Modeling Group Behavior of Pedestrian Crowd

Peng Wang **Timo Korhonen**

In order to investigate how individuals interact and form the collective pattern of crowd, microscopic models have been developed in recent decades, in which an individual's behavioral/psychological status can be modeled and simulated. A well-known model is called social-force model based on Newtonian dynamics. Inspired by the social-force model, this paper presents a new model to characterize group behavior in crowd dynamics. We essentially introduce a new form of group force to characterize social relationship among individuals, and opinion dynamics is further integrated in the model to study how self-organized phenomena emerges from interaction of individuals. The cognition and behavior of individuals are further modeled with environmental stimuli to form a closed-loop process. This method has been partly tested in FDS+Evac, and the testing results justifies it contributes to modeling group behavior in crowd evacuation.

I. INTRODUCTION

How individual behavior leads to emergence of group-level organization is a fascinating research topic, and it is commonly deemed as an inter-discipline involving complex systems, physics, computer science and nonlinear systems. Various models have been proposed to investigate such a system, such as cellular automaton, lattice gas model, and so forth. Around 2000 an innovative pedestrian model was introduced by physical scientists (Helbing and Molnar, 1995; Helbing, Farkas and Vicsek, 2000; Helbing et al., 2002), and it was named by social-force model. This model has been widely accepted and mainly used in simulation of crowd evacuation in the past decade, and it has been incorporated into several advanced egress simulators, such as Fire Dynamics Simulator with Evacuation (Korhonen et. al., 2008, Korhonen, 2017), Pedsim, SimWalk, MassMotion (Oasys, 2018), VisWalk, Maces (Pelechano and Badler, 2006) and Menge.

Very interestingly, certain psychological factors are characterized in the model such as "desired velocity" and "social force," and to some extent, these concepts are not physical entities because they reflect people's opinions and exist in human mind. Thus, the model is not typically within the scope of physics study, but in an interdisciplinary domain. In fact, the psychological concept of stress contributes to investigating a pedestrian's response and adaption to the environment, and the social-force model is hereby extended to characterize the interplay between individuals and their surroundings. In this report we will focus on how an individual interacts with the surrounding individuals, and group dynamics will be discussed in detail. We will introduce a new form of group force which contributes to form groups in pedestrian motion. The force is an extension of the traditional social-force, and it describes the social relationship among individuals.

The rest of the paper is organized as below. Section 2 briefly reviews the mathematical formula of the social force model. In Section 3 the group force is presented in an exponential form. In Section 4 we introduce how group dynamics is formed and adapted based on the group force, and the concluding remarks are presented in Section 5.

Peng Wang previously studied in the Department of Electrical and Computer Engineering, University of Connecticut, Storrs, USA,

II. About the Social-Force Model

The social-force model presents psychological forces that drive pedestrians to move as well as keep a proper distance with others. In this model an individual's motion is motivated by a self-driving force $\boldsymbol{f}_i^{drv}$ and resistances come from surrounding individuals and facilities (e.g., walls). Especially, the model describes the social-psychological tendency of two individuals to keep proper interpersonal distance (as called the social-force) in collective motion, and if people have physical contact with each other, physical forces are also taken into account. Let $\boldsymbol{f}_{ij}$ denote the interaction from individual j to individual i, and $\boldsymbol{f}_{iw}$ denote the force from walls or other facilities to individual i. The change of the instantaneous velocity $\boldsymbol{v}_i(t)$ of individual i is given by the Newton Second Law:

$$m_i \frac{d\boldsymbol{v}_i(t)}{dt} = \boldsymbol{f}_i^{drv} + \sum_{j(\neq i)} (\boldsymbol{f}_{ij}^{soc} + \boldsymbol{f}_{ij}^{phy}) + \sum_w (\boldsymbol{f}_{iw}^{soc} + \boldsymbol{f}_{iw}^{phy})$$

$$m_i \frac{d\boldsymbol{v}_i(t)}{dt} = \boldsymbol{f}_i^{drv} + \sum_{j(\neq i)} \boldsymbol{f}_{ij} + \sum_w \boldsymbol{f}_{iw} + \boldsymbol{\xi}_i \tag{1}$$

where m_i is the mass of individual i, and $\boldsymbol{\xi}_i$ is a small fluctuation force. Furthermore, the self-driving force $\boldsymbol{f}_i^{drv}$ is specified by

$$\boldsymbol{f}_i^{drv} = m_i \frac{\boldsymbol{v}_i^0(t) - \boldsymbol{v}_i(t)}{\tau_i}, \tag{2}$$

This force describes an individual tries to move with a desired velocity $\boldsymbol{v}_i^0(t)$ and expects to adapt the actual velocity $\boldsymbol{v}_i(t)$ to the desired velocity $\boldsymbol{v}_i^0(t)$ within a certain time interval τ_i. In particular, the desired velocity $\boldsymbol{v}_i^0(t)$ is the target velocity existing in one's mind while the actual velocity $\boldsymbol{v}_i(t)$ characterizes the physical speed and direction being achieved in the reality. The gap of $\boldsymbol{v}_i^0(t)$ and $\boldsymbol{v}_i(t)$ implies the difference between the human subjective wish and realistic situation, and it is scaled by a time parameter τ_i to generate the self-driving force. This force motivates one to either accelerate or decelerate, making the realistic velocity $\boldsymbol{v}_i(t)$ approaching towards the desired velocity $\boldsymbol{v}_i^0(t)$. This mathematical description of the self-driving force could be dated back to the Payne-Whitham traffic flow model (Payne, 1971; Whitham, 1974). Sometimes $\boldsymbol{v}_i^0(t)$ is rewritten as $\boldsymbol{v}_i^0(t) = v_i^0(t)\boldsymbol{e}_i^0(t)$, where $v_i^0(t)$ is the desired moving speed and $\boldsymbol{e}_i^0(t)$ is the desired moving direction. In a similar manner, we also have $\boldsymbol{v}_i(t) = v_i(t)\boldsymbol{e}_i(t)$ where $v_i(t)$ and $\boldsymbol{e}_i(t)$ represent the physical moving speed and direction, respectively.

The interaction force of pedestrians consists of the social-force $\boldsymbol{f}_{ij}^{soc}$ and physical interaction $\boldsymbol{f}_{ij}^{phy}$. i.e., $\boldsymbol{f}_{ij} = \boldsymbol{f}_{ij}^{soc} + \boldsymbol{f}_{ij}^{phy}$. The social-force $\boldsymbol{f}_{ij}^{soc}$ characterizes the social-psychological tendency of two pedestrians to stay away from each other, and it is given by

$$\boldsymbol{f}_{ij}^{soc} = A_i \exp\left[\frac{(r_{ij} - d_{ij})}{B_i}\right]\boldsymbol{n}_{ij} \quad \text{or} \quad \boldsymbol{f}_{ij}^{soc} = \left(\lambda_i + (1-\lambda_i)\frac{1+\cos\varphi_{ij}}{2}\right) A_i \exp\left[\frac{(r_{ij} - d_{ij})}{B_i}\right]\boldsymbol{n}_{ij} \tag{3}$$

where $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$ are positive constants, which affect the strength and effective range about how two pedestrians are repulsive to each other. The distance of pedestrians i and j is denoted by d_{ij} and the sum of their radii is given by r_{ij}. $\boldsymbol{n}_{ij}$ is the normalized vector which points from pedestrian j to i. The geometric features of two pedestrians are illustrated in Figure 1. In practical simulation, an anisotropic formula of the social-force is widely applied where Equation (3) is scaled by a function of λ_i. The angle φ_{ij} is the angle between the direction of the motion of pedestrian i and the direction to pedestrian j, which is exerting the repulsive force on pedestrian i. If $\lambda_i = 1$, the social force is symmetric and $0 < \lambda_i < 1$ implies that the force is larger in front of a pedestrian than behind. This anisotropic formula assumes that pedestrians move forward, not backward, and thus we can differ the front side from the backside of pedestrians based on their movement.

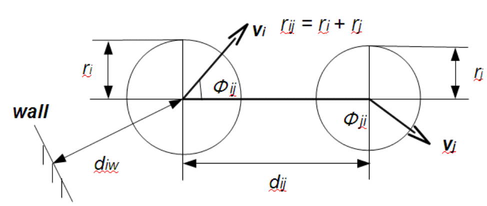


Figure 1. A Schematic View of Two Pedestrians (Equation 3)

The physical interaction $\boldsymbol{f}_{ij}^{phy}$ describes the physical interaction when pedestrians have body contact, and it is composed by an elastic force that counteracts body compression and a sliding friction force that impedes relative tangential motion of two pedestrians. Both of them are valid only when $r_{ij}>d_{ij}$. In Helbing, Farkas and Vicsek, 2000 the interaction force is repulsive. The model may also include an attraction force in its original version (Helbing and Molnar, 1995, Helbing et al., 2002 Korhonen, 2017). The interaction of a pedestrian with obstacles like walls is denoted by $\boldsymbol{f}_{iw}$ and is treated analogously, i.e., $\boldsymbol{f}_{iw} = \boldsymbol{f}_{iw}^{soc} + \boldsymbol{f}_{iw}^{phy}$. Here $\boldsymbol{f}_{iw}^{soc}$ is also an exponential term and $\boldsymbol{f}_{ij}^{phy}$ is the physical interaction when pedestrians touch the wall physically.

By simulating many such individuals in collective motion, several scenarios in crowd movement were demonstrated, and one is called the "faster-is-slower" effect. This scenario was observed when a crowd pass a bottleneck doorway, and it shows that increase of desired speed (i.e., $|\boldsymbol{v}_i^0(t)|$) can inversely decrease the collective speed of crowd passing through the doorway. Another paradoxical phenomenon is called "freezing-by-heating," and it studies two groups of people moving oppositely in a narrow passageway, and the simulation shows that increasing the fluctuation force in Equation (1) can also cause blocking in the counter-flow of pedestrian crowd. Other spatio-temporal patterns include herding effect, oscillation of passing directions, lane formation, dynamics at intersections and so forth.

The validation of the model has been partly conducted based on data sets from real-world experiments. The method of validation involves comparing the simulation of the model with associated observations drawn from video-based analysis (Johansson, Helbing and Shukla, 2007; Johansson et al., 2008).

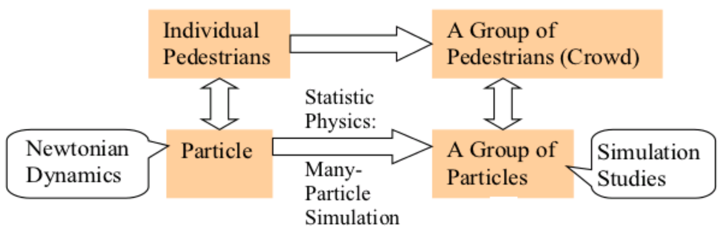


Figure 2. From Particle Dynamics to Crowd Simulation
The Framework of Many-particle Simulation in Helbing, Farkas, and Vicsek, 2000

III. GROUP SOCIAL FORCE

This section will present the a new mathematical formula of group force. The group force is used to combine individuals in certain circumstances, and the model is contributes to modeling group behavior of crowd.

Another important issue is that the social force may also include attractions to describe people's tendency to keep together and form groups in motion. In Helbing and Molnar, 1995 and Helbing et al., 2002 attractions were presented, but separate from the social force. In our model attraction and repulsion are within the same social context: repulsion makes people to keep proper distance while attraction makes them cohesive and form groups. The resulting interaction force is either repulsive or attractive, and it contributes to modeling group dynamics in crowd behavior. The group force is defined as below.

$$\boldsymbol{f}_{ij}^{soc} = A_{ij}(d_{ij}^0 - d_{ij})\exp\left[\frac{(d_{ij}^0 - d_{ij})}{B_{ij}}\right]\boldsymbol{n}_{ij} \quad \text{or} \quad \boldsymbol{f}_{ij}^{soc} = \left(\lambda_i + (1-\lambda_i)\frac{1+\cos\varphi_{ij}}{2}\right)A_{ij}(d_{ij}^0 - d_{ij})\exp\left[\frac{(d_{ij}^0 - d_{ij})}{B_{ij}}\right]\boldsymbol{n}_{ij} \tag{4}$$

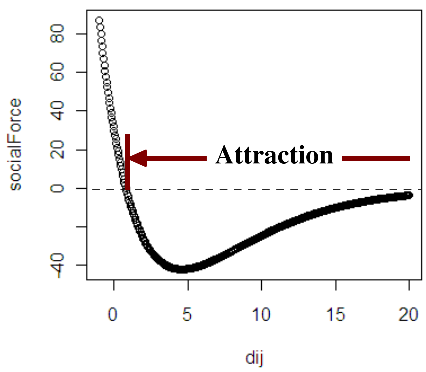
Similar to desired velocity $\boldsymbol{v}_i^0$, the desired distance d_{ij}^0 is the target distance in one's mind, specifying the distance that one expects to adapt oneself with others. The physical distance d_{ij} is the distance achieved in the reality. The gap of d_{ij}^0 and d_{ij} implies the difference between the subjective wish in one's mind and objective feature in the reality. Similar to $\boldsymbol{v}_i^0$- $\boldsymbol{v}_i$, as an indication of time-related stress concerning emergencies, d_{ij}^0 - d_{ij} is an indication of interpersonal stress related to the social composition of crowd. Such stress depends on the intrinsic social characteristics of the crowd, not directly related to the emergency situation. Here A_i and B_i are parameters as introduced before, and $\boldsymbol{n}_{ij}$ is the normalized vector which points from pedestrian j to i. The social force also functions in a feedback manner to make the realistic distance d_{ij} approaching towards the desired distance d_{ij}^0. A difference is that $\boldsymbol{v}_i^0$ and $\boldsymbol{v}_i$ are vectors while d_{ij}^0 and d_{ij} are scalars.

The mathematical characteristics of Equation (4) is discussed as below.

When d_{ij} is sufficiently large, the social force tends to be zero so that individual i and individual j have no interaction. This trend is the same as the repulsive social force as given by Equation (3). If d_{ij} is comparable to d_{ij}^0, interaction of individual i

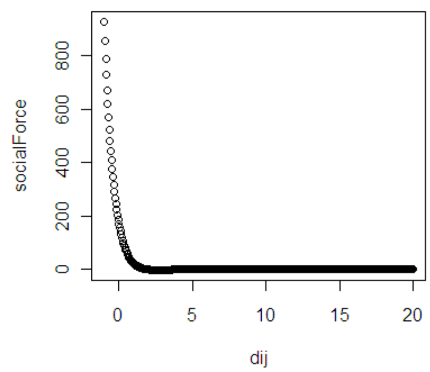
and j comes into existence. If $d_{ij}^0 < d_{ij}$, the social force is attraction whereas it is repulsion if $d_{ij}^0 < d_{ij}$. The attraction reaches the maximal when $d_{ij}^0 - d_{ij} = B_{ij}$, and the maximal is $A_{ij} B_{ij} \exp(1)$. The desired distance d_{ij}^0 makes the curve move horizontally with a certain interval. The curve shape is affected by parameter A_{ij} and B_{ij}. A_{ij} is a linear scaling factor which affects the strength of the force whereas B_{ij} determines the effective range of the interaction. In an addition, the force is approximated by a linear form when $d_{ij}^0 \approx d_{ij}$, and such a linear approximation will be useful in our later discussion.

Two plots of Equation (4) are given as below: Figure 2(a) shows that individual i is attracted by individual j when they are sufficiently close, and this suggests that individual i is probably familiar with j. Figure 2(b) does not imply such relationship and their interaction is mainly repulsion, implying that they are almost strangers.



$d_{ij}^0 = 0.8m$ $A_{ij} = 30$ N $B_{ij} = 3.8$m

(a) Familiarity



$d_{ij}^0 = 1.8m$ $A_{ij} = 10$ N $B_{ij} = 0.8$m

(b) Stranger

Figure 2. Social-force from individual *j* to individual *i*

In the above curve the negative segment represents attraction (See Equation 3 and 7), or as called cohesive social-force. In contrast the positive segment denotes repulsion, or as called repulsive social-force. In an addition, when two individuals are strangers, there is almost no attraction as shown in Figure 2(b), and their interaction force is mainly repulsive.

As below we present a diagram to describe the interplay between individuals and their surroundings and it illustrates how the desired velocities and distance interact with physical velocities and distance in a feedback loop.

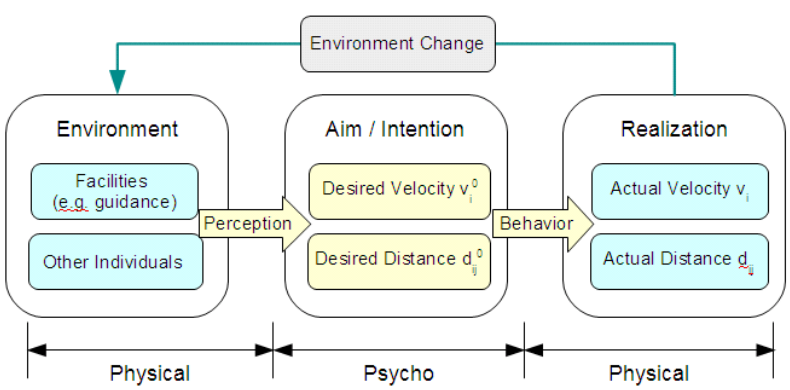


Figure 3. Perception and Behavior in a Feedback Mechanism

In order to elaborate the above group force especially from the psychological perspective, we have the following remark.

(a) Proxemics and Desired Interpersonal Distance

The first remark refers to proxemics and interpersonal distance. In Hall, 1963 and 1966 the study of interpersonal distance was named by proxemics, and it was defined as "the interrelated observations and theories of man's use of space as a specialized elaboration of culture." In brief, proxemics suggests that people surround themselves with a "bubble" of personal space that they claim as their own region, and they normally feel stressed when their personal space is invaded by others. Our personal space protects us from too much arousal and helps us keep comfortable when we interact with surrounding people. When the interpersonal distance is smaller than the desired, it is reasonable to assume that repulsion comes into being in this situation, and repulsion increases when the distance further decreases. This theory justifies the assumption of repulsive social-force in Equation (3). However, the repulsion is not related to physical size of two people (i.e., r_{ij}), but the social relationship, culture and occasions. Comparing social force with self-driving force, we suggest that there should be a subjective concept of desired distance d_{ij}^0 in the pedestrian model, and it replaces r_{ij} in Equation (3). Here d_{ij}^0 is the target distance that individual i expects to maintain with individual j. This distance describes the desired interpersonal distance when people interact, and it is a function of the social relationship of individual i and j as well as the culture and social occasions.

(b) Social Norms and Proxemics

The interpersonal distance reflects a kind of social norms, and it is also redefined in different cultures. For example, in a crowded train or elevators, although such physical proximity is psychologically disturbing and uncomfortable, it is accepted as a social norm of modern life. Also, it is also known that the male and female commonly keep larger distance in public place in Muslim culture than other cultures.

Because people may follow different social norms and expect different proximal distance in different occasions, , d_{ij}^0 is occasion-dependent and it is not a static concept. For example in elevators or entrance of a passageway, people usually accept smaller interpersonal distances than in other occasions. The accepted proximal distance is much smaller than other cases. Thus, d_{ij}^0 could be scaled down with a smaller value in these places. A straightforward method is scaling down each element d_{ij}^0 when people stay together in these places. In brief d_{ij}^0 should be a function in both temporal and spatial sense and it varies temporally and spatially. In addition, culture is another important factor but it is a little beyond the scope of our discussion.

IV. Group Dynamics

Individuals in a group exhibit some degree of social cohesion based on their relationship or inter-dependencies and they are more than a simple collection or aggregate of individuals. Based on Equation (4), considering a group composed by n individuals, the social relationship of the group members is described by a $n \times n$ matrix D^0, of which the element is d_{ij}^0. In a similar way, there are $n \times n$ matrices A and B, and the elements are A_{ij} and B_{ij}, respectively. Generally speaking, D^0, A and B are asymmetrical, implying that Newton 3rd law does not hold for social force.

$$\boldsymbol{D^0} = [d_{ij}^0]_{n\times n} \qquad \boldsymbol{A} = [A_{ij}]_{n\times n} \qquad \boldsymbol{B} = [B_{ij}]_{n\times n} \tag{5}$$

The group force is specified by the matrices D^0, A and B, and the method has been tested in FDS+Evac. A testing result is illustrated in Figure 4, where the built environment is from an example in Pan et. al., 2007. Several groups are identified in the simulation. Some small groups merge into a large group and regrouping may occur at intersections or at bottlenecks when many groups meet there. In sum, grouping behavior is not a static concept in our model, but an adaptive feature. Structure of groups change dynamically, resulting in a self-organized phenomenon during the movement.

Based on Equation (2) and (7), a typical kind of social relationship is described as the leader-and-follower group, where there is a kind of individual whose behavior is mainly motivated by himself, and if others would like to follow them, they become leaders in a group. Thus, if individual i is the leader in a group, his motion is mainly motivated by the self-driving force (i.e., desired velocity). Follower is the one whose behavior is mainly motivated by others. Thus, if individual i is the follower in a group, his motion is mainly governed by group force, and the self-driving force is secondary. As mentioned before imbalance (asymmetry) of d_{ij}^0 and d_{ji}^0 will contribute to model leadership in crowd behavior. If i is a leader, d_{ji}^0 is much smaller than d_{ij}^0. As a result, the leader will attract his surrounding people, but not easily be attracted by them. In brief an individual's motion can be classified into two types. One type of motion is mainly motivated by the self-driving force, not by surrounding people, and thus is called active motion. The other type of motion is motivated by surrounding people, and is called passive motion. In general, an individual's motion is a combination of both types, but we can differentiate such two types in simulation and identify whether one's motion is dominated by either active or passive type.

In sum, imbalance (Asymmetry) of d_{ij}^0 and d_{ji}^0 will contribute to model leadership in crowd behavior. If i is a leader, d_{ji}^0 is much smaller than d_{ij}^0. As a result, the leader will attract his surrounding people, but not easily be attracted by them. As a result, such active and passive types of motion form the pattern of leader and follower in crowd dynamics.

As shown in Figure 4 we use the color bar to observe the magnitude of the cohesive force in simulation (Forney, 2018). An individual in active motion usually moves in the front of a group. Individuals in passive motion are followers in the group and they usually move behind the leaders. The leaders are under smaller cohesive force than the followers.

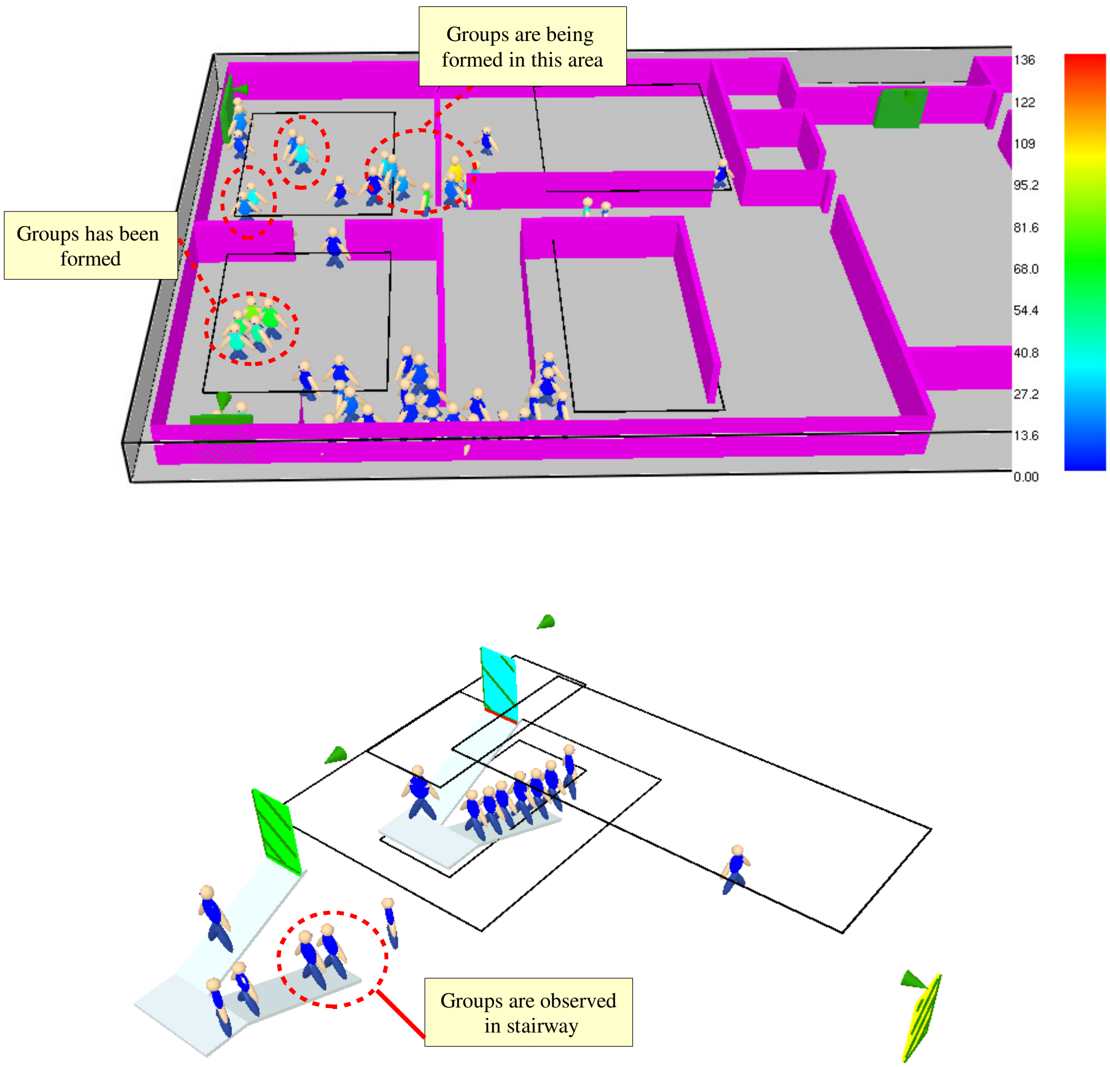


Figure 4. Simulation of Group Dynamics in FDS+Evac (The color bar indicates the magnitude of the group social force given by Equation (7)).

About group dynamics we have the following two remarks regarding psychological study.

In evacuation simulation Equation (7) and (8) can be better explained by flight-or-affiliation effect in psychological studies. The self-driving force motivates one to flee while the cohesive social force makes one affiliated with others. This effect agrees with social attachment theory in psychological study (Mawson, 2007; Bañgate et al., 2017). The social attachment theory suggests that people usually seek for familiar ones (e.g., friends or parents) to relieve stress in face of danger, and this is rooted from our instinctive response to danger in childhood when a child seek for the parents for shelter. Affiliated with familiar and trust individuals relieves our stress. Thus, different from the fight-or-flight response (Cannon, 1932), the modified social model well agrees with the flight-or-affiliation effect. Therefore, the pedestrian model presented above is especially useful to model crowd behavior in pre-movement stage in crowd evacuation (Sorensen, 1991, Kuligowski, 2009). In brief, when the alarm or hazard is detected, people usually do not head to exits immediately, but go to find their friends or trust ones to form groups. Such grouping effect usually delays the movement towards exits, and thus is called pre-movement stage. Thus, the new model contributes to modeling the crowd behavior in pre-movement period and will be useful to investigate how the initial delay is formed and influenced by the group dynamics.

APPENDIX

!! Group force is added. Here I declare a 2D matrix when the number of EVAC lines is determined. Users may initialize or modify the matrix by using EVAC Namelist, and this matrix characterizes the social relationship of agents.

!! The total number of groups is NPC_EVAC, then DFAC AFAC and BFAC are in dimension of NPC_EVAC*NPC_EVAC.

!! DFAC: DFactor(I, IE)   AFAC: AFactor(I, IE)   BFAC: BFactor(I, IE)

!! I: Index of the current agent (outer loop)   IE: Index of the other agent (inner loop)

!! The following Fortran code is in the loop where social force is computed.

!! GROUP_FORCE is a boolean variable which enables the group dynamics in computation.

```
FCG_X = 0.0_EB
FCG_Y = 0.0_EB
IF (GROUP_FORCE) THEN
   TIM_DIST = MAX(0.001_EB,SQRT((X_TMP(2)-X_TMP(5))**2 + (Y_TMP(2)-Y_TMP(5))**2))
   FCG_X = (X_TMP(2)-X_TMP(5))*HR_A*AFAC*COSPHIFAC*EXP( -(TIM_DIST-( R_TMP(2)+R_TMP(5) )
               *DFAC)/HR_B/BFAC)/TIM_DIST*(( R_TMP(2)+R_TMP(5) )*DFAC-TIM_DIST)
   FCG_Y = (Y_TMP(2)-Y_TMP(5))*HR_A*AFAC*COSPHIFAC*EXP( -(TIM_DIST-( R_TMP(2)+R_TMP(5) )
               *DFAC)/HR_B/BFAC)/TIM_DIST*(( R_TMP(2)+R_TMP(5) )*DFAC-TIM_DIST)
END IF

!HR_A_CF
!HR_B_CF
HR%FX_Group = HR%FX_Group + FCG_X
HR%FY_Group = HR%FY_Group + FCG_Y
```

References

[1] J. Bañgate, J. Dugdale, C. Adam, E. Beck, “A Review on the Influence of Social Attachment on Human Mobility During Crises,” WiPe Paper – T2-Analytical Modelling and Simulation Proceedings of the 14th ISCRAM Conference, Albi, France, May 2017.

[2] W. B. Cannon, Wisdom of the Body. United States: W.W. Norton & Company, 1932.

[3] G. P. Forney, “Smokeview, A Tool for Visualizing Fire Dynamics Simulation Data, Volume I: User’s Guide”, NIST Special Publication 1017-1 6th Edition, National Institute of Standards and Technology, Gaithersburg, MA, June 2018, 188 p.

[4] E. T. Hall. "A System for the Notation of Proxemic Behavior," American Anthropologist. Vol. 65, No. 5, pp. 1003–1026, 1963.

[5] E. T. Hall, The Hidden Dimension. Anchor Books. 1966.

[6] D. Helbing, I. Farkas, and T. Vicsek, “Simulating Dynamical Features of Escape Panic,” Nature, Vol. 407, pp. 487– 490, 2000.

[7] D. Helbing, I. Farkas, P. Molnar, T. Vicsek, “Simulation of pedestrian crowds in normal and evacuation situations,” in: Schreckenberg, M., Sharma, S.D. (Eds.), Pedestrian and Evacuation Dynamics, pp. 21–58. 2002.

[8] D. Helbing and P. Molnar, “Social force model for pedestrian dynamics,” Physical Review E, Vol. 51, No. 5, pp. 4282-4286, 1995.

[9] D. Helbing, A. Johansson, H. Z. A-Abideen, “Dynamics of crowd disasters: A empirical study,” Physical Review E, 75: 046109, 2007.

[10] A. Johansson, D. Helbing, and P. K. Shukla, Specification of the social force pedestrian model by evolutionary adjustment to video tracking data, Advances in Complex Systems, Vol. 10, pp. 271-288, 2007.

[11] A. Johansson, D. Helbing, H. Z. A-Abideen, and S. Al-Bosta, “From crowd dynamics to crowd safety: A video-based analysis,” Advances in Complex Systems, Vol. 11, No. 4, pp. 497-527, 2008.

[12] T. Korhonen, Technical Reference and User's Guide for Fire Dynamics Simulator with Evacuation, (FDS+Evac, FDS 6.5.3, Evac 2.5.2), VTT Technical Research Center of Finland, 2017.

[13] T. Korhonen, S. Hostikka, S. Heliövaara, and H. Ehtamo, "FDS+Evac: An Agent Based Fire Evacuation Model", Proceedings of the 4th International Conference on Pedestrian and Evacuation Dynamics, February 27–29, 2008, Wuppertal, Germany.

[14] T. I. Lakoba, D. J. Kaup, N. M. Finkelstein, "Modifications of the Helbing-Molnár-Farkas-Vicsek Social Force Model for Pedestrian Evolution," Simulation, Vol. 81, Issue 5, pp. 339-352, May 2005.

[15] A. R. Mawson, "Mass Panic and Social Attachment, The Dynamics of Human Behavior," Ashgate. Chap. 10, pp. 113–119, 2007.

[16] K. McGrattan, S. Hostikka, R. McDermott, J. Floyd, C. Weinschenk and K. Overholt, "Fire Dynamics Simulator, User's Guide", NIST Special Publication 1019 6th Ed., National Institute of Standards and Technology, Gaithersburg, MA, 2018, 367 p.

[17] Oasys Inc., The Verification and Validation of MassMotion for Evacuation Modelling, Issue 01, January 2018.

[18] F. Ozel, "Time Pressure and Stress as a Factor During Emergency Egress," Safety Science, Vol. 38, pp. 95-107, 2001.

[19] H. J. Payne, Models of freeway traffic and control, in Mathematical Models of Public Systems, Vol. 1 of Simulation Councils Proc. Ser., pp. 51-60, 1971.

[20] N. Pelechano and N. I. Badler, "Modeling Crowd and Trained Leader Behavior during Building Evacuation," IEEE Computer Graphics and Applications, Vol. 26, No. 6, pp. 80-86, 2006.

[21] J. H. Sorensen, "When Shall We Leave? Factors Affecting the timing of evacuation departures". In: *International Journal of Mass Emergencies and Disasters* 9.2, pp. 153–165. 1991.

[22] P. Wang, "Understand Social Force Model in Psychological Principles of Crowd Behavior," arXiv:1605.05146 [physics.soc-ph].

[23] P. Wang, P. B. Luh, S. C. Chang and J. Sun, "Modeling and Optimization of Crowd Guidance for Building Emergency Evacuation," Proceedings of the 2008 IEEE International Conference on Automation Science and Engineering (CASE 2008), Washington, D.C., pp. 328 – 334, August 2008.

[24] G. B. Whitham, Linear and nonlinear waves, John Wiley and Sons, New York, 1974.