

# Trees: Data Structure #4



# Data Structure Comparisons: Efficiency

Operation / Data Structure	Add	Remove	Get	Report: GetNext GetPrev
<b>Sparse Array</b>	<b>Instantaneous.</b> Assign to index slot	<b>Instantaneous.</b> Remove from index slot.	<b>Instantaneous.</b> Assign to index slot.	<b>Slow:</b> Walk thru all index spaces
<b>Orderable Array</b>	<b>Slow.</b> Insert → pushback	<b>Slow:</b> Remove → move down	<b>Fast.</b> Binary search to item	<b>Fast.</b> Binary Search to current item
<b>Linked List</b>	<b>Slow:</b> “walk” ½ entries	<b>Slow:</b> “walk” ½ entries	<b>Slow:</b> “walk” ½ entries	<b>Instantaneous</b> Move to adjacent entry
<b>Hash Table</b>	<b>Very Fast.</b> Hash key, assign to Bucket. Append	<b>Very Fast.</b> Hash Key, find Bucket, Delete	<b>Very Fast.</b> Hash Key, find Bucket. Get.	<b>Unsupported.</b> No “ordering”

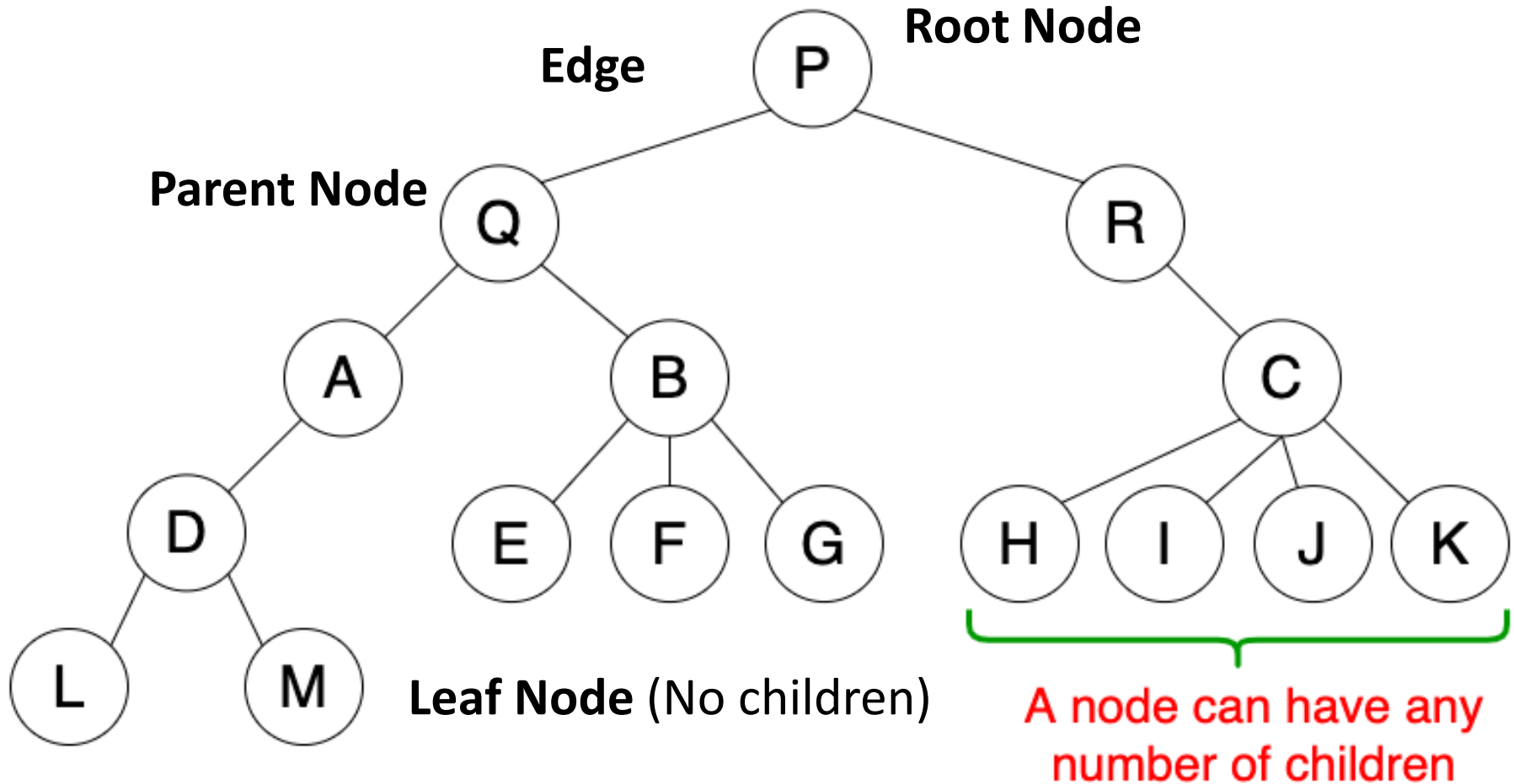
# Data Abstraction Comparisons: Operations

Data Structures (Internal Classes)	Data Abstraction	External Interface
Linked List + Node Structure + Iterator or Ordered Array	Deque	Linked List Iterator (next/previous)
Ordered Array or Deque	LIFO FIFO	Stack (push / pop) Queue (push / pop)
Hash Table Ordered Array Buckets	Associative Array	Store element with key Get element by key Remove element by key

# Data Abstraction Comparisons: API

Data Abstraction	Underlying Data Structure	Add / Delete	Ordering
Deque	Ordered Array or LinkedList	Insert / Remove	By time of insert (front and back) getNext/getPrev
Stack and Queue	Ordered Array or Deque	Push / Pop	By time of insert
Associative Array	Sparse Array Hash Table	Insert / Remove	N/A

# Data Abstraction: General Data Tree: Unordered



**Node Depth:** # edges until Root Node

<Nodes A, B, C all at Depth Level 2>

**Tree Height:** Largest depth of any node

**What is something this data structure might most effectively represent?**



# A Directory / Folder Hierarchy

## Posix: Directory Walking & Recursion

**Hierarchy cpp file Printer – prints location of all C++ code files in Directory Hierarchy**

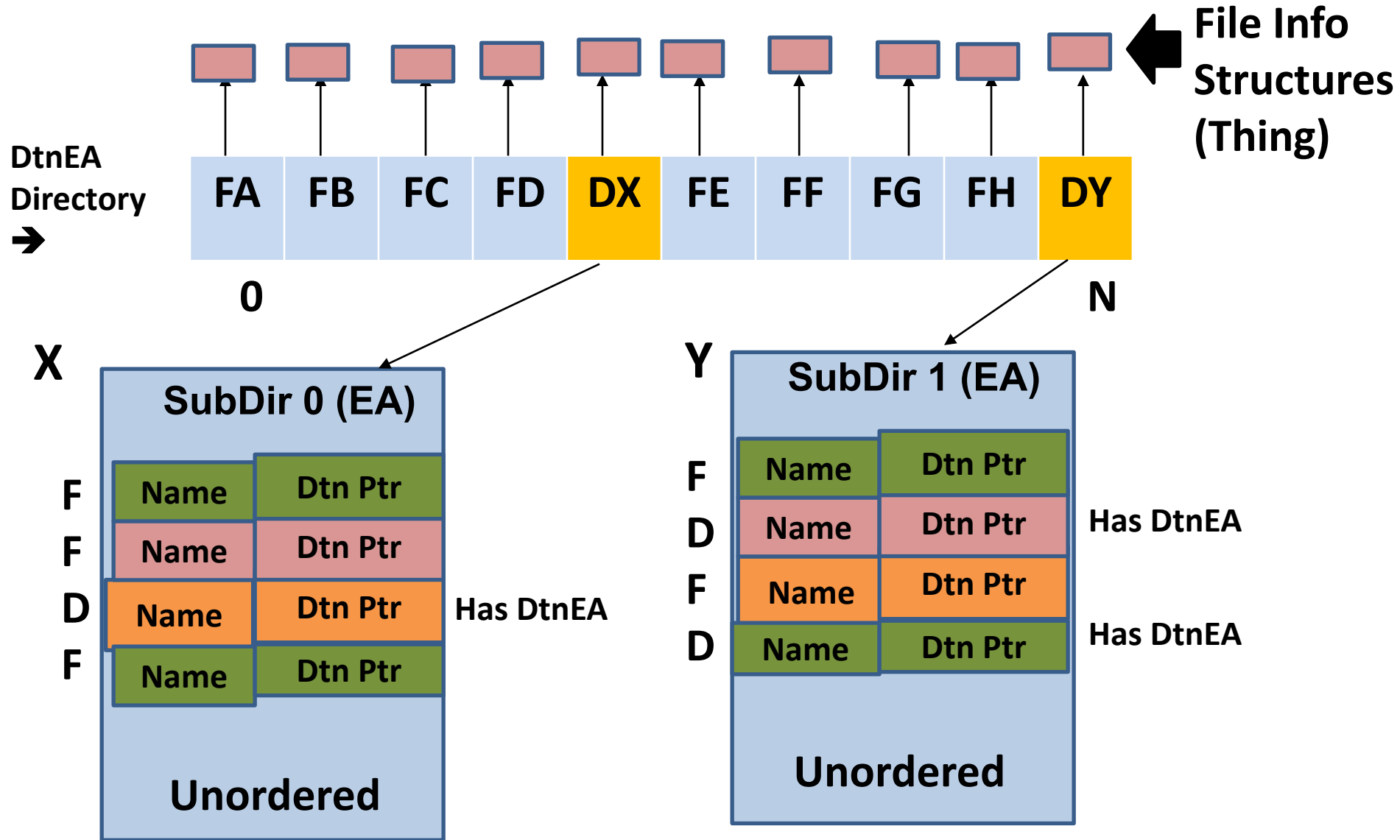
```
void fhp (DirectoryPath)
{ // Pseudocode (obviously)
  Open provided Directory // opendir (returns an array of entries)
  While (more entries) // Ignore everything not a cpp file or subdirectory
    Read next entry // readdir (returns an entry)
    if (entryType == "cpp" file) Print ("DirectoryPath/entryname")
    if (entryType == subdirectory) // Recursively invoke self
      fhp ("DirectoryPath/entryName") // Print all cpp files in subdirectory hierarchy
  }
  // All cpp files in or under the provided Directory have been printed
};
```

# Directory Tree “Node”

```
struct Dtn {  
    Dtn * parent; // Owing Directory Ptr (or NULL if Root Directory)  
    string name; // ID in “parent” (or empty if Root)  
  
    FileInfo *finfo; // Struct with logical block assignments, RW owners ...  
                    // (i.e. Ptr to the “Thing”)  
  
    DtnEA *contents;  
        // If Directory:  
            //Ptr to (non-ordered) EA of Dtn’s  
                // Subdirectory Dtn’s  
                // File Dtn’s  
        // If File:  
            // NULL  
};
```

**➔ All Files and empty Directories are Leaf Nodes**

# Directory Tree Hierarchy





# Binary Trees: Nodes have 0,1,2 Children

```
class ThingNode {
```

```
private:
```

```
ThingNode *par; // Ptr to Parent or 0 (root)
```

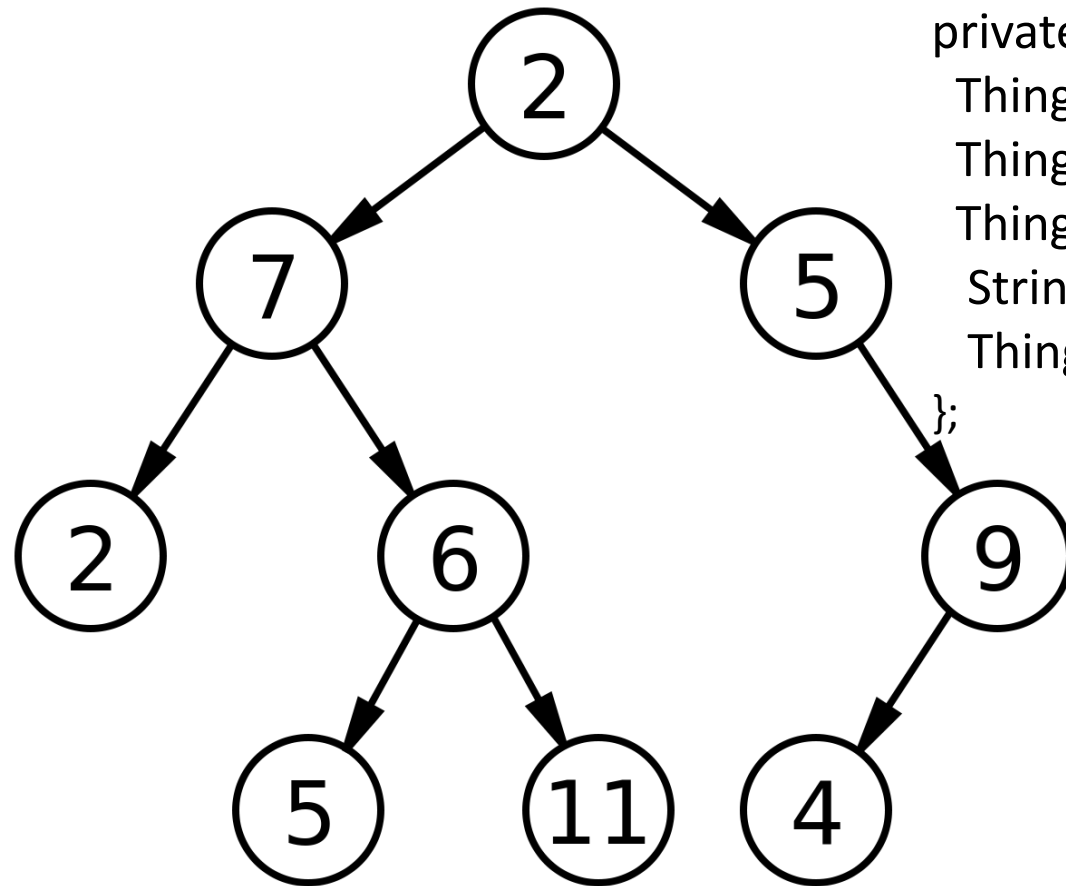
```
ThingNode *lc; // Ptr to Left Child or 0
```

```
ThingNode *rc; // Ptr to Right Child or 0
```

```
String id; // Comparator (the #'s here)
```

```
Thing *thing; // Ptr to Thing represented
```

```
};
```



**Up to 2 Successors  
(and for all but the  
Root Node) a  
Parent**

**Unsorted Tree: How can you find the node with Identity you wanted?**

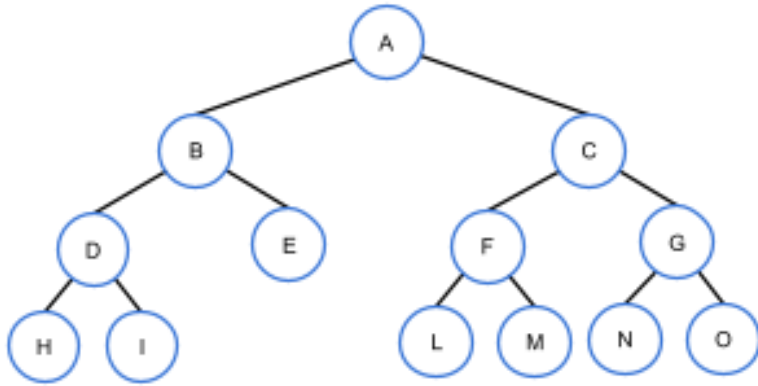
Ex: What path do you go down searching for the Node with ID 4??



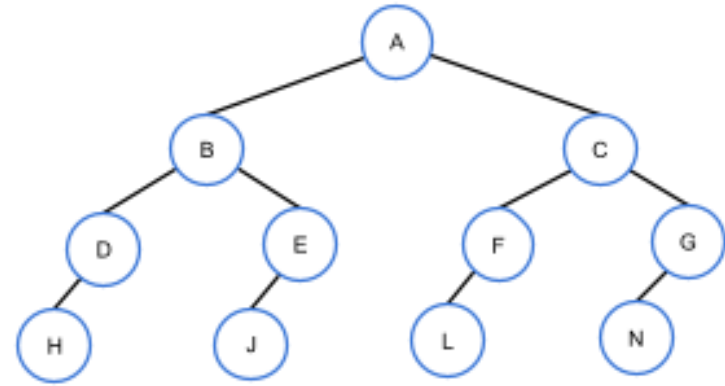
# Binary Tree Terminology

- **Node Depth:** # edges until Root Node
- **Tree Height:** Largest depth of any node
- **Root Node:** Topmost Node, if none, Tree is empty
- **Leaf Node:** Possessing neither a right or left “child”
- **Full Tree:** Every Node has 0 or 2 children
- **Complete Tree:** All levels except last contain all possible children, All Leaf Nodes must be on LHC
- **Perfect Tree:** Each non-Leaf node has 2 children, all Leaf nodes at same depth

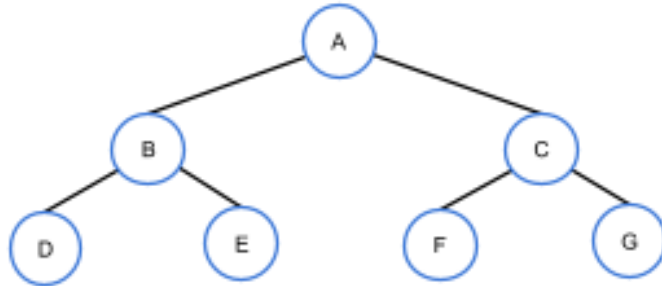
# Tree Example



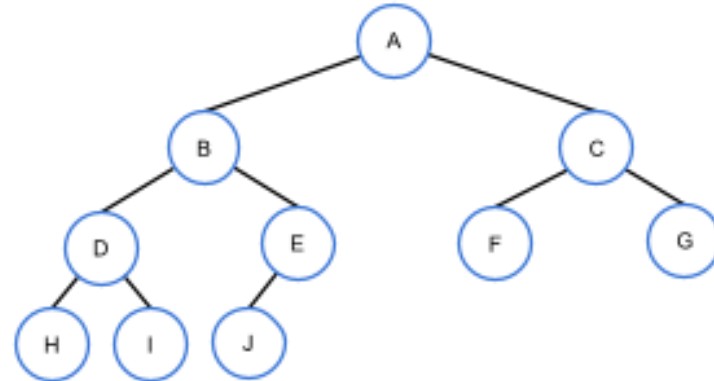
Tree 1



Tree 2



Tree 3

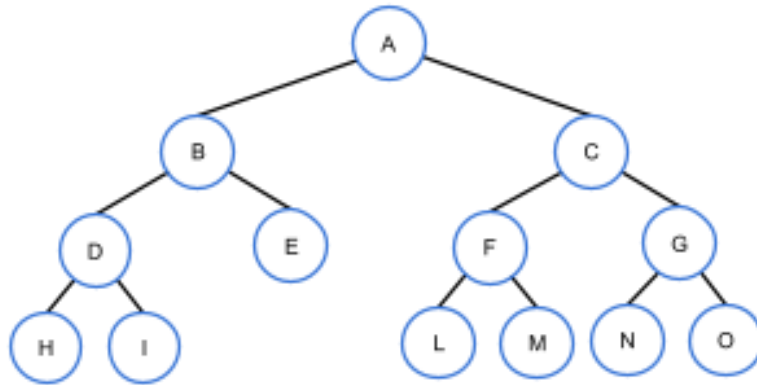


Tree 4

**A. Full Complete Perfect / B. Not Full, Not Complete Not Perfect**  
**C. Not Full Complete Not Perfect / D. Full Not Complete Not Perfect**

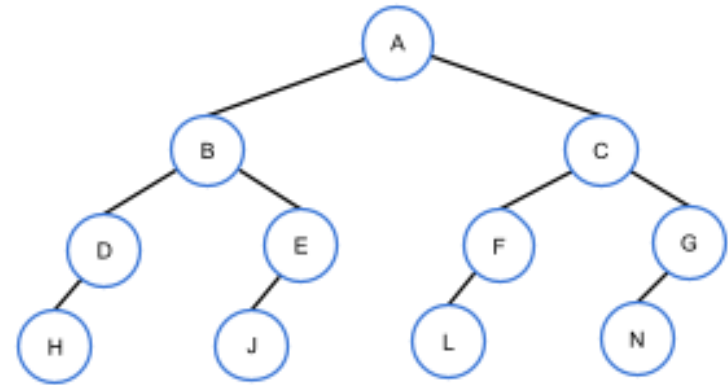


# Tree Example



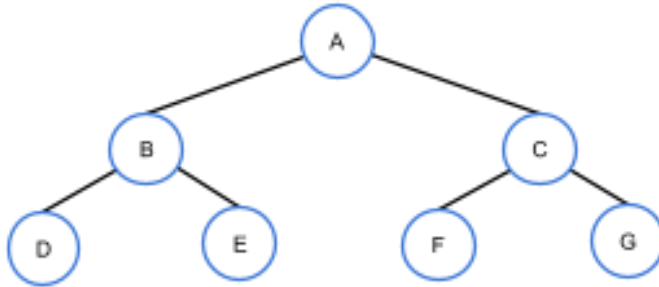
**D**

Tree 1



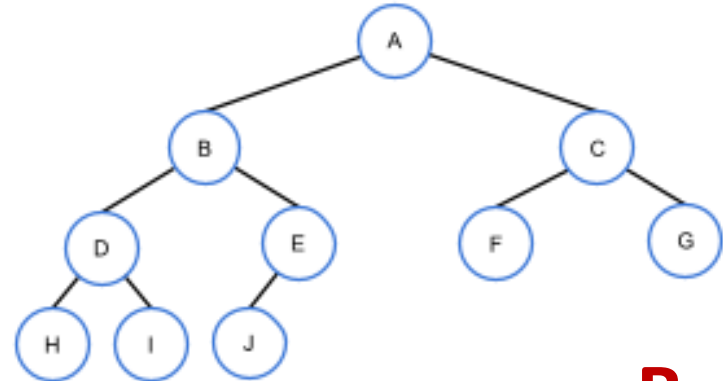
**C**

Tree 2



**A**

Tree 3



**B**

Tree 4

A. Full Complete Perfect / B. Not Full, Not Complete Not Perfect  
C. Not Full Complete Not Perfect / D. Full Not Complete Not Perfect

# Binary Search Tree (BST): (Ordered)

## Placing a new Node:

If the current node has a Left-Handed Child (LHC)

The ID of the LHC will be  $\leq$  the ID of the current Node

If the current node has a Right-Handed Child (RHC)

The ID of the RHC will be  $>$  the ID of the current Node

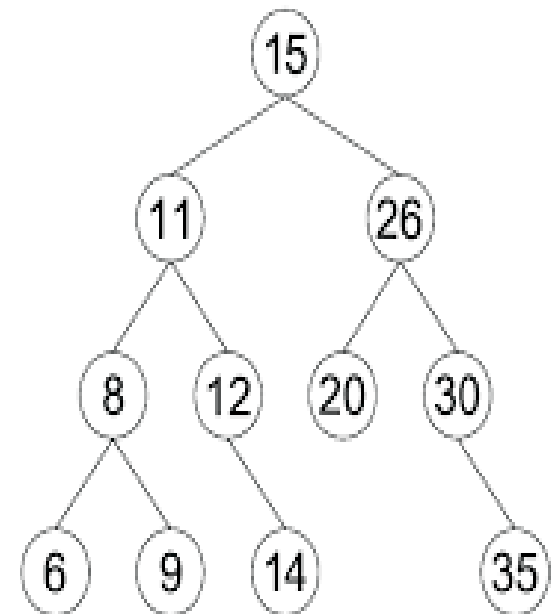
## Implies:

**No node under LHC will be  $>$  Parent**

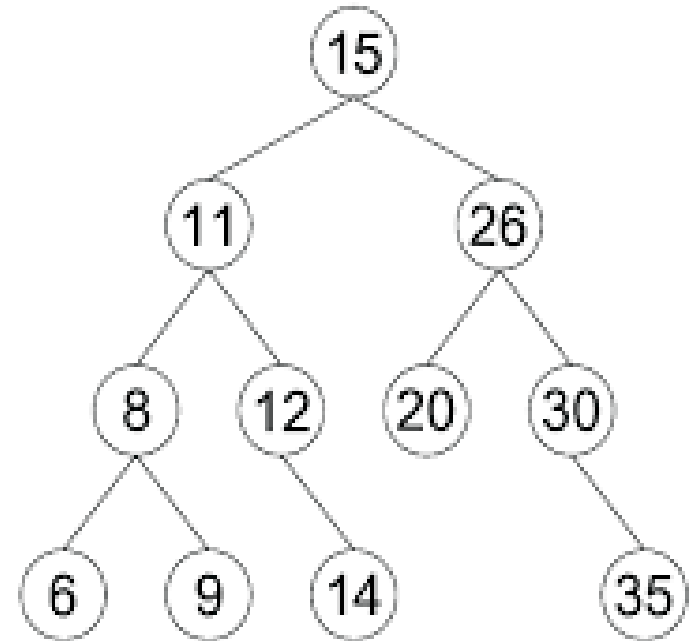
**No node under RHC will be  $<$  Parent**

Q. Where could Node ID=9 have been placed?  
( RHC of 8, LHC of 12, LHC of 20, LHC of 30)

Q. What path do you go down searching for Node ID=9??



# Equivalent Binary Search Trees:



**“Seeded by ID’s in order:**

**15,26,30,11,20,12,35,14,8,9,6**

**Max Node Depth = 3**

**Many different Binary Search Trees can be “seeded” by supplying identical elements in different order:**

**26,35,12,15,14,9,11,30,6,20,8 Max Node Depth = ??**

**(But things could be a lot worse ...)**



# EXAMPLE OF A POORLY BALANCED TREE

A VERY different looking Binary Search Tree will result if the IDs of the elements being seeded are ordered.

Ex: 10,20,30,40,50,60,70

Max Node Depth = ??

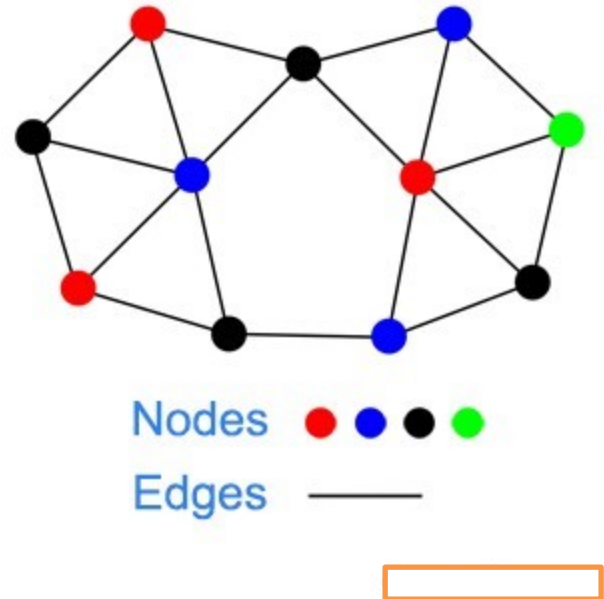
The BST becomes a “Unichild” Linked List!!!

→ “Randomize” the insertion order or  
hash the Element IDs before insertion



# A brief review of “Nodes”

- **Accessible through index // Externally Ordered**
  - Sparse Array, Orderable Array
  - Pointer to object is enough
- Hash Table (no nodes)
- Linked List





# Orderable Array

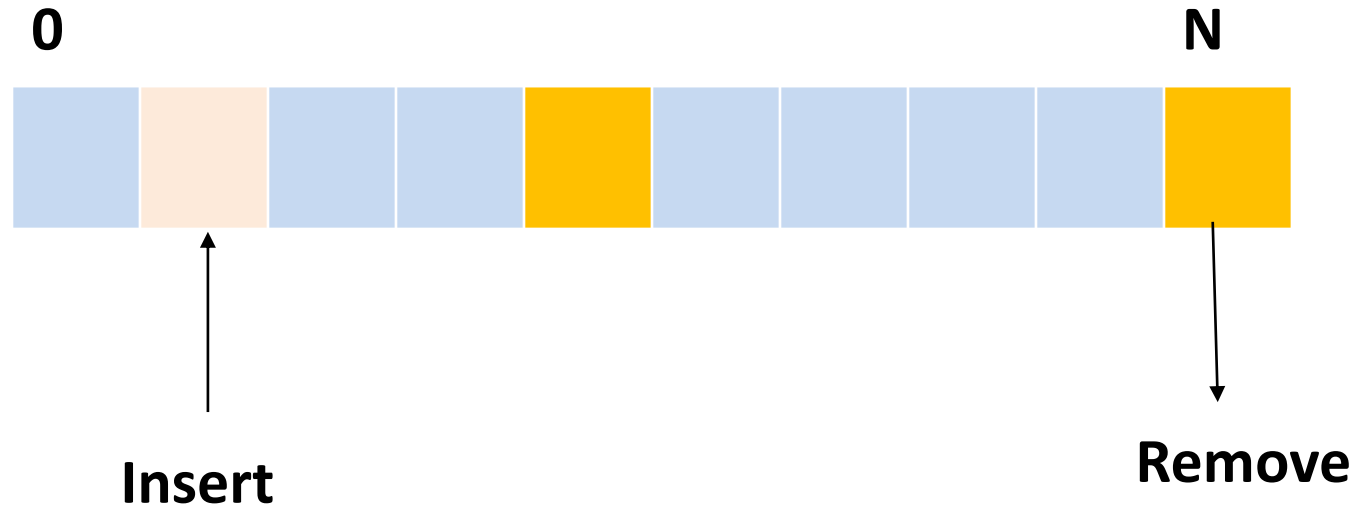


## Orderable Array

Ordering maintained by client (externally maps key → Location Index)

- Object Key unknown to collection class
- “Neighbors” do not have to be linked to
- **No Connecting Nodes!!**

# Sparse Array (empty spaces okay)

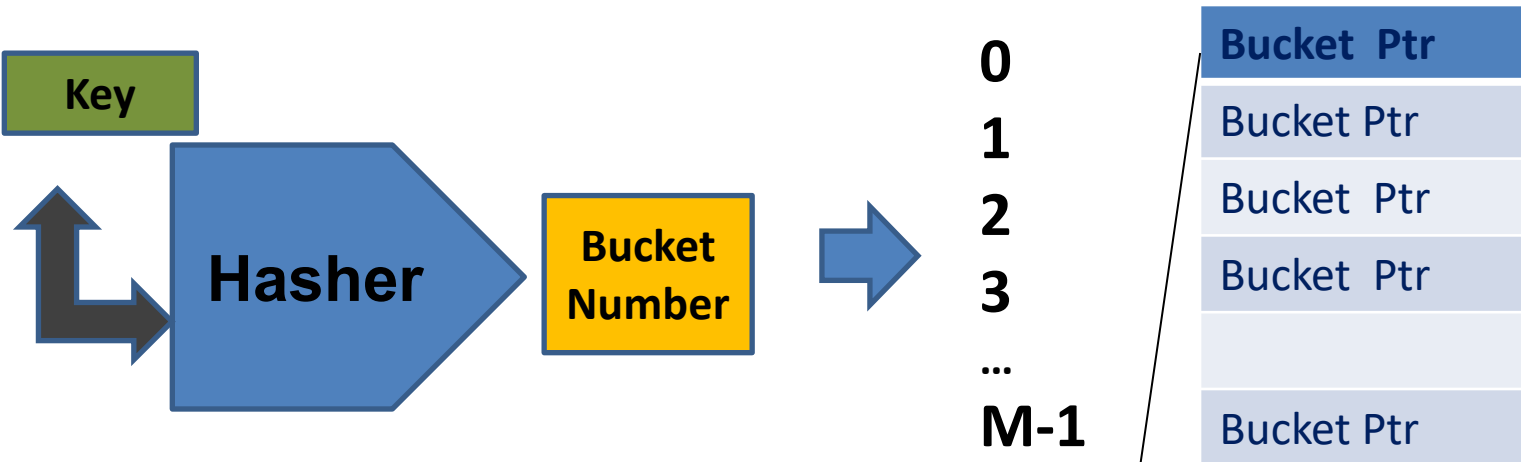


## Sparse Array

Ordering determined by client (uses key directly as Location Index)

- ➔ Object Key unknown to collection class
- ➔ “Neighbors” do not have to be linked to
- ➔ **No Connecting Nodes!!**

# Hash Table



Key hashed → Bucket index  
Collection class “knows” Key

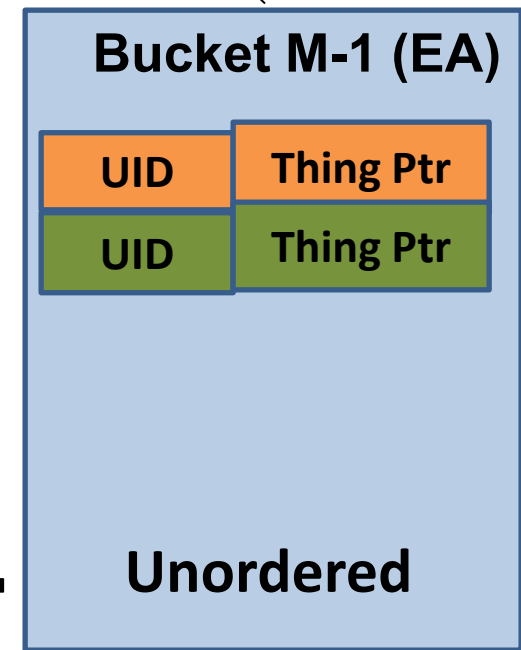
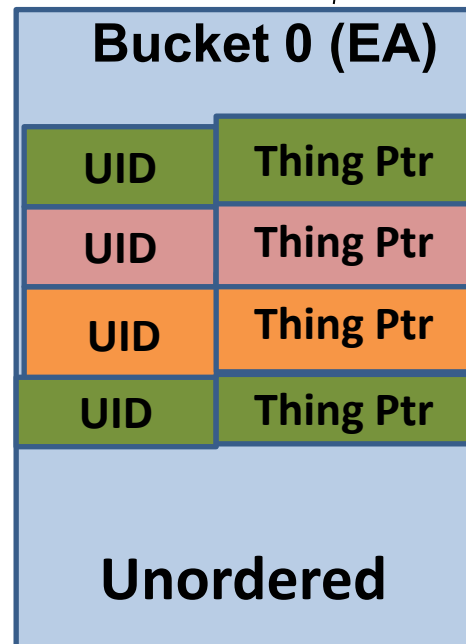
**But:**

**No ordering possible**

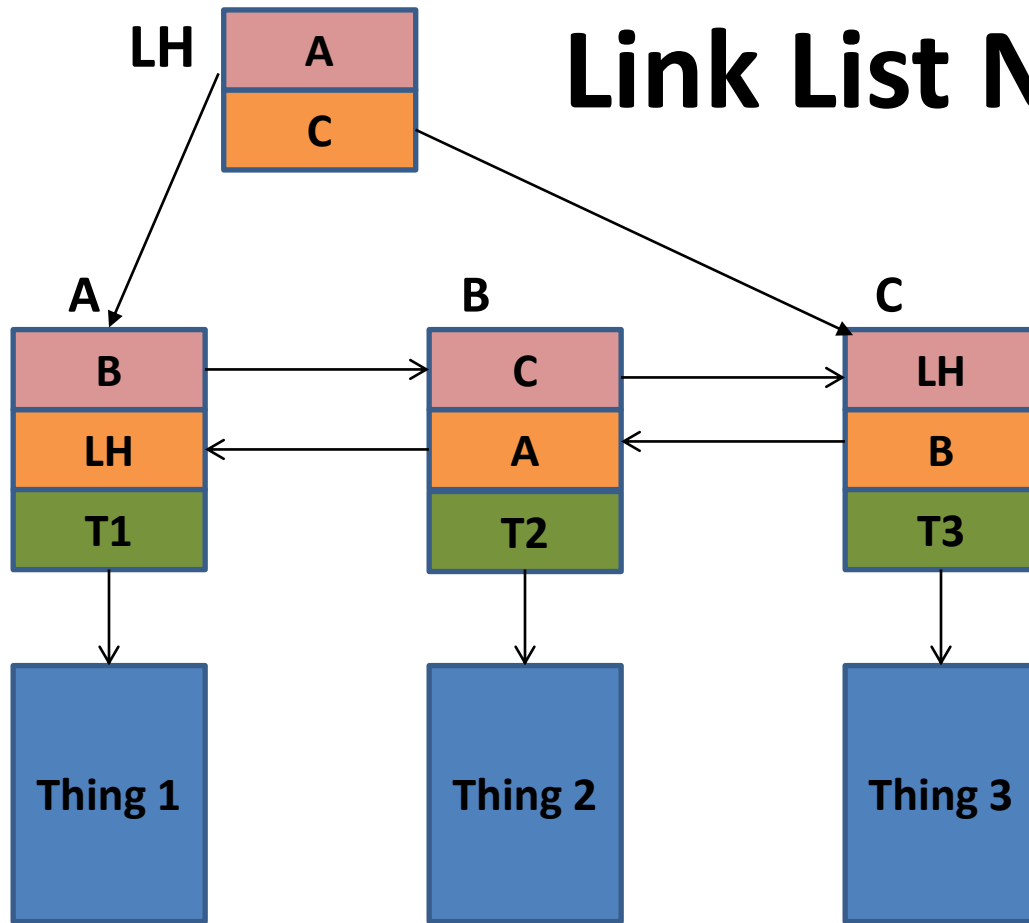
→ **No next/previous Links**

→ **Collect ID/Obj Ptr Structs**

→ **No Connecting Nodes!!**



# Link List Nodes



Class ThingNode  
{ // Internal List class to support  
// a Thing object in a Linked List  
private:

```
ThingNode *fp;  
ThingNode *bp;  
Thing * tp;
```

➔ Thing collected in a Linked List but  
doesn't know it!

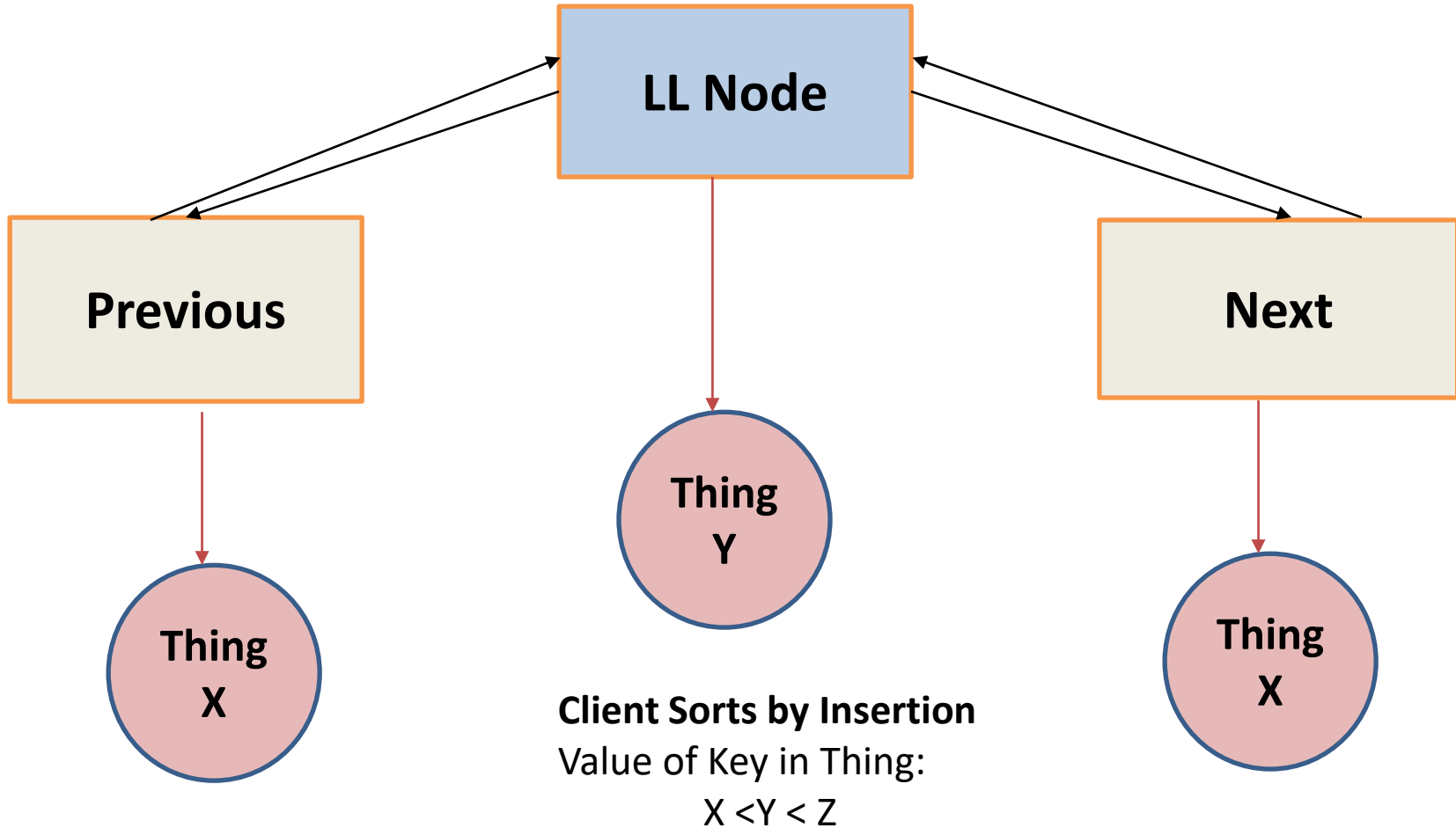
Positional Ordering determined by Client

➔ Object Key unknown to collection class

➔ “Neighbors” **HAVE** to be linked to

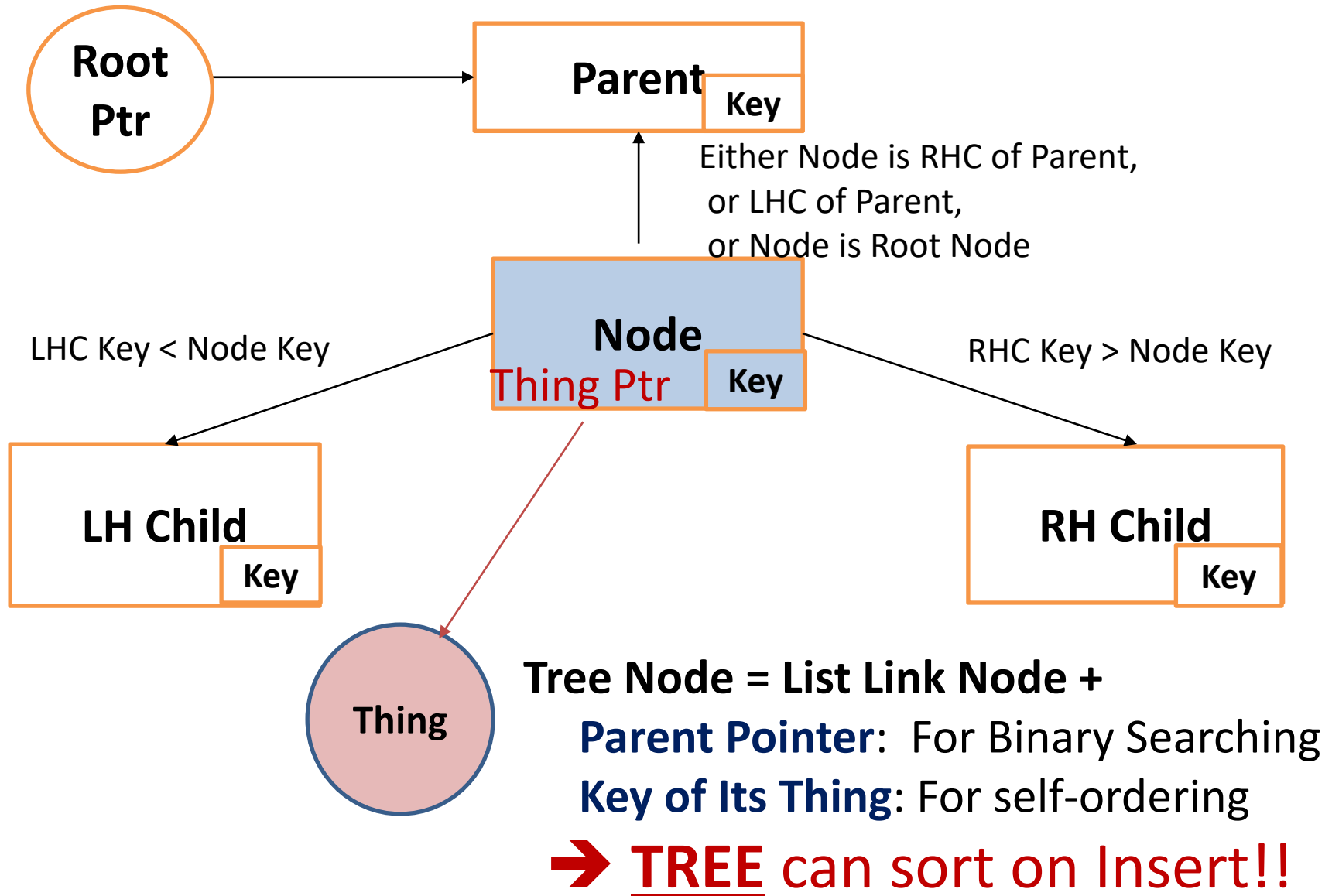
➔ **Connecting Nodes required**

# Link List Node Structure



➔ If you have ANY Node, you can walk the entire List

# BST Tree Node Structure



➔ If you have ANY Node, you can walk the entire Tree!!

# Node Interface

```
class Node { // Node created by BSL, interface used by BST, other Nodes
private:
    Node *parent; Node *lhc; Node *rhc; // Some could be Null
    String key; Thing *t; //Key and Pointer to Thing carried by this node
    BST *myTree; // Could also be static (class level) variable
public:
    Node(Node* parent, String key, Thing *tp, BST *bst);
// No Children at Node Creation (BST Needed to change Tree Root Ptr)
    Node (Node&); ~Node();
    String getKey(); Thing* getThing();
    Node* getRHC(), getLHC(), getParent();
    void setRHC(Node*), setLHC(Node*), setParent(Node*);
};
```

➔ **What is the problem with this Node Interface?**



# What is the problem with Node Interface?

```
class Node { // Node created by BST, interface used by BST, other Nodes
private:
    Node *parent; Node *lhc; Node *rhc; // Some could be Null
    String key; Thing *t; //Key and Pointer to Thing carried by this node
    BST *myTree; // Could also be static (class level) variable
public:
    Node(Node* parent, String key, Thing *tp, BST *bst);
// No Children at Node Creation (BST Needed to change Tree Root Ptr)
    Node (Node&); ~Node();
    String getKey(); Thing* getThing();
    Node* getRHC(), getLHC(), getParent();
    void setRHC(Node*), setLHC(Node*), setParent(Node*);
};
```

➔ It's all “sets” and “gets”!! So ...





# Node is a Private “Linking” Structure which is manipulated by its BST!

```
struct ThingNode { // Node created by BST, data used by BST
{
    Node* parent; // Ptr to parent node (0 if Root)
    Node*lhc;     // Ptr to Left Hand Child
    Node *rhc;    // Ptr to Right Hand Child
    String key;   // Object UID. Determines ordering
    Thing*Thing; // Pointer to the Thing with that key
};
```

**→ No “per-node” local or single global BST ptrs needed  
because the self-contained BST does all the work**

# BST Interface: Non-Iterator

**// Note: BST does NOT remember Client's current position in Tree.**

**Node\* toRoot;     // Pointer to Root Node. Null if Tree empty**

Node\* getRoot ():             // Get Root Node or Null (empty Tree)

Node\* getNode (Key):     // Get Node with that Key

      // Move currNode ptr.

Node \*getNext (Node \*); // Get Next (higher) Node or NULL (at end)

Node\*getPrior (Node \*); // Get Prior (lower) Node or NULL (at start)

      // Start at Root Node

Node\* insert (Node\*); // Utilizes Node Keys

Node\* remove (Key); // Remove Node with that key. Return Next or NULL

Node\* remove (Node \*); // Remove Node pointed to. Return Next

      // Initially start at Supplied Node

void printSelf (Node\*); // Print key and Thing data

void printAll (Node\*); // Print key and Thing data + all nodes below

unsigned int countNodes (Node \*); // Return total Nodes below this

unsigned int getHeight (Node\*); // Return Max Height of this node

# BST Iterator: External

```
// BST API : "Item" Struct is <Key / Object Ptr>
    Iterator* BST::begin (); // Get Iterator (Set to Root Node)
    BST*bst; Node*; currNode; // Tree & Current position in Tree (hidden)
// Iterator API: Leaves Current Position of Iterator unchanged
    Item* getRoot(); // Get Root Item (i.e. "Get First")
    Item* getThis(); // Get current Item
    Item* getNext (); // Get Next (higher) Item or NULL (at end)
    Item* getPrior (); // Get Prior (lower) Item or NULL (at start)
    Item* remove (); // Remove & return current Item, advance position
// Initially start at Tree Root. Changes Current Position of Iterator
    Item* getItem (Key): // Gets Item with that Key (Position at item)
    Node* insert (Item* add); // Add item to Tree (Position at item)
    Node* remove (Key); // Remove Item from Tree (Position at next item)
// Start at Current Position of Iterator. Does not change it
    void printSelf (); // Print Item data
    void printAll (); // Print Item data + all Items below
    unsigned int countNodes (); // Return total # Items below this in Tree
    unsigned int getHeight (); // Return Max Height of current Item
```

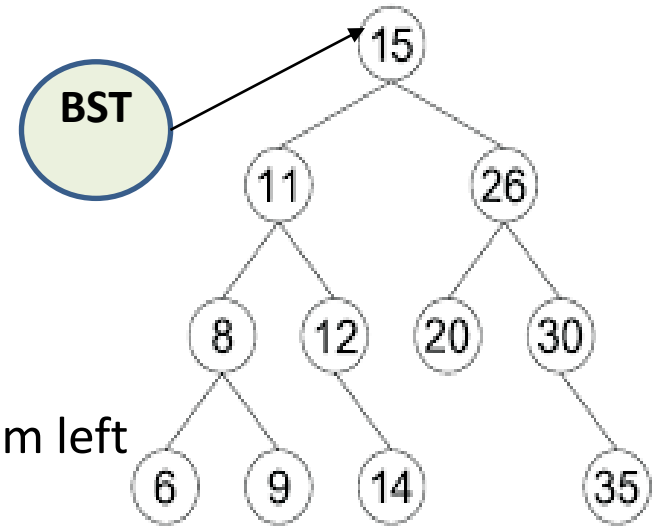
# Binary Search Tree Operations: Details

(Most are algorithmic / Some are Recursive)

- Count Nodes
- Get Height (longest path to bottom) of Node
- Ordered printout of all Tree Entry Values
- Get next Node
- **Get Previous Node**
- Search for Node
- Insert Leaf Node
- Remove Node

# Count Nodes

**Count all Nodes below this point in Tree**



```
int BST::countNodes (Node*sNode) {  
    int count = 0;  
    { // If LHC, recursively call self for LHC. Gets to bottom left  
        // Return implies entire LH Hierarchy counted  
        // If RHC, recursively call self for RHC. Gets to bottom right  
        // Return implies entire RH Hierarchy counted  
        // Add 1 to RH count + LH count, and return
```

```
    if (sNode→lhc)  
        sum+= countNodes (sNode→lhc);  
    if (sNode→rhc)  
        sum+= countNodes (snode→rhc);  
    return (sum+1);  
};
```

**Called by Iterator::countNodes();**

**BST::countNodes(Node\*)** is a simple recursive tree-walking routine.

**There will be more!**

# Get Height

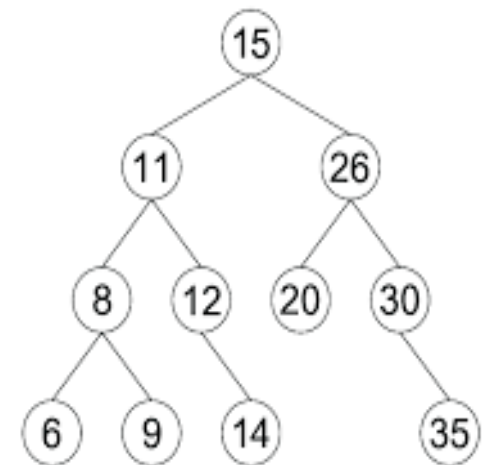
## Strategy:

If LHC, invoke self there. Save value +1

If RHC, invoke self there. Save value +1.

Return height

```
int BST::getHeight( Node* top) { // Return Longest path from Root
    //Increment larger of left / right paths. Report 0 if Leaf
    int lhgt = rhgt = report = 0;
    if (lhc)
        lhgt =getHeight (lhc) +1;
    if (rhc)
        rhgt = getHeight (rhc) +1;
    // Report back 1 higher than L/R top height
    report = ((lhgt > rhgt) ? lhgt : rhgt);
    return (report);
}
```



# Ordered Print

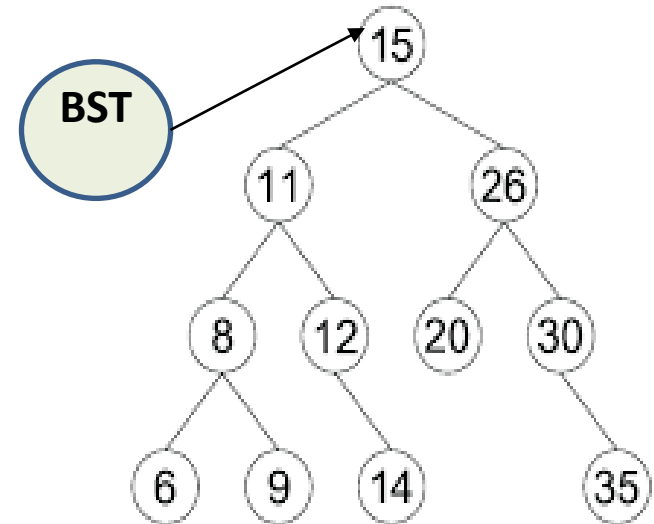
**Print all Key Values below this point**

## Strategy:

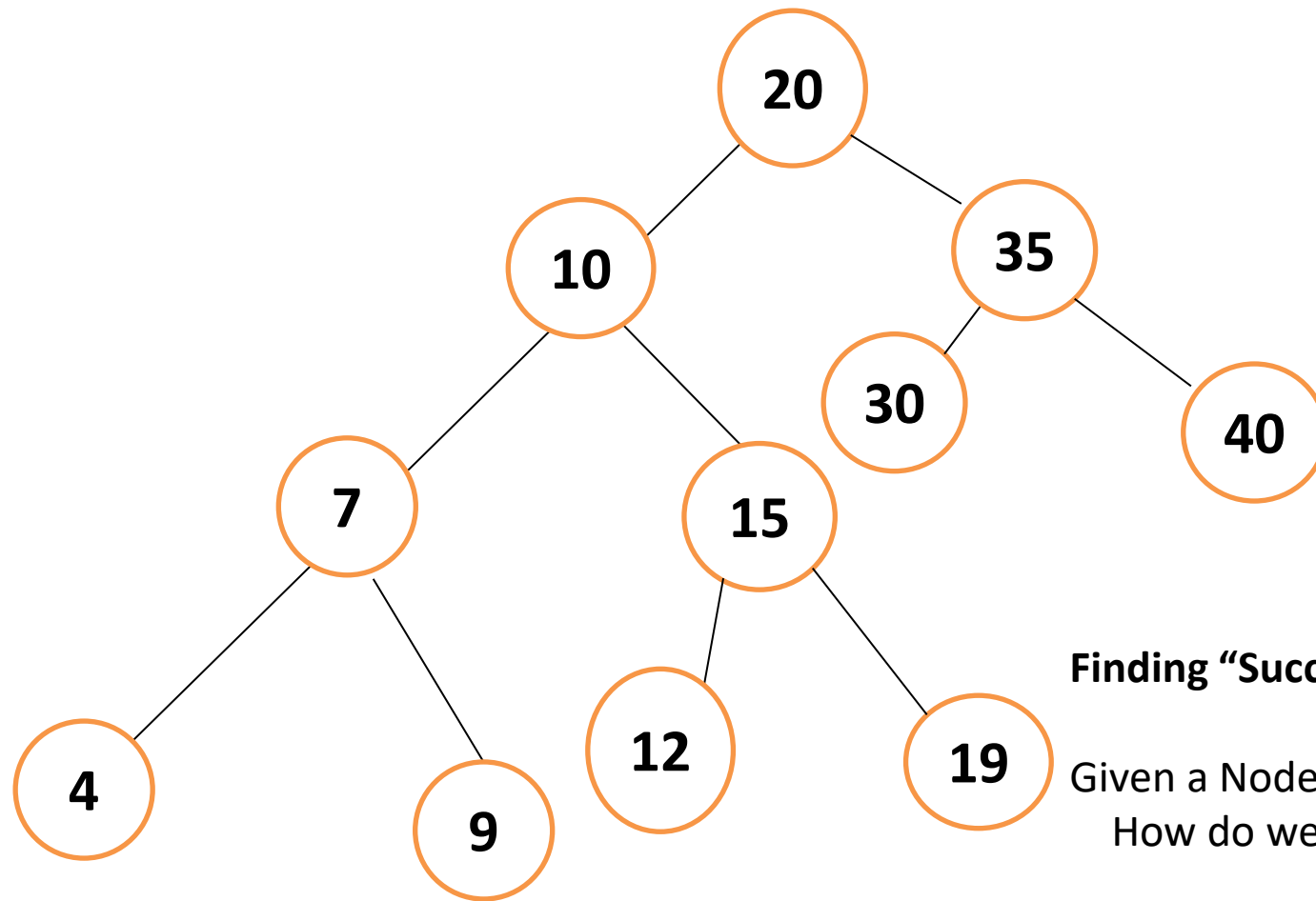
1. If LHC, recursively call this routine for it.  
<Entire Bottom Left printed>
2. Print own Key ... you are next
3. If RHC, recursively call this routine for it.  
<Entire Bottom Right printed>

## Method:

```
void BST::printSelf (Node*sNode)
{
    if (sNode->lhc) printSelf (sNode->lhc);
        cout < sNode->key < endl;
    if (sNode->rhc) printSelf (snode->rhc);
};
```



# Get Next Node



**Finding “Successor” node (if exists):**

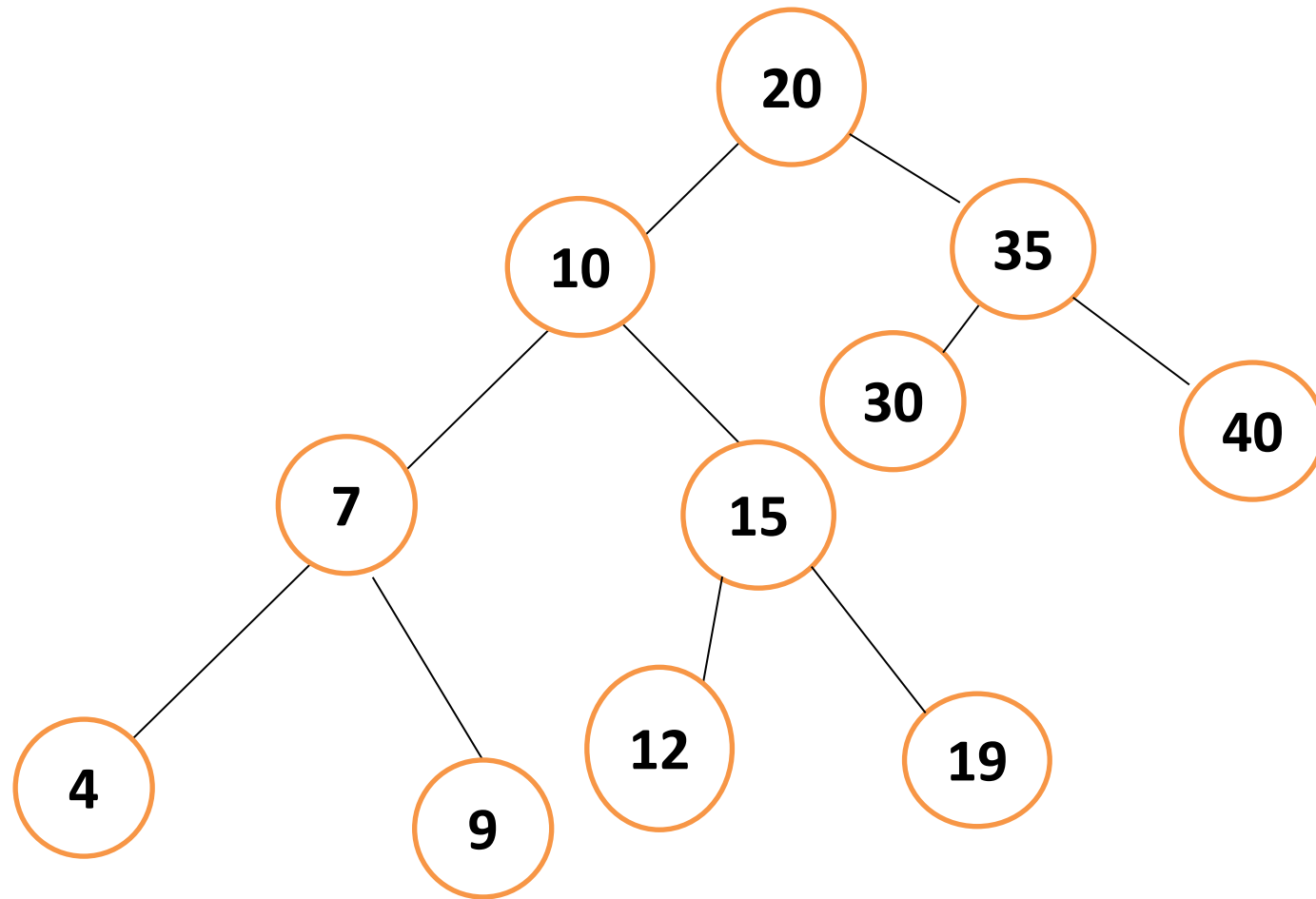
Given a Node on the tree:  
How do we find the “next node”?

Starting at:  
Node 10?  
Node 19?





# Get Next Strategy



Next Node Starting at:

(10)? **If RHC exists, Go RHC, walk down all LHC's to Leaf (12)**

(19)? **Else, Walk-up Parents till Parent X LHC of GParent Y. Use Y (20)**

# Get Next Node Code

**Finding “Successor” node (if exists):**  
(Non-recursive)

```
Node* BST::getNext (Node* from) { // Returns next node
```

```
    Node *current = NULL;
```

```
    If (from->RHC) { // If RHC, find leftmost leaf (15→20)
```

```
        currNode = RHC
```

```
        while (currNode→LHC) { currNode = LHC ; }
```

```
        return (currNode )
```

```
//No RHC. Go up tree until parent found where child was LHC. Return Parent
```

```
    while (currNode != toRoot) { // Ensure not root
```

```
        if (currNode == Parent→RHC) { currNode = Parent }; (14→15)
```

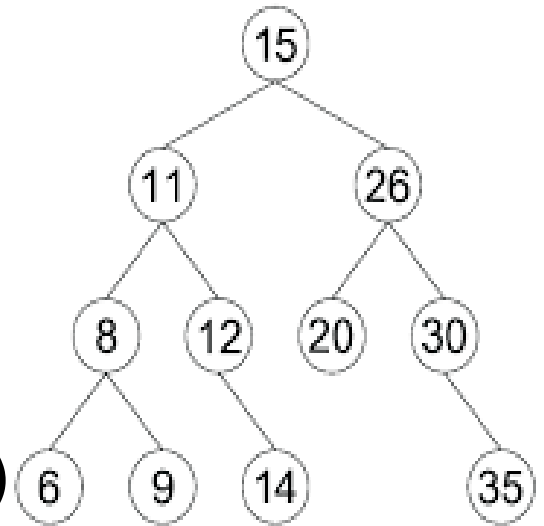
```
    };
```

```
// If currNode is Root node, this node was RHC to everything INCLUDING Root (35)
```

```
// So when the walk is over, the return is Root Parent (Null)
```

```
    return (currNode→Parent);
```

```
}
```

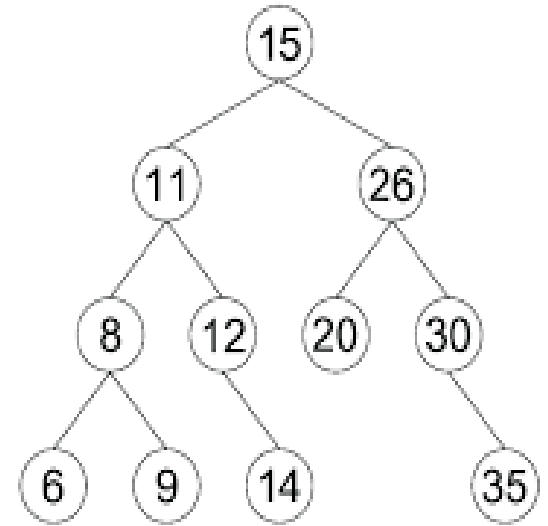


# Homework: Get Previous

## Deliverables:

1. List of “Treewalking Rules
2. Code for:

Node\* BST::getprevious (Node\* from)



## Finding the “Preceding” node (if exists):

Node\* getPrevious (Node\* from) { // Returns previous node or Null (if lowest value)  
// Null should be returned only for Node 6 in example

Node 15 → 14

Node 20 → 15

# Search For

## Strategy (Find node == Supplied Key)

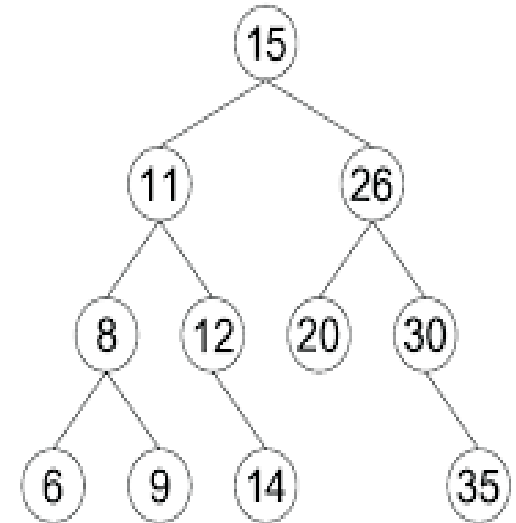
<On initial call, Node Ptr = Root>

How do we find:

Node 9?

Node 20?

Node 28?



*Assumes prior check made for empty Tree (in which case, failure is immediate)*

*Otherwise. search is invoked with startN = Root*

# Search For: Strategy

## Strategy (Find node == Supplied Key)

<On initial call, Node Ptr = Root>

If key matches this node's key

Set success

Else if LHC and key < LHC key

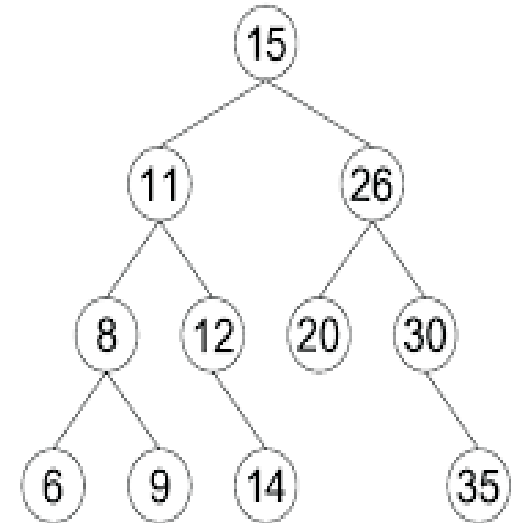
Recursive call at LHC

Else if RHC and key > RHC Key

Recursive call at RHC

Else (A non-matching key, but can't proceed)

Set failure



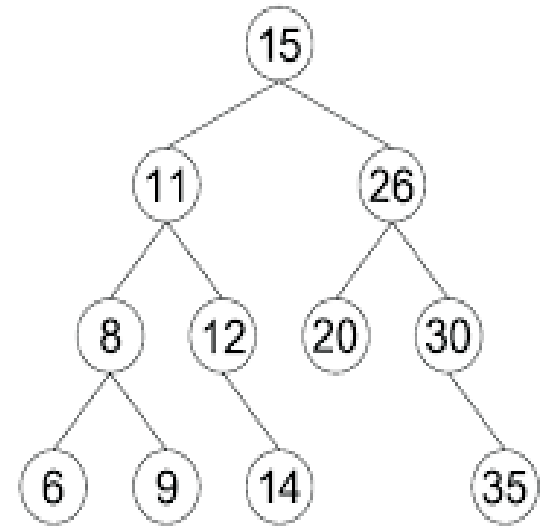
*Assumes prior check made for empty Tree (in which case, failure is immediate)*

*Otherwise. search is invoked with startN = Root*

# Elements looked at = BST Height +1  
(log 2 of N)

# Search For: Code

```
Node *search (Node *startN, int desKey)
{ // On initial call, Node Ptr = Root.
  // Object Key is ASSUMED integer
  Node *rval = NULL; // Assume failure
  Node *currN = startN;
  if (currN->key == desKey) {
    rval = currN; } // Node found. Return Ptr
  else if ((currN->LHC) && (desKey < currN->key))
  { // Recursive call to LHC
    rval = search (currN->LHC, desKey);}
  else if (currN->RHC) && (desKey > currN->key))
  { // Recursive call to RHC
    rval = search (currN->RHC, desKey); }
  else
  { // A non-matching Node and we cannot proceed
    // rval already set to failure
    } return (rval);
}
```



*Assumes prior check made for empty Tree (in which case, failure is immediate)*

*Otherwise. search is invoked with startN = Root*

# Insert Leaf Node

## Strategy (Make each new Node a Leaf Node)

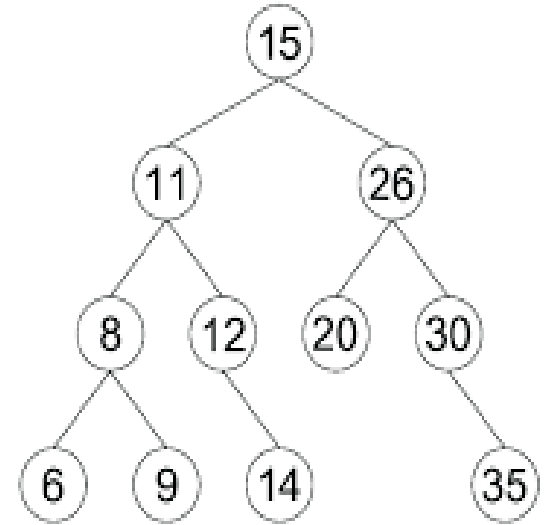
- <No 2 Nodes with same Key>
- <May “unbalance” the Tree>

How do we insert:

Node 13?

Node 22?

Node 9?



*Assumes prior check made for empty Tree (in which case, newN is made Root)*

*Otherwise insert is invoked with startN = Root*

*Do for Start 15 Insert 10.  
<Where is 10>*

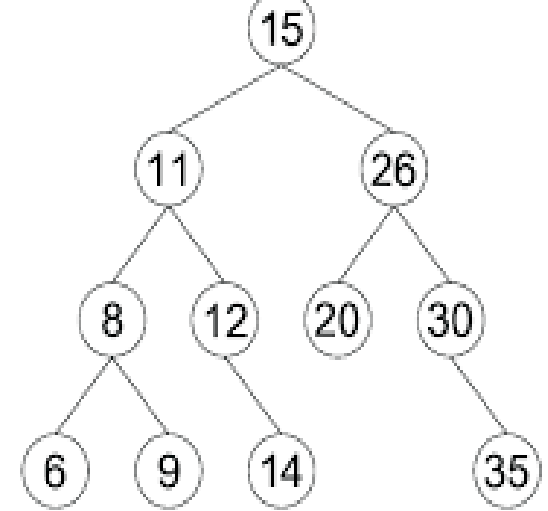


# Insert Leaf Node: Strategy

## Strategy (Make each new Node a Leaf Node)

- <No 2 Nodes with same Key>
- <May “unbalance” the Tree>
- <Call invoked on “top” node>

```
if LHC and Given Key < Top Node's Key
    Recursive call at LHC
else if RHC and Given Key > Top Node's Key
    Recursive call at RHC
else // Can't descend further
    If Given Key < Supplied Node's Key
        Insert Given Node as LHC of Top Node
    if Given Key > Supplied Node's Key
        Insert Given Node as RHC of Top Node
    else // Given Key == Top Node's Key
        Return error (no duplicate keys allowed)
```



*Assumes prior check made for empty Tree (in which case, newN is made Root)*

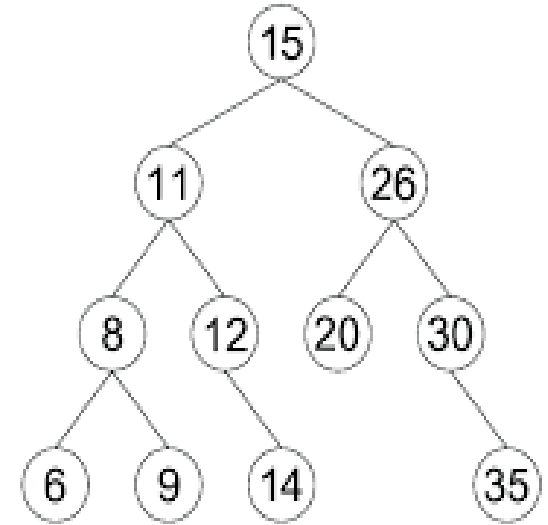
*Otherwise insert is invoked with startN = Root*

*Do for Start 15 Insert 10.*



# Insert Leaf Node: Code

```
Node *insert (Node *startN, int desKey Node *newN)
{ // Fails (returns Null) if deskey node already exists
  Node *currN = startN;
  if ((currN→LHC) && (desKey < currN→key))
  { // Recursive call to LHC
    rval = insert (currN->LHC, desKey, newN); }
  else if (currN→RHC) && (desKey < currN→key))
  { // Recursive call to RHC
    rval = insert (currN->RHC, desKey, newN); }
  else { // Can't descend further.
    // Insert as leaf child unless key match
    newN->parent = currN;
    rval = newN; // Assume success
    if (desKey < currN→key)
    { currN→LHC = newN; newN→parent = currN; }
    else if (desKey > currN→key)
    { currN→RHC = newN; newN→parent = currN; }
    else // Keys MATCH!!
      rval = NULL }}
  return (rval); };
```

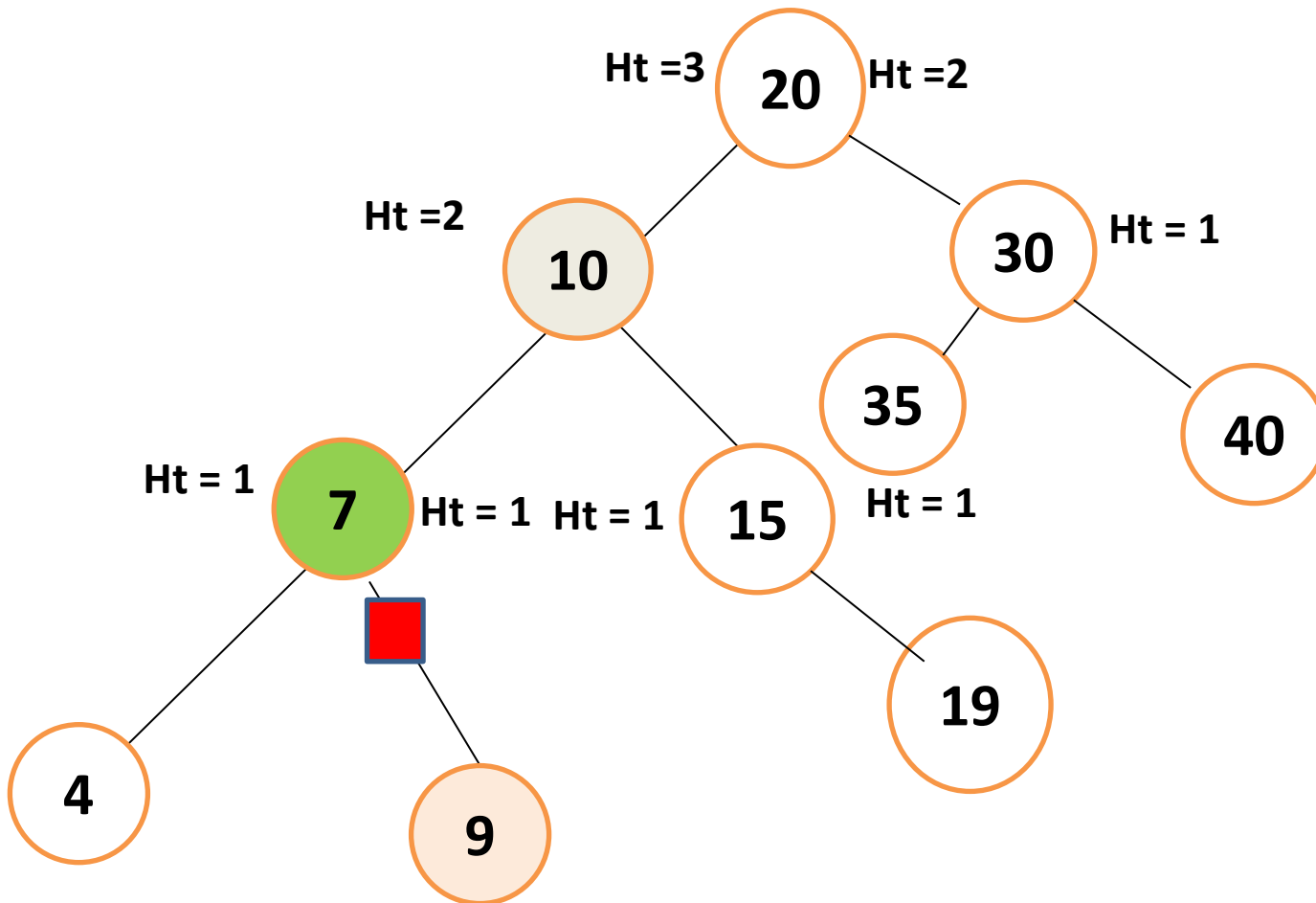


*Assumes prior check made for empty Tree (in which case, newN is made Root)*

*Otherwise insert is invoked with startN = Root*

*Do for Start 15 Insert 10.*

# Deletion of Leaf (9)



# Remove Leaf

```
Node *remove (Node *tnode) { // Check children
    Node* rc = tnode→rhc, lc = tnode→lhc, repl = NULL;
    Node *par = tnode→parent, *w1 = NULL, *w2 = NULL;
```

```
// 4 cases need to be dealt with. We are removing
```

```
    // A leaf node
```

```
    // A node with only a RHC
```

```
    // A node with only a LHC
```

```
    // A node with both LHC and RHC (nasty)
```

```
if ((!rc) && (!lc)) // Neither child of node being removed exists: → Leaf
```

```
{ // Leaf Node (ex: 9)
```

```
    if (!par) // Root node being deleted is only Node!
```

```
        toRoot = NULL; // BST is empty
```

```
    // Clear out Parent pointer to this node
```

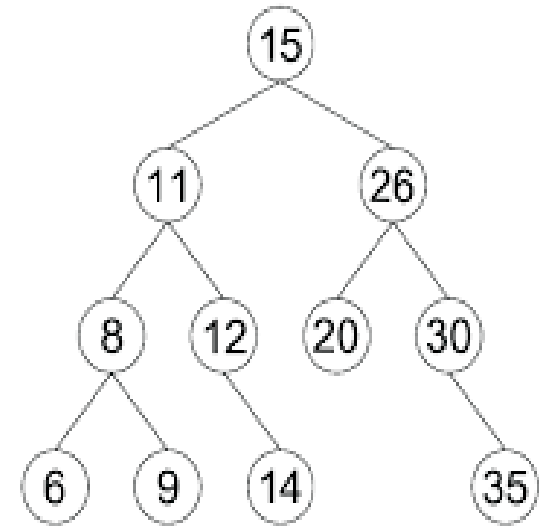
```
    else if (par→lhc == tnode)
```

```
        par→lhc = 0;
```

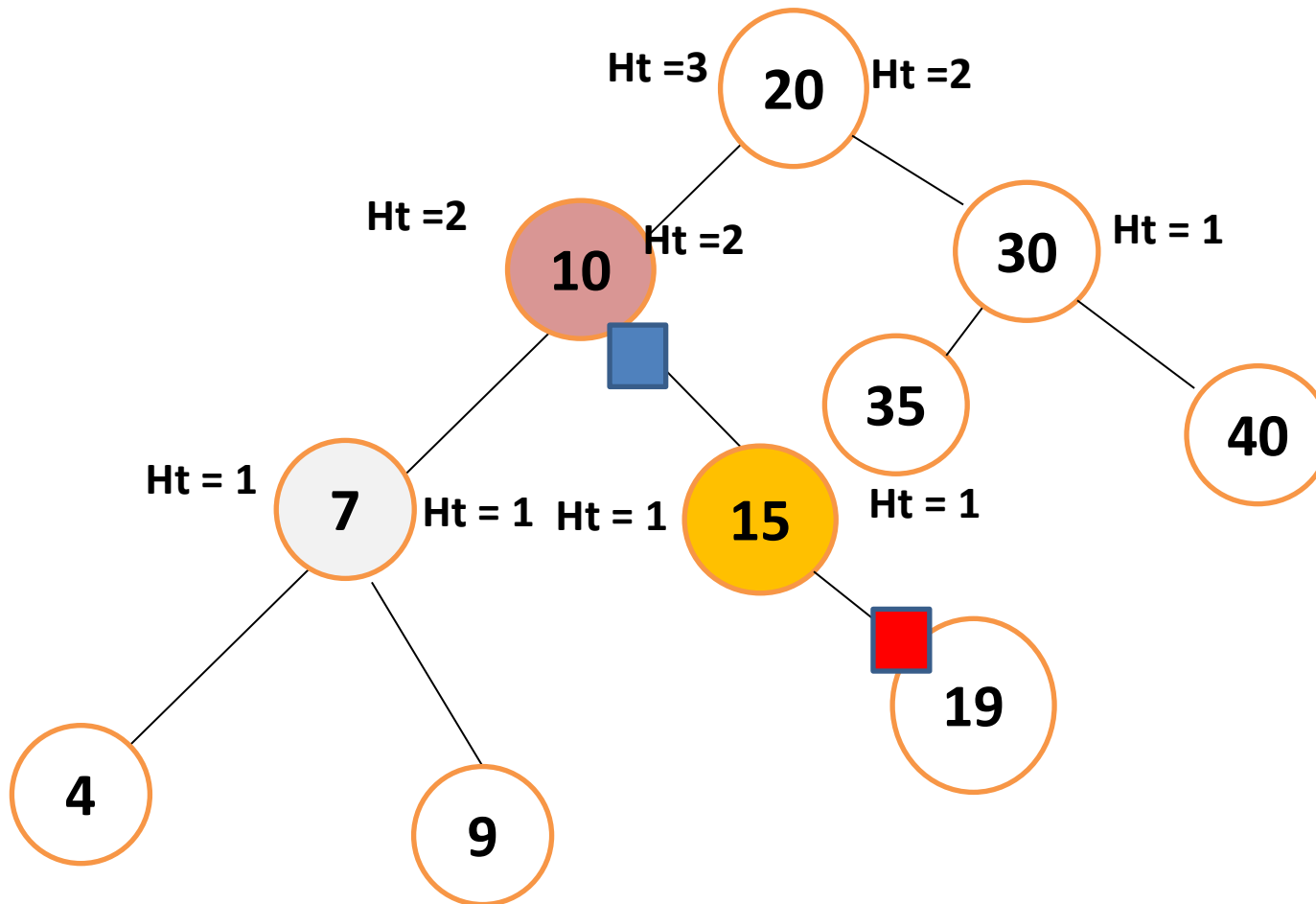
```
    else
```

```
        par→rhc = 0;
```

```
}
```

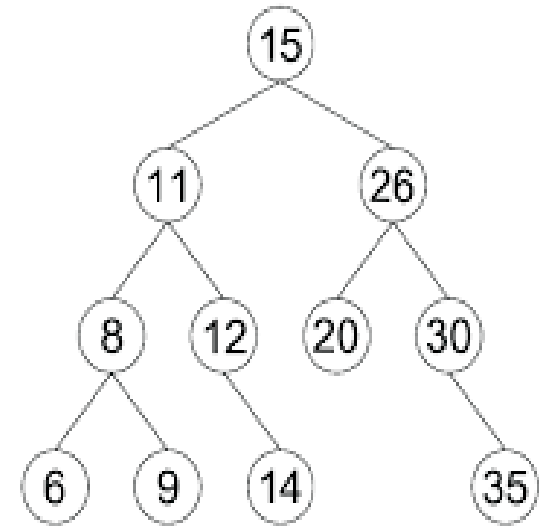


# Deletion with Only RHC (15)

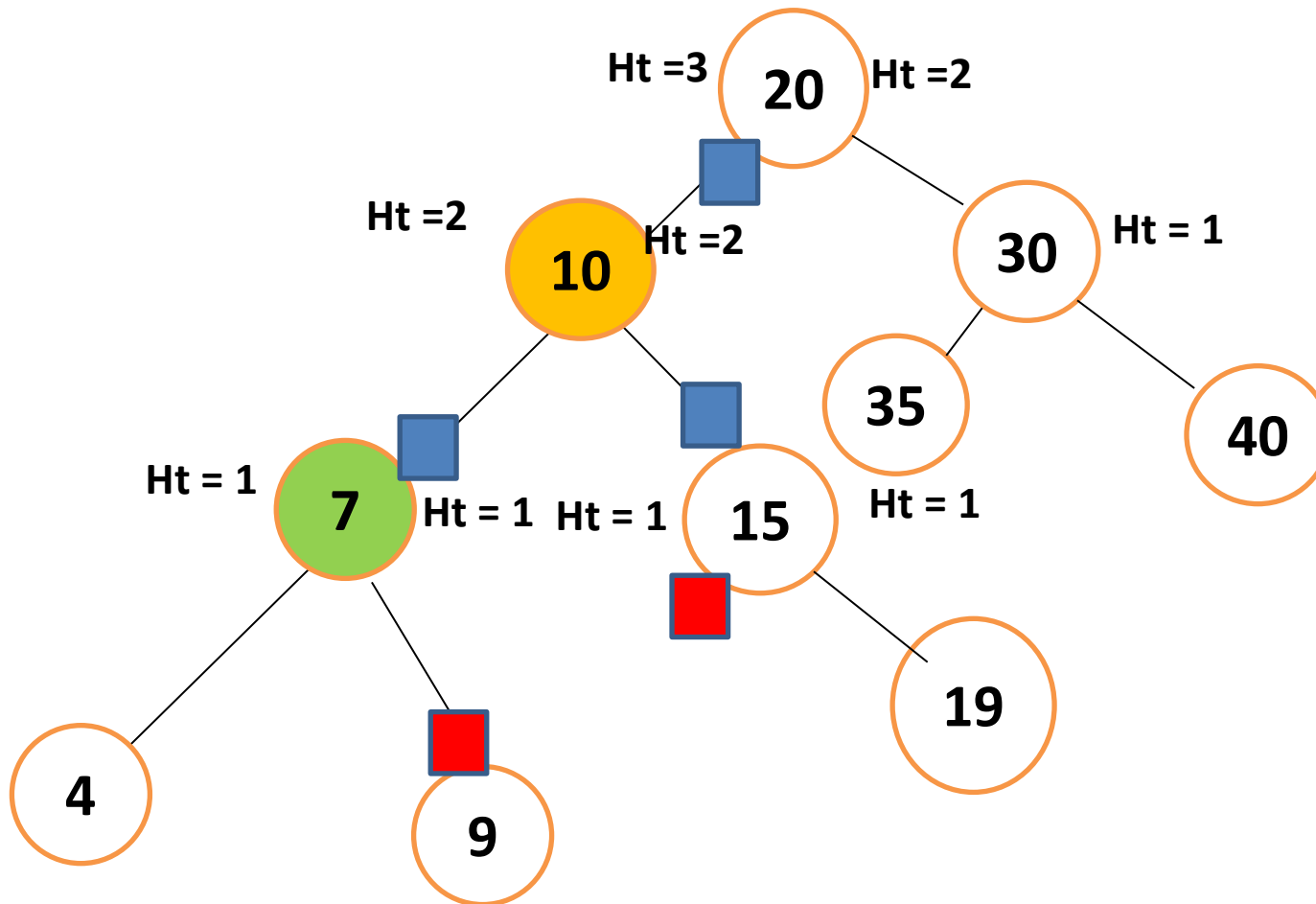


# Remove Node with only 1 Child: Strategy

```
else if ((rc) && (!lc)) { // Only RHC (30)
    if (!par) { //Root is lowest Node in Tree
        toRoot = rc; rc→parent = NULL;}
    else { // Advance RHC
        rc→parent = par; // Set 35 Par = 26 and
        if (par→rhc == tnode) par→rhc = rc; // 26 RC = 35
        else
            par→lhc = rc;
    }
}
else if ((!rc) && (lc)) { // Only LHC
    if (!par) { //Root is highest Node in Tree
        toRoot = lc; lc→parent = NULL;}
    else { // Advance LHC
        lc→parent = par; // Replace appropriate Child Ptr
        if (par→rhc == tnode) par→rhc = lc;
        else par→lhc = lc; }}
```



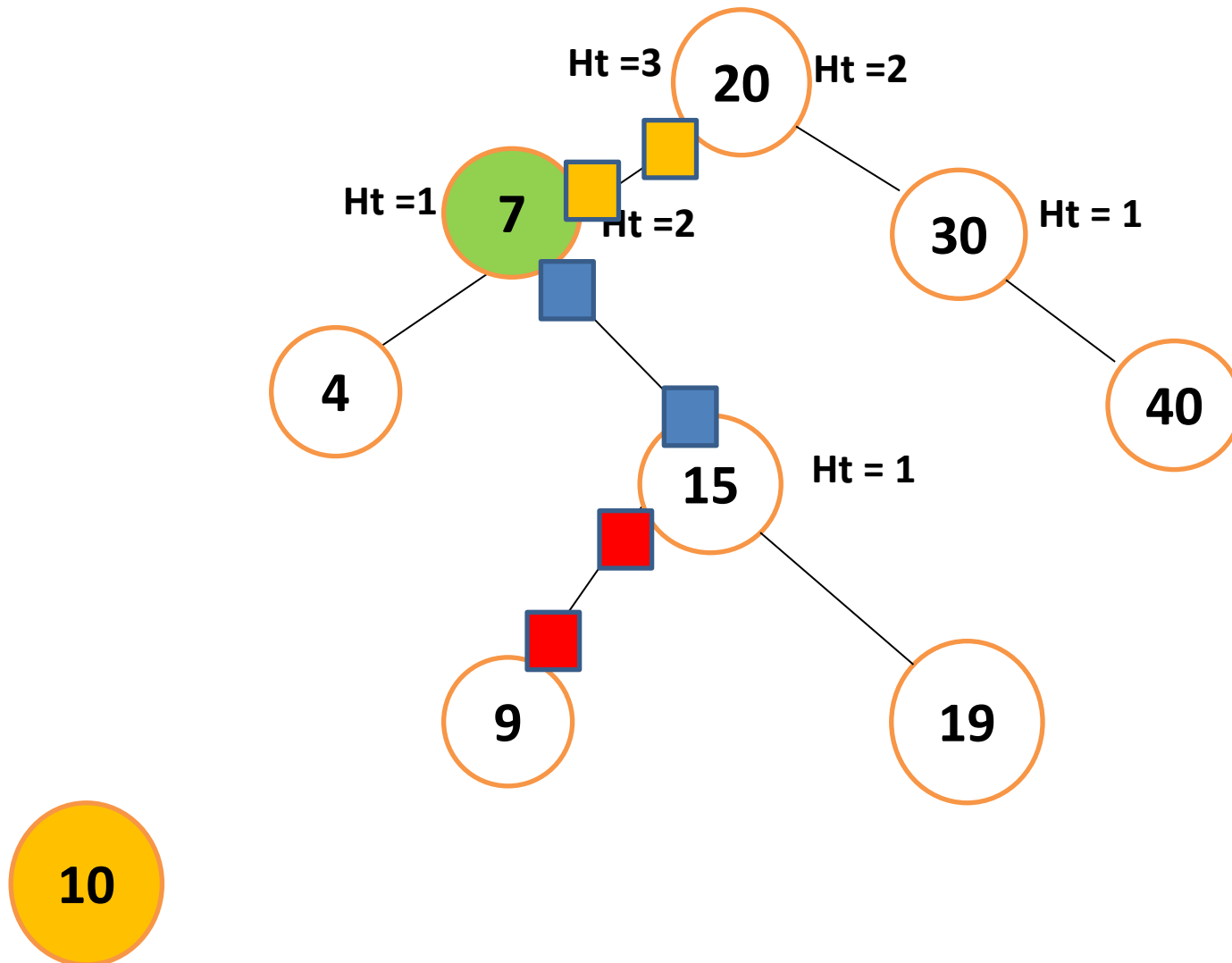
# Deletion of Node with LHC & RHC (10)



# Remove Node with both children

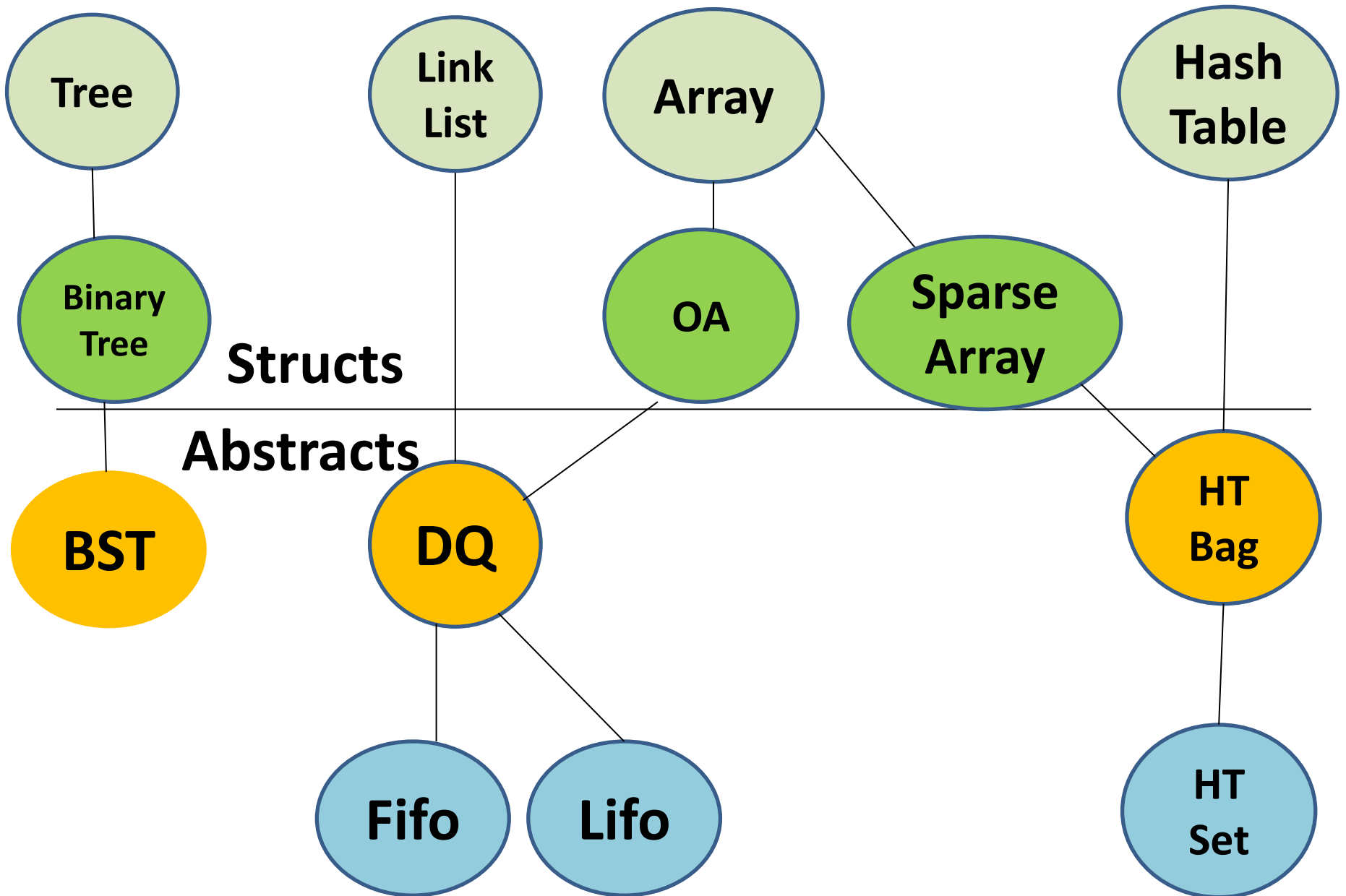
```
else { // Deleted Node has both RHC and LHC (Node 10)
// tnode = 10, rc = 15, lc = 7, par = 20
// Use highest node under LHC as replacement (7)
    repl = lc; w1 = repl→rhc; // Original repl RC reattached later
    tnode→rc→parent = repl; repl→rc = tnode→rc; // repl RC = tnode RC
    if (!par) {toRoot = repl; , repl→parent= NULL;} //New Root
    else { // repl parent = tnode parent
        repl→parent = par; (20)
        if (par→lhc== tnode) par→lhc = repl; // (20 LC = 7)
        else par→rhc = repl; }
// Set RH hierarchy of deleted node as RH of replacement
    repl→rhc = tnode→rhc;
    if (repl→rhc == NULL) repl→rch = w1; // Deleted node had no RC. Reattach orig RC
    else if (w1) { // Repl had an original RC, hook as LC to lowest LC of first RC
        w2 = repl→rhc;
        while (w2→lhc) { w2 = w2→lhc; } // W2 > repl original RHC , has no LC
        w2→lhc = w1;
        w1→parent = w2;
    }
    return (tnode); };
```

# Post Deletion of Node with LHC & RHC





# Relationship of Collections



# Data Structure Comparisons: Retrievals

(assume N elements in collection)

Data Structure Collection Type	Retrieval Given “Key”	Ordered Retrievals getNext(),getPrev()	Maintaining Order
Ordered Array	Excellent. Binary Search for Element $O(\log N)$	Excellent $O(1)$	Fair. Ins / Rem ops require bulk moves $O(N/2)$
Linked List (Deque)	Fair. Search through half of elements $O(N/2)$	Excellent $O(1)$	Fair. Search through half of elements $O(N/2)$
Associative Array (Hash Table Set)	Excellent. Depends on Entries / Buckets. $O(1)$	Not Possible	N/A
BST (Ordered, unbalanced)	Good to Excellent Depends on balancing. Ranges from $O(\log N)$ to $O(N)$	Excellent (Walk Tree nodes) $O(1)$	Excellent ignoring Tree Balancing. $O(\log N)$

# Data Abstraction Comparisons: Updates

(assume N elements in collection)

Underlying Structure	Data Abstraction	Search	Add / Delete (after search)	Ordering
Array	Ordered Array	Binary Search $O(\log N)$	Involves push/pull of multiple elements $O(N/2)$	User determined
Linked List or Ordered Array	DeQueue	Run through $\frac{1}{2}$ elements $O(N/2)$	Change a few pointers $O(1)$	User Determined
Hash Table	Associative Array	Instantaneous $O(1)$	Instantaneous by supplied Key $O(1)$	N/A
Tree	Binary Search Tree	Depends upon balance) $O(\log N) - O(N)$	Add as Leaf Delete (relink) $O(1)$	By Key
Tree	AVL Tree	Binary Search $O(\log N)$	Requires rotations $O(1)$	By Key

# Questions?



# Review: Data Collection Structures

- **Orderable Array**

- Extending initial size via reallocate is **slow** ( $2N + \text{malloc}$ )
- Access to element via index is **instantaneous**
- Find element by key is  **$\text{Log}_2 N$**  (using Binary Search)
- Insert & Remove are **slow** ( $2N = N$ )

- **Dequeue**

- Extending initial size **instantaneous** (independent of  $N$ )
- **Find** element by key **slow** (walk chain  $N/2 = N$ )
- **Insert / Remove** by Key is slow (precede with **Find**)
- **next / previous** is **instantaneous**

- **Hash Table**

- Extending initial size **instantaneous** (independent of  $N$ )
- Find element by key **instantaneous** (hash time)
- **No ordering possible**