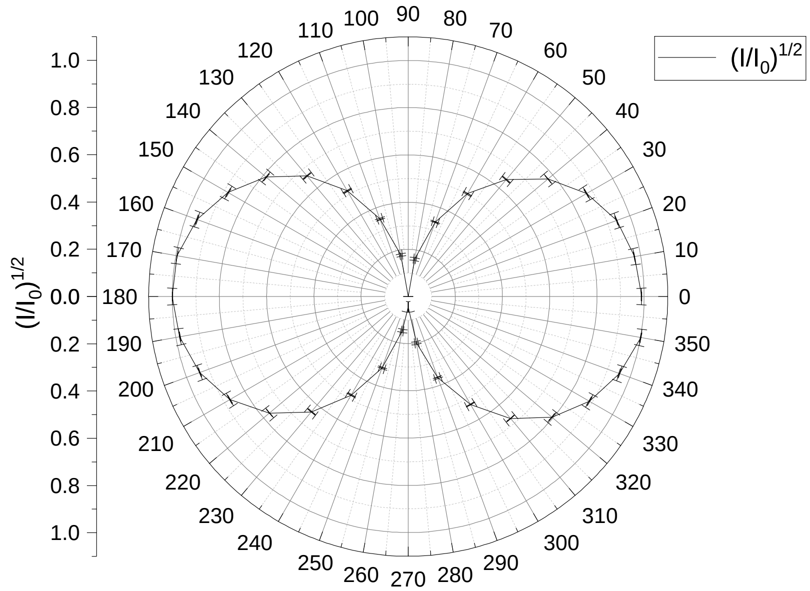

Figure 7: The graph of $\sqrt{I/I_0}$ vs. θ in polar coordinate when angle is 0°

3.5 The angle of the quarter wave plate is 45°

Table 10 is the original data for this part. I tried to apply a linear fit with the data and get a graph of $\sqrt{I/I_0}$ vs. θ. Table 11 is the data I used to perform the fit and I got Figure 9. In Figure 9, we can read the slope(the ratio of $\sqrt{\frac{I}{I_0}}$ and θ) is -0.00014 ± 0.00007. This is quite close to zero. However, as the Pearson's r and R-Square here is not close to 1, the linear fit does not work quite well.

4 Discussion

4.1 The demonstration of Malus's law

The verification of Malus's law is quite successful. From Figure 5, we can conclude that

$$\frac{I}{I_0 \cos^2 \theta} = 1.01 \pm 0.01.$$

The expected value of this ration is 1, which is within the range of the confidence interval. The uncertainty mainly comes from the error of the universal meter,

Rotation angle of 1/4-wave plate: 20[°]			
Maximum Electric Current I_0		$0.446 \pm 0.001[\mu\text{A}]$	
$\theta[°] \pm 2[°]$	$I[\mu\text{A}] \pm 0.001[\mu\text{A}]$	$\theta[°] \pm 2[°]$	$I[\mu\text{A}] \pm 0.001[\mu\text{A}]$
0	0.387	180	0.382
10	0.337	190	0.320
20	0.277	200	0.265
30	0.213	210	0.208
40	0.154	220	0.151
50	0.112	230	0.107
60	0.083	240	0.081
70	0.077	250	0.074
80	0.097	260	0.088
90	0.128	270	0.127
100	0.189	280	0.180
110	0.248	290	0.237
120	0.304	300	0.301
130	0.352	310	0.371
140	0.410	320	0.420
150	0.438	330	0.430
160	0.445	340	0.446
170	0.416	350	0.425

Rotation angle of the quarter wave plate: 70°	
$\theta[°] \pm 2[°]$	23
$I[\mu\text{A}] \pm 0.001[\mu\text{A}]$	0.481

Table 8: Measurement data for the 1/4-wave plate(rotation angle 20° and 70°).

as I suppose based on this result. Therefore, the intensity of the transmitted light is directly proportional to the incident light intensity and the square of the cosine of the angle of the transmission direction of the polarizer and the analyzer. i.e.,

$$I_{0,light} = I_0 \cos^2 \theta$$

4.2 Linearly Polarized Light and Half Wave Plate

When I rotate the half wave plate for a full round, the light intensity will vanish twice. Also, if I rotate the analyzer, it will also vanish twice. This is because the transmission light is linearly polarized. The light intensity will vanish when the light vector is orthogonal to the direction of transmission allowed by the analyzer, and it has two direction in a full round. The transmission light after a linearly polarized light is incident to a half wave plate will have a light vector rotated 2α, where α is the angle between the polarizing axis and the optical axis of the plate.

$\sqrt{\frac{I}{I_0}}$	Uncertainty	$\theta[°]$	Uncertainty[°]
0.9315	0.0016	0	2
0.8693	0.0016	10	2
0.7881	0.0017	20	2
0.6911	0.0018	30	2
0.588	0.002	40	2
0.501	0.002	50	2
0.431	0.003	60	2
0.416	0.003	70	2
0.466	0.002	80	2
0.536	0.002	90	2
0.6510	0.0019	100	2
0.7457	0.0017	110	2
0.8256	0.0016	120	2
0.8884	0.0016	130	2
0.9588	0.0016	140	2
0.9910	0.0016	150	2
0.9989	0.0016	160	2
0.9658	0.0016	170	2
0.9255	0.0016	180	2
0.8470	0.0016	190	2
0.778	0.0017	200	2
0.6829	0.0018	210	2
0.582	0.002	220	2
0.490	0.002	230	2
0.426	0.003	240	2
0.407	0.003	250	2
0.444	0.003	260	2
0.534	0.002	270	2
0.6353	0.0019	280	2
0.7290	0.0017	290	2
0.8215	0.0017	300	2
0.9121	0.0016	310	2
0.9704	0.0016	320	2
0.9819	0.0016	330	2
1.0000	0.0016	340	2
0.9762	0.0016	350	2

Table 9: The data for the polar coordinate plot when angle is 20°

The result of the linear fit tells us that

$$\frac{\Delta\theta}{\theta} = 2.01 \pm 0.06.$$

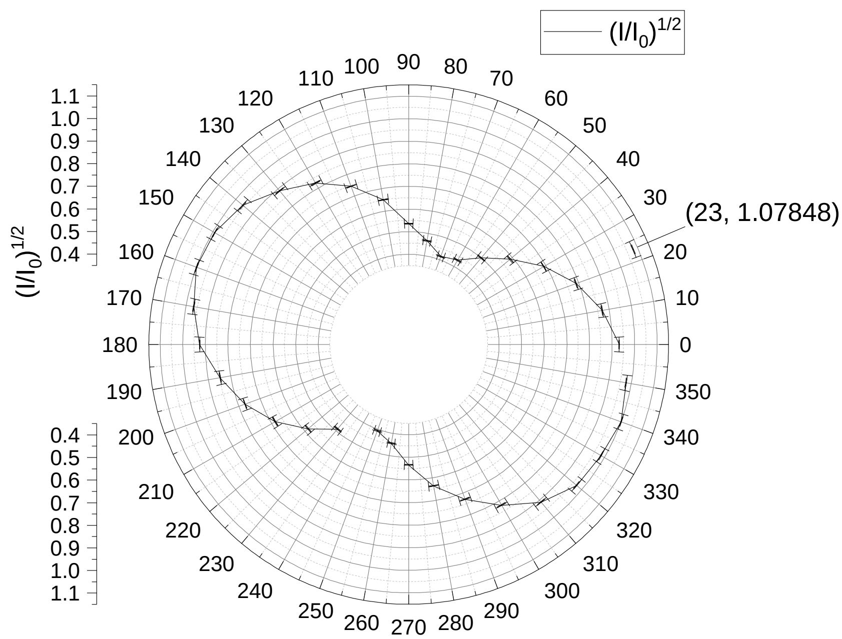

Figure 8: The graph of $\sqrt{I/I_0}$ vs. θ in polar coordinate when rotation angle is 20°

The expected ratio is 2, which is in the confidence interval. So the experiment is quite successful. The uncertainty mainly came from the error of the experiment apparatus, including the universal meter and the protractor on the analyzer and the plate.

4.3 Linearly Polarized Light through Quarter-wave Plate when angle is 0°

In this section, we let a linearly polarized light incident onto a quarter wave plate with angle 0°. From Figure 7, we can see that there are two angles where the intensity goes to zero. This means that the transmission light is also a linearly polarized light. However, as we did not rotate the quarter wave plate, we cannot find out the change of the direction of the light vector of the incident light.

Rotation angle of 1/4-wave plate: 45[°]			
Maximum Electric Current I_0		$0.291 \pm 0.001[\mu\text{A}]$	
$\theta[°] \pm 2[°]$	$I[\mu\text{A}] \pm 0.001[\mu\text{A}]$	$\theta[°] \pm 2[°]$	$I[\mu\text{A}] \pm 0.001[\mu\text{A}]$
0	0.224	180	0.220
10	0.236	190	0.227
20	0.248	200	0.235
30	0.263	210	0.249
40	0.274	220	0.260
50	0.285	230	0.275
60	0.291	240	0.278
70	0.285	250	0.278
80	0.282	260	0.275
90	0.278	270	0.269
100	0.274	280	0.260
110	0.258	290	0.247
120	0.244	300	0.238
130	0.230	310	0.232
140	0.222	320	0.225
150	0.214	330	0.221
160	0.215	340	0.219
170	0.217	350	0.215

Table 10: Measurement data for the 1/4-wave plate(rotation angle 45°).

4.4 Linearly Polarized Light through Quarter-wave Plate when angle is 20°

When the quarter wave plate is rotated for 20°, the transmission light becomes elliptically polarized. When I rotate the analyzer for a full round, there is no point where the intensity vanishes. In Figure 8, I have included an extra point, which is marked with its coordinate. This point is a datum where the rotation angle of the quarter wave plate is 70° and the analyzer at a random angle θ. It should be on the trajectory of the graph. However, as we can see, it is not. A possible explanation for this is that the quarter wave plate we used in lab is not symmetric as it should be. Or it may be the outside light like the torch that interfered the single measurement when rotation angle is θ.

4.5 Linearly Polarized Light through Quarter-wave Plate when angle is 45°

When the quarter wave plate is rotated for 45°, the transmission light should be circularly polarized, which means that when we rotate the analyzer, the intensity of the transmission light should not change. However, this is not

$\sqrt{\frac{I}{I_0}}$	Uncertainty	$\theta[°]$	Uncertainty[°]
0.877	0.002	0	2
0.901	0.002	10	2
0.923	0.002	20	2
0.951	0.002	30	2
0.970	0.002	40	2
0.990	0.002	50	2
1.000	0.002	60	2
0.990	0.002	70	2
0.984	0.002	80	2
0.977	0.002	90	2
0.970	0.002	100	2
0.942	0.002	110	2
0.916	0.002	120	2
0.889	0.002	130	2
0.873	0.002	140	2
0.858	0.002	150	2
0.860	0.002	160	2
0.864	0.002	170	2
0.869	0.002	180	2
0.883	0.002	190	2
0.899	0.002	200	2
0.925	0.002	210	2
0.945	0.002	220	2
0.972	0.002	230	2
0.977	0.002	240	2
0.977	0.002	250	2
0.972	0.002	260	2
0.961	0.002	270	2
0.945	0.002	280	2
0.921	0.002	290	2
0.904	0.002	300	2
0.893	0.002	310	2
0.879	0.002	320	2
0.871	0.002	330	2
0.868	0.002	340	2
0.860	0.002	350	2

Table 11: The data for linear fit in Section 3.5

the case. I applied linear fit in Figure 9. Although the slope it gives is

$$\frac{\sqrt{\frac{I}{I_0}}}{\theta} = 0.00014 \pm 0.00007,$$

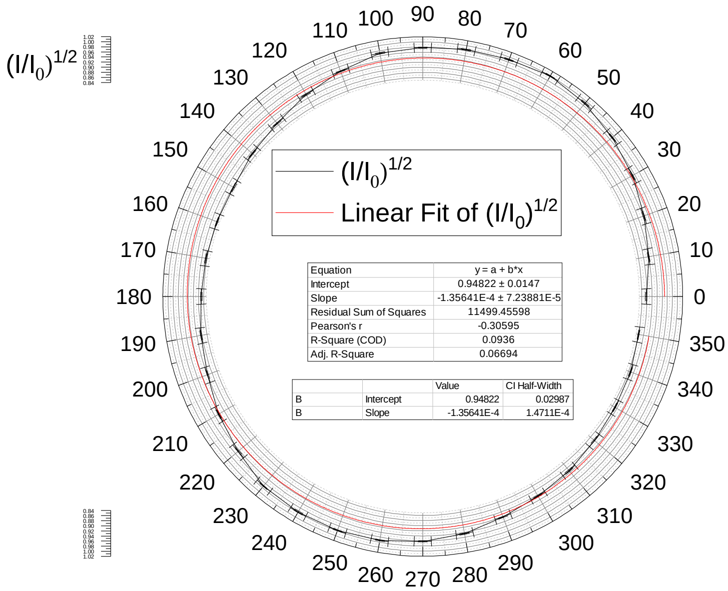

Figure 9: The linear fit of $\sqrt{\frac{I}{I_0}}$ vs. θ

which is quite close to zero, as we have expected, the Pearson's r for this linear fit is -0.306, which makes the linear fit not so valid. The reasons may include the following:

1. The quarter wave plate is not ideal. In some direction the light is blocked because of the plate's inner structure.

2. The thickness of the plate does not strictly makes the phase difference to be $\pm\frac{\pi}{2}$, probably because of grinding.

3. We did not set the angle of rotation to be exactly 45°, there are some uncertainties on the quarter-wave plate's protractor we did not take into consideration during the experiment.

5 Conclusion

In this lab section, we have studied the polarization of light. We first did an experiment to verify Malus's Law. Then we extensively studied the bahaviour of linearly polarized light passing through a half-wave or quarter wave plate. The result of the linearly polarized light is quite satisfactory, but the result concerning circularly polarized light is not what we have expected. This is probably due to some issues about the quarter wave plate we used in lab.

6 Reference

1. Qin Tian, Cao Jianjun, Lin Xinyu, Mateusz Krzyzosiak, “Exercise 4 - lab manual [rev. 2.7]”.

A Uncertainty Analysis

A.1 The uncertainty of I/I_0 and $\cos^2\theta$ in Section 3.1

The uncertainty for a single current is 0.001[μA], and the uncertainty for I/I_0 is

$$\begin{aligned} u_{I/I_0} &= \sqrt{\left[\frac{\partial}{\partial I}\left(\frac{I}{I_0}\right)\right]^2 u_I^2 + \left[\frac{\partial}{\partial I_0}\left(\frac{I}{I_0}\right)\right]^2 u_{I_0}^2} \\ &= \frac{u_I}{I_0}\sqrt{1+\left(\frac{I}{I_0}\right)^2}. \end{aligned}$$

For example, when $I = I_0 = 0.780$[μA],

$$u_{I/I_0} = \frac{0.001}{0.780}\sqrt{1+1} = 0.0018.$$

The uncertainty for a single angle is 2[°], and it is equal to $u_\theta = \pi/90 = 0.035$[rad].

$$\begin{aligned} u_{\cos^2\theta} &= \left|\frac{\mathrm{d}(\cos^2\theta)}{\mathrm{d}\theta}u_\theta\right| \\ &= 2\sin\theta\cos\theta u_\theta. \end{aligned}$$

For example, when $\theta = 10$[°],

$$u_{\cos^2\theta} = 2\times\sin(10^\circ)\cos(10^\circ)\times 0.035 = 0.012.$$

A.2 The uncertainty of $\sqrt{I/I_0}$ in Section 3.3, 3.4 & 3.5

The uncertainty for the measurement of I and I_0 are both 0.001[μA]. So the uncertainty of $\sqrt{I/I_0}$ is

$$\begin{aligned} u_{\sqrt{I/I_0}} &= \sqrt{\left[\frac{\partial}{\partial I_0}\left(\sqrt{\frac{I}{I_0}}\right)u_{I_0}\right]^2 + \left[\frac{\partial}{\partial I}\left(\sqrt{\frac{I}{I_0}}\right)u_I\right]^2} \\ &= u_I\sqrt{\frac{1}{4I_0I}+\frac{I}{4I_0^3}} \end{aligned}$$

For example, when $I = 0.224$[μA] and $I_0 = 0.291$[μA],

$$u_{\sqrt{I/I_0}} = 0.001\times\sqrt{\frac{1}{4\times0.291\times0.224}+\frac{0.224}{4\times0.291^3}} = 0.002.$$

B Data Sheet

The data sheets of this lab is attached to this report.