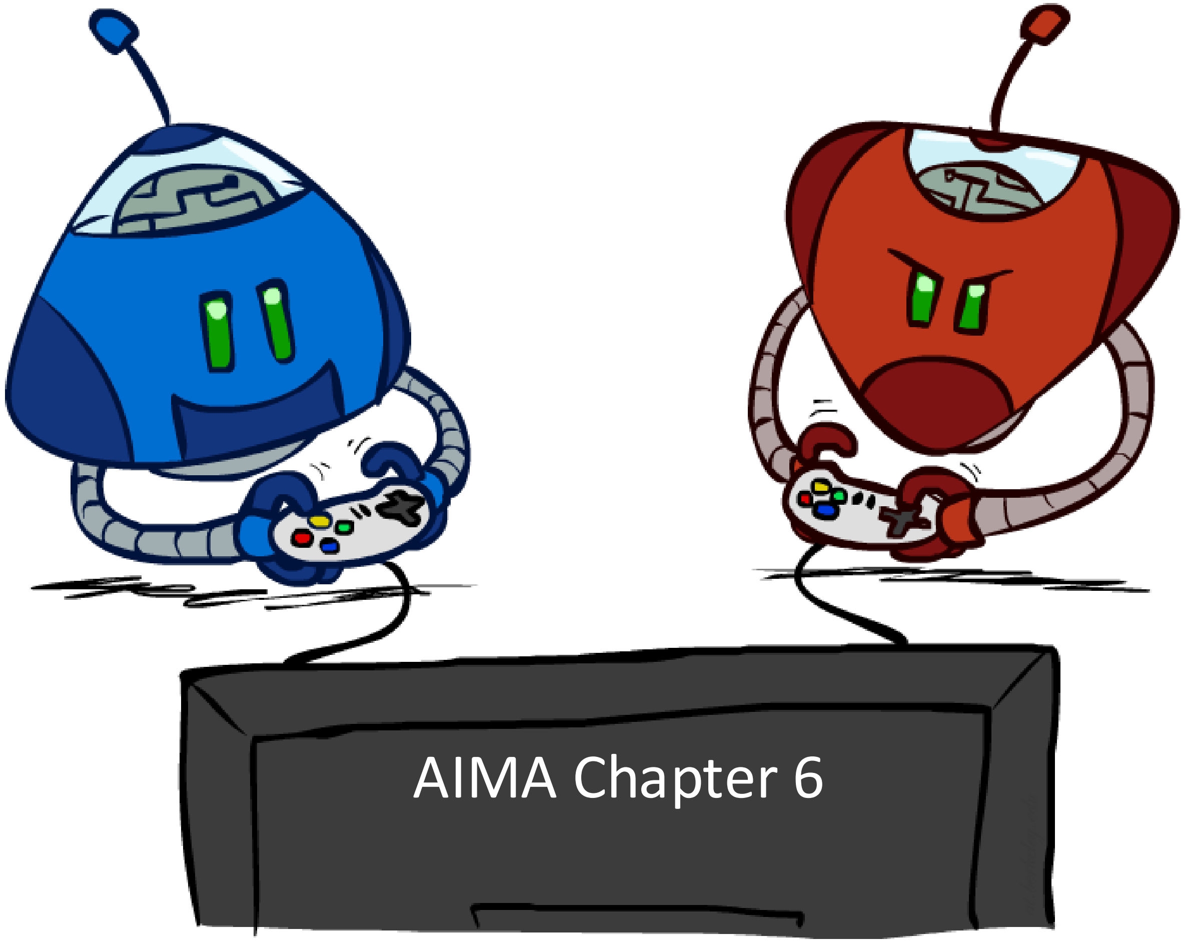
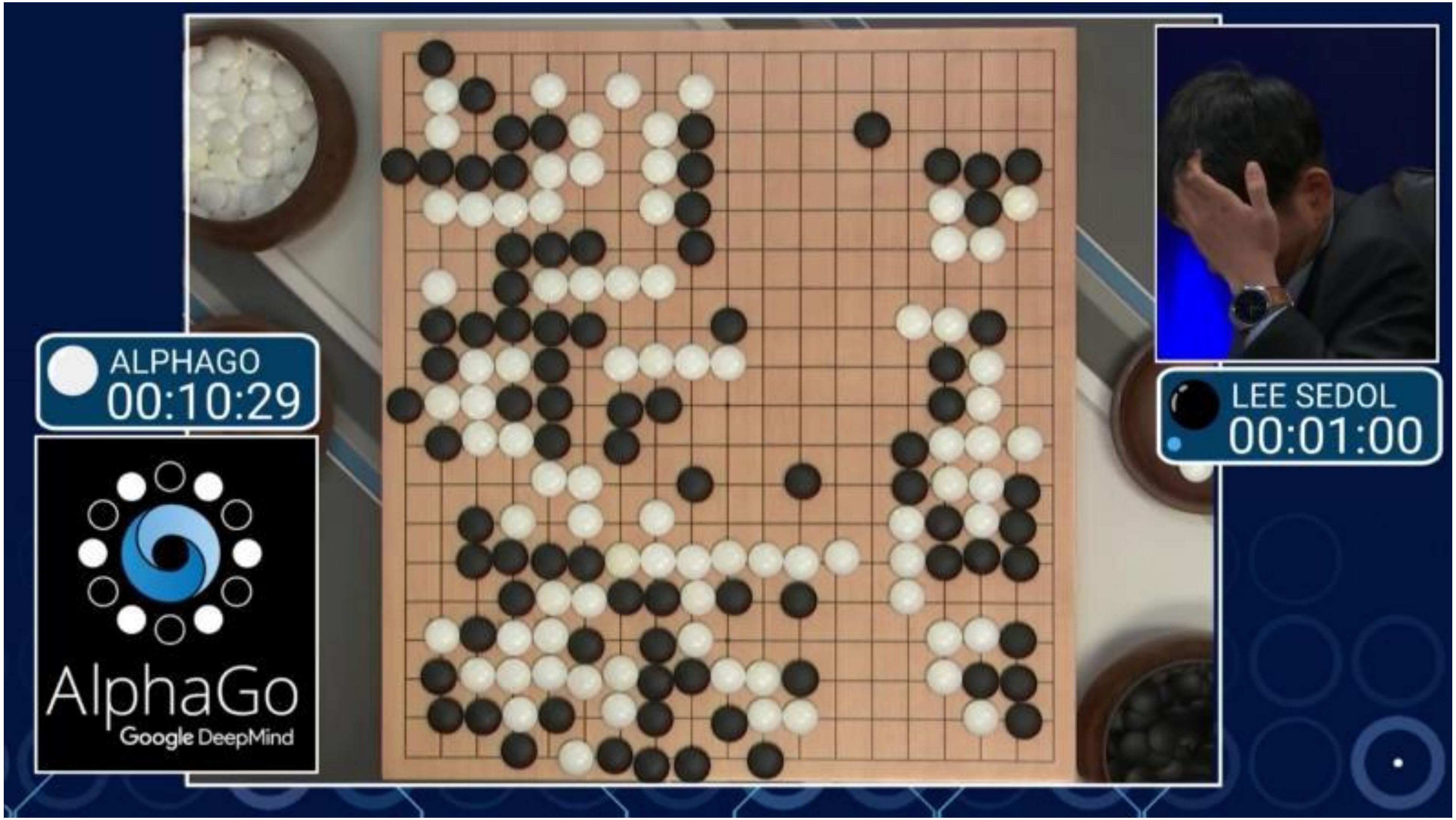
Adversarial Search



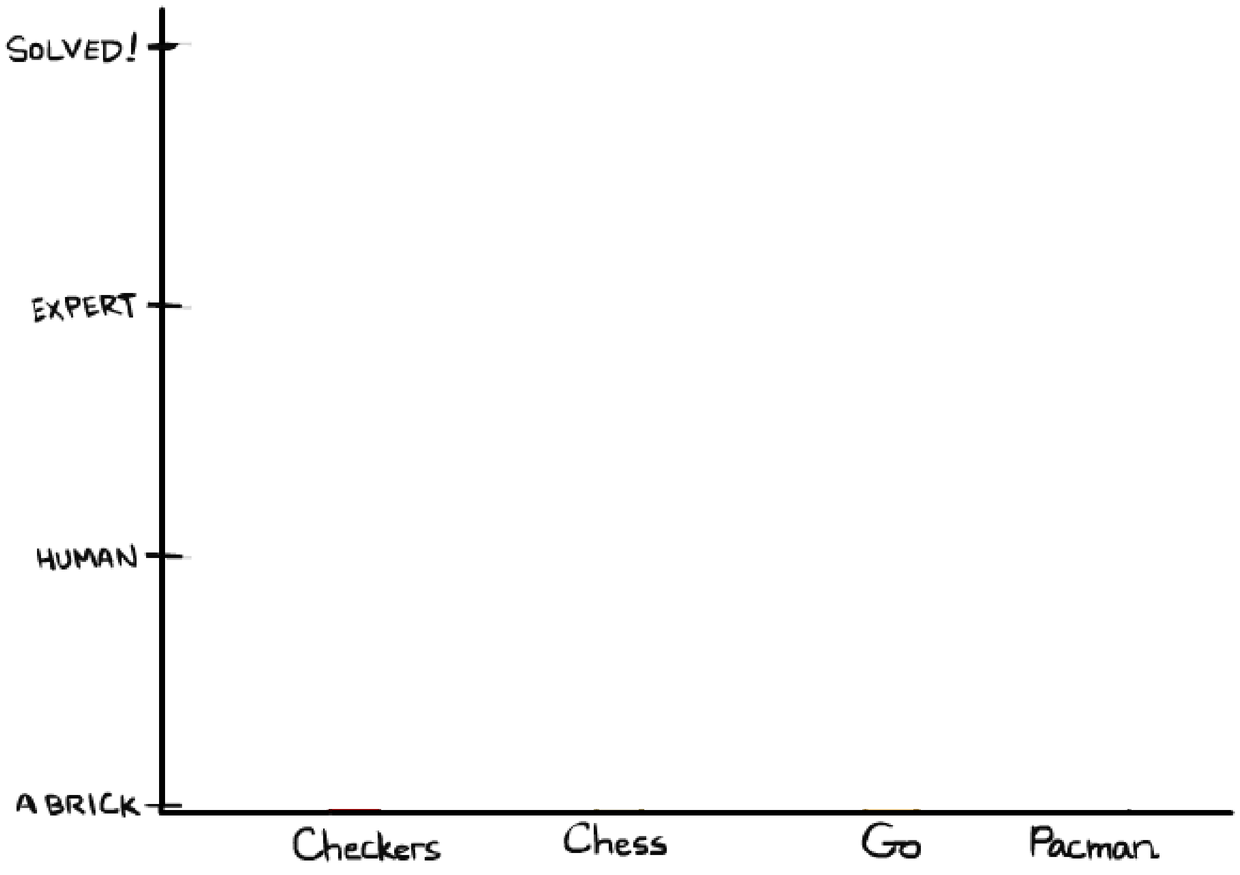
[Adapted from slides by Dan Klein and Pieter Abbeel at UC Berkeley (ai.berkeley.edu).]

AlphaGo: the most well-known AI?



Game Playing State-of-the-Art

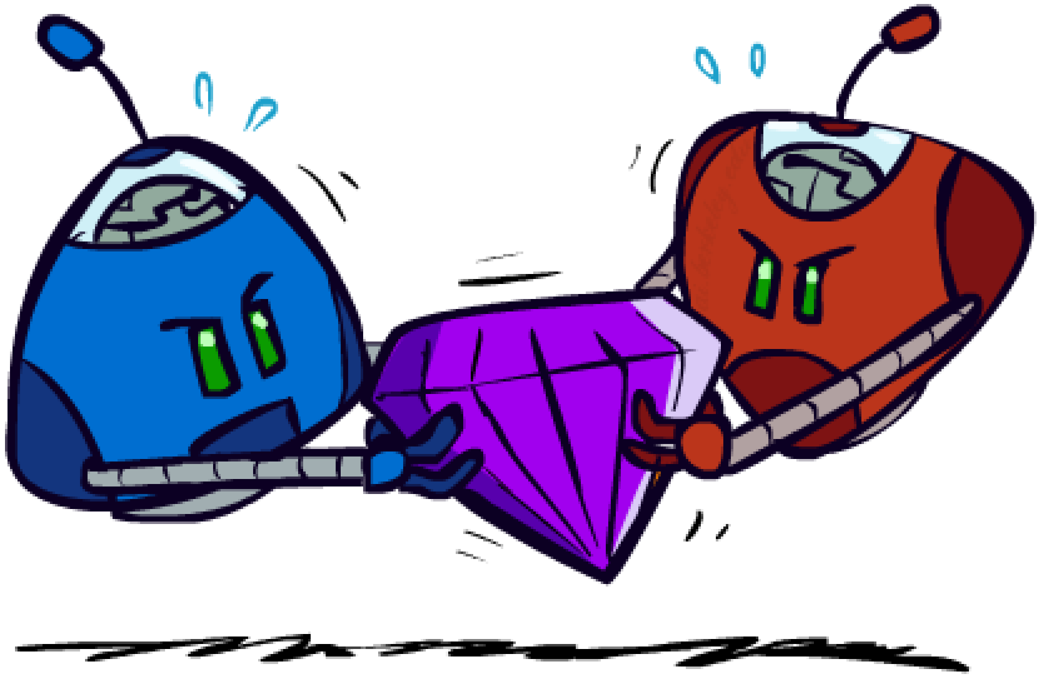
- **Checkers:** 1950: First computer player. 1994: First computer champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame. 2007: Checkers solved!
- **Chess:** 1997: Deep Blue defeats human champion Gary Kasparov in a six-game match. Deep Blue examined 200M positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 ply. Current programs are even better, if less historic.
- **Go: 2016: Alpha GO defeats human champion! Uses Monte Carlo Tree Search, learned evaluation function.**
- **Pacman**



Outline

- Adversarial Games
- Adversarial Search
- Resource Limits
- Game Tree Pruning
- Uncertain Outcomes
- Other Game Types

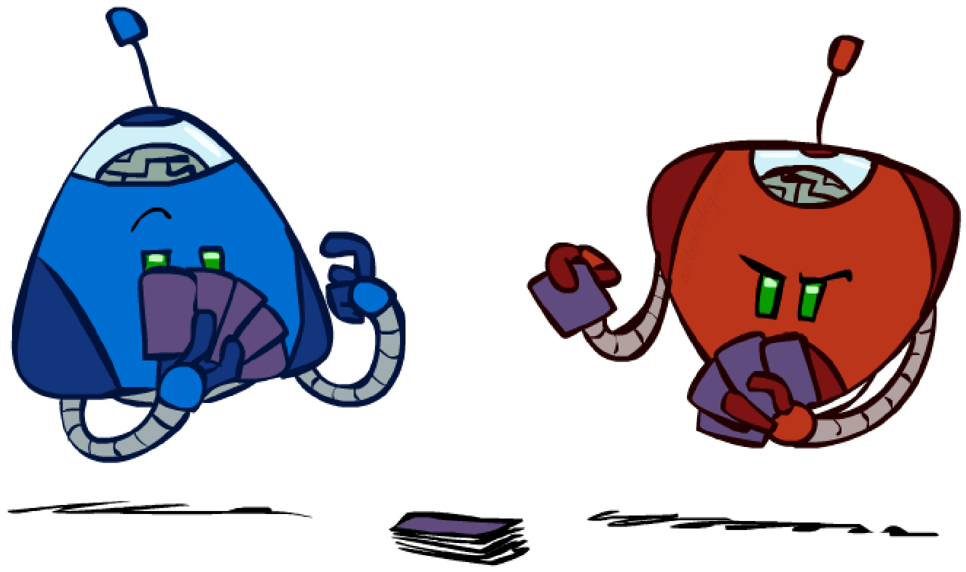
Adversarial Games



Types of Games

- Many different kinds of games!

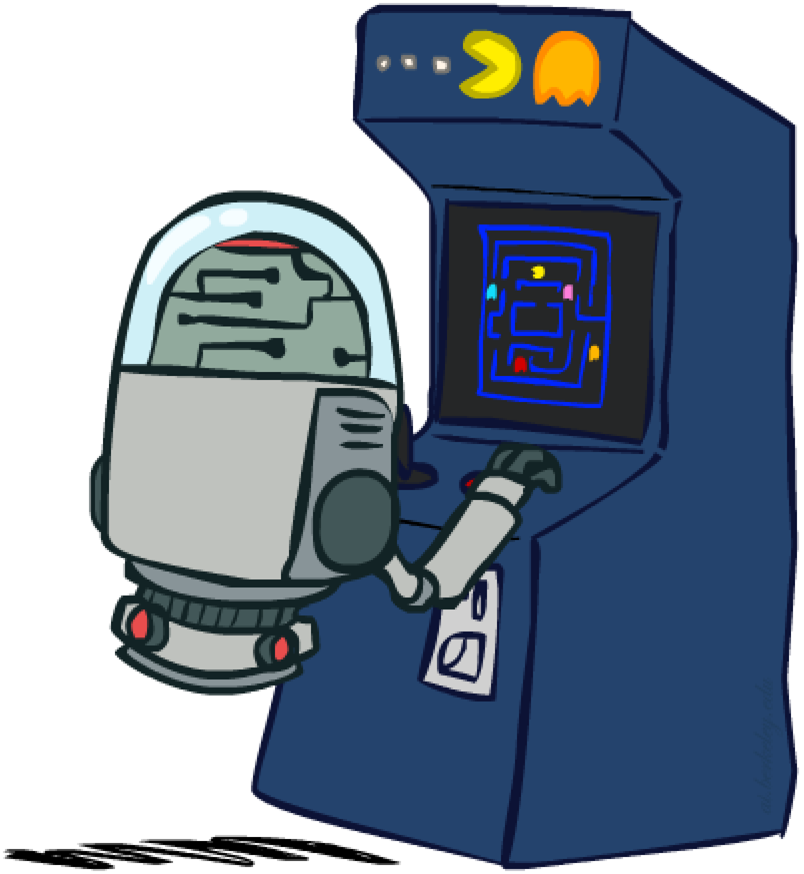
- Differences:
  - Deterministic or stochastic?
  - One, two, or more players?
  - Zero sum?
  - Perfect information (can you see the state)?



- Want algorithms for calculating a strategy (policy) which recommends a move from each state

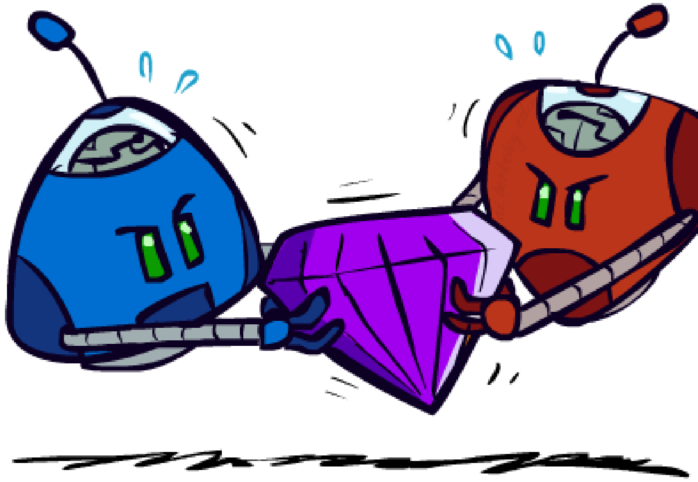
Deterministic Games

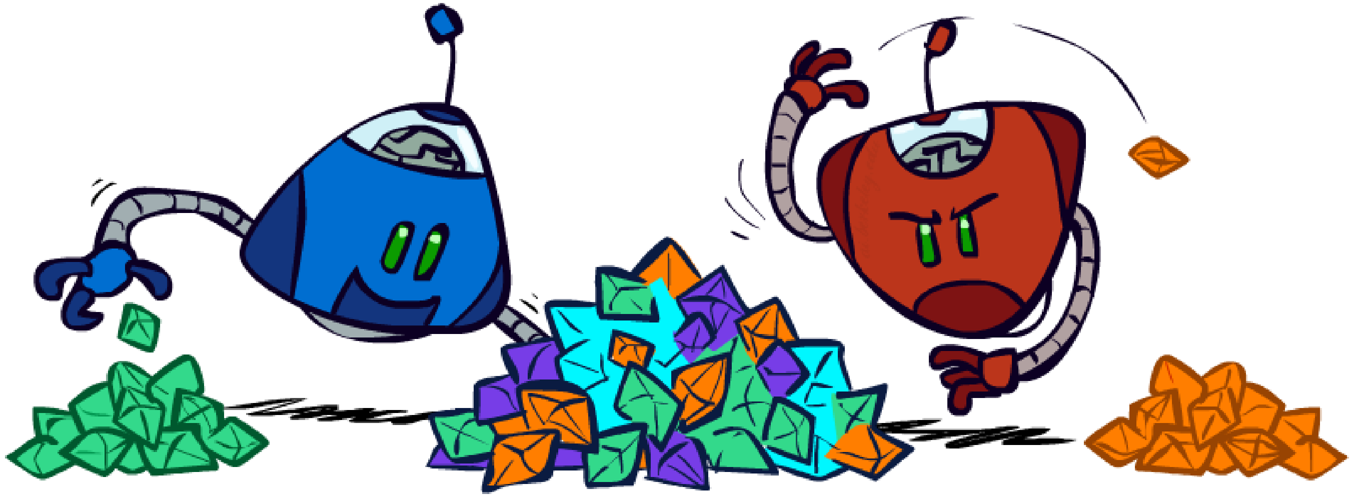
- Many possible formalizations, one is:
  - States: S (start at s_0)
  - Players: P={1...N} (usually take turns)
  - Actions: A (may depend on player / state)
  - Transition Function: SxA $\rightarrow$ S
  - Terminal Test: S $\rightarrow$ {t,f}
  - Terminal Utilities: SxP $\rightarrow$ R



- Solution for a player is a policy: S $\rightarrow$ A

Zero-Sum Games

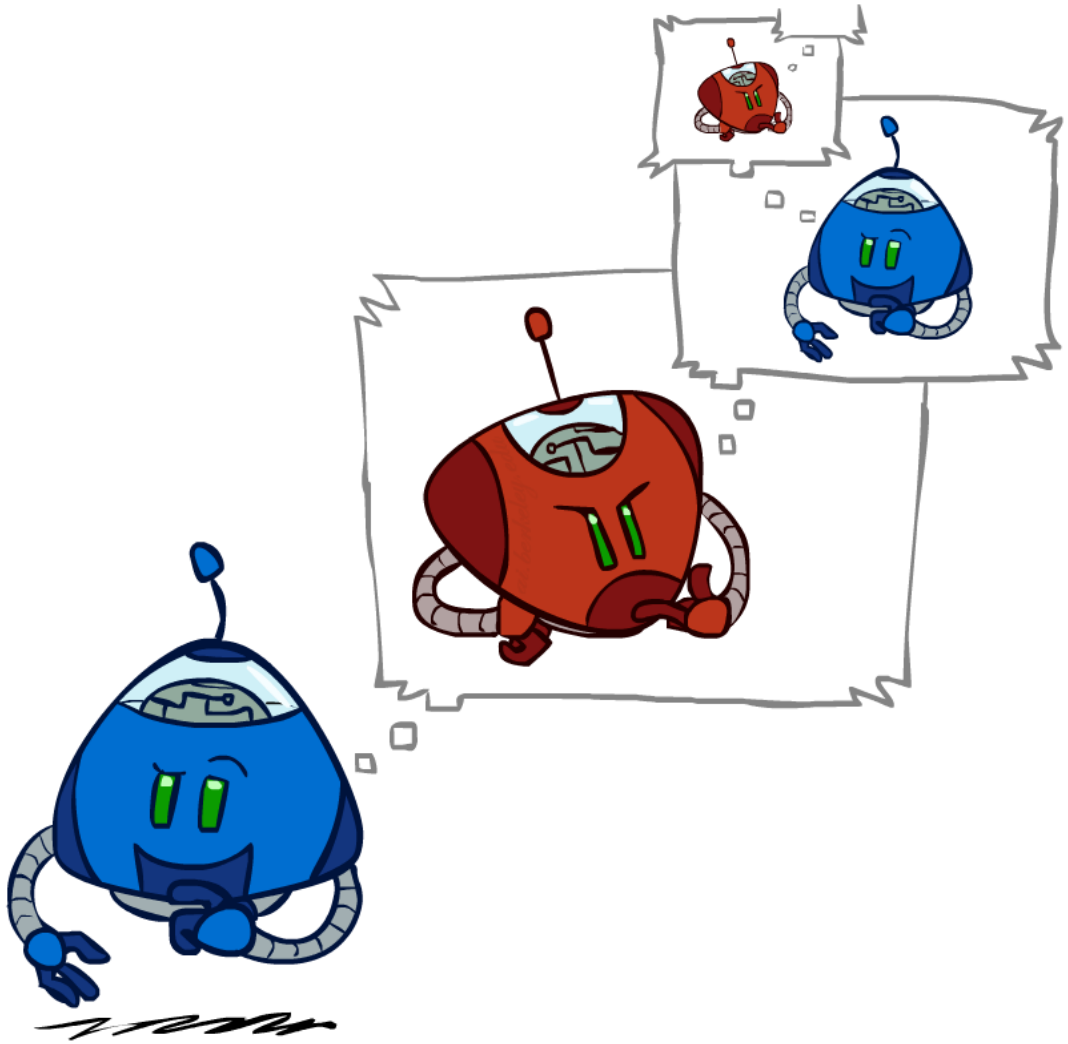




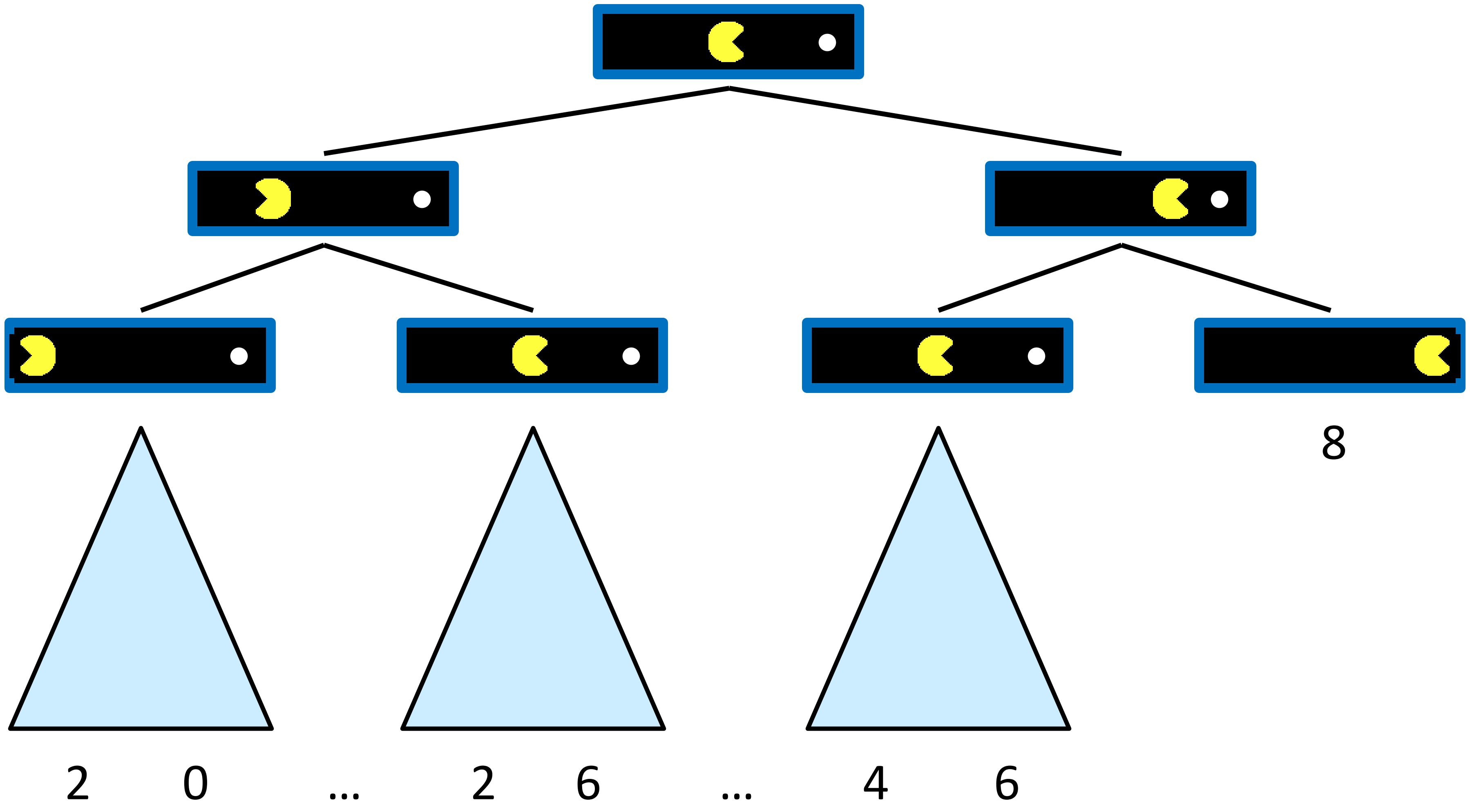
- Zero-Sum Games
  - Agents have opposite utilities (values on outcomes)
  - Lets us think of a single value that one maximizes and the other minimizes
  - Adversarial, pure competition

- General Games
  - Agents have independent utilities (values on outcomes)
  - Cooperation, indifference, competition, and more are all possible
  - More later on non-zero-sum games

Adversarial Search



Single-Agent Trees



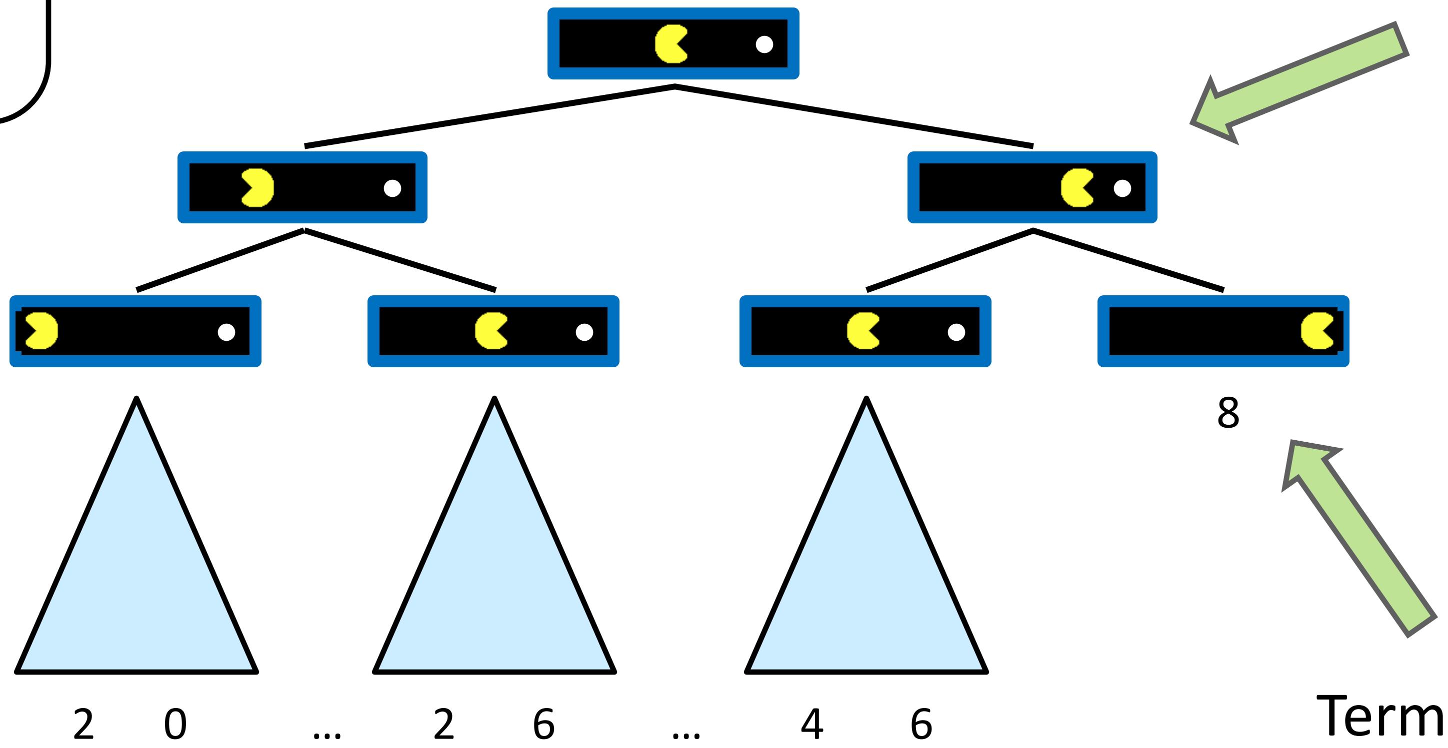
Value of a State

Value of a state:
The best achievable outcome (utility) from that state

Policy: the agent should choose an action leading to the state with the largest value

Non-Terminal States:

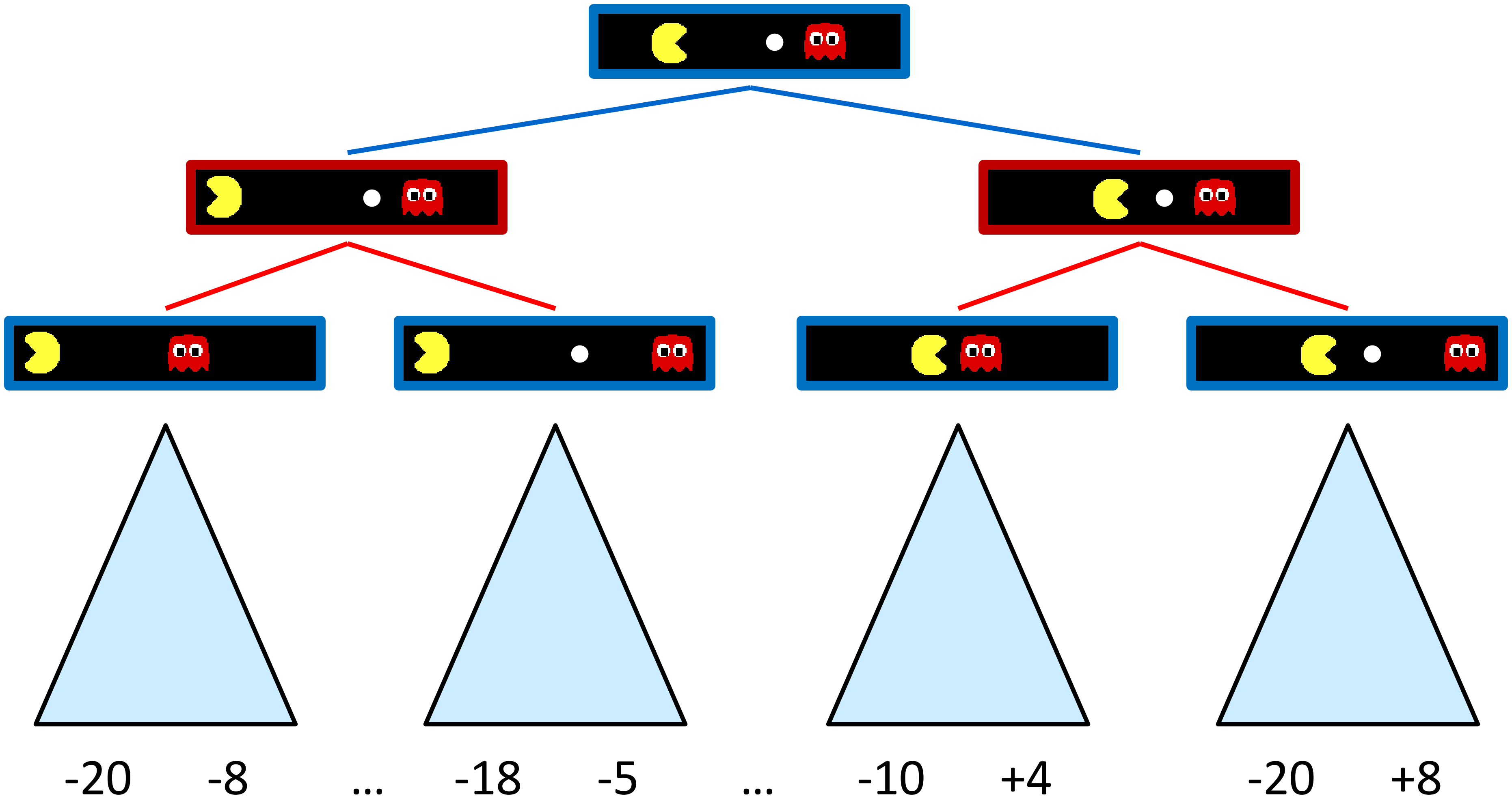
$$V(s) = \max_{s' \in \text{children}(s)} V(s')$$



Terminal States:

$$V(s) = \text{known}$$

Adversarial Game Trees



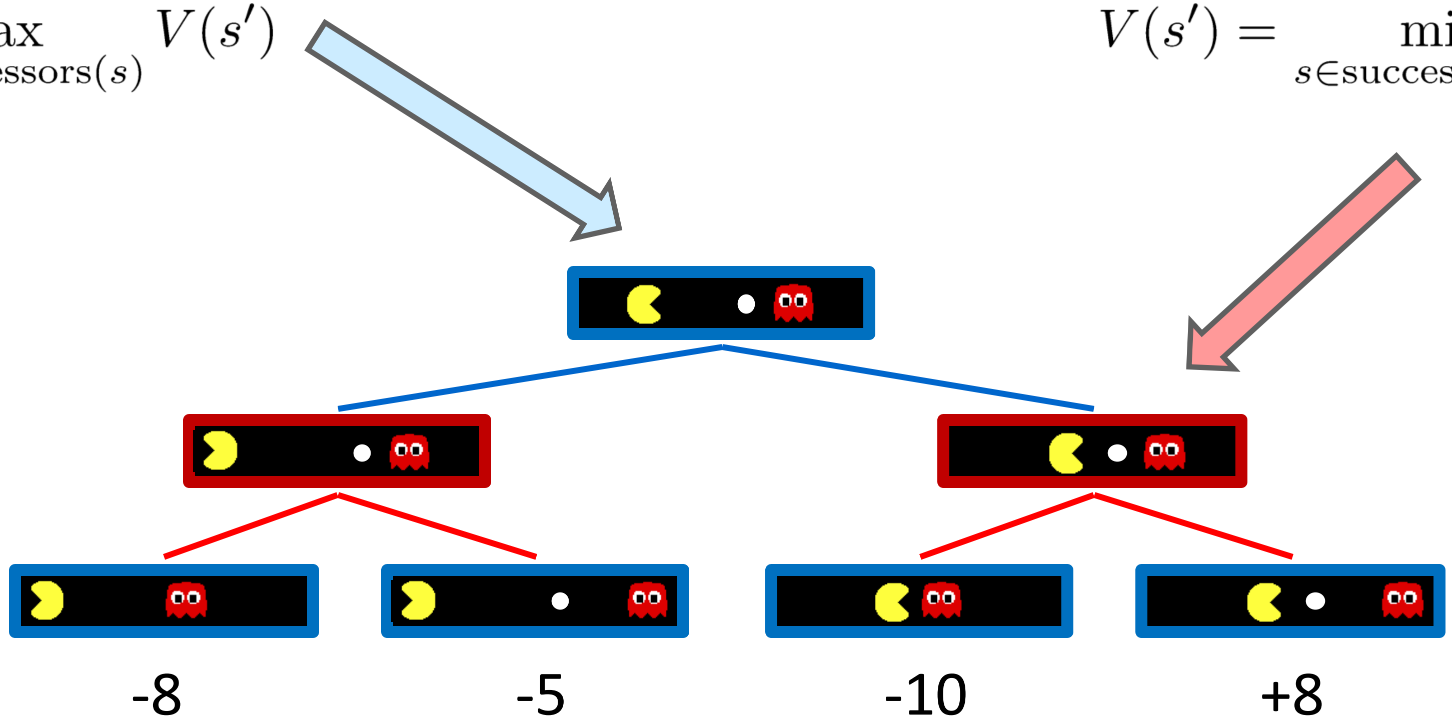
Minimax Values

States Under Agent's Control:

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

States Under Opponent's Control:

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

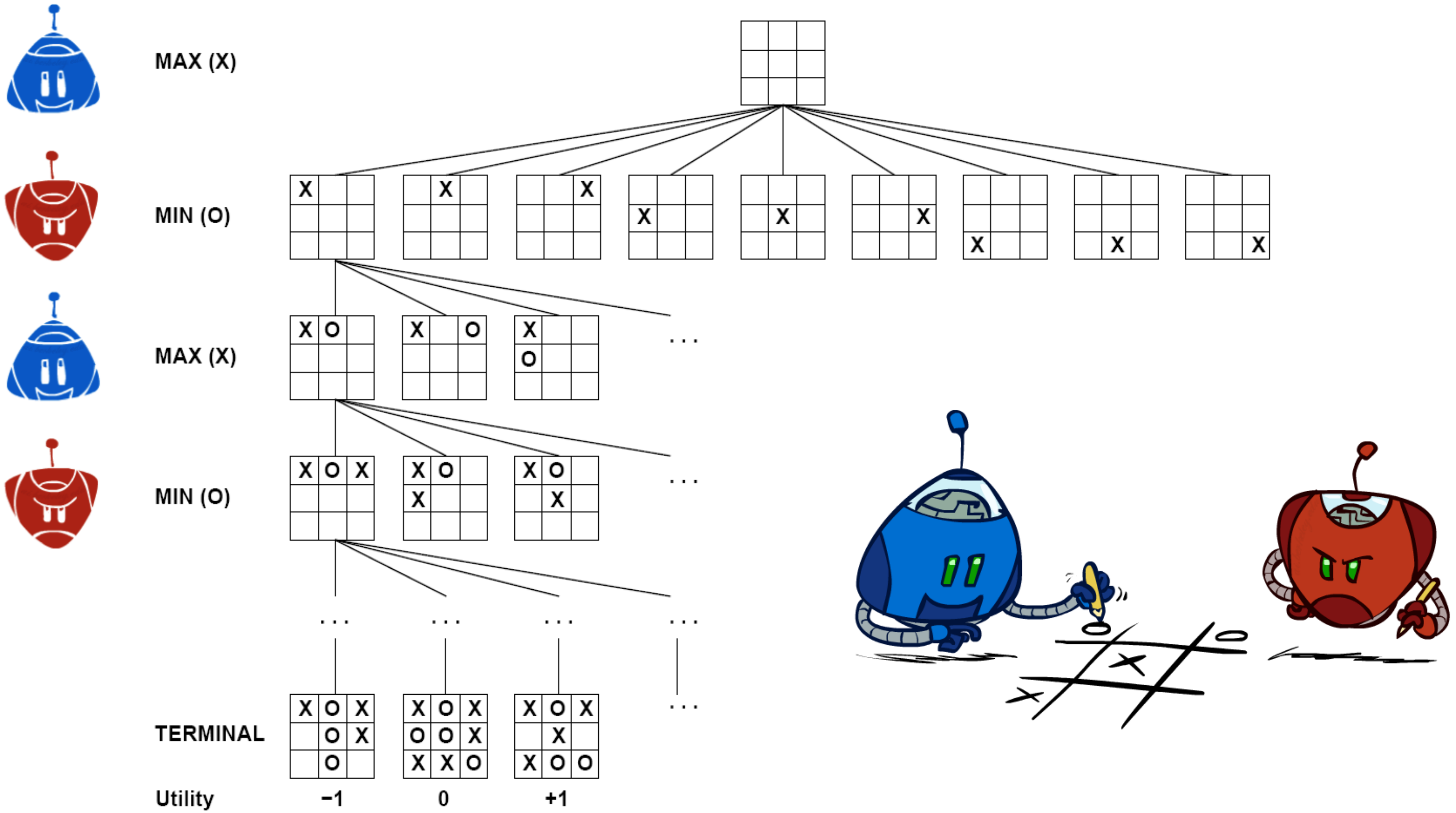


Policy: the agent should choose an action leading to the state with the largest value

Terminal States:

$$V(s) = \text{known}$$

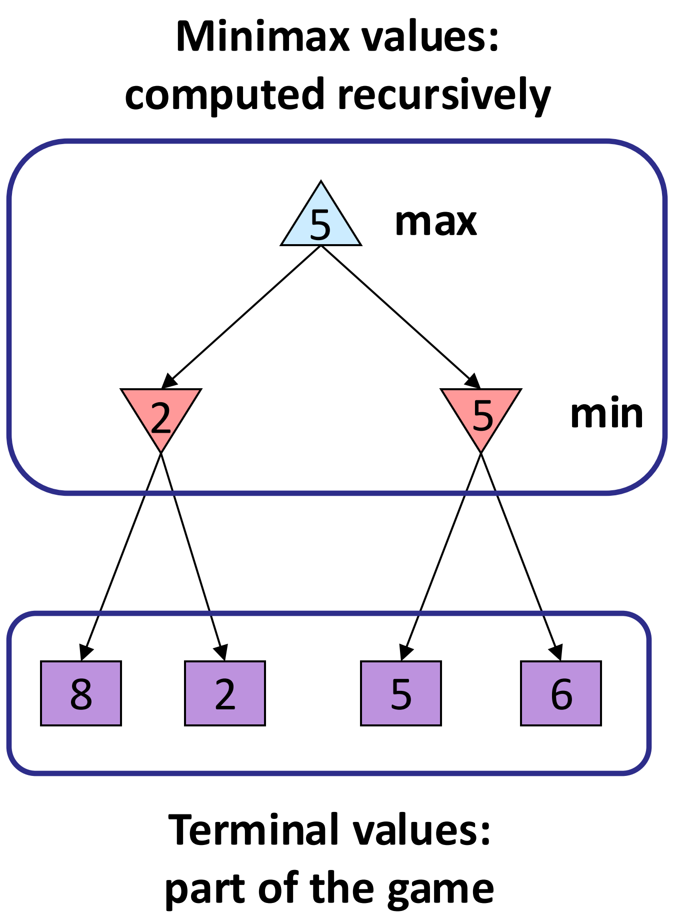
Tic-Tac-Toe Game Tree



Adversarial Search (Minimax)

- Deterministic, zero-sum games:
  - Tic-tac-toe, chess, checkers
  - Players alternate turns
  - One player maximizes result
  - The other minimizes result

- Minimax search:
  - A state-space search tree
  - Compute each node's minimax value: the best achievable utility against a rational (optimal) adversary



Minimax Implementation

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, min-value(successor))
    return v
```

```
def min-value(state):
    initialize v = +∞
    for each successor of state:
        v = min(v, max-value(successor))
    return v
```

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

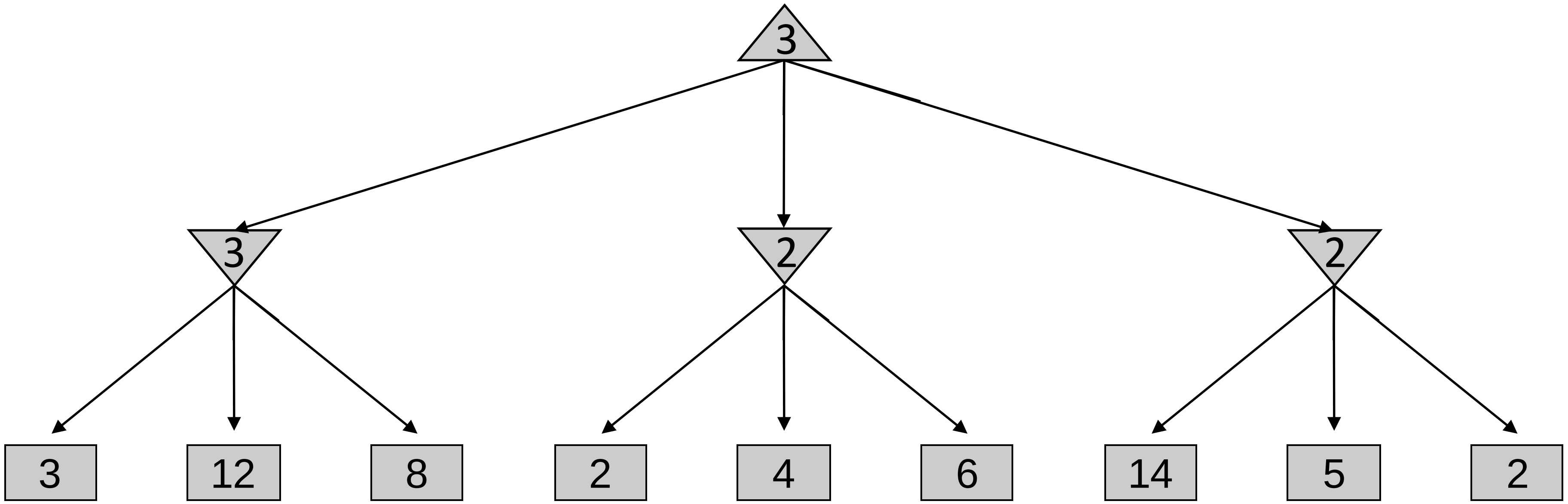
Minimax Implementation

```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is MIN: return min-value(state)
```

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

```
def min-value(state):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor))
    return v
```

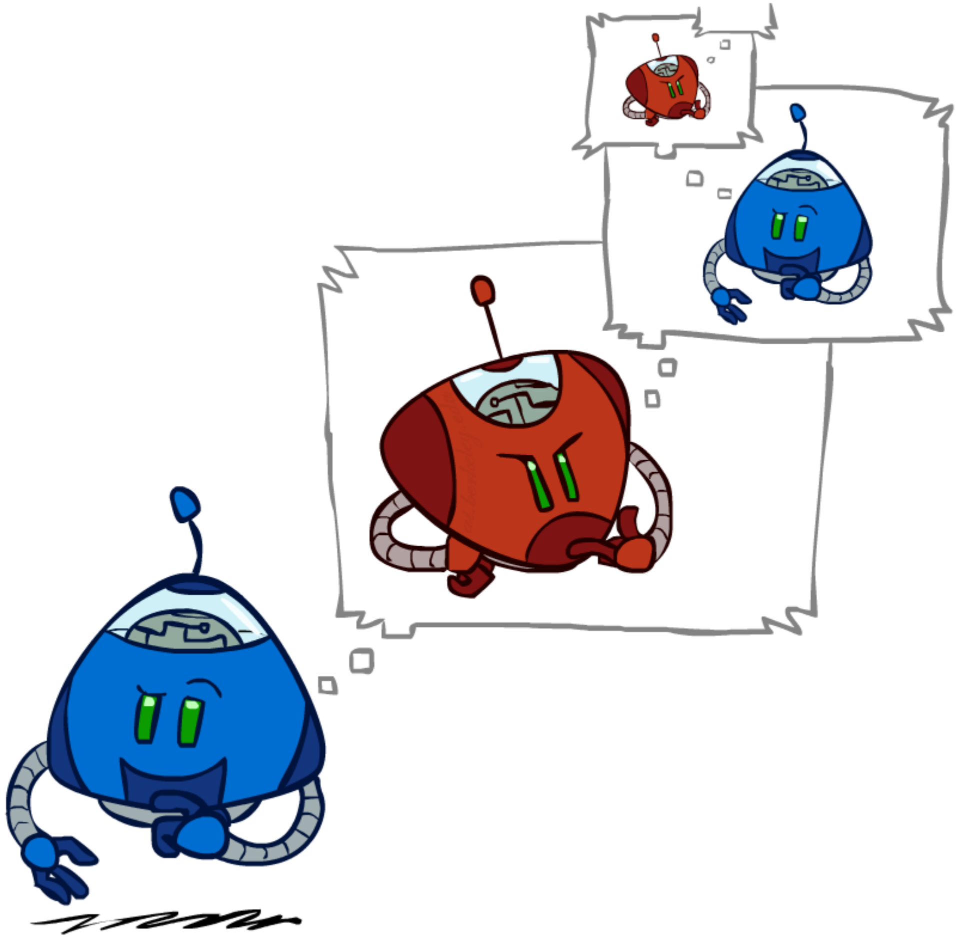
Minimax Example



Minimax Efficiency

- How efficient is minimax?
  - Just like (exhaustive) DFS
  - Time: $O(b^m)$
  - Space: $O(bm)$

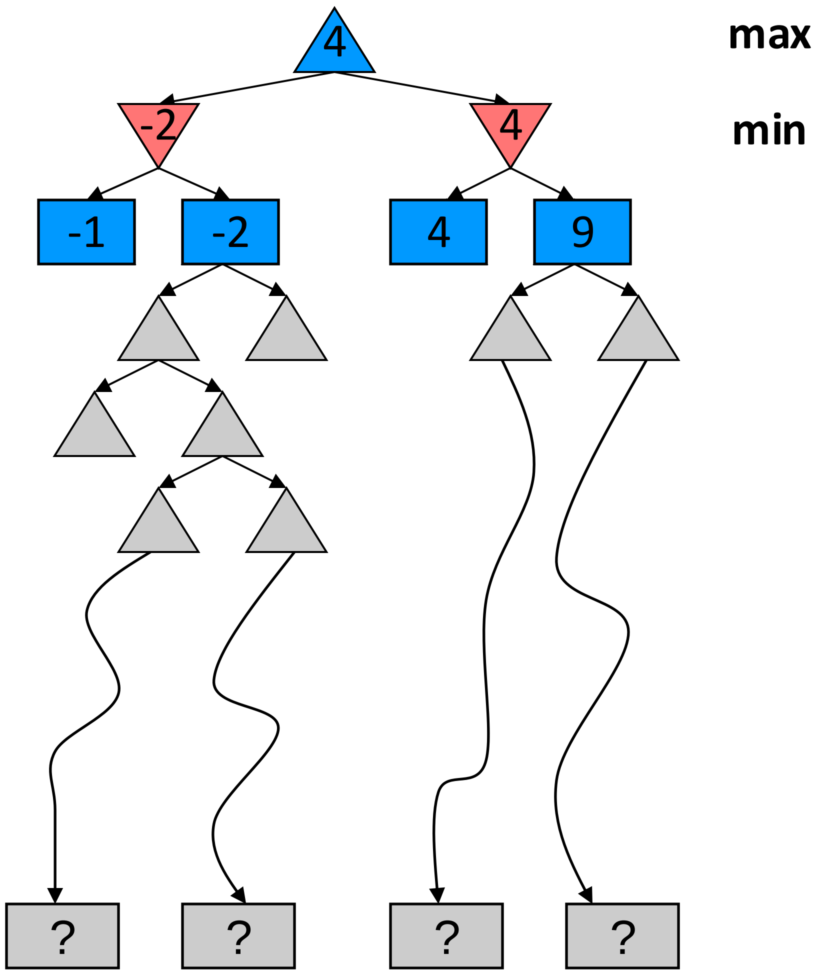
- Example: For chess, $b \approx 35$, $m \approx 100$
  - Exact solution is completely infeasible
  - But, do we need to explore the whole tree?



Resource Limits

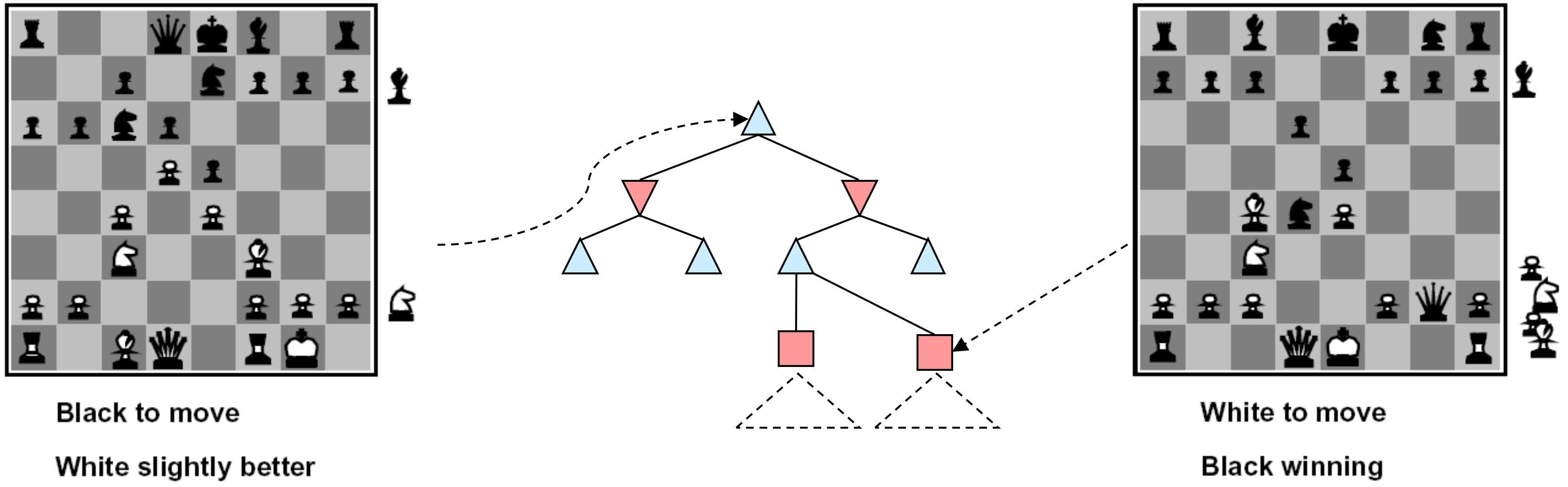
Resource Limits

- Problem: In realistic games, cannot search to leaves!
- Solution: Depth-limited search
  - Instead, search only to a limited depth in the tree
  - Replace terminal utilities with an evaluation function for non-terminal positions
- Example:
  - Suppose we have 100 seconds, can explore 10K nodes / sec
  - So can check 1M nodes per move
  - α-β reaches about depth 8 – decent chess program
- Guarantee of optimal play is gone
- More depth makes a BIG difference



Evaluation Functions

- Evaluation functions score non-terminals in depth-limited search



- Ideal function: returns the actual minimax value of the position
- A simple solution in practice: weighted linear sum of features:

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

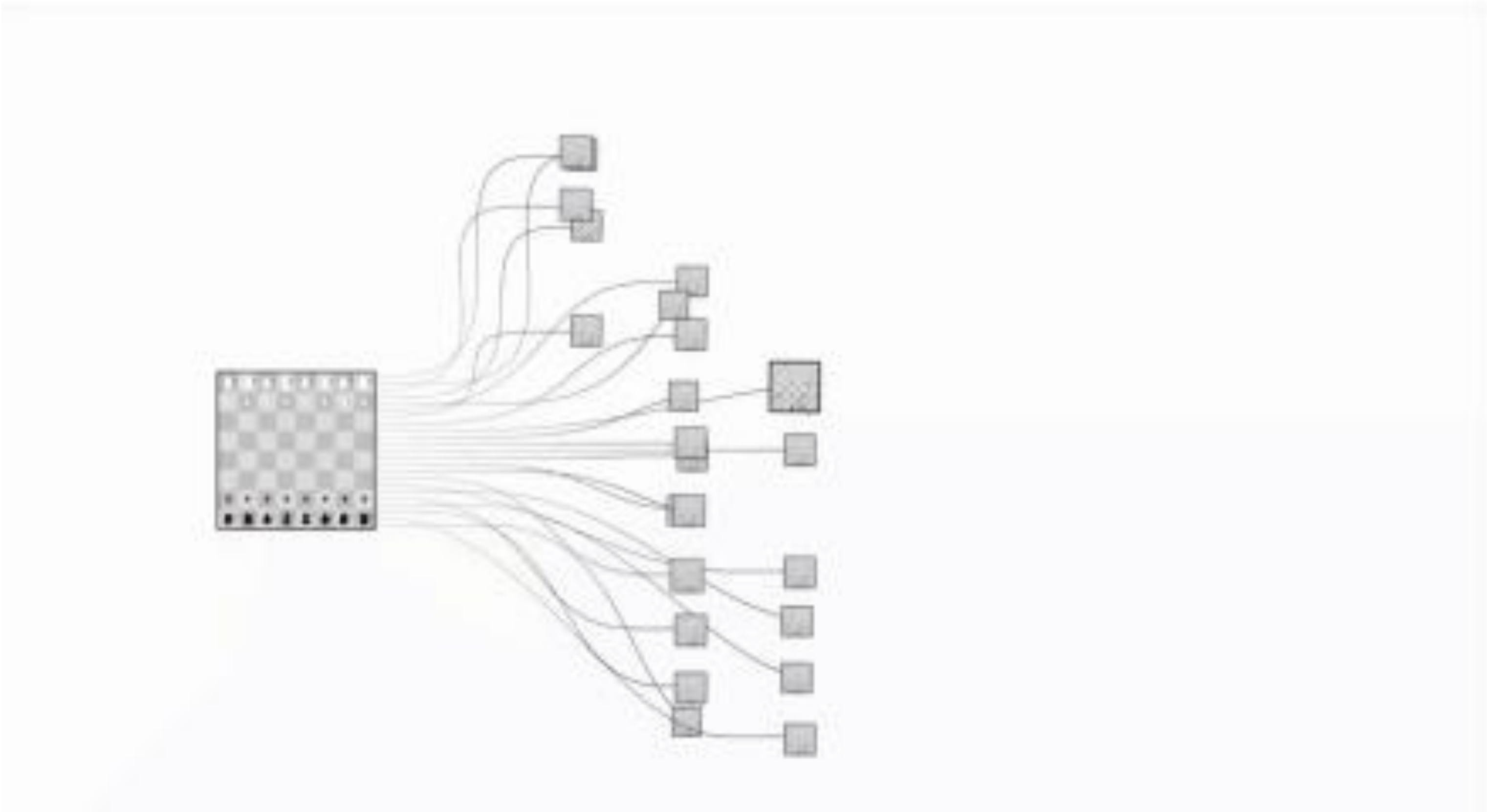
- e.g. $f_1(s)$ = (num white queens – num black queens), etc.

Evaluation Functions

- Recent advances
  - Monte Carlo Tree Search
    - Randomly choose moves until the end of game
    - Repeat for many many times
    - Evaluate the state based on these simulations, e.g., the winning rate
  - Convolutional Neural Network (value network in AlphaGo)
    - Trained from records of game plays to predict a score of the state

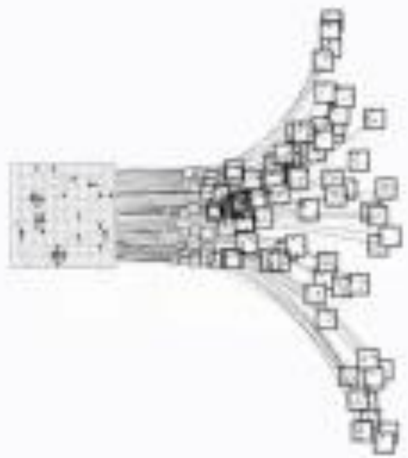
Branching Factor

- Chess



Branching Factor

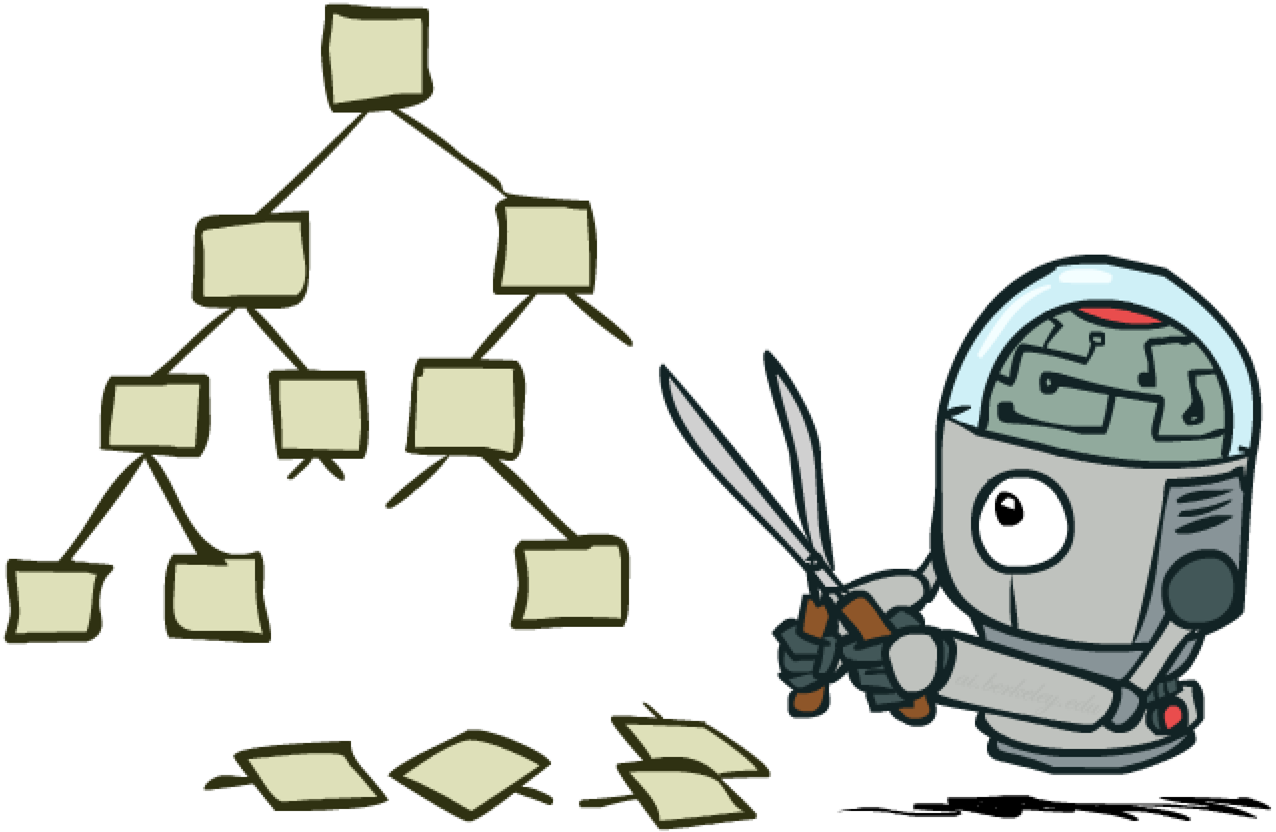
- Go



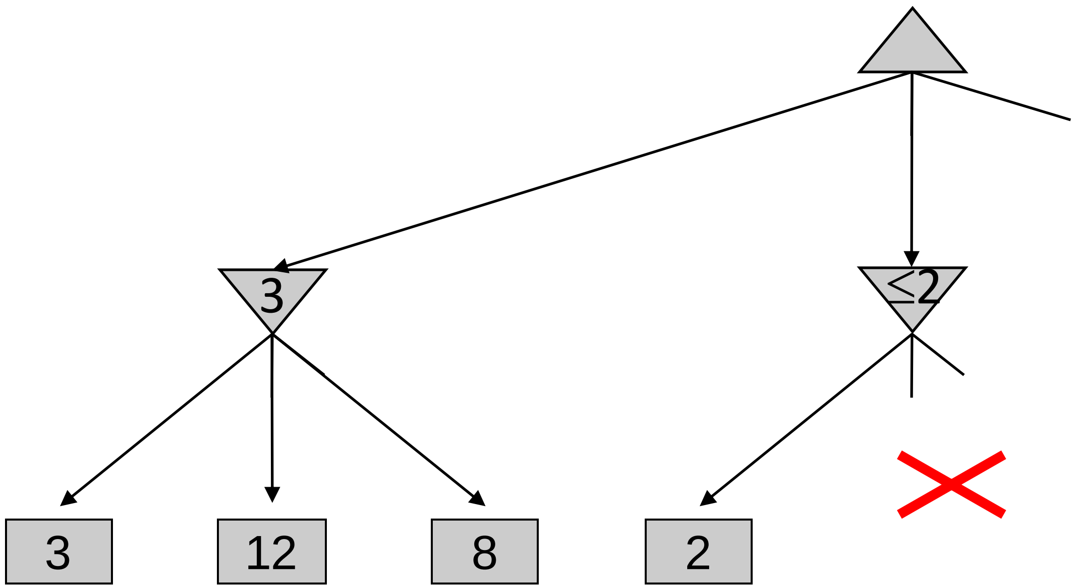
Branching Factor

- Go has a branching factor of up to 361
- Idea: limit the branching factor by considering only good moves
  - AlphaGo uses a Convolutional Neural Network (policy network)
    - Trained from records of game plays
    - Trained using reinforcement learning
      - AlphaGo Zero uses RL only

Game Tree Pruning

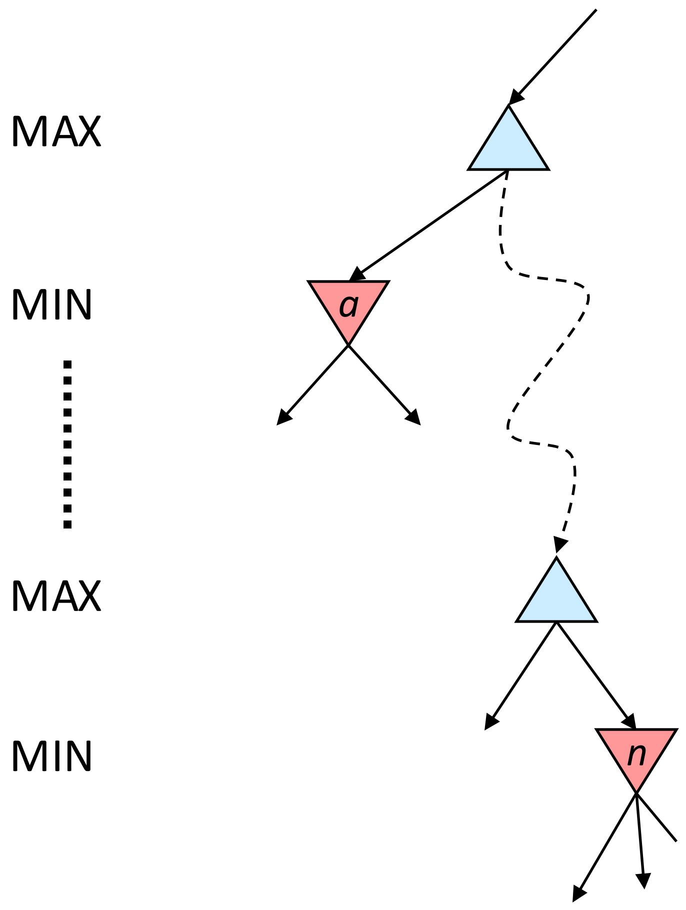


Minimax Pruning



Alpha-Beta Pruning

- General configuration (MIN version)
  - We're computing the MIN-VALUE at some node *n*
  - We're looping over *n*'s children, so *n*'s estimate is decreasing
  - Let *a* be the best value that MAX can get at any choice point along the current path from the root
  - **If *n* becomes worse than *a*, then we can stop considering *n*'s other children**
  - Reason: if *n* is eventually chosen, then the nodes along the path shall all have the value of *n*, but *n* is worse than *a* and hence the path shall not be chosen at the MAX



- MAX version is symmetric

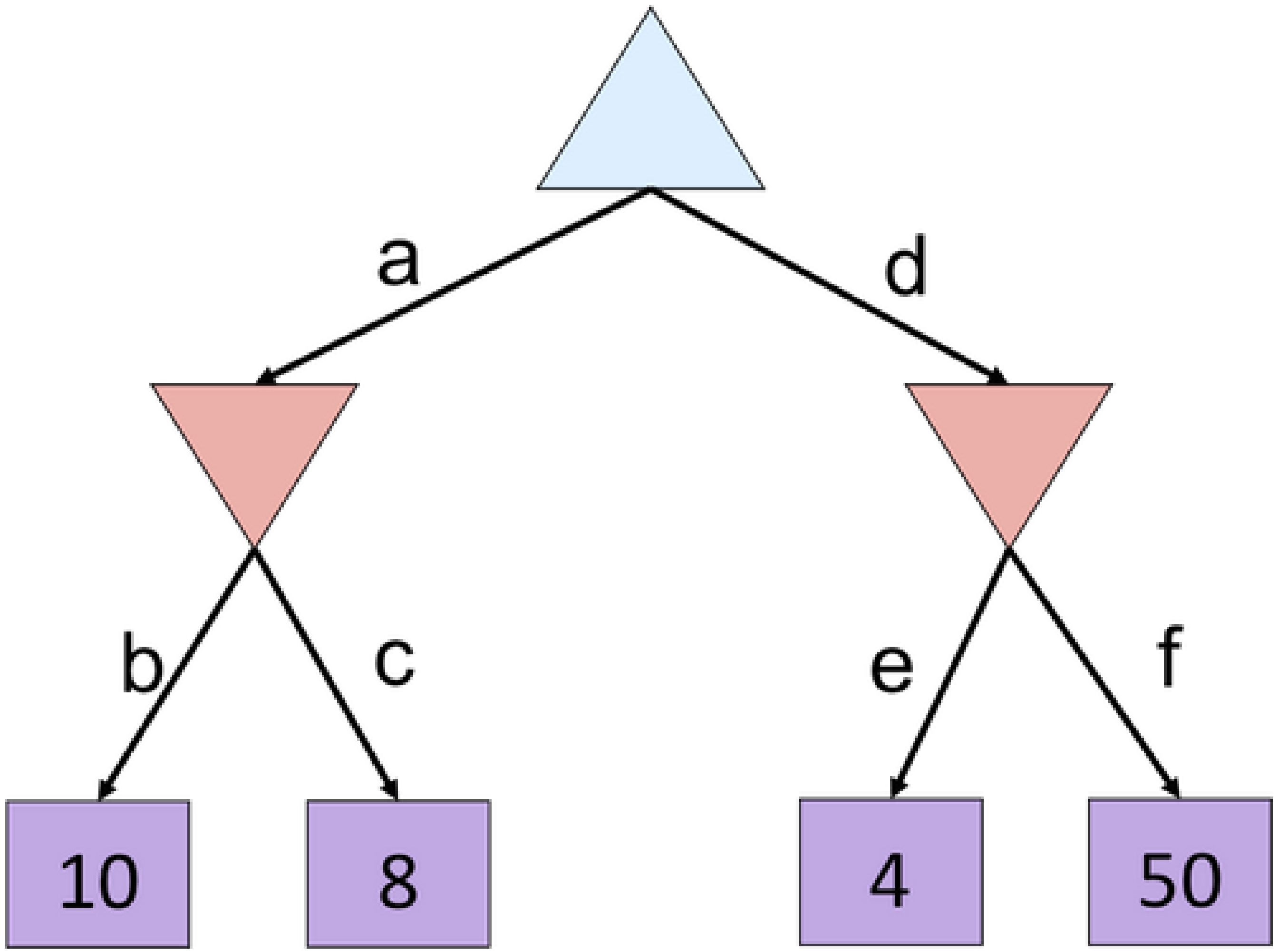
Alpha-Beta Implementation

α: MAX's best option on path to root
β: MIN's best option on path to root

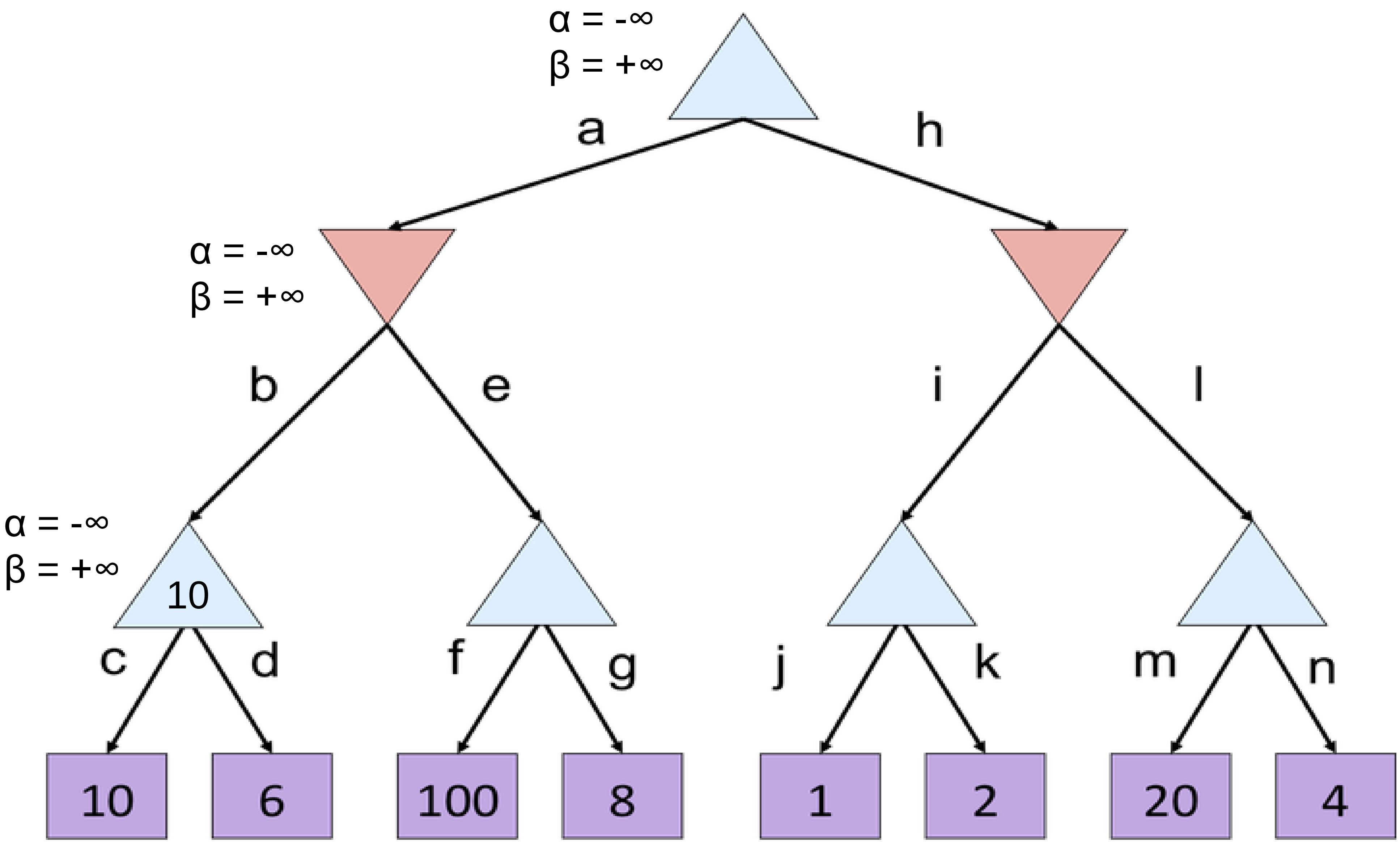
```
def max-value(state, α, β):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor, α, β))
        if v ≥ β return v
        α = max(α, v)
    return v
```

```
def min-value(state , α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α return v
        β = min(β, v)
    return v
```

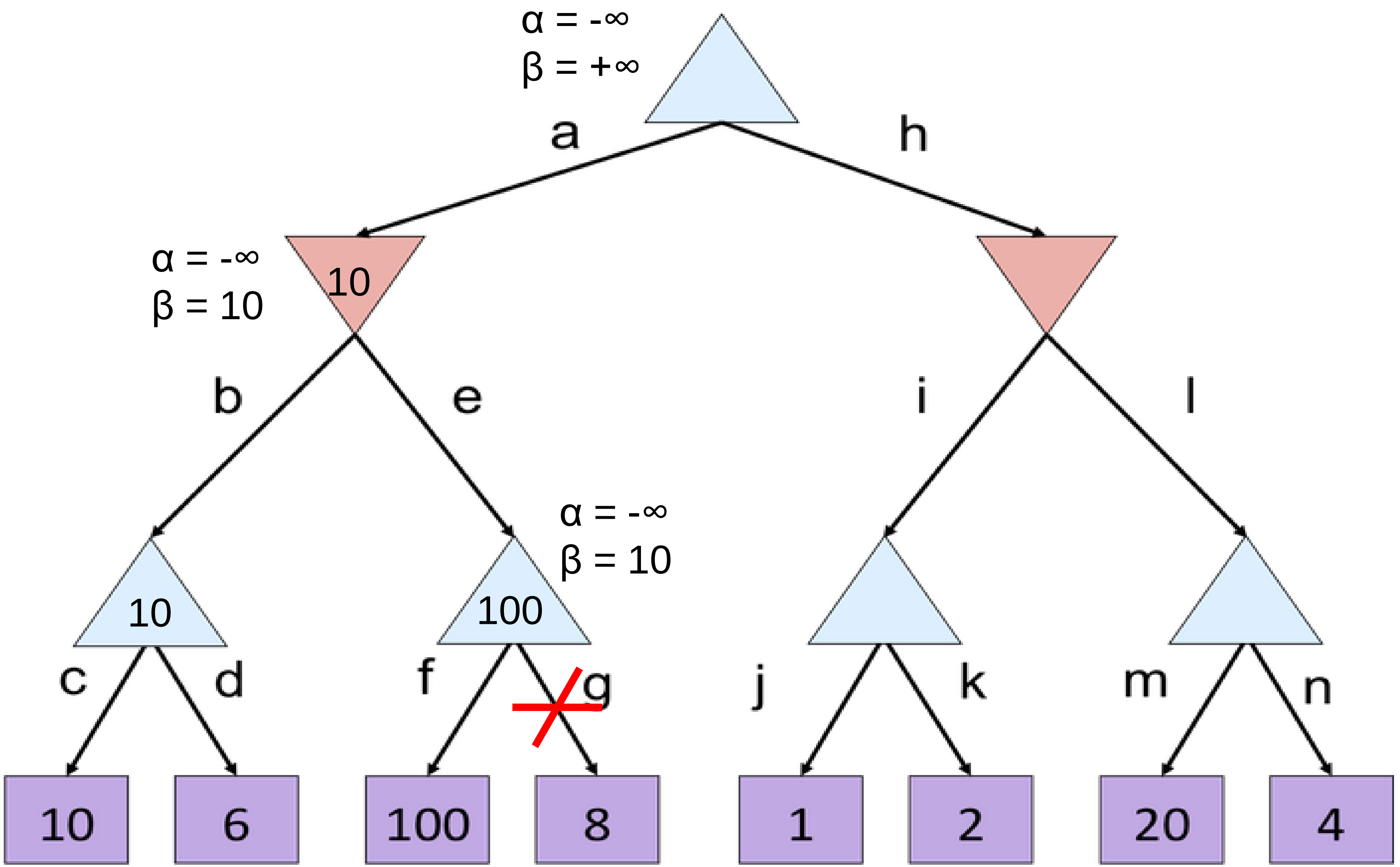
Alpha-Beta Example



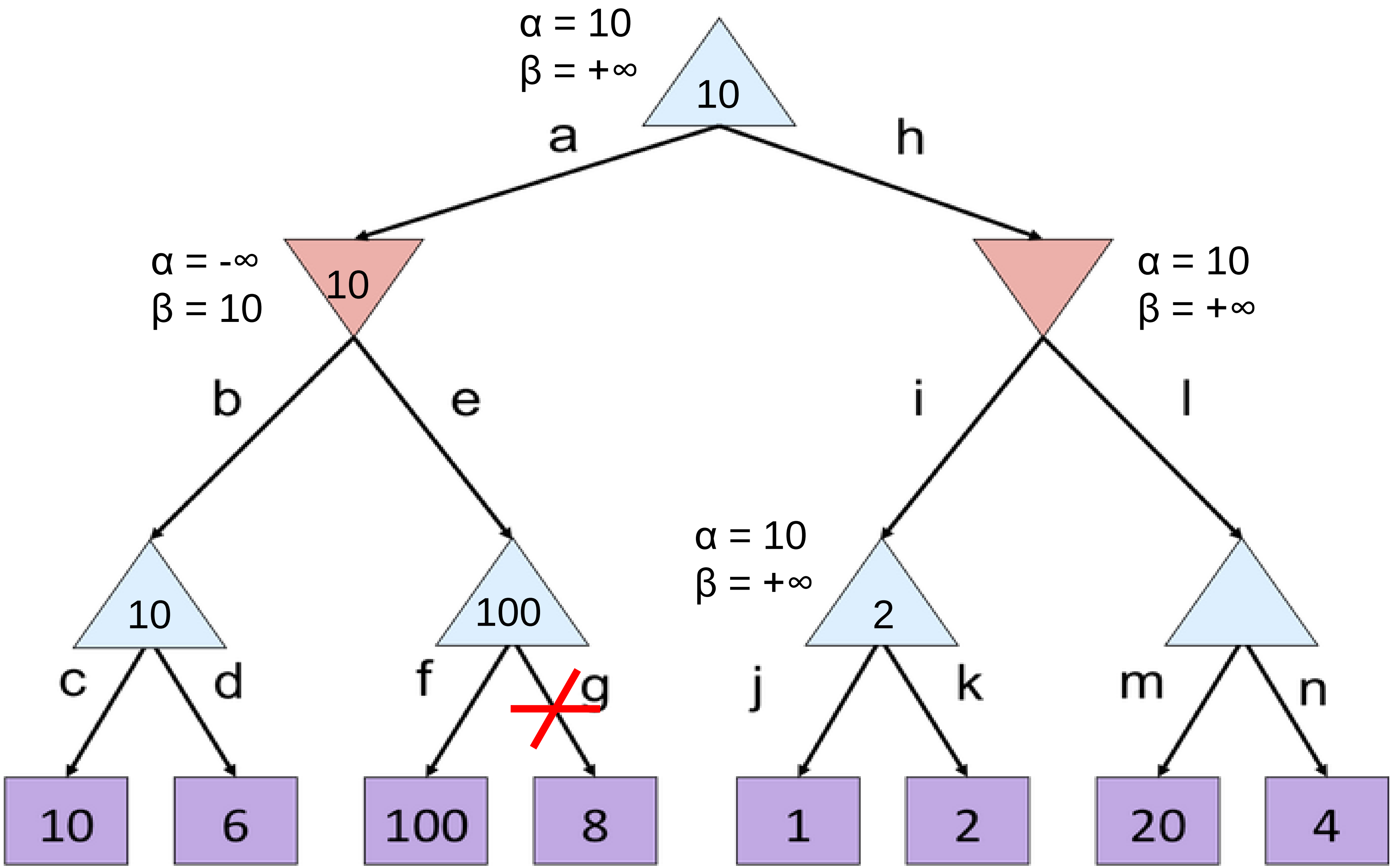
Alpha-Beta Example 2



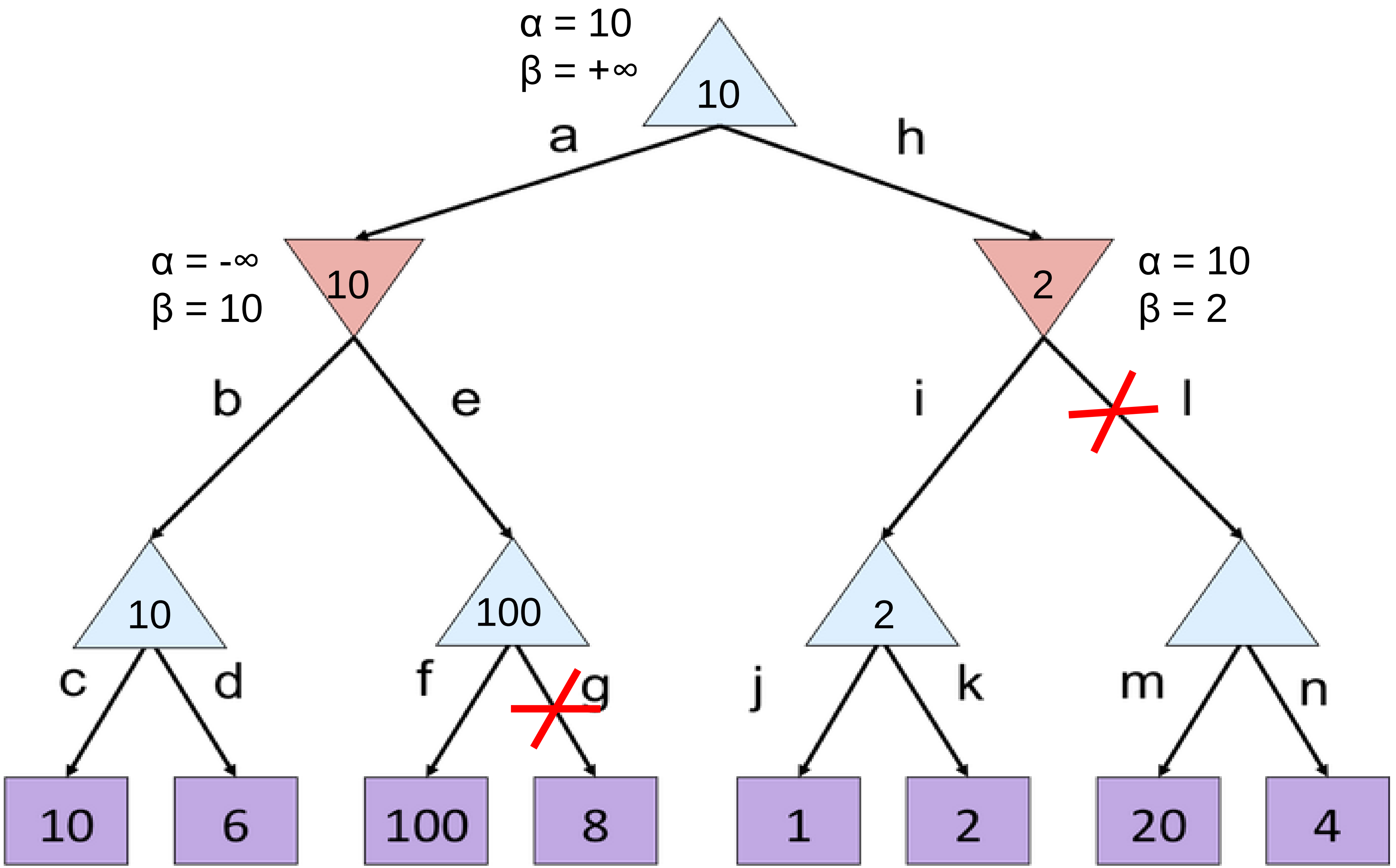
Alpha-Beta Example 2



Alpha-Beta Example 2



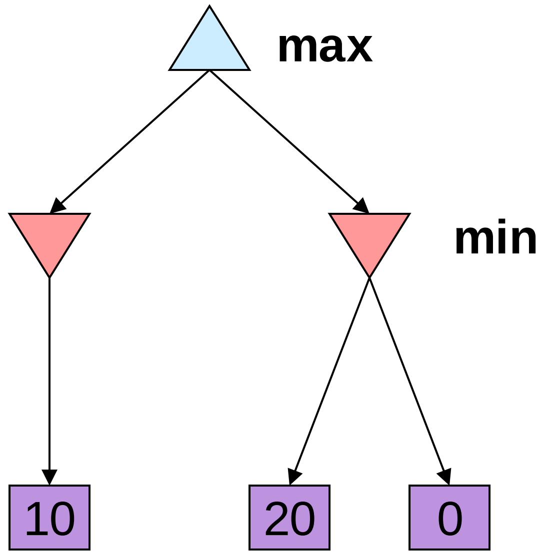
Alpha-Beta Example 2



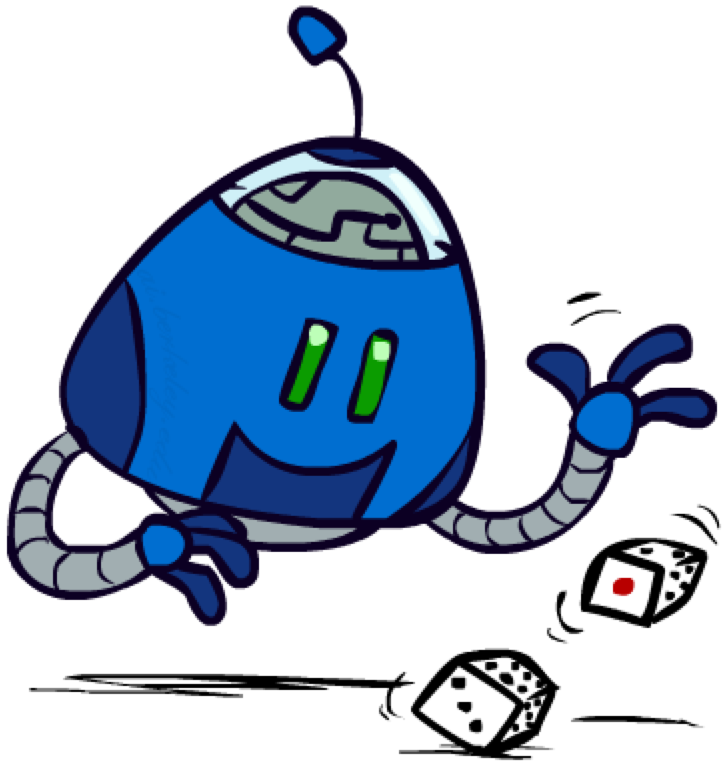
Alpha-Beta Pruning Properties

- Good child ordering improves effectiveness of pruning

- With "perfect ordering":
  - Time complexity drops to $O(b^{m/2})$
  - Doubles solvable depth!

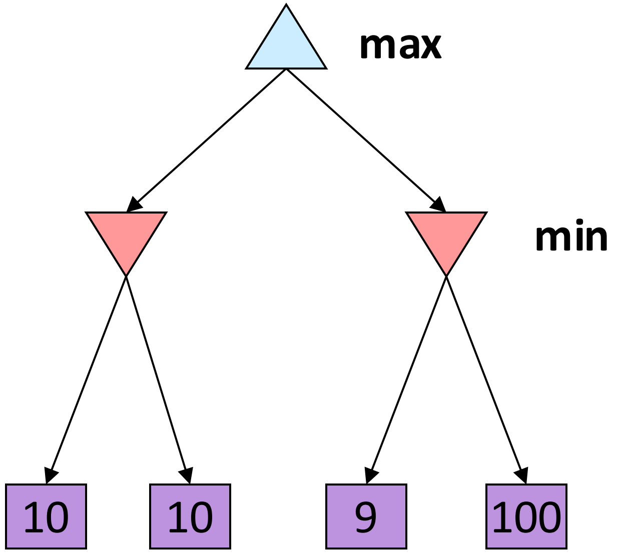


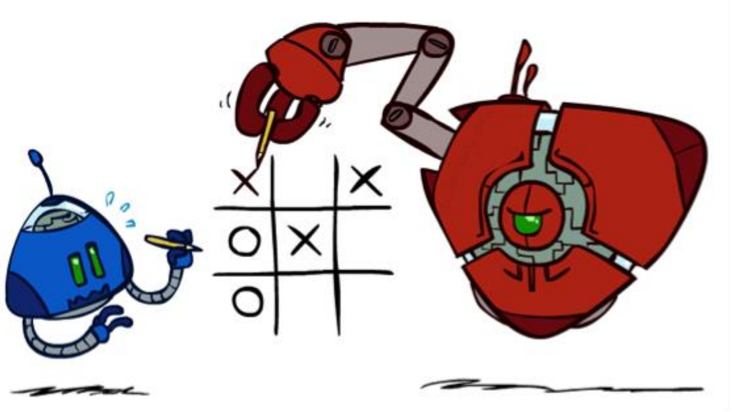
Uncertain Outcomes



Worst-Case vs. Average Case

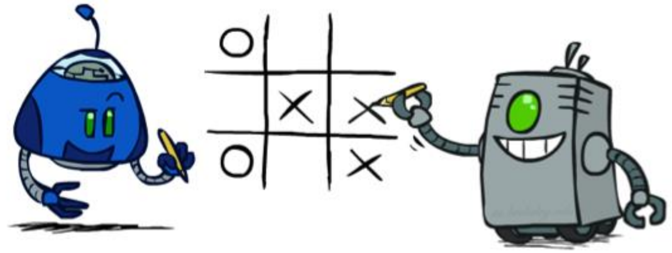
- The hidden assumption behind minimax
  - Your opponent is rational and smart

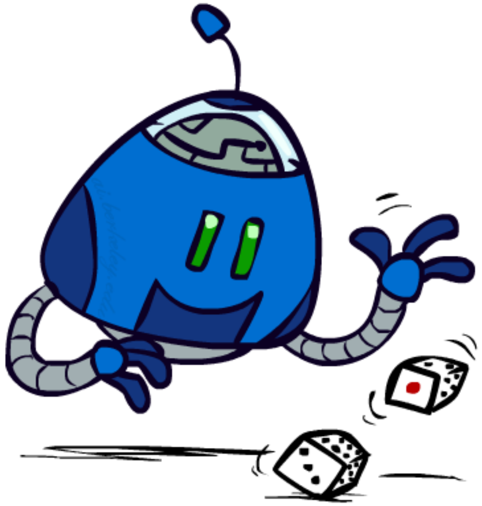




Worst-Case vs. Average Case

- What if…
  - Unpredictable opponents
    - E.g., the ghosts respond randomly
  - Explicit randomness
    - E.g., rolling dice
  - Actions can fail
    - E.g., when moving a robot, wheels might slip





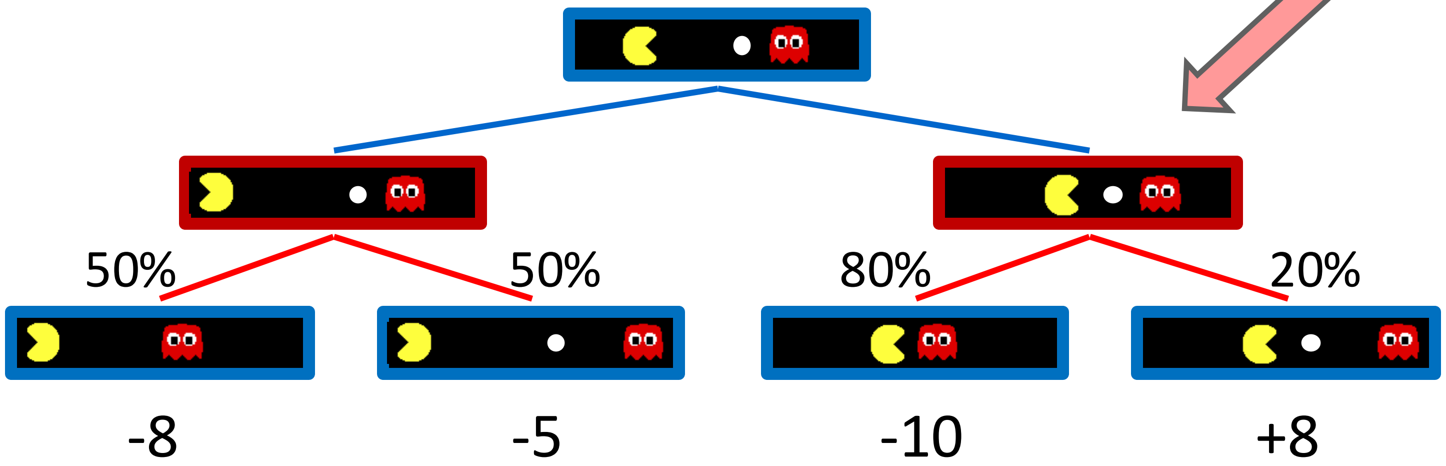
State Values

States Under Agent's Control:

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

States Under Opponent's Control:

$$V(s') = \sum_{s \in \text{successors}(s')} P(s) \times V(s)$$

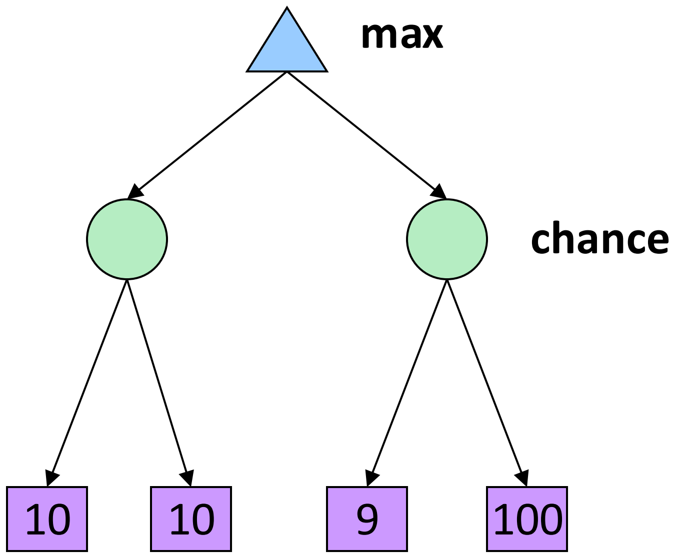


Terminal States:

$$V(s) = \text{known}$$

Expectimax Search

- Expectimax search: compute the average score under optimal play
  - Max nodes as in minimax search
  - Chance nodes are like min nodes but the outcome is uncertain
  - Calculate their expected utilities, i.e. taking weighted average (expectation) of children



- Later, we'll learn how to formalize the underlying uncertain-result problems as Markov Decision Processes

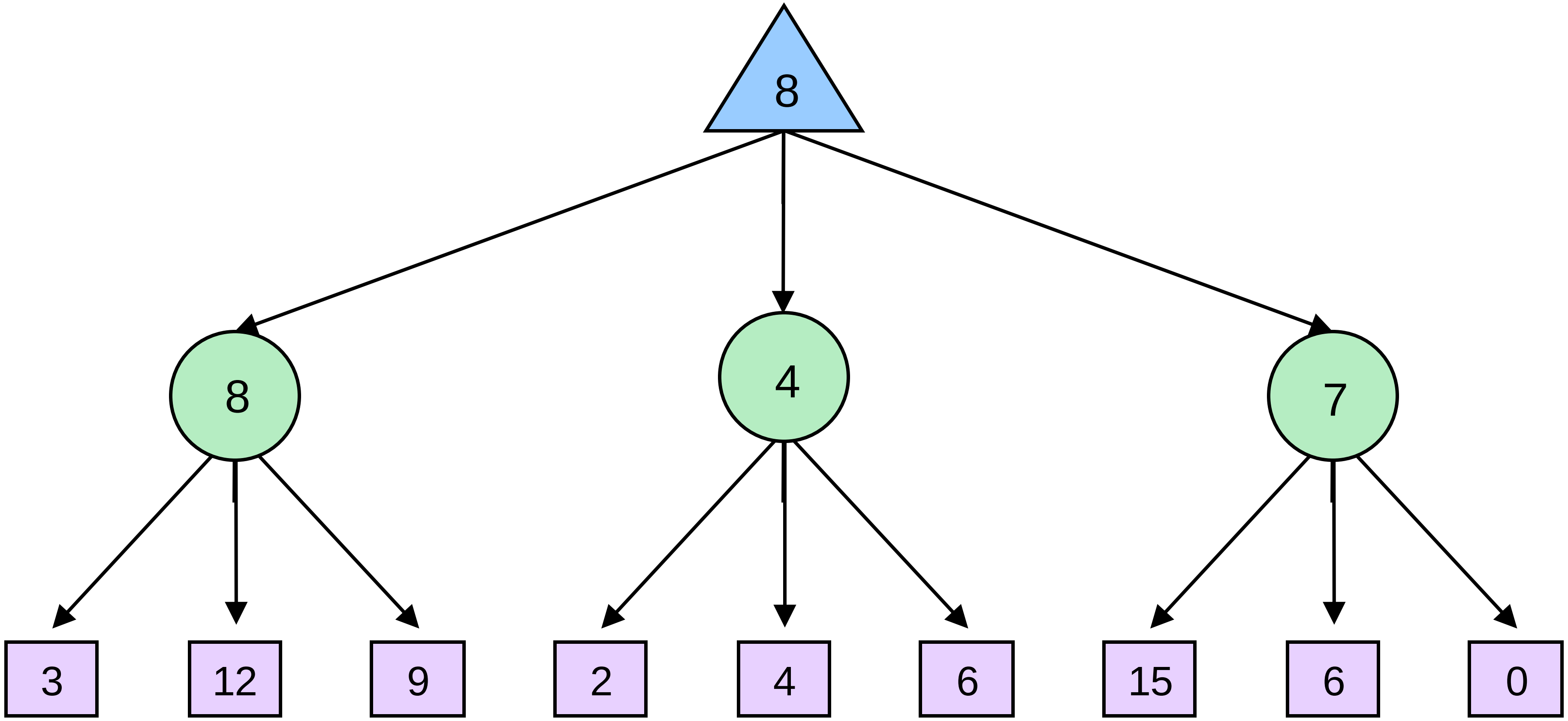
Expectimax Pseudocode

```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is EXP: return exp-value(state)
```

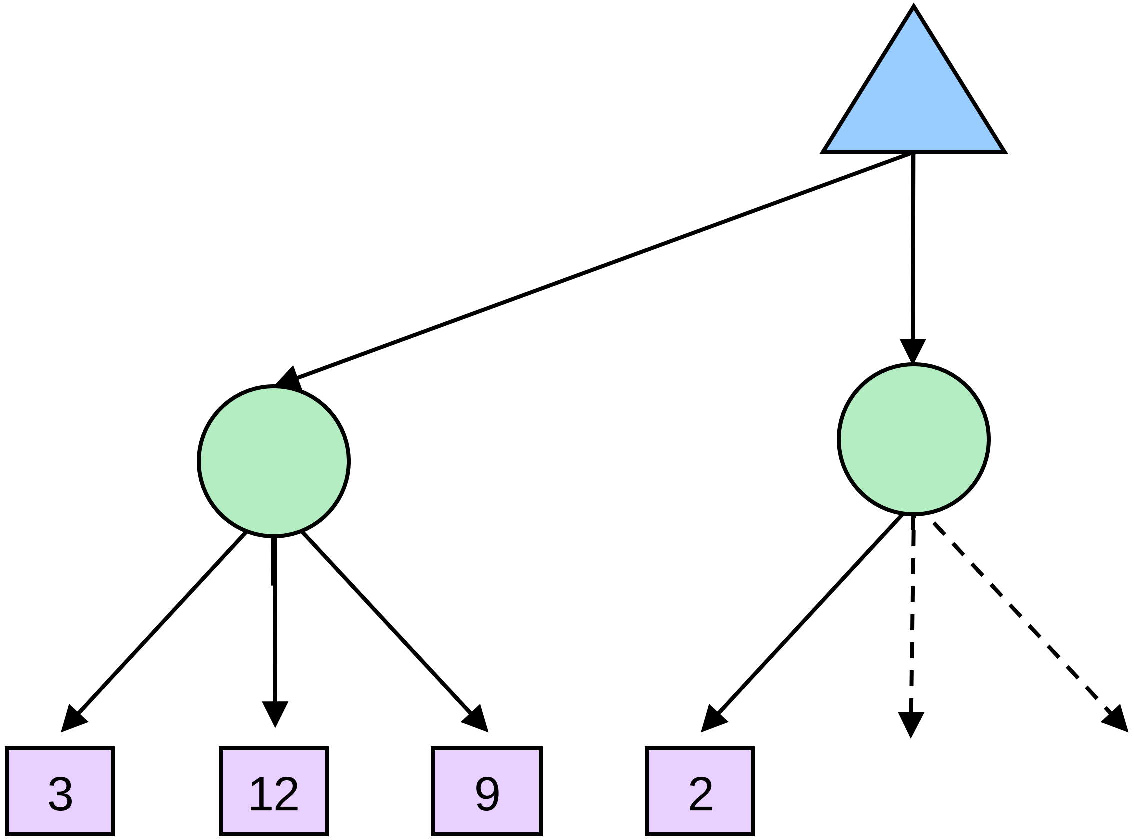
```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

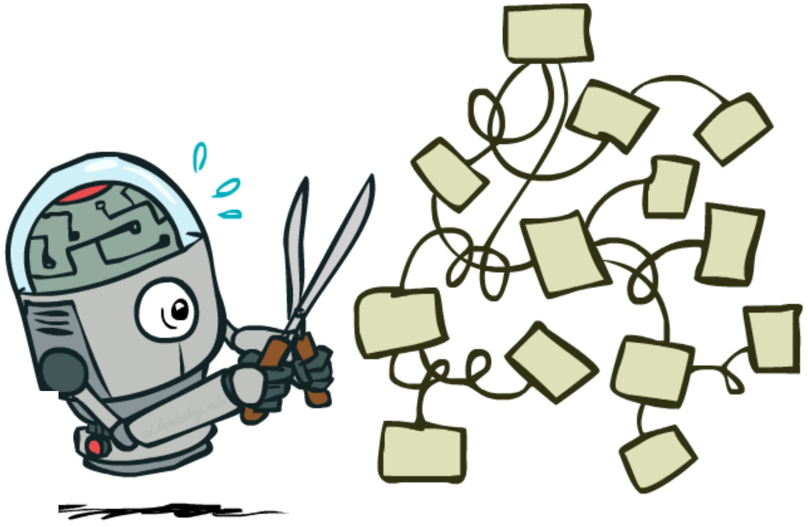
```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
```

Expectimax Example



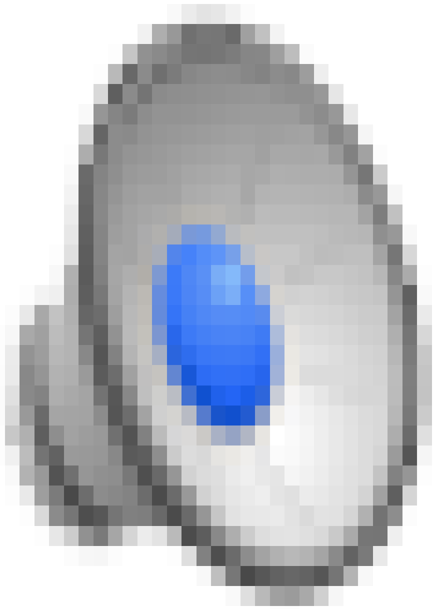
Expectimax Pruning?





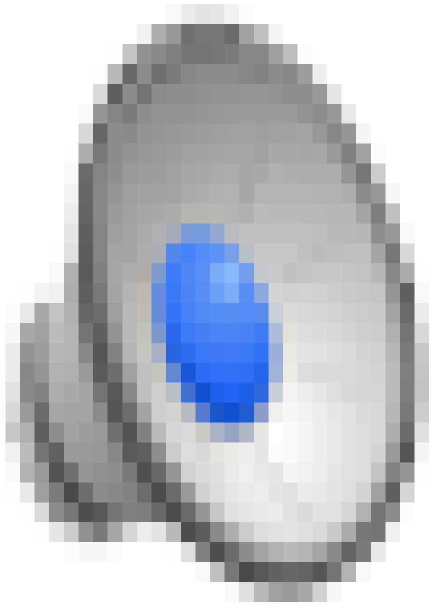
Video of Demo Minimax vs Expectimax (Min)

The game:
+10 for eating each dot
+500 for eating all the dots
-500 for being eaten
-1 for each move



Video of Demo Minimax vs Expectimax (Exp 1)

The game:
+10 for eating each dot
+500 for eating all the dots
-500 for being eaten
-1 for each move

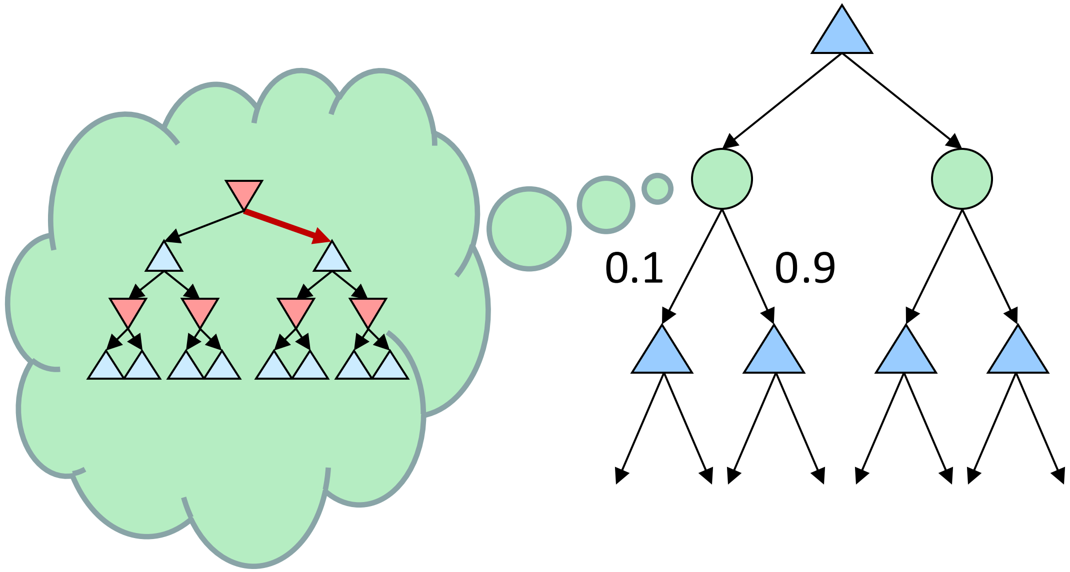


Video of Demo Minimax vs Expectimax (Exp 2)

The game:
+10 for eating each dot
+500 for eating all the dots
-500 for being eaten
-1 for each move

Quiz: Informed Probabilities

- Let's say you know that your opponent is actually running a depth 2 minimax, using the result 80% of the time, and moving randomly otherwise
- Question: What tree search should you use?



- Answer: Expectimax!
  - To figure out EACH chance node's probabilities, you have to run a simulation of your opponent
  - This kind of thing gets very slow very quickly
  - Even worse if you have to simulate your opponent simulating you…

Modeling Assumptions

The Dangers of Optimism and Pessimism

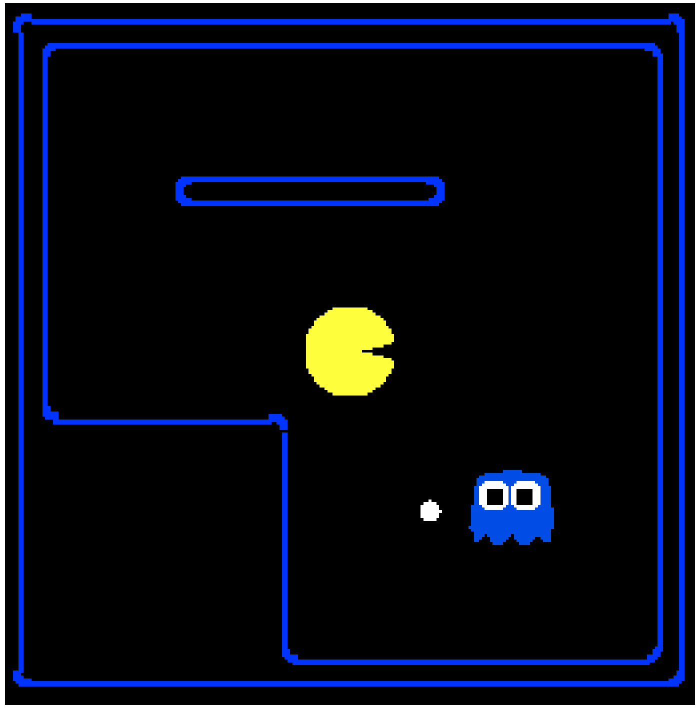
Dangerous Optimism

Assuming chance when the world is adversarial

Dangerous Pessimism

Assuming the worst case when it's not likely

Assumptions vs. Reality



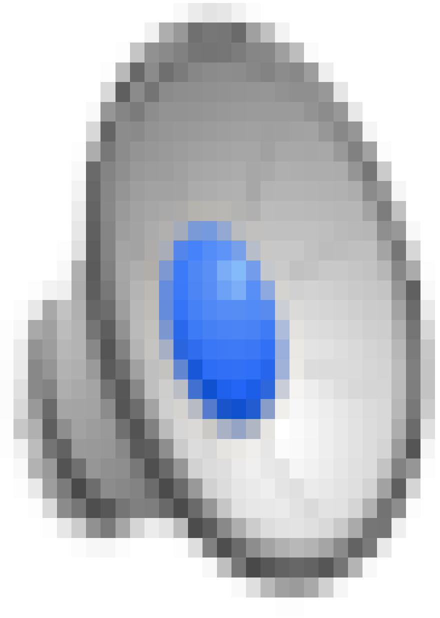
	Adversarial Ghost	Random Ghost
Minimax Pacman		
Expectimax Pacman		

Results from playing 5 games

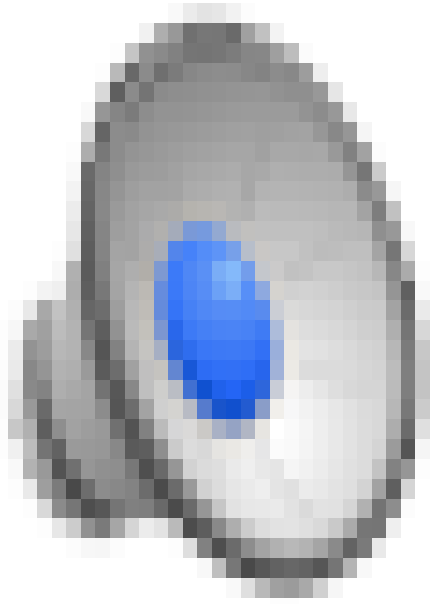
Pacman used depth 4 search with an eval function that avoids trouble
Ghost used depth 2 search with an eval function that seeks Pacman

Video of Demo World Assumptions
Random Ghost – Expectimax Pacman

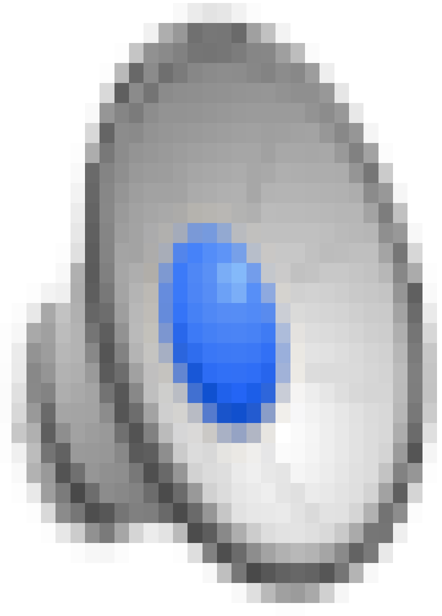
Video of Demo World Assumptions
Adversarial Ghost – Minimax Pacman



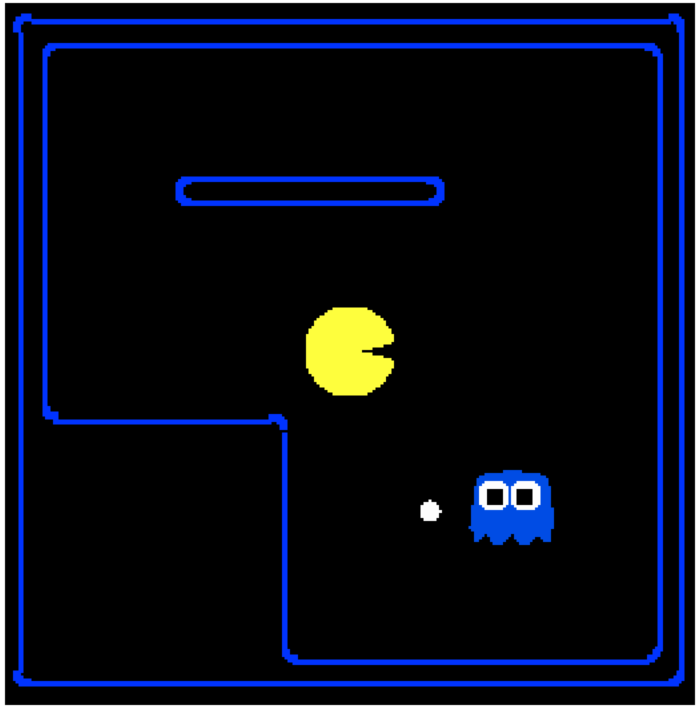
Video of Demo World Assumptions
Adversarial Ghost – Expectimax Pacman



Video of Demo World Assumptions
Random Ghost – Minimax Pacman



Assumptions vs. Reality

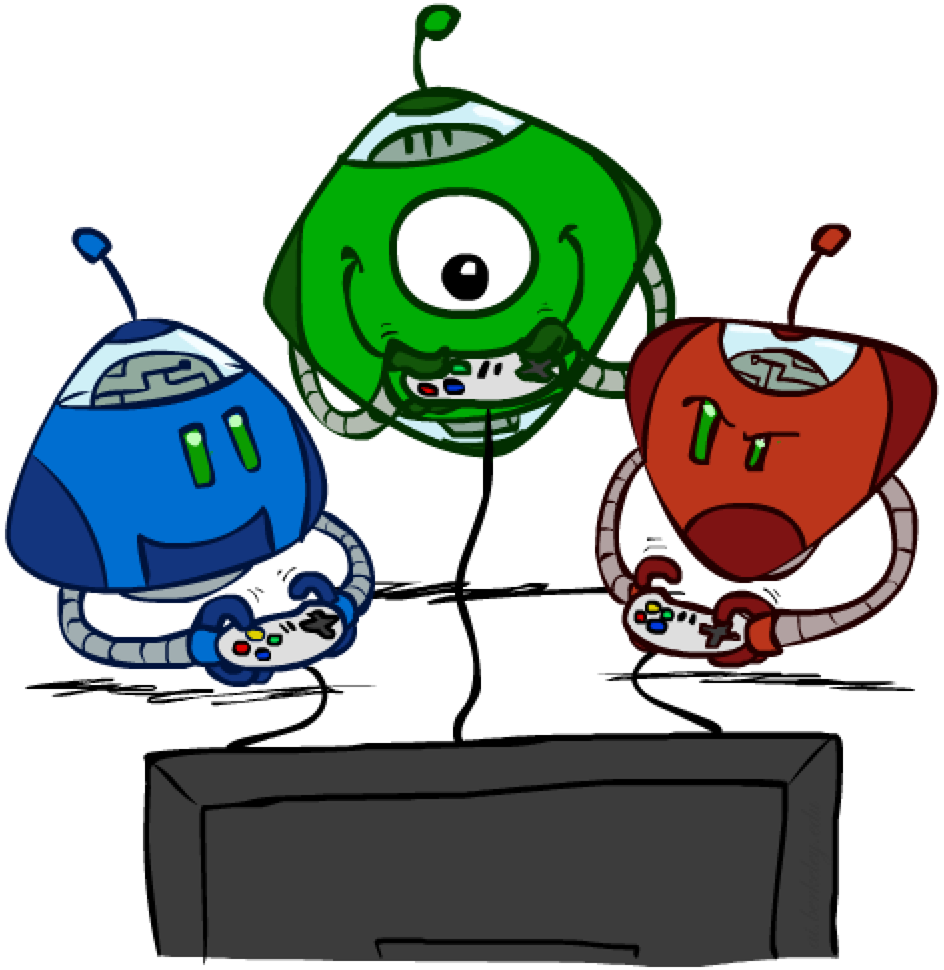


	Adversarial Ghost	Random Ghost
Minimax Pacman	Won 5/5 Avg. Score: 483	Won 5/5 Avg. Score: 453
Expectimax Pacman	Won 1/5 Avg. Score: -303	Won 5/5 Avg. Score: 503

Results from playing 5 games

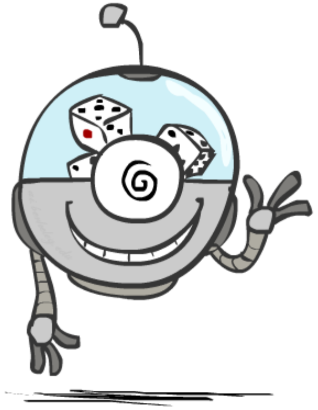
Pacman used depth 4 search with an eval function that avoids trouble
Ghost used depth 2 search with an eval function that seeks Pacman

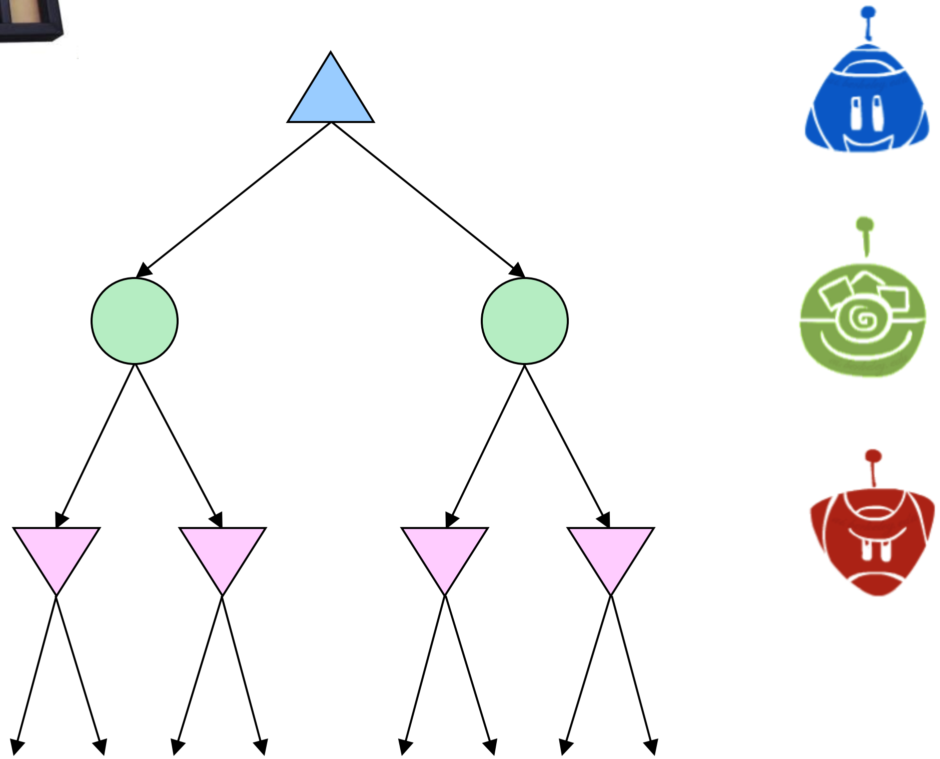
Other Game Types



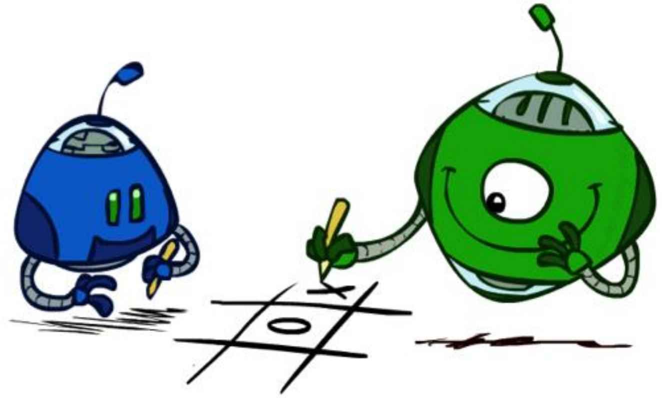
Mixed Layer Types

- Backgammon
- Expectiminimax
  - Environment is an extra “random agent” player that moves after each min/max agent
  - Each node computes the appropriate combination of its children

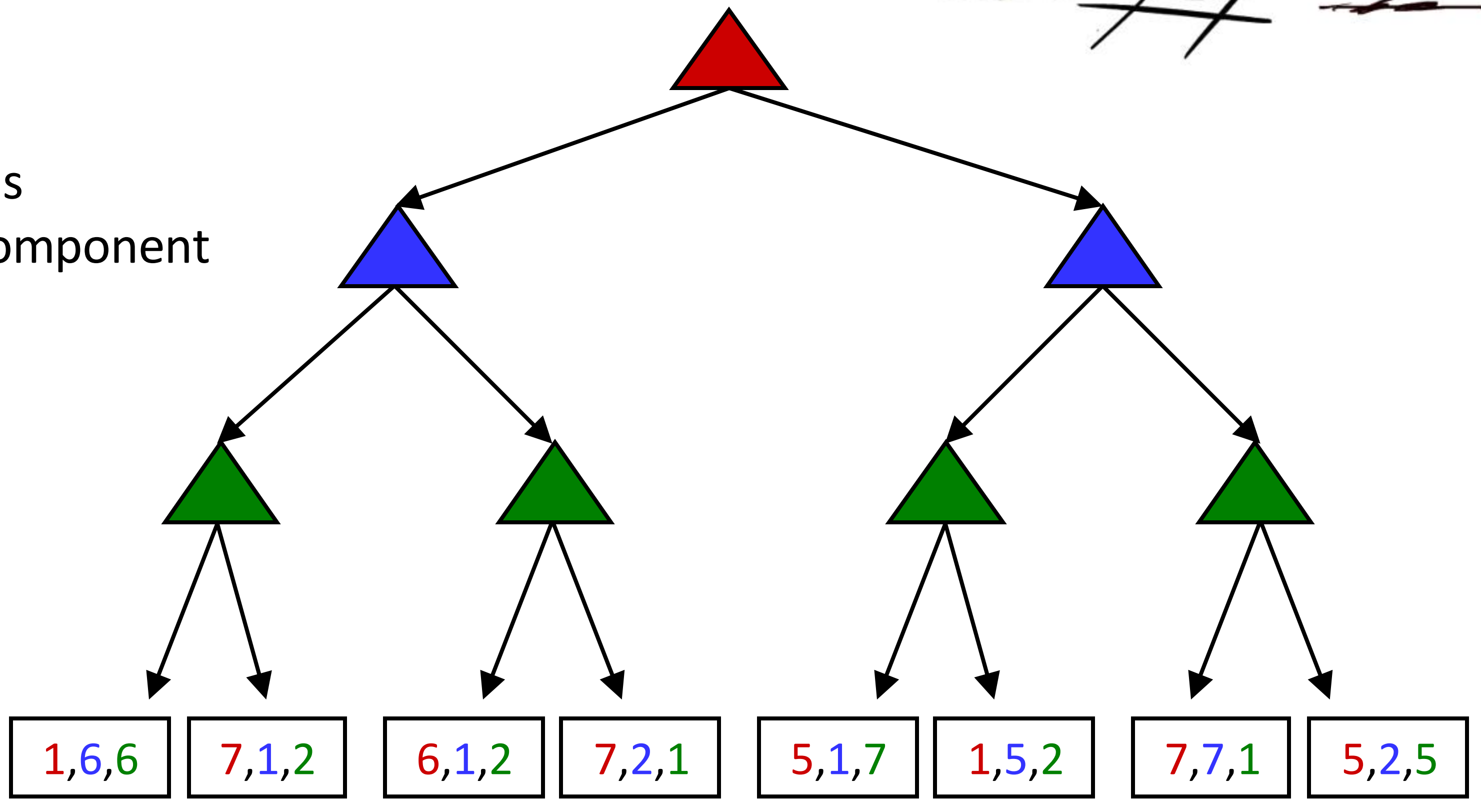


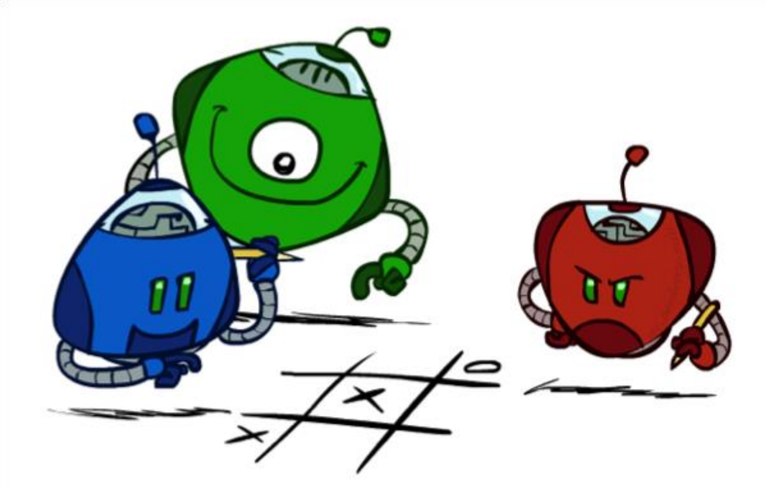


Multi-Agent Utilities

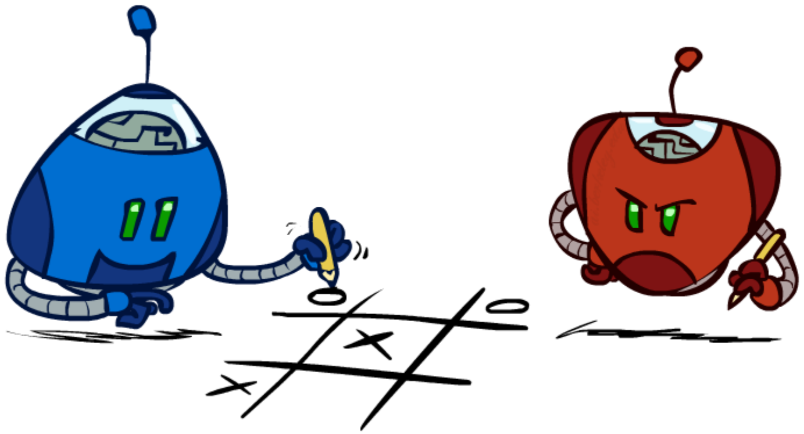


- What if the game is not zero-sum, or has multiple players?
- Generalization of minimax:
  - Terminals have utility tuples
  - Node values are also utility tuples
  - Each player maximizes its own component
  - Can give rise to cooperation and competition dynamically…





Summary



- Adversarial Games
- Adversarial Search
  - Minimax
- Resource Limits
  - Depth-limited search, limiting branching factor
- Game Tree Pruning (alpha-beta pruning)
- Uncertain Outcomes
  - Expectimax
- Other Game Types