Computer Simulations and Risk Assessment – Lecture 9

Fall 2019

Brandeis International Business School

Course Information - Schedule

Class Date	Text Chapters
Aug. 30, 2019 – L1	• Course Introduction/Python Installation • Introduction to Quantitative Finance Career • Python basics
Sep. 6, 2019 – L2	• Advanced Python Topics
Sep. 13, 2019 – L3	• Advanced Python Topics
Sep. 20, 2019 – L4	• Sourcing and handling Data • Stylized financial data analysis using Python
Sep. 27, 2019 – L5	• Value at Risk
Oct. 4, 2019 – L6	• Conditional Value at Risk (Expected Shortfall) + Mid-term Review
Oct. 11, 2019	• Mid-term
Oct. 18, 2019 – L7	• Modeling Volatility I
Oct. 25, 2019 – L8	• Modeling Volatility II
Nov. 1, 2019 – L9	• Practical application case Studies I
Nov. 8, 2019 – L10	• Practical application case Studies II
Nov. 15, 2019 – L11	• Back Testing + Conditional risk prediction
Nov. 22, 2019 – L12	• Research project presentation
Dec. 6, 2019 – L13	• Final Review

Lecture 9 – Outline

Sample risk optimization strategies

- Design of Risk-Parity strategies
- Hierarchical Risk Parity: Machine learning + Risk Parity

Process to develop Risk – Based Investment Strategies

1. Develop risk model to forecast Volatility/Risk of a portfolio, e.g.
$$\sigma_{p,t}^2 = \boldsymbol{W'}\Sigma\boldsymbol{W} \text{ or VaR}$$
2. Define risk-based investment objectives, e.g.
Tracking error minimization $TE^2 = \boldsymbol{W'_a}\Sigma\boldsymbol{W_a}$
or equal risk contribution: $w_i \frac{(\boldsymbol{\Sigma w})_i}{\sigma_p^2} = \sigma_{p,t}^2/\text{N}$
3. Allocate weights to different assets based on their risk contribution through optimizations

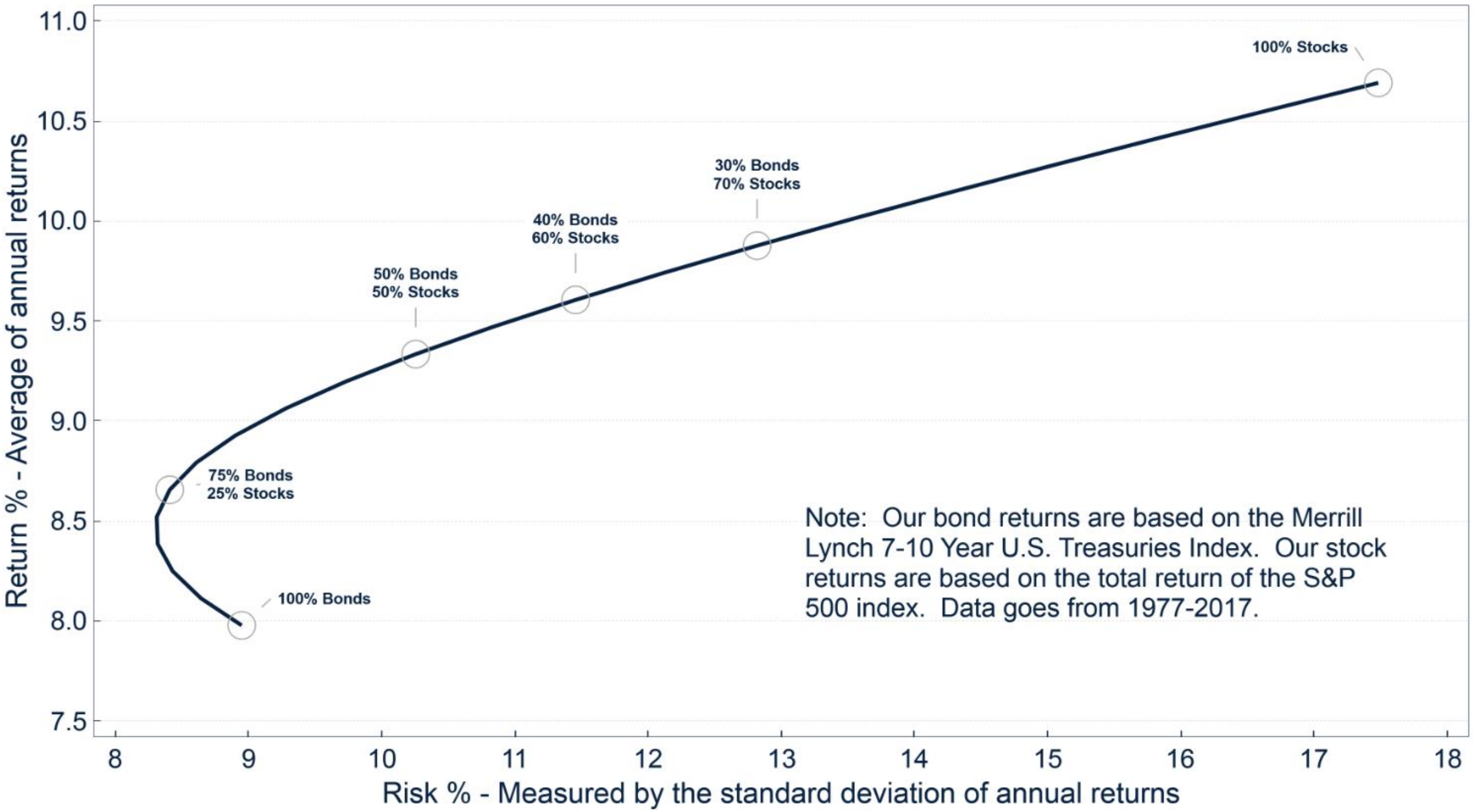
Risk-Parity Strategies

Risk-Parity Strategies

- Concept behind Risk-Parity Strategies
- Design of Risk-Parity strategies
- Example implementing Risk-Parity Strategies

Concept behind Risk-Parity

- Historical returns/risks for different asset classes have been different. E.g., equities vs. bonds



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Concept behind Risk-Parity

- Classical asset allocation has tilted towards higher allocations to equities because their perceived higher returns over a long enough period of time
- But the reality is bonds have meaningfully higher Sharpe Ratio then equities. I.e., per unit of risk, bonds have offered higher returns
- This contributed to the development of the Risk-Parity idea: How about we scale up the risk of the bond portfolio so it has the same risk level as the equity portfolio? This way it is an apple to apple comparison between bonds and equities
- Results: The return of the scaled up bond portfolio is higher than equity portfolio with similar risk
- How: leverage up the bond portfolio (by borrowing money to buy more bonds)

Development of the Risk – Parity Strategy

- Volatility of a portfolio

$$\sigma_{p,t}^2 = \boldsymbol{W}'\Sigma\boldsymbol{W} = \sum_{i=1}^{N}\sum_{j=1}^{N} w_{i,t}w_{i,t}\rho_{i,j,t}\sigma_{ij,t}\sigma_{ji,t} = w_1^2\sigma_1^2 + 2w_1w_2\rho\sigma_1\sigma_2 + w_2^2\sigma_2^2$$

- The risk contribution by each asset is

$$w_i\frac{\partial\sigma_p}{\partial w_i} = w_i\frac{(\boldsymbol{\Sigma w})_i}{\sigma_p}$$

- The proportion of each asset's risk contribution to the portfolio:

$$w_i\frac{(\boldsymbol{\Sigma w})_i}{\sigma_p^2}$$

- If risk contribution by each asset is set to be b_i, then the risk parity strategy can be designed by

$$\min_{\boldsymbol{w}} \sum_{\boldsymbol{i=1}}^{\boldsymbol{N}}\left(w_i\frac{(\boldsymbol{\Sigma w})_i}{\boldsymbol{\sigma_p^2}} - b_i\right)^{\boldsymbol{2}} \text{ with } b_i = 1/N \text{ for risk parity}$$

Risk Parity Strategy Formula for two assets

- Volatility of a portfolio

$$\sigma_{p,t}^2 = \boldsymbol{W}'\Sigma\boldsymbol{W} = w_1^2\sigma_1^2 + 2w_1w_2\rho\sigma_1\sigma_2 + w_2^2\sigma_2^2$$

- The risk contribution by each asset is

$$w_1 \frac{\partial\sigma_p}{\partial w_1} = w_1 * 0.5 * (w_1^2\sigma_1^2 + 2w_1w_2\rho\sigma_1\sigma_2 + w_2^2\sigma_2^2)^{-\frac{1}{2}} *$$

$$(2w_1\sigma_1^2 + 2 * w_2\rho\sigma_1\sigma_2) =$$

$$w_1 * (w_1\sigma_1^2 + w_2\rho\sigma_1\sigma_2)/\sigma_p = w_1 \frac{(\boldsymbol{\Sigma w})_1}{\sigma_p}$$

- The proportion of the first asset's risk contribution to the portfolio:

$$w_1 \frac{\partial\sigma_p}{\partial w_1} / \sigma_p = w_1 \frac{(\boldsymbol{\Sigma w})_1}{\sigma_p^2}$$

Risk – Parity Strategy

- Target risk contribution objective

$$\min_{\boldsymbol{w}} \sum_{i=1}^{N} \left(w_i \frac{(\boldsymbol{\Sigma w})_i}{\sigma_p^2} - b_i \right)^2$$

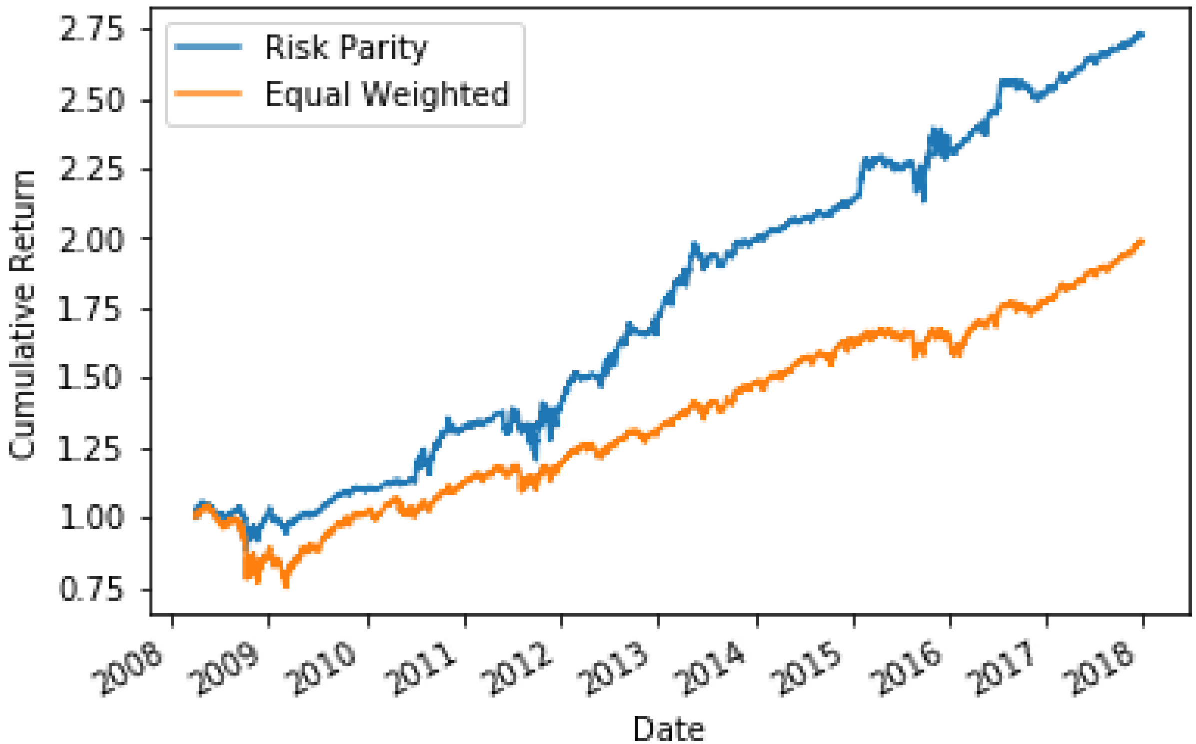
- Where
  - $\boldsymbol{\Sigma}$ is forecasted next period covariance matrix
  - σ_p is the total portfolio volatility
  - b_i is the targeted percent risk contribution by each of the asset, if set to 1/N then the strategy is of the risk-parity type
- Constraints:
  - Minimum weights for each asset class

Risk – Parity Strategy Example

- Two assets: S&P500 and Barclays Aggregates
- Rolling time window = 90 days
- Monthly rebalancing
- Use both exponentially weighted and simple moving average model to calculate covariance matrix
- Past 10 years of historical daily data
- With or without leverage: sum(W) = 1 or sum(W) = 2
- λ=0.94

Risk – Parity Strategy Example (MA)

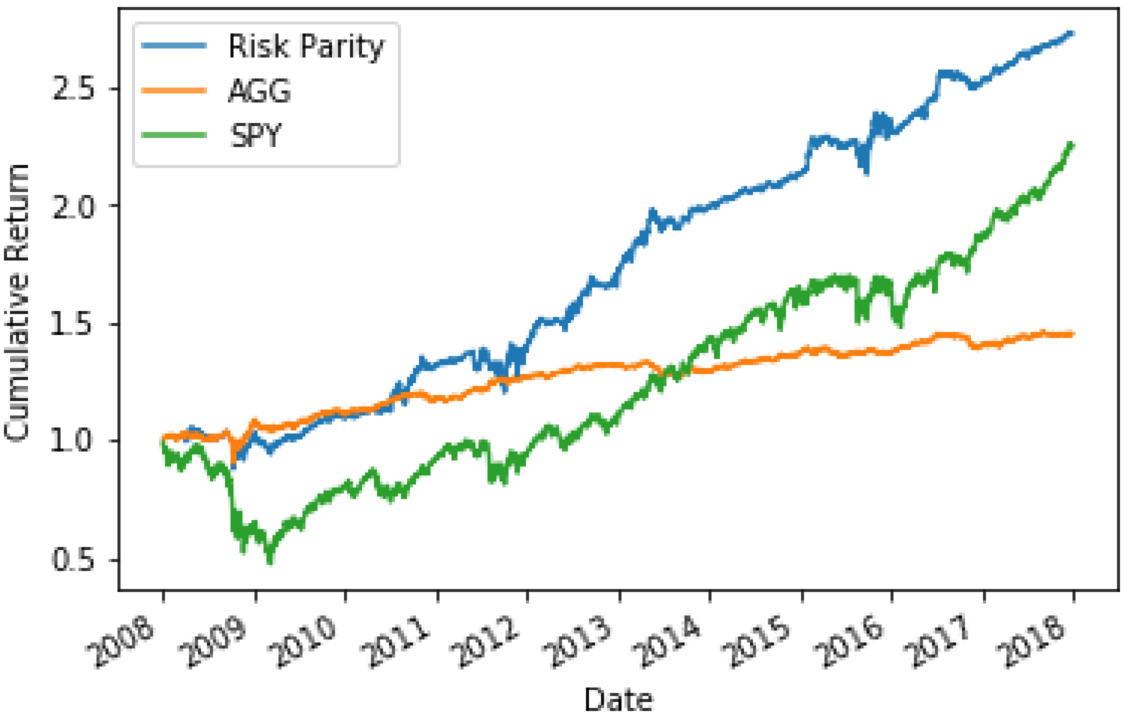
- Risk parity portfolio has both higher returns and lower volatility and downside risk
- Thus provide better risk-adjusted returns



Sharpe ratio of strategies:
risk_parity 1.094
equal_wted 0.711

Risk – Parity Strategy Example (MA)

- Risk parity portfolio trying to harness the best of the two underlying assets
- Risk parity portfolio has higher Sharpe ratio than each underlying assets



Sharpe ratio of strategies:

risk_parity	1.094
AGG	0.734021
SPY	0.413695

Risk – Parity Strategy Example (MA)

- Area plot of historical weights of the two assets
- It is possible one asset has 100% of weights even under the 'equal' risk contribution constraint
- Reason: contribution of one asset is negative because of low asset vol.+ negative correlation

cov_assets =

	AGG	SPY
AGG	0.000016	-0.000028
SPY	-0.000028	0.000207

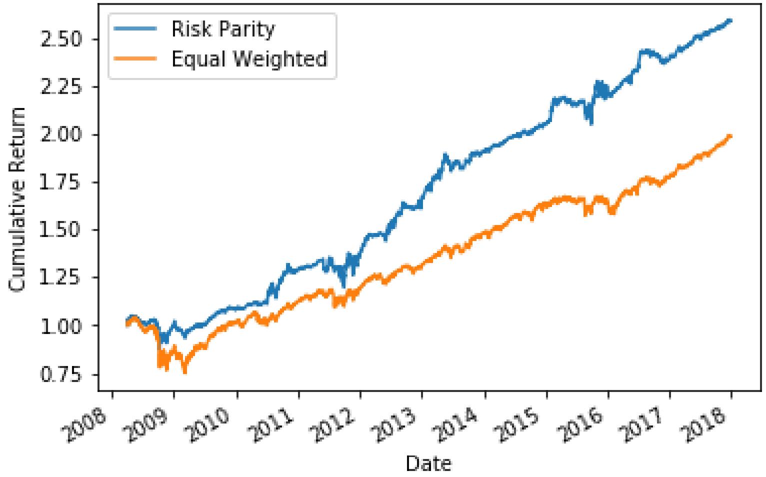
W=[0.5, 0.5]

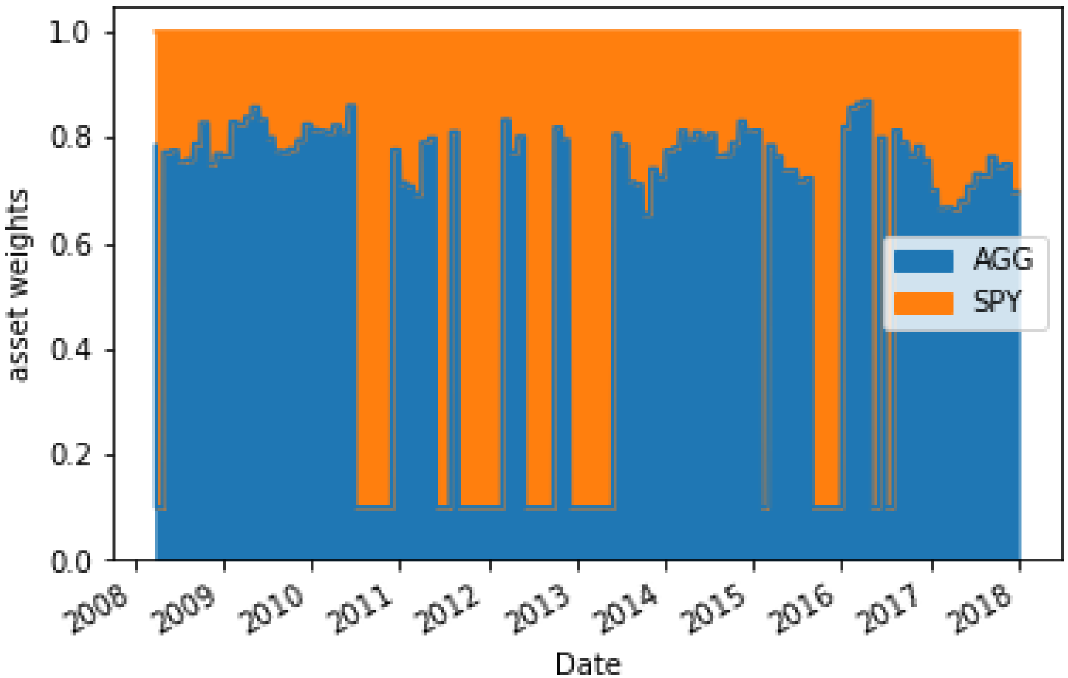
The risk contribution by each asset is

$$w_i \frac{(\boldsymbol{\Sigma w})_i}{\sigma_p} = [-0.00046,\ 0.0069]$$

Risk – Parity Strategy Example (MA) with min wts

- Adding the min_wts of 10% for each asset class helped to improve the Sharpe Ratio



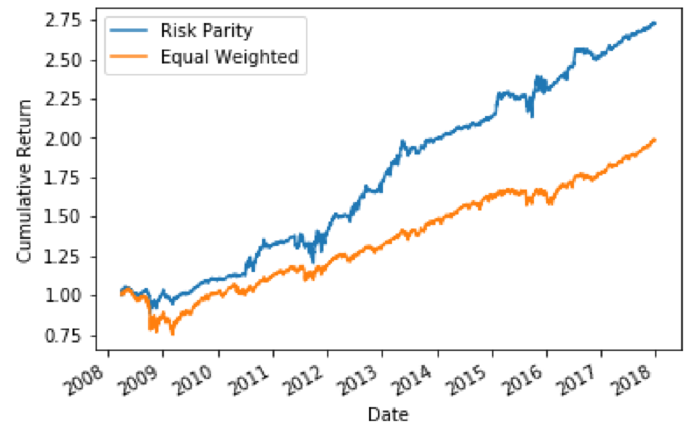


Sharpe ratio of strategies:

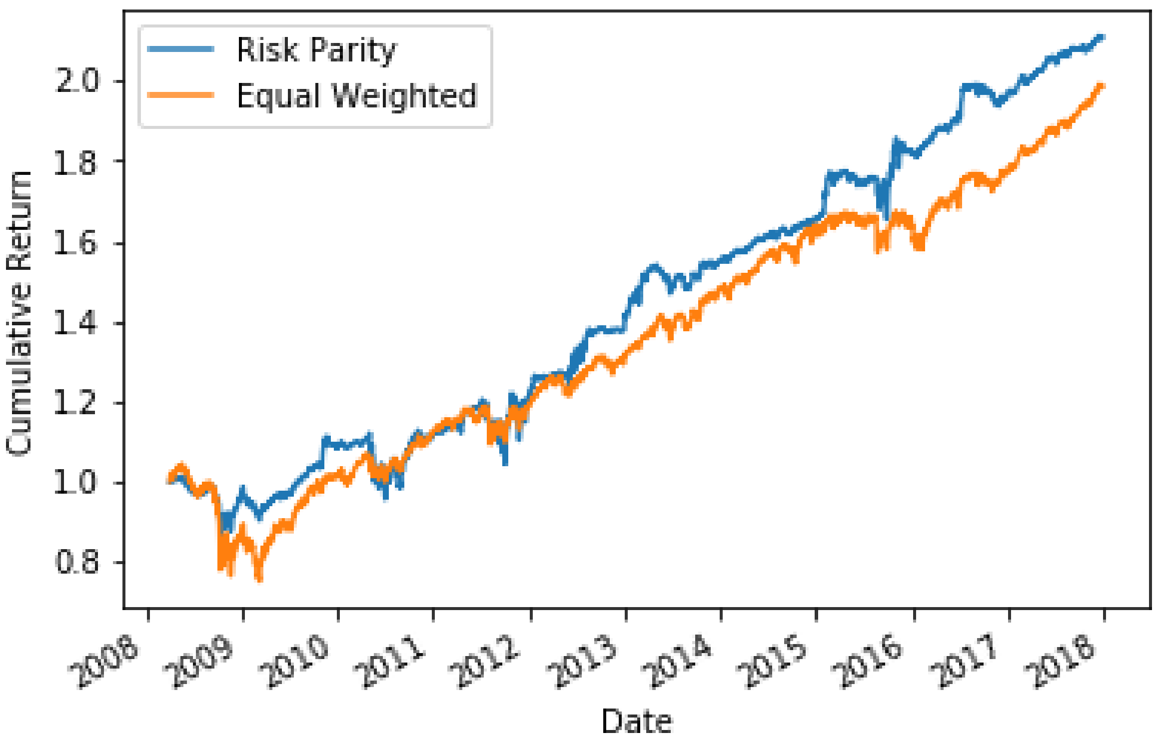
risk_parity w/o min wts	1.094
risk_parity with min wts	1.115

Risk – Parity Strategy Example (EWMA)

- Risk parity portfolio has both higher returns and lower volatility and downside risk
- Thus provide better risk-adjusted returns



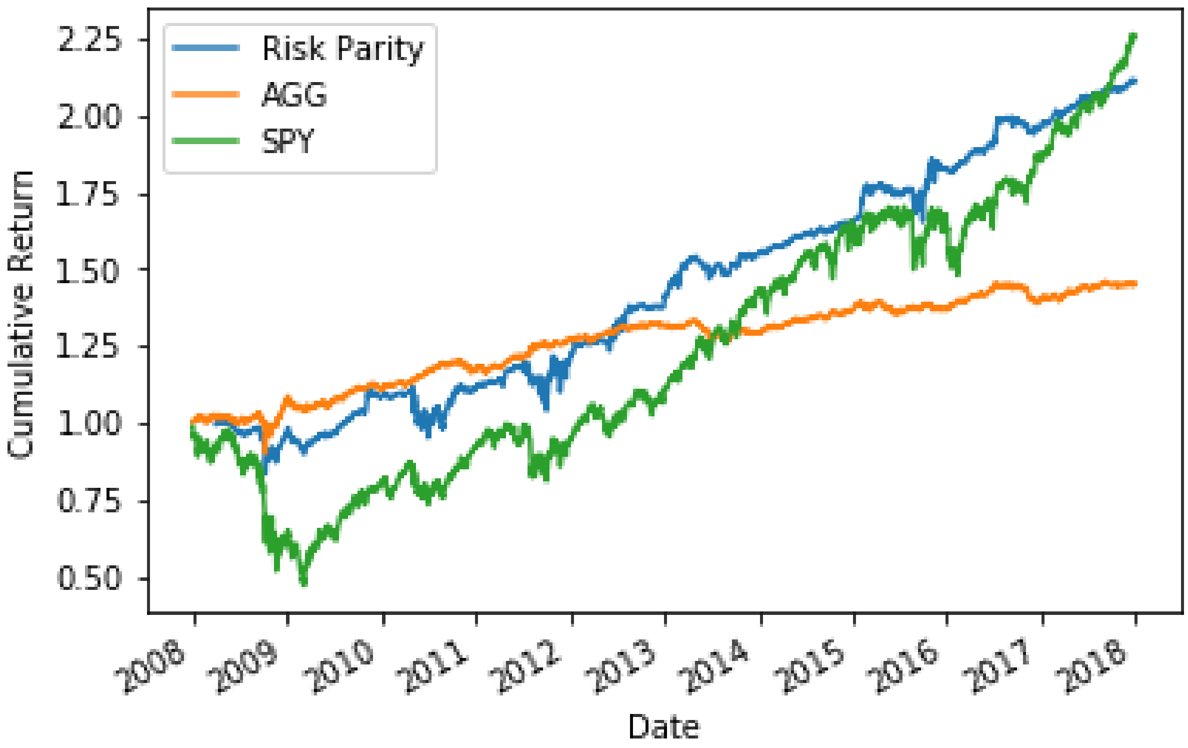
Sharpe ratio of strategies:
risk_parity 1.094
equal_wted 0.711



Sharpe ratio of strategies:
risk_parity 0.801
equal_wted 0.711

Risk – Parity Strategy Example (EWMA)

- Risk parity portfolio trying to harness the best of the two underlying assets
- Risk parity portfolio has higher Sharpe ratio than each underlying assets



Sharpe ratio of strategies:

risk_parity	0.801
AGG	0.734021
SPY	0.413695

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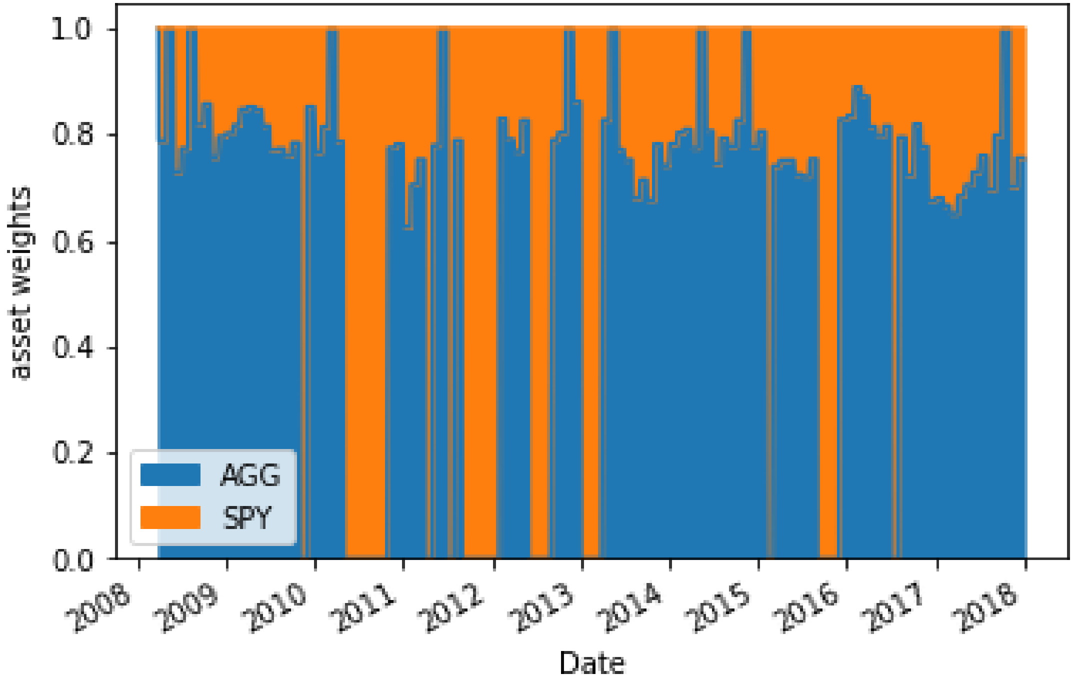
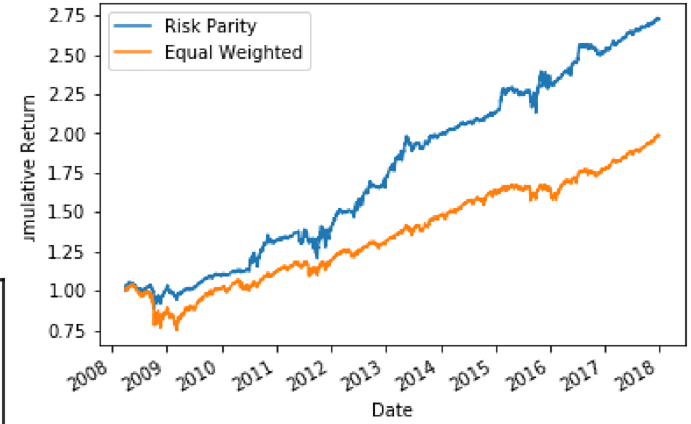
Risk – Parity Strategy Example (EWMA)

- Area plot of historical weights of the two assets

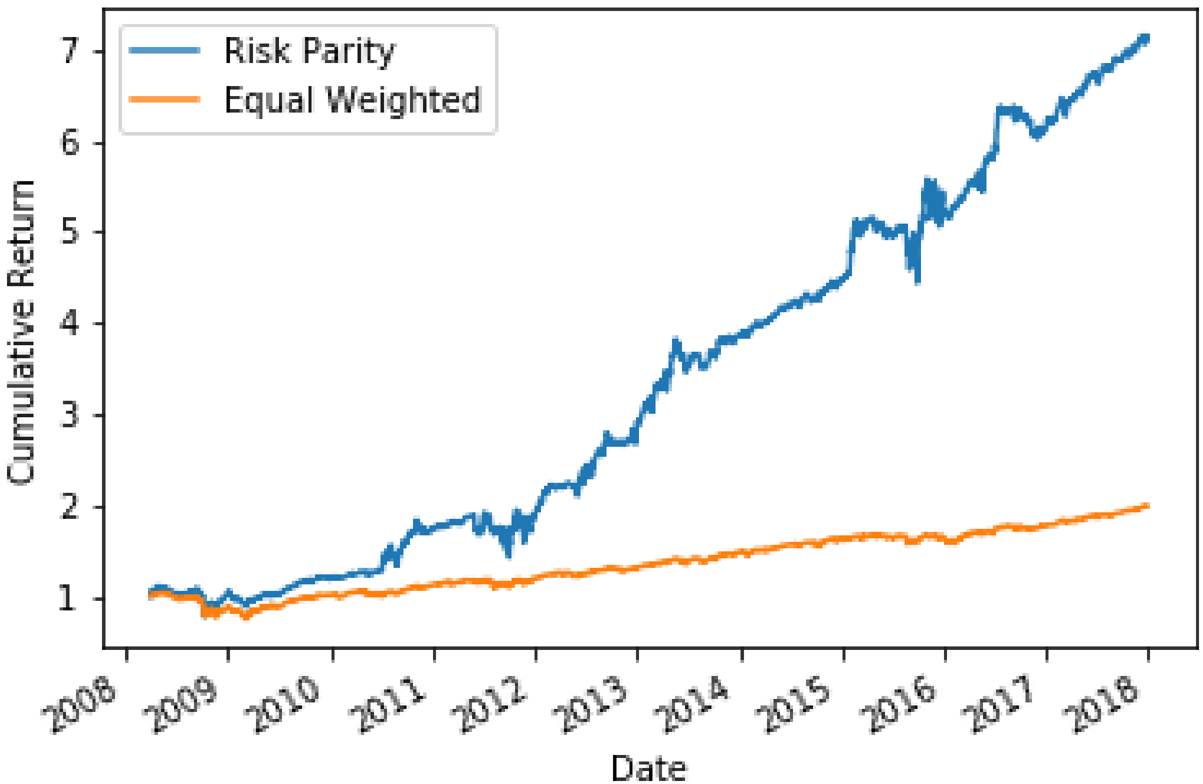


Risk – Parity Strategy Example (leverage)

- Leverage increases the total cumulative returns



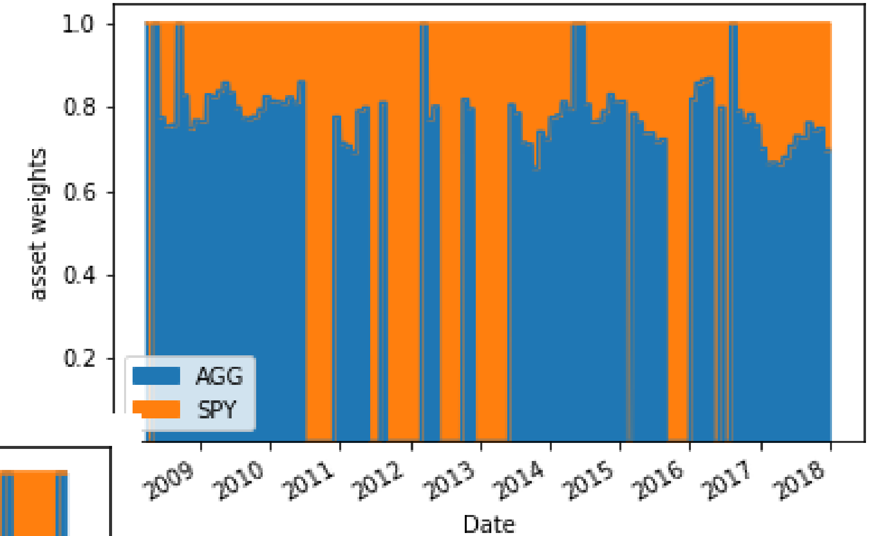
Sharpe ratio of strategies:
risk_parity 1.094
equal_wted 0.711

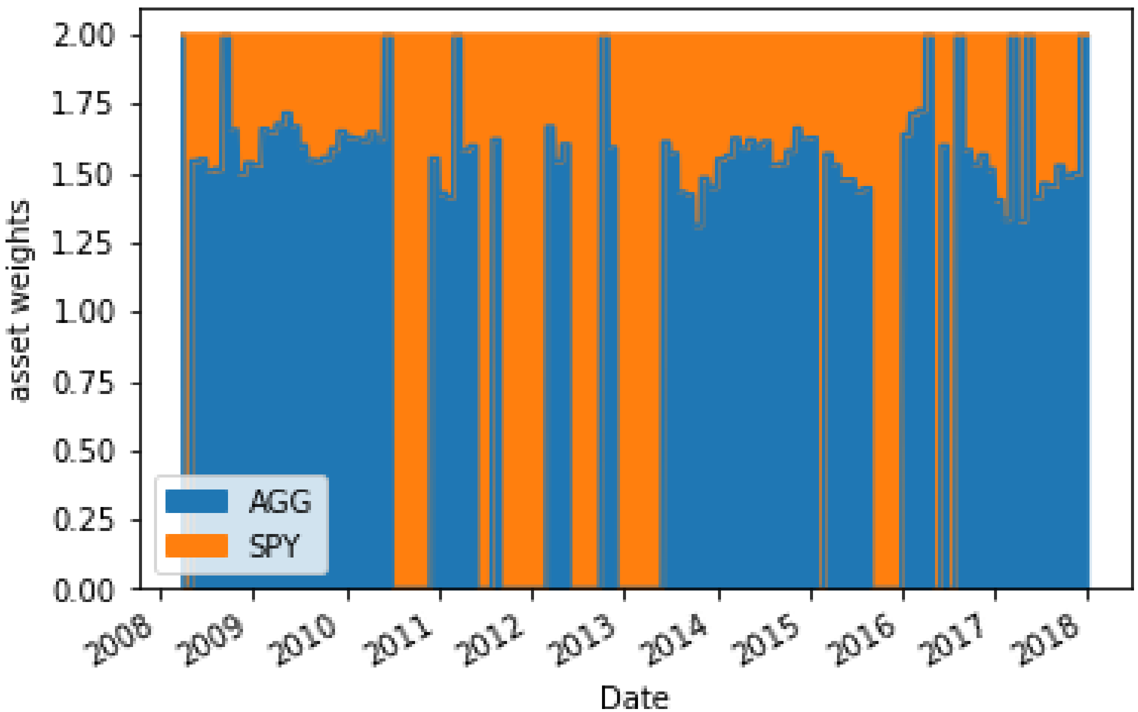


Sharpe ratio of strategies:
risk_parity 1.126
equal_wted 0.711

Risk – Parity Strategy Example (leverage)

- Area plot of historical weights of the two assets





Hierarchical Risk-Parity Strategies

Hierarchical Risk-Parity Strategies

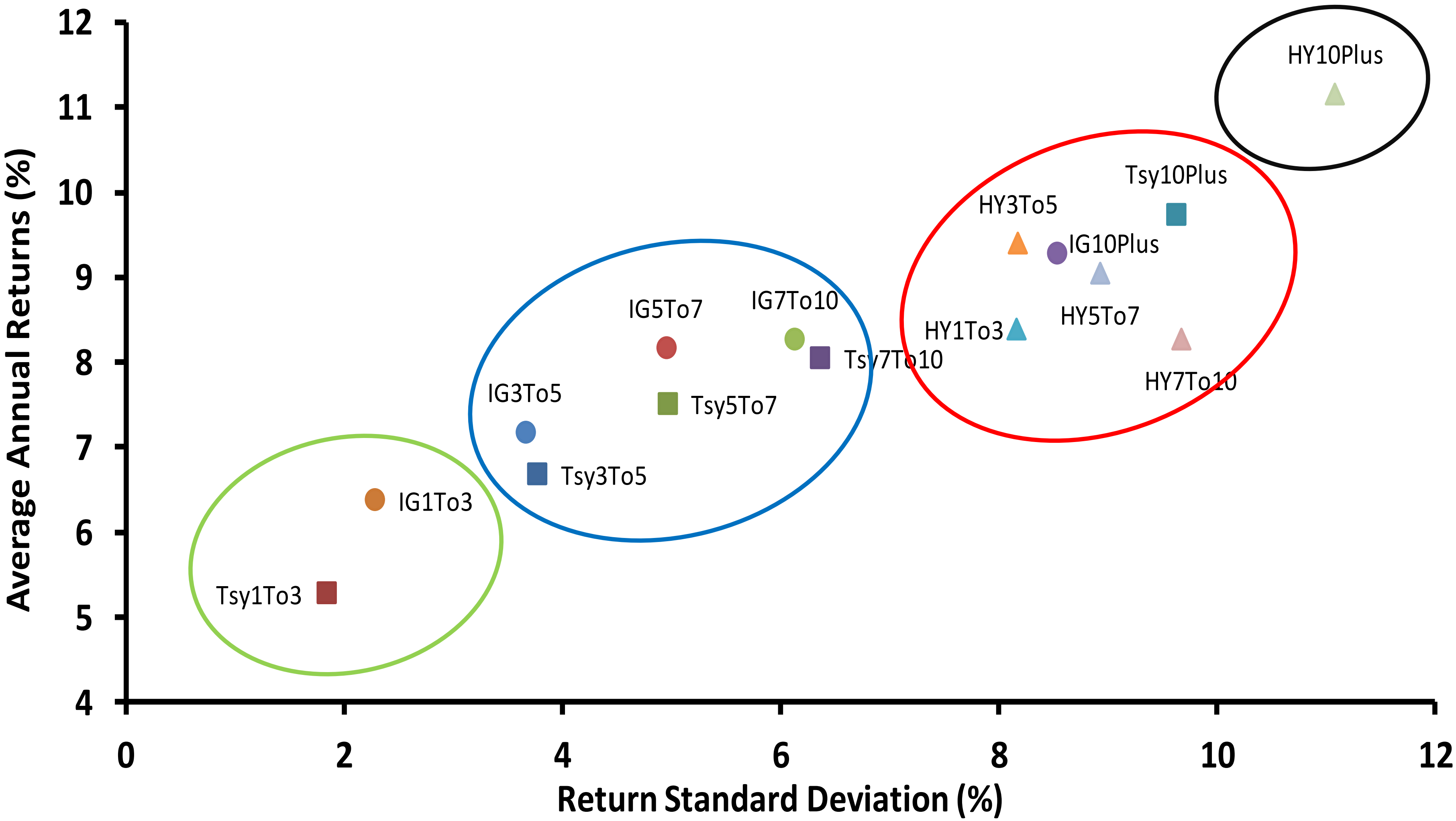
- Motivation behind Hierarchical Risk-Parity Strategies
- Example implementing Hierarchical Risk-Parity Strategies

Motivations behind Hierarchical Risk-Parity

- Asset allocation is very often done across many different types of assets that may form clusters. E.g., you are allocating to 10 different stock funds and 5 different bond funds
- Classical Risk-Parity may not work well under this construct because it favors the group with highest number of managers/funds (e.g., the 10 stock funds will end up having a higher total weights than the 5 bonds funds just because their numbers)
- Solution: create clustering groups then allocate risk equally/proportionally across the clustering groups first, then within the groups

Clustering of Assets

- Assets risk/return behavior exhibit clustering behavior



Hierarchical Clustering

- Hierarchical clustering is a type of unsupervised machine learning algorithm used to cluster unlabeled data points with similar characteristics.
- There are two types of hierarchical clustering: Agglomerative and Divisive.
- Agglomerative: data points are clustered using a bottom-up approach, starting with each individual data point in its own cluster
- Divisive: top-down approach where all the data points are treated as one big cluster to begin and the clustering process involves dividing the one big cluster into several small clusters

Clustering Algorithm

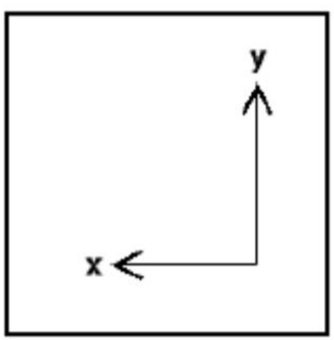
- Steps to Perform Hierarchical Clustering
  1. At the start, treat each data point as one cluster. Therefore, the number of clusters at the start will be K, while K is an integer representing the number of data points.
  2. Form a cluster by joining the two closest data points resulting in K-1 clusters. A distance measure is used to judge the closeness between objects
  3. Form more clusters by joining the two closest clusters resulting in K-2 clusters.
  4. Repeat the above three steps until one big cluster is formed.
  5. Once single cluster is formed, dendrograms are used to divide into multiple clusters depending upon the problem. We will study the concept of dendrogram in detail in an upcoming section.

Measurement of Distance Between Clusters

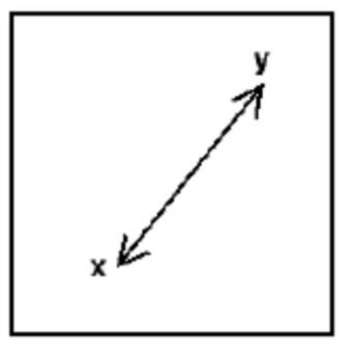
- The distance between clusters can be Euclidean or Manhattan distance.

$$d = \sum_{i=1}^{n} |x_i - y_i|$$

$$|x_1 - x_2| + |y_1 - y_2| \qquad (d) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Manhattan



Euclidean

Distance between assets using the correlation matrix

- To perform the hierarchical clustering of the assets, the following steps are needed
  1. Define the distance matrix based on the correlation matrix.
  2. Compute the linkage matrix for assets in the investment universe.
  3. Group the assets by the linkage (quasi-diagonalization of correlation matrix).
- As a result, the correlation or covariance matrix can be rearranged to be very close to a diagonal matrix.

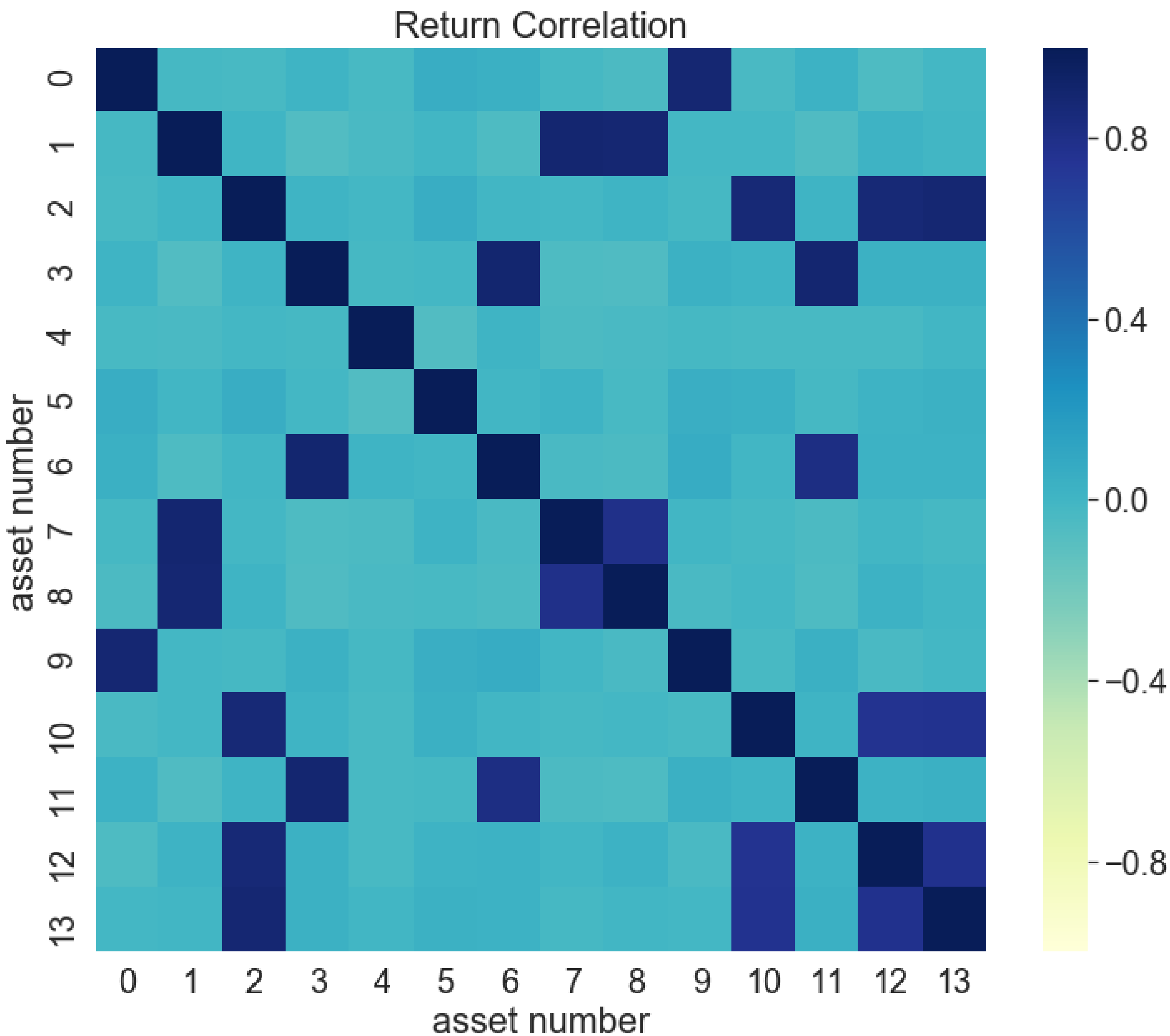
Distance between clusters using correlations

- The distance between clusters can be Euclidean or Manhattan.
- $Distance = \sqrt{\frac{1-Corr}{2}} = Dist = \begin{cases} 0\ when\ corr = 1, shortest\ distance \\ 1\ when\ corr = -1, longest\ distance \end{cases}$

distCorr - DataFrame

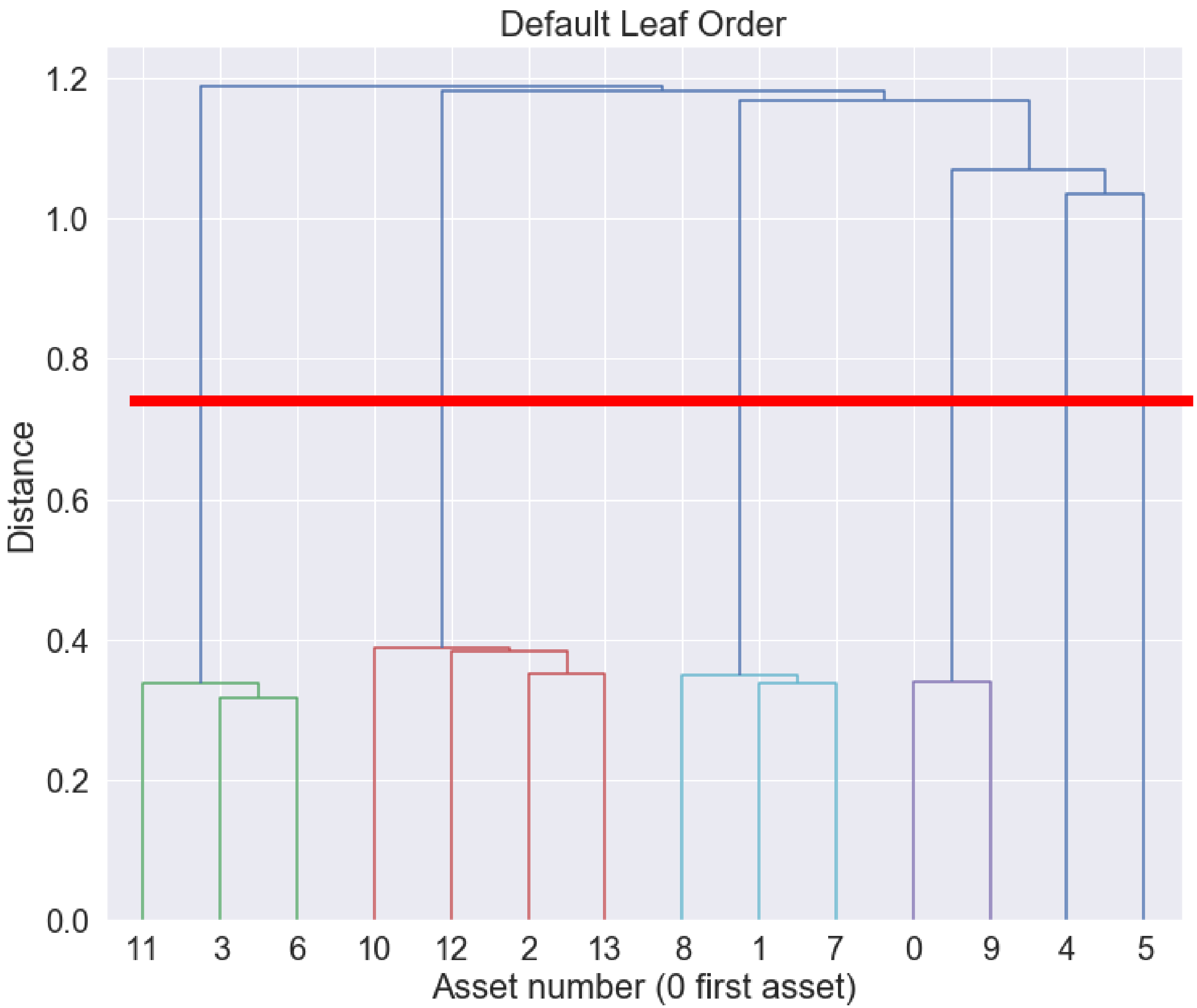
Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	0	0.713774	0.717578	0.702359	0.717101	0.684817	0.690761	0.712715	0.721526	0.240377	0.719964	0.697741	0.72414	0.710345
1	0.713774	0	0.706261	0.73059	0.720199	0.707997	0.725281	0.230982	0.238022	0.71187	0.711375	0.728301	0.700999	0.70808
2	0.717578	0.706261	0	0.703404	0.711209	0.686909	0.707185	0.710935	0.702406	0.712901	0.255896	0.702633	0.253574	0.234932
3	0.702359	0.73059	0.703404	0	0.714862	0.710196	0.21799	0.724639	0.726693	0.695747	0.703141	0.231091	0.693768	0.693962
4	0.717101	0.720199	0.711209	0.714862	0	0.7295	0.70352	0.722139	0.71887	0.715042	0.716699	0.715373	0.716959	0.708278
5	0.684817	0.707997	0.686909	0.710196	0.7295	0	0.708922	0.699804	0.716421	0.688024	0.690518	0.713	0.699952	0.693845
6	0.690761	0.725281	0.707185	0.21799	0.70352	0.708922	0	0.720028	0.720807	0.683196	0.708878	0.302708	0.697732	0.697804
7	0.712715	0.230982	0.710935	0.724639	0.722139	0.699804	0.720028	0	0.321615	0.709236	0.714181	0.722522	0.707726	0.714648
8	0.721526	0.238022	0.702406	0.726693	0.71887	0.716421	0.720807	0.321615	0	0.719721	0.711048	0.725945	0.696441	0.707792
9	0.240377	0.71187	0.712901	0.695747	0.715042	0.688024	0.683196	0.709236	0.719721	0	0.717266	0.690737	0.720196	0.71053
10	0.719964	0.711375	0.255896	0.703141	0.716699	0.690518	0.708878	0.714181	0.711048	0.717266	0	0.702449	0.346638	0.337181
11	0.697741	0.728301	0.702633	0.231091	0.715373	0.713	0.302708	0.722522	0.725945	0.690737	0.702449	0	0.696047	0.691967
12	0.72414	0.700999	0.253574	0.693768	0.716959	0.699952	0.697732	0.707726	0.696441	0.720196	0.346638	0.696047	0	0.334644
13	0.710345	0.70808	0.234932	0.693962	0.708278	0.693845	0.697804	0.714648	0.707792	0.71053	0.337181	0.691967	0.334644	0

Correlation matrix before applying clustering algorithm



Applying Hierarchical Clustering - *dendrogram*

- The vertical height of the dendrogram shows the Euclidean distances between points.
- Each horizontal line is a threshold that defines the minimum distance required to be a separate cluster.
- Draw a horizontal line that crosses some of the longest vertical lines – it helps to set the number of unique clusters - 6



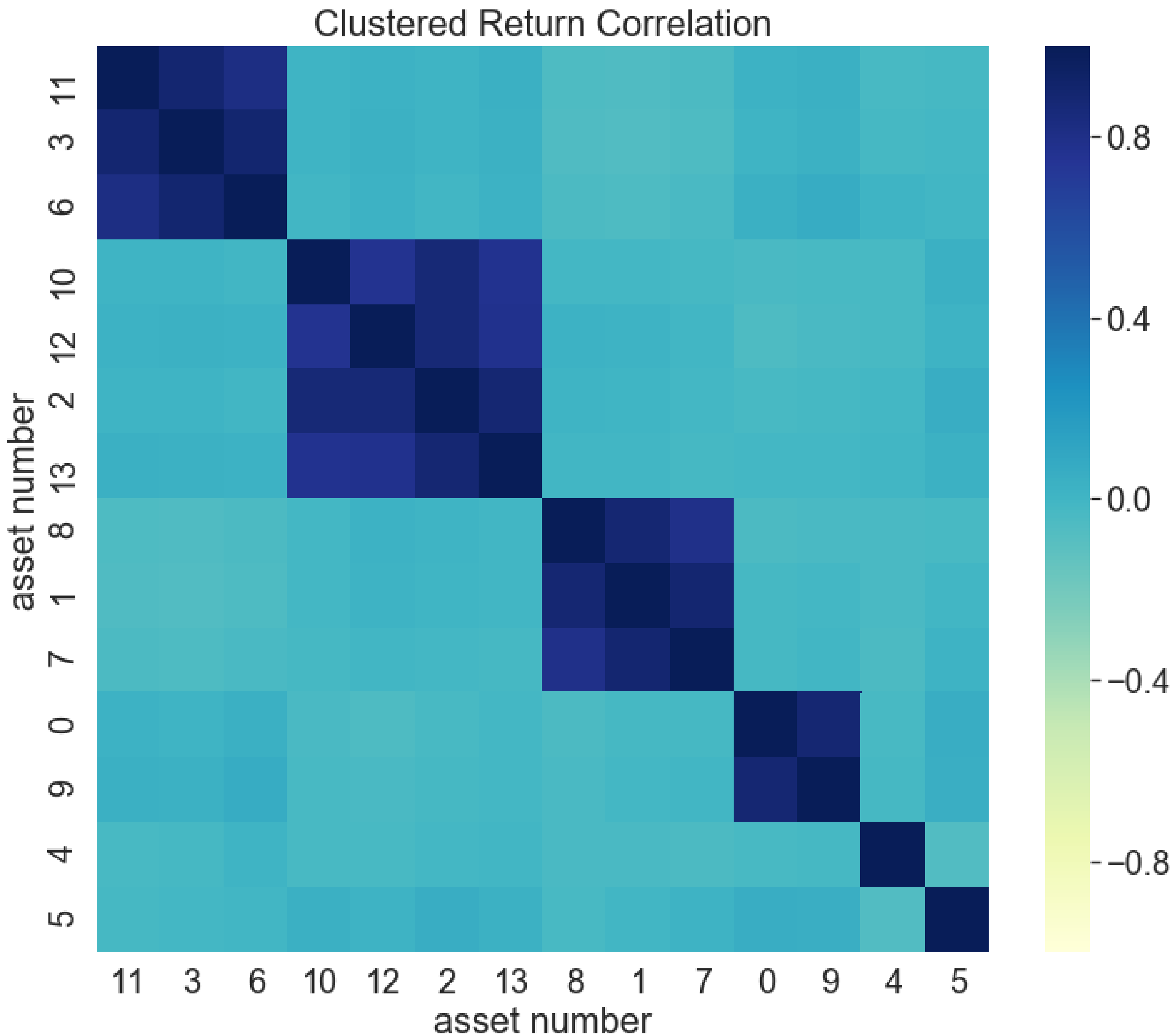
Python Code

Machine Learning technique: clustering

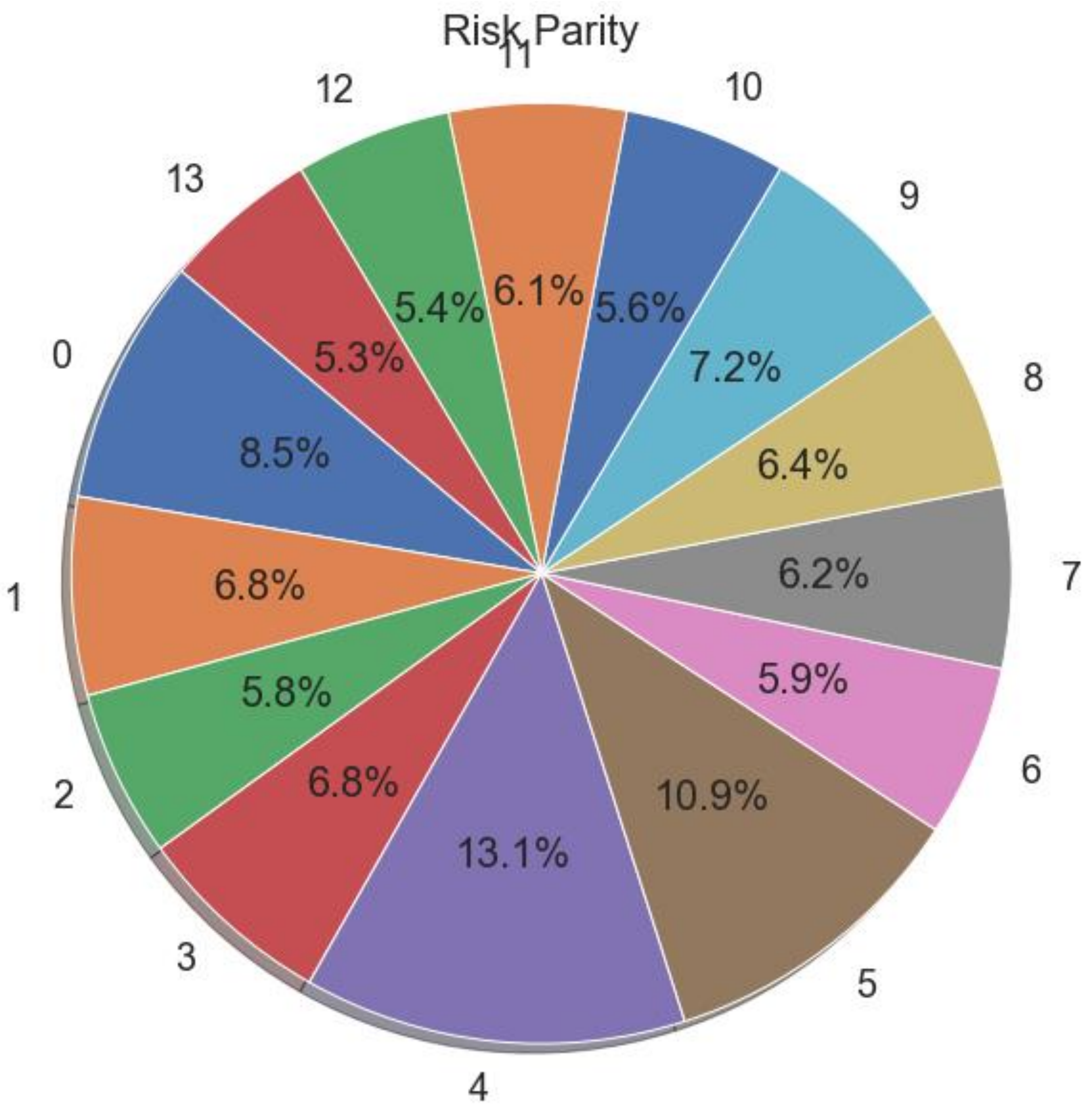
```
import scipy.cluster.hierarchy as shc
from sklearn.cluster import AgglomerativeClustering

link = shc.linkage(distCorr, method='single', metric='euclidean)
dend = shc.dendrogram(link, ax=ax)
cluster = AgglomerativeClustering(n_clusters=6, affinity='euclidean',
        linkage='single')
```

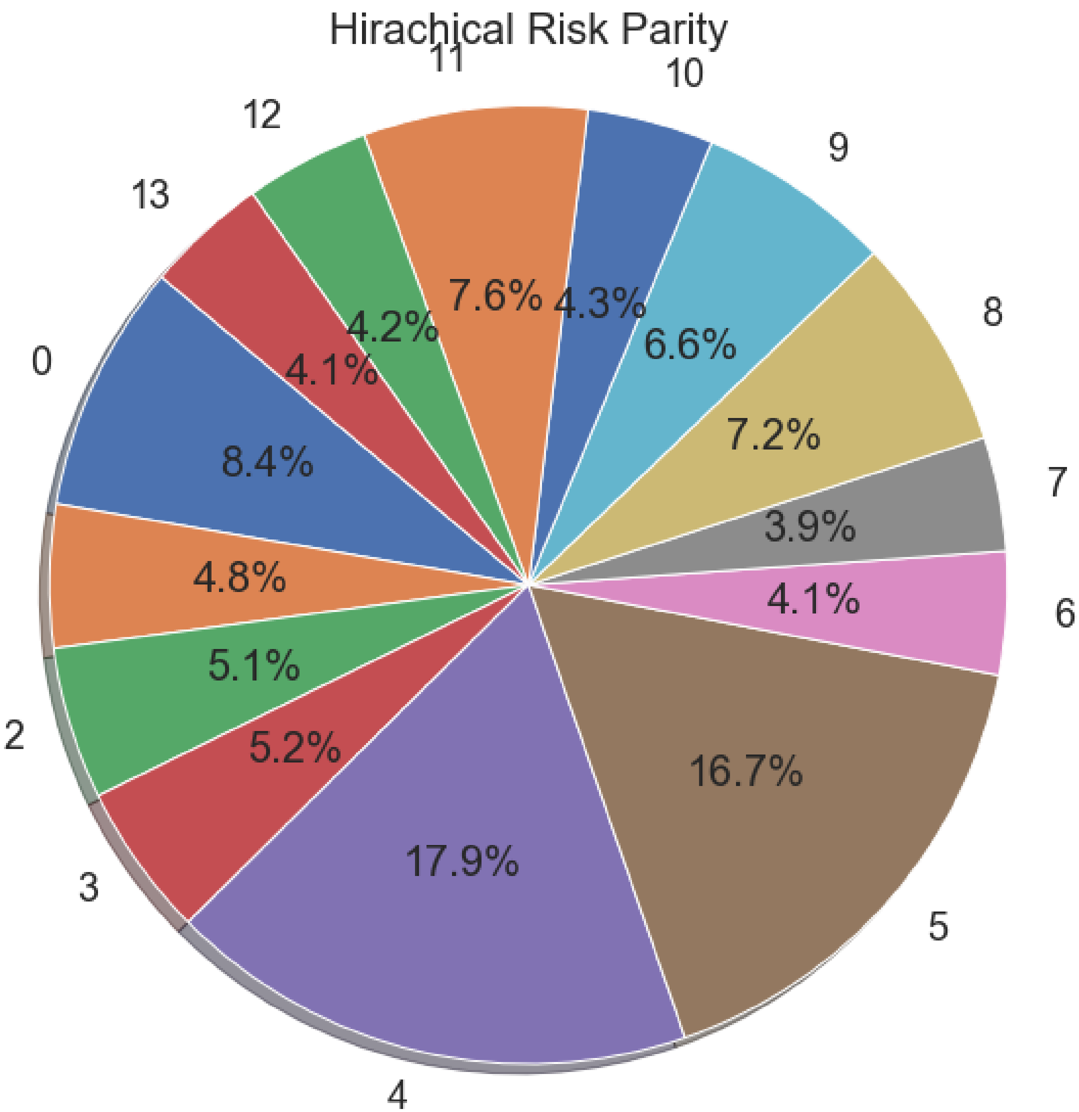
Correlation matrix after clustering



Asset allocation results – Risk Parity



Asset allocation results – Hierarchical Risk Parity



Observations

- The most significant changes to asset allocation, RP vs. HRP:
  - Asset 4 and 5's weights increased meaningfully under HRP. Reason: 4 and 5 not clustered with any other assets, favored more by HRP because it recognize their unique diversification benefits