Valuation of Debt Securities and Interest Rate Derivatives in the World of CVA and DVA

CVA - credit valuation adjustment
DVA - debt valuation adjustment

Key Parameters:

- Expected Exposure to default loss
- Probability of Default
- Recovery Rate

Objective: Obtain fair value prices for debt securities and derivatives given the extent of credit risk.

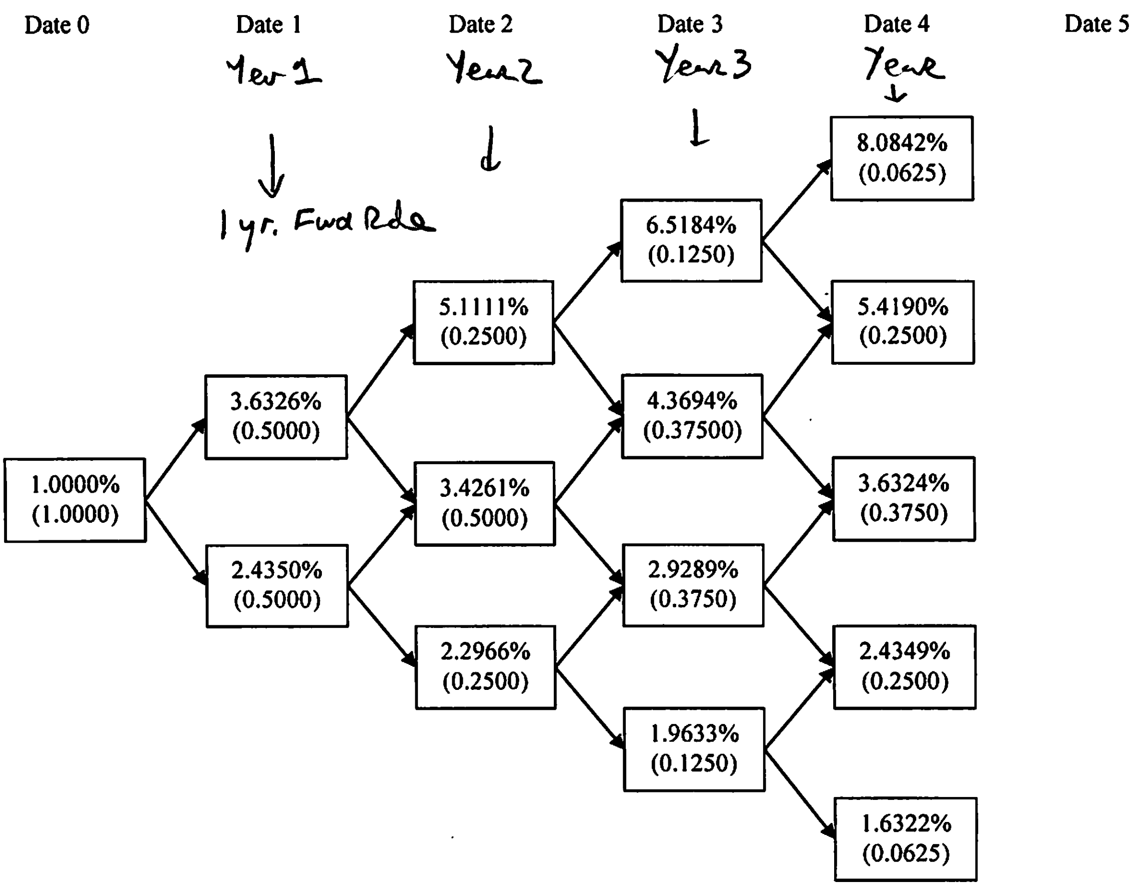
Section I. Forward Rate Binomial Tree Model

Assume: 1-year rate
Underlying Bonds make annual coupon payments

Model: Kalotay-Williams-Fabozzi (KWF)
Ho-Lee
Hull-White
Black-Karasinski
Black-Derman-Toy

See the

Exhibit 1: Binomial Forward Rate Tree for 20% Volatility



Expected Forward Rate	3.0338%	3.5650%	3.7971%	3.9329%

Assume 20% volatility

For all nodes
Prob up = 50%
Prob down = 50%

Note: these are based on "risk-free" government bonds.

Exhibit 2: Underlying Risk-Free Benchmark Coupon Rates, Prices, Discount Factors, Spot Rates, and Forward Rates

Date	Coupon Rate	Price	Discount Factor	Time Frame	Spot Rate	Time Frame	Forward Rate
1	1.00%	100.000	0.990099	0 x 1	1.0000%		
2	2.00%	100.000	0.960978	0 x 2	2.0101%	1 x 2	3.0303%
3	2.50%	100.000	0.928023	0 x 3	2.5212%	2 x 3	3.5512%
4	2.80%	100.000	0.894344	0 x 4	2.8310%	3 x 4	3.7658%
5	3.00%	100.000	0.860968	0 x 5	3.0392%	4 x 5	3.8766%

Par-Curve for benchmark bonds out to 5 years
Note: Coupon Rate = YTM because Par Bond
No accrued interest.

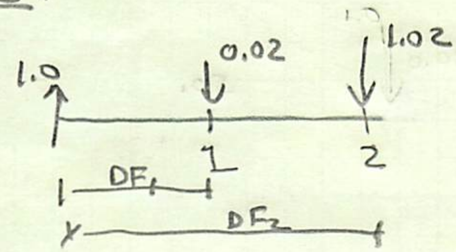
Bootstrap Discount Factors

DF (discount factor) = PV (present value) of 1 unit of currency received at a future date.

$$DF_1 = \frac{1}{(1+r_1)^1} = \frac{1}{1+0.01} = \frac{1}{1.01} = \underline{0.990099 = DF_1}$$

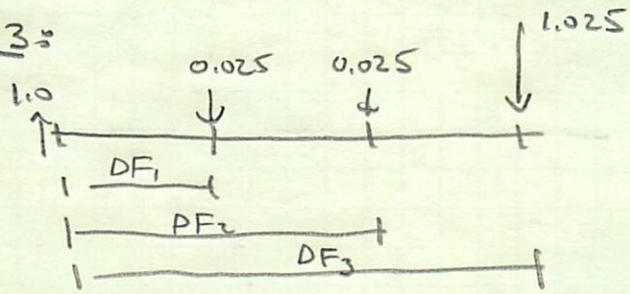
Note: Implicitly $\underline{DF_0 = 1}$

<u>Date 2:</u>



$$1.0 = (0.02)(0.990099) + (1.02)\,DF_2 \qquad \underline{DF_2 = 0.960978}$$

<u>Date 3:</u>



$$1.0 = (0.025)(0.990099) + (0.025)(0.960978) + (1.025)(DF_3)$$

$$\underline{DF_3 = 0.928023}$$

For par bonds:

$$DF_n = \frac{1 - CR_n \sum_{j=1}^{n-1} DF_j}{1 + CR_n}$$

where,

CR = coupon rate

Implied Spot Rates (or Zero-Coupon)

$$DF_n = \left(\frac{1}{1 + spot_{0,n}} \right)^n$$

or

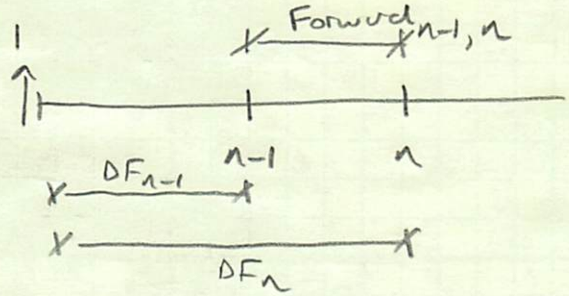
$$spot_{0,n} = \left(\frac{1}{DF_n} \right)^{1/n} - 1$$

For the 5-year "0×5" implied spot rate

$$Spot(0,5) = \left(\frac{1}{0.860968} \right)^{1/5} - 1 = 0.030392$$

$$S_5 = 3.0392\ \%$$

Implied Forward Rates



$$\frac{1}{DF_{n-1}} \left(1 + Forward_{n-1,n} \right)^1 = \frac{1}{DF_n}$$

$$Forward_{n-1,n} = \frac{DF_{n-1}}{DF_n} - 1$$

1 year forward rate, 3 years forward

fwd $3y1y \Rightarrow Forward_{3,4} \Rightarrow f_{3y1y}$

$$f_{3y,1y} = \frac{DF_3}{DF_4} - 1 = \frac{0.928023}{0.894344} - 1$$

$$f_{3y1y} = 0.037658 = 3.7658\ \%$$

Note that the implied forward rates are lower than the expected forward rates.

Implied $\text{fwd}_{4,5}$ = 3.8766%

Expected $\text{fwd}_{4,5}$ = 3.9329%

This occurs because:

KWF assumes lognormal forward rates thus no negative rates that Ho-Lee and Hull-White can have due to assumptions of a normal forward rates

KWF is an "arbitrage-free" term structure model which must be "calibrated" to assure that the benchmark bond matches the market price

$$\left(\underset{\text{Implied}}{\text{Forward Rate}}\right) * \exp\left\{-\frac{\sigma^2 T}{2} \pm \sigma\sqrt{T}\right\}$$

for $\sigma = 0.020$ $T = 1$

Initial Trial See Exhibit 3

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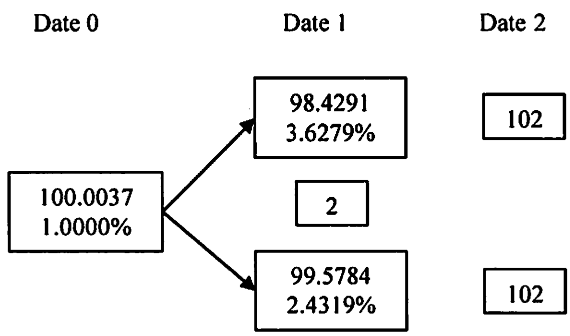
Exhibit 3: Calibrating the Forward Rates on the Binomial Tree for Date 1

Upper Panel: The Initial Test

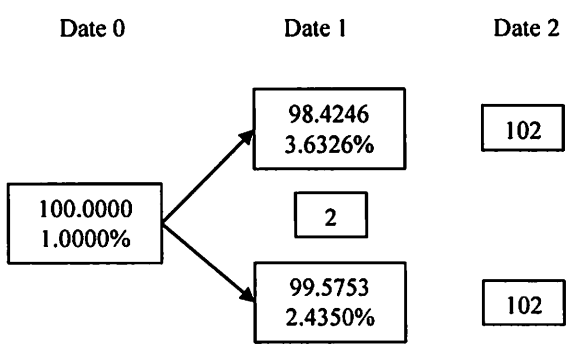


Calculations:

$$\frac{102}{1.036279} = 98.4291 \qquad \frac{102}{1.024319} = 99.5784$$

$$\frac{[2 + (0.50 * 98.4291 + 0.50 * 99.9784)]}{1.010000} = 100.0037$$

Lower Panel: The Final Calibration



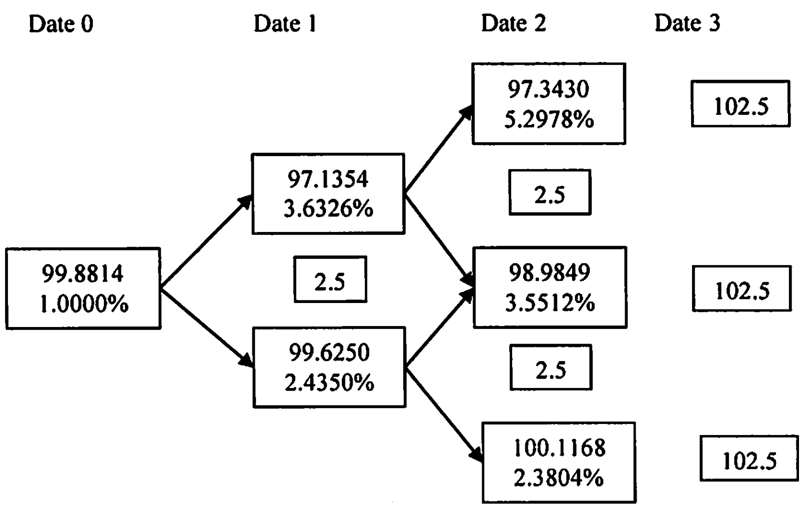
Calculations:

$$\frac{102}{1.036326} = 98.4246 \qquad \frac{102}{1.024350} = 99.5753$$

$$\frac{[2 + (0.50 * 98.4246 + 0.50 * 99.9753)]}{1.010000} = 100.0000$$

Exhibit 4: Calibrating the Forward Rates on the Binomial Tree for Date 2

Upper Panel: The Initial Test



Lower Panel: The Final Calibration

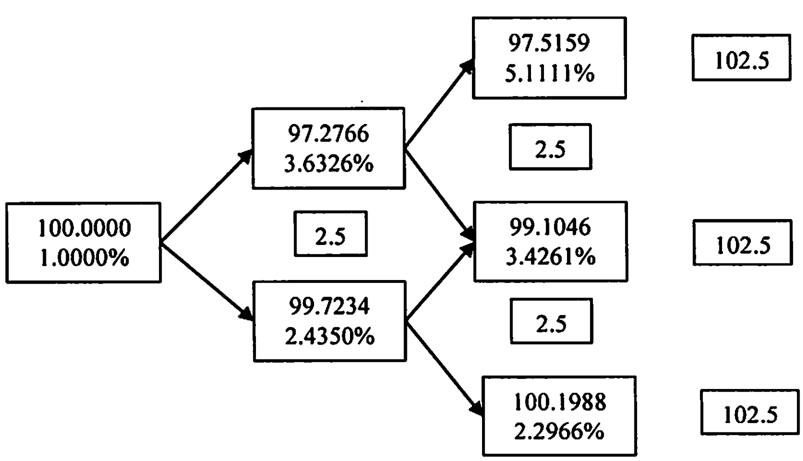


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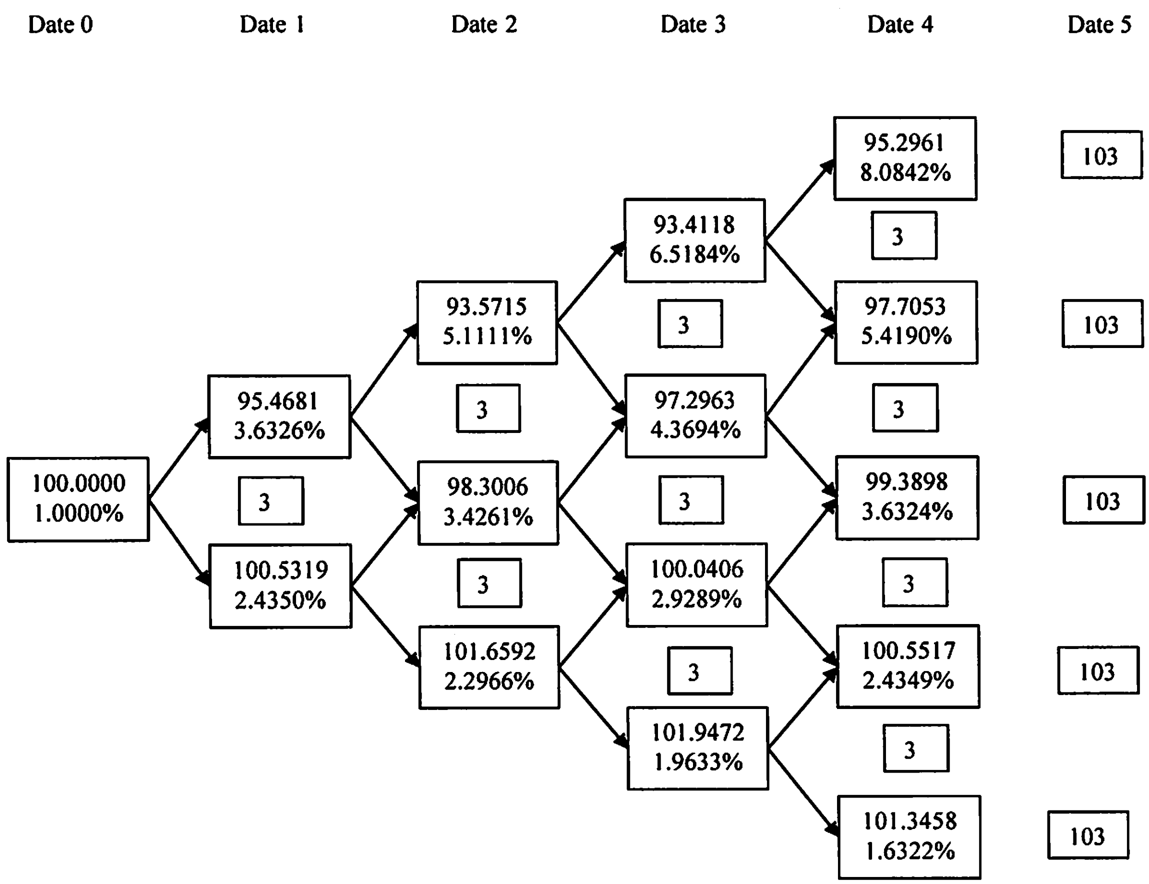
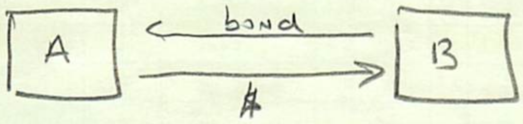
Exhibit 5: Valuation of the 3%, 5-Year, Annual Coupon Payment Benchmark Bond



Calibrated to implied forward rates :

Unilateral Credit Risk

- credit risk to only 1 party in a transaction



bond issuer

$\boxed{A}$ asset to the owner of the bond

$\boxed{B}$ liability to the issuer of the bond

To the asset holder, the fair value of the debt security or derivative writer

$$\text{Value}^{\text{asset}} = VND - CVA$$

where VND = value assuming no default
riskless value

CVA depends on the credit risk of the issuer of the debt security or derivative writer

Credit risk is captured by:

- probability of default
- recovery rate if default occurs
- expected exposure if default occurs

In general,

$$CVA = \sum_{t=1}^{T} (\text{Expected Exposure}_t)(1 - \text{recovery rate}_t) * (\text{Default Probability}_{t-1,t})(DF_t)$$

$\Rightarrow$ (1 − recovery rate) = loss severely

$\Rightarrow$ (Expected Exposure)(1 − recovery rate) = LGD
loss given default

$\Rightarrow$ Default Probability$_{t-1,t}$ = Prob($\tau > t \mid \tau > t-1$)
= hazard rate

note that there is no distinction on when from $(t-1, t)$ default occurs

and that the realization that impact occurs at t, the same time that recovery rate is applied.

For the issuer,

$$\text{Value}^{\text{Liability}} = -(\text{VND} - \text{DVA})$$
$$= -\text{VND} + \text{DVA}$$

⊖ negative implies liability

⊕ positive implies asset

Note that with unilateral credit risk

$\text{CVA} = \text{DVA}$ differ only in perspective

$$\text{Value}^{\text{Asset}} + \text{Value}^{\text{Liability}} = \text{VND} - \text{CVA} + (-\text{VND} + \text{DVA}) = 0$$

$$\text{CVA} = \text{DVA}$$

Note financial assets = financial liabilities in terms of economics, transactions, and aggregation across many assets/liabilities.

- Note that accounting rules might have different results, e.g.,

  investors carry assets at market value
  Issuers carry liabilities at book value

Example: Newly Issued Fixed Rate Corporate Bond

See Exhibit 6

Note: The sum of the CVAs for the various dates is the overall CVA.

- CVA of 2.3172 summarizes the credit risk on the bond in terms of present value at date 0.

- Also, the credit risk can be summarized as the yield on the benchmark bond having the same time-to-maturity.

  Corporate Bond ytm = 3.50% 5-yr bond

  Government Bond ytm = 3.00% 5-yr bond

  Annual credit spread = 50 bps

- Z-spread

$$100 = \frac{3.5}{(1+0.01+z)} + \frac{3.5}{(1+0.020101+z)^2} + \frac{3.5}{(1+0.025212+z)^3} + \frac{3.5}{(1+0.028310+z)^4} + \frac{103.5}{(1+0.030392+z)^5}$$

trial-error ⇒ Z-spread = 50.65 bps

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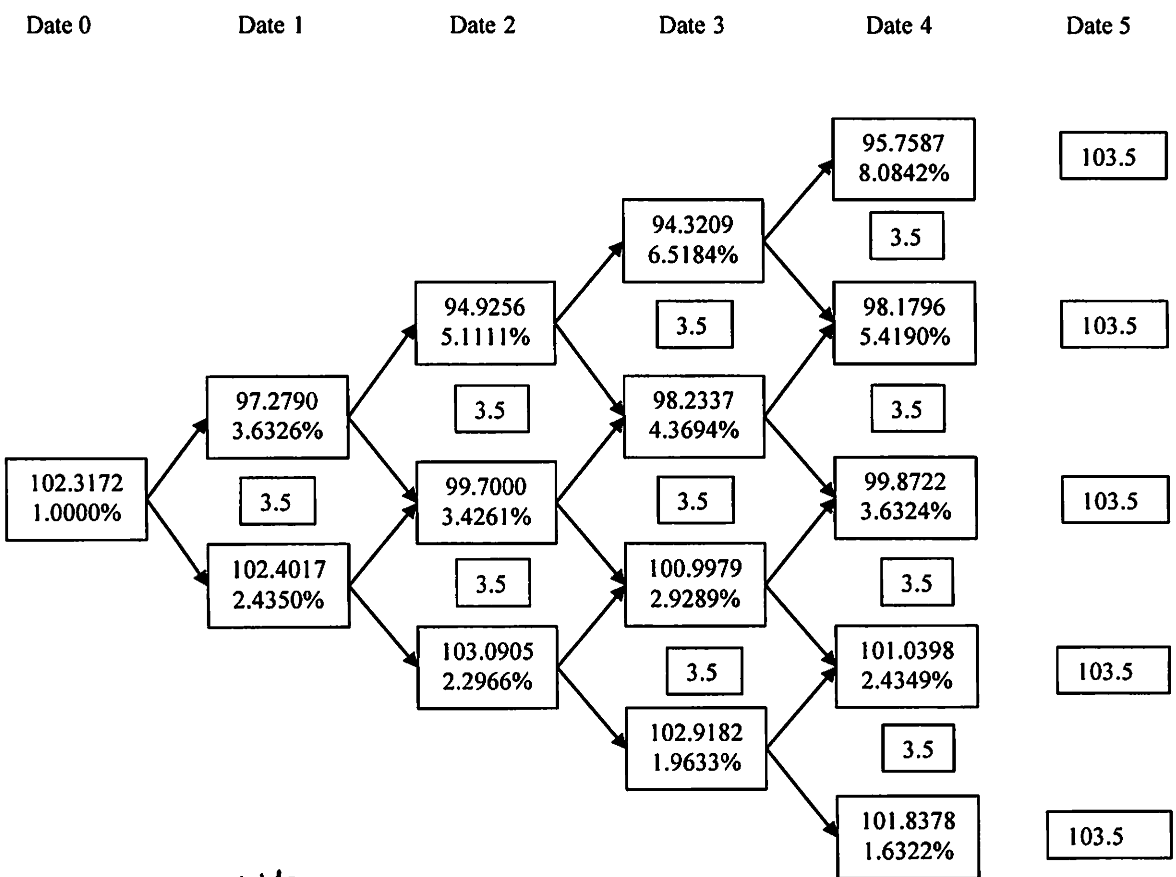
Exhibit 6: Valuation of a Newly Issued, 3.50%,

5-Year, Annual Coupon Payment Corporate Bond



we need the term structure

(1- 40%)

hazard rate

riskless df

	Expected Exposure	Loss Severity	Probability of Default	Discount Factor	CVA/DVA
Date 0					
Date 1	103.3404	60%	0.80805%	0.990099	0.4961
Date 2	102.8540	60%	0.80805%	0.960978	0.4792
Date 3	102.8667	60%	0.80805%	0.928023	0.4628
Date 4	103.1067	60%	0.80805%	0.894344	0.4471
Date 5	103.5000	60%	0.80805%	0.860968	0.4320
					2.3172

Fair Value = 102.3172 – 2.3172 = 100.0000

Components of a Corporate Bond Yield

See Exhibit 7

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Exhibit 7: Components of a Corporate Bond Yield