Trading Credit Curves

- Modeling Defaultable Bonds
- Modeling Credit
- Default Probabilities & CDS Pricing
- Shape of Credit Curves
- Forwards in Credit

- Drivers of PnL in Curve Trades
- Curve Trading Strategies
- Risk Measurements

Valuing Defaultable Bonds

- Zero-Coupon Bonds (Non-defaultable)

  Let $B(t,T)$ = today (time $= t$) price of a riskless zero coupon bond that pays out \$1 at a future time T

  Let $R(t,T)$ = continuously compounded yield to maturity of the above $B(t,T)$ bond

  so

$$B(t,T) = \exp\left\{-R(t,T)\,(T-t)\right\}$$

Note: we can view $B(t,T)$ as the time value of money, and thus

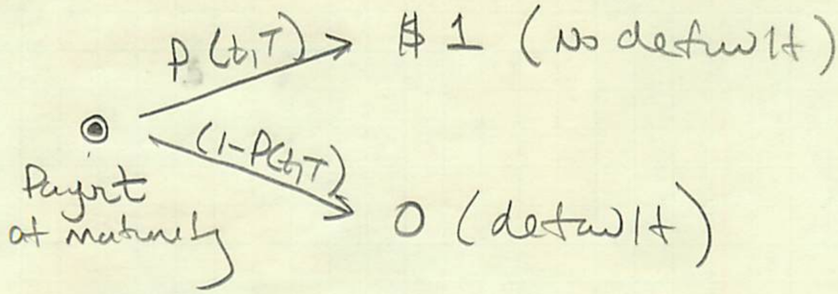
$$B(t,T) < 1\ , \text{ when } t < T$$

- Now consider a bond which is defaultable (risky)

$$\text{Prob}(\tau > t) > 0$$

where τ = time to default (stopping time)

At maturity T, relative to today, t



where $P(t,T)$ = probability of survival from t to T

$(1-P(t,T))$ = probability of default from t to T

Letting $\bar{B}(T,T)$ = risky bond's payout ($\Lambda(T)$)

or,

$$E_t\left[\bar{B}(T,T)\right] = P(t,T)*1 + (1-P(t,T))*0$$
$$= P(t,T)$$

where $E_t[\cdot]$ = expectation formed on the basis of information available at time t, given the survival probability $P(t,T)$.

Now, let's discount $E_t\left[\bar{B}(T,T)\right]$ to the present time; two ways:

1) Discounting at a higher rate,

$$\bar{B}(t,T) = 1 \cdot \exp\left\{-\left(R(t,T)+S(t,T)\right)(T-t)\right\}$$
$$= B(t,T)\exp\left\{-S(t,T)(T-t)\right\}$$

where, $S(t,T) > 0$

2) Artificial probability ('measure')

$$\bar{B}(t,T) = \exp\left\{-R(t,T)(T-t)\right\}\left[Q(t,T)*1 + (1-Q(t,T))*0\right]$$
$$= B(t,T)\,Q(t,T)$$

where, $(1-Q(t,T))$ is the 'probability' attached to a default by the bond issuer.

Note that

$$(1 - Q(t,T)) \neq (1 - P(t,T))$$

artificial prob. of default $\neq$ actual prob. of default

because investors are 'risk averse'

most investor's prefer the sure thing.

$\therefore$

$$B(t,T)\,E_t\left[\bar{B}(T,T)\right] > \bar{B}(t,T)$$

$$B(t,T)\,P(t,T) > B(t,T)\,Q(t,T)$$

$$P(t,T) > Q(t,T)$$

actual probability of survival > artificial probability of survival

if investors are risk neutral (indifferent about risk), then,

$$P(t,T) = Q(t,T)$$

thus,

$$B(t,T)Q(t,T) = \bar{B}(t,T) \qquad (1)$$

Risk Neutrality implies that all investors are indifferent regarding risk. Also, it is a necessary condition for the arbitrage condition of pricing financial instruments.

Coupon-paying Defaultable Bonds

For a two-period bond,

$$V^B(t, t+2) = \bar{B}(t, t+1)\, C + \bar{B}(t, t+2)(1 + C)$$

where $V^B(t, t+2)$ = value of the bond

also,

$$V^B(t, t+2) = B(t, t+1)\, Q(t, t+1)\, C + B(t, t+2)\, Q(t, t+2)(1 + C)$$

Generalizing over a multi-period bond

$$V^B(t, T_N) = \left[\sum_{i=1}^{N} \big(B(t, T_i)\, Q(t, T_i)\, C \big) + B(t, T_N)\, Q(t, T_N) \right] F$$

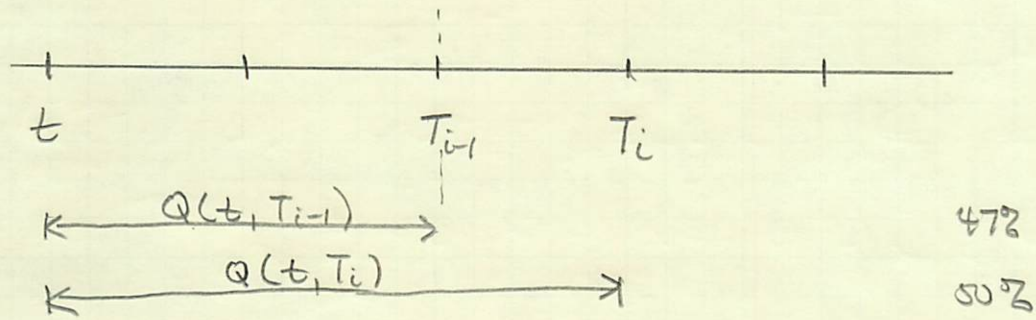
where, C = coupon rate

F = face value

Non Zero Recovery

Let X = recovery value of a defaultable bond with $X < F$

Assuming X is non-random



for default in the time span $[T_{i-1}, T_i)$

discounted recovery between $[T_{i-1}, T_i)$

$$= B(t, T_{i-1})\left[Q(t, T_{i-1}) - Q(t, T_i) \right] X$$

over all multi-periods

$$V^{Rec}(t) = \sum_{i=1}^{N} B(t, T_i)\left[Q(t, T_{i-1}) - Q(t, T_i) \right] X$$

and thus finally,

$V^B(t)$ = probability weighted (coupons + face value + recovery value)

Risky Bond spreads

Recall from page 3,

$$\bar{B}(t,T) = \exp\{-R(t,T)(T-t)\} Q(t,T)$$
$$= \exp\{-[R(t,T) + S(t,T)](T-t)\}$$

which yields,

$$Q(t,T) = \exp\{-S(t,T)(T-t)\}$$

Recovery Rates

TABLE 15.1
Historical Recovery Value Statistics (1970–1999)[a]

Seniority/ security	Min	1st quartile	Median	Mean	3rd quartile	Max	Std dev
Sr. sec. loans	15.00	60.00	75.00	69.91	88.00	98.00	23.47
Eq. trust bds	8.00	26.25	70.63	59.96	85.00	103.00	31.08
Sr. sec. bds	7.50	31.00	53.00	52.31	65.25	125.00	25.15
Sr. unsec. bds	0.50	30.75	48.00	48.84	67.00	122.60	25.01
Sr. sub. bds	0.50	21.34	35.50	39.46	53.47	123.00	24.59
Sub. bds	1.00	19.62	30.00	33.17	42.94	99.13	20.78
Jr. sub. bds	3.63	11.38	16.25	19.69	24.00	50.00	13.85
Pref. stocks	0.05	5.03	9.13	11.06	12.91	49.50	9.09

Source: Moody's Investors Service

[a]Prices of defaulted instruments approximately one month after default, expressed as a percent of the instrument's par value. Abbreviations: Eq. = equipment, Sr. = senior, sec. = secured, sub. = subordinated, Jr. = junior, pref. = preferred, bds = bonds.

Modeling Credit

$$Q(t,T) \equiv Prob_t\left[\tau > T \,/\, \tau > t\right]$$

where $Q(t,T) \equiv$ probability of survival

Forward Default Probabilities

$$1 - Q(t,U) \equiv Prob_t\left[\tau \le U \,/\, \tau > t\right]$$

$$Prob_t\left[T < \tau < U \,|\, \tau > t\right]$$

$$= Prob_t\left[\tau > T \,/\, \tau > t\right] - Prob_t\left[\tau > U \,/\, \tau > t\right]$$

$$= Q(t,T) - Q(t,U)$$ $\Rightarrow$ unconditional forward probability

Note: Using Bayes' rule:

$$Prob_t\left[\tau > U \,|\, \tau > T\right] = \frac{Prob_t\left[\tau > U \,|\, \tau > t\right]}{Prob_t\left[\tau > T \,|\, \tau > t\right]}$$

$$= \frac{Q(t,U)}{Q(t,T)}$$

and thus,

$$Prob_t\left[\tau \le U \,|\, \tau > T\right] = 1 - Prob_t\left[\tau > U \,|\, \tau > T\right]$$

$$= 1 - \frac{Q(t,U)}{Q(t,T)} = \frac{Q(t,T) - Q(t,U)}{Q(t,T)}$$

Forward Default Rates

Key Concept!!

default rate $H(t,T) \equiv \dfrac{\text{risk Neutral default probability}}{\text{length of time horizon}}$

$$H(t,T) \equiv \frac{\mathrm{Prob}_t\left[\tau \leq T \mid \tau > t\right]}{T-t} = \frac{1 - Q(t,T)}{T-t}$$

$$H(t,T,U) \equiv \frac{\mathrm{Prob}_t\left[\tau \leq U \mid \tau > T\right]}{U-T}$$

$$= \frac{Q(t,T) - Q(t,U)}{Q(t,T)} \; \frac{1}{U-T}$$

if $U \equiv T + \Delta T$

$$H(t,T,T+\Delta T) = -\frac{Q(t,T+\Delta t) - Q(t,T)}{\Delta t} \cdot \frac{1}{Q(t,T)}$$

time-t instantaneous forward default rate:

$$h(t,T) \equiv \lim_{\Delta T \to 0} H(t,T,T+\Delta T)$$

$$h(t,T) = -\frac{\partial Q(t,T)}{\partial T} \; \frac{1}{Q(t,T)}$$

finally

$$-\int_T^{\bar{U}} h(t,v)\,dv = +\int_T^{\bar{U}} \frac{\partial Q(t,v)}{Q(t,v)}\,dv$$

$$= \ln Q(t,v)\Big|_T^{\bar{U}}$$

$$= \ln \frac{Q(t,\bar{U})}{Q(t,T)}$$

thus,

$$\exp\left\{-\int_T^{\bar{U}} h(t,v)\,dv\right\} = \frac{Q(t,\bar{U})}{Q(t,T)}$$

or,

$$\mathrm{Prob}_t\left[\tau > \bar{U} \mid \tau > T\right] = \frac{Q(t,\bar{U})}{Q(t,T)} = \exp\left\{-\int_T^{\bar{U}} h(t,v)\,dv\right\}$$