# Computer Simulations and Risk Assessment – Lecture 11

**Fall 2019**

**Brandeis International Business School**

# Course Information - Schedule

| Class Date | Text Chapters |
|---|---|
| Aug. 30, 2019 – L1 | • Course Introduction/Python Installation<br>• Introduction to Quantitative Finance Career<br>• Python basics |
| Sep. 6, 2019 – L2 | • Advanced Python Topics |
| Sep. 13, 2019 – L3 | • Advanced Python Topics |
| Sep. 20, 2019 – L4 | • Sourcing and handling Data<br>• Stylized financial data analysis using Python |
| Sep. 27, 2019 – L5 | • Value at Risk |
| Oct. 4, 2019 – L6 | • Conditional Value at Risk (Expected Shortfall) + Mid-term Review |
| Oct. 11, 2019 | • Mid-term |
| Oct. 18, 2019 – L7 | • Modeling Volatility I |
| Oct. 25, 2019 – L8 | • Modeling Volatility II |
| Nov. 1, 2019 – L9 | • Practical application case Studies I |
| Nov. 8, 2019 – L10 | • Practical application case Studies II |
| Nov. 15, 2019 – L11 | • Back Testing + Conditional risk prediction |
| Nov. 22, 2019 – L12 | • Research project presentation |
| Dec. 6, 2019 – L13 | • Final Review |

# Lecture 11 – Outline

**Back-testing**

- Back-testing and examples

**Conditional Risk**

- How to set historical regime flags
- Predict next period risk using conditional risk metrics

**Copulas**

- Theoretical background and process
- Properties of different types of Copulas using an example

# Research Project

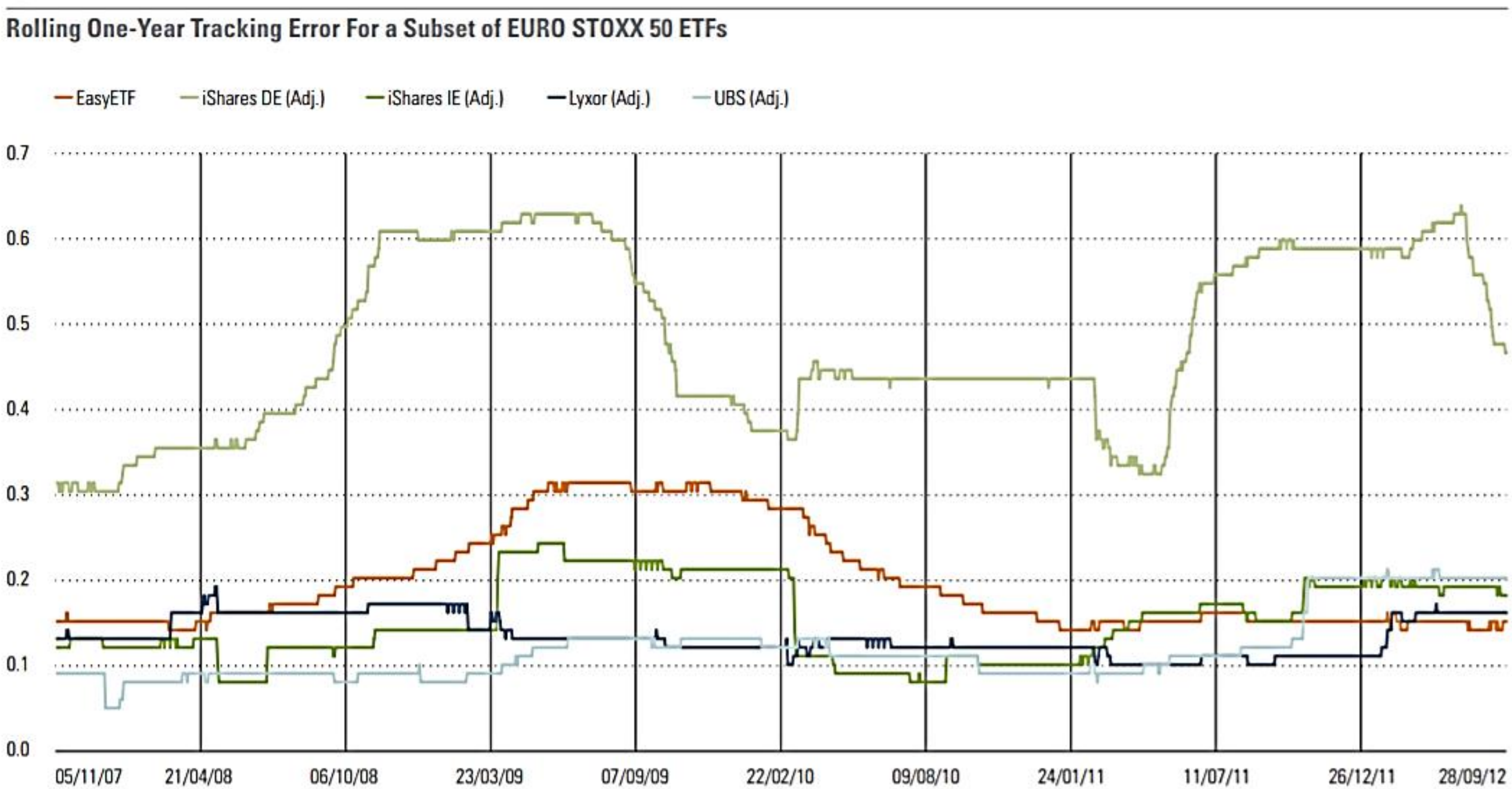
- Project 1: The objective is to create weights for the ETF so you can keep both in-sample and out-of-sample TE to a acceptable level
- Parameters to change to reduce out-of-sample TE:
  - the number of stock,
  - Type of model, e.g., MA vs. EWMA
  - Change the training/test data set length somewhat (the key is the distribution of returns between the split is similar)
- Dynamic optimization: changing weights dynamically by incorporating latest return data

# Research Project

- Presentation should discuss:
  - Index/benchmark/asset chosen and thinking behind
  - Results obtained
  - What interesting things we can learn from your research?
  - What interesting properties we can learn from the assets you picked?
  - Try to use charts to illustrate whenever possible!
  - About 10 minutes per team so not too many slides

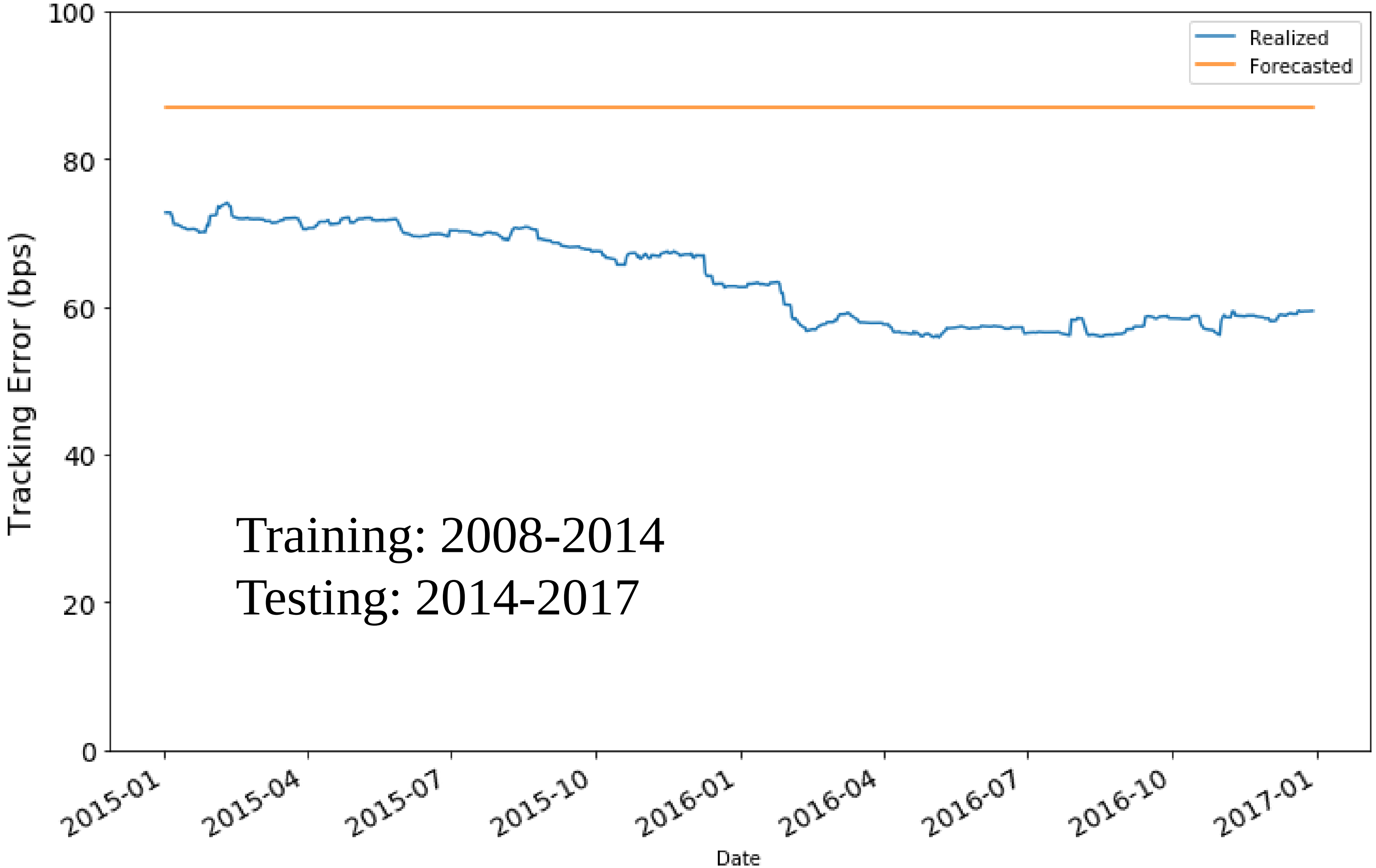
- Data submission should be clean! Make sure other users can run your code

# Research project: Tracking Error




Source: MorningStar

# Tracking Error Comprison – DJI30 Example



**Back Testing**

- **Back Testing basics**
- **Market risk testing per regulation**
- **Examples**

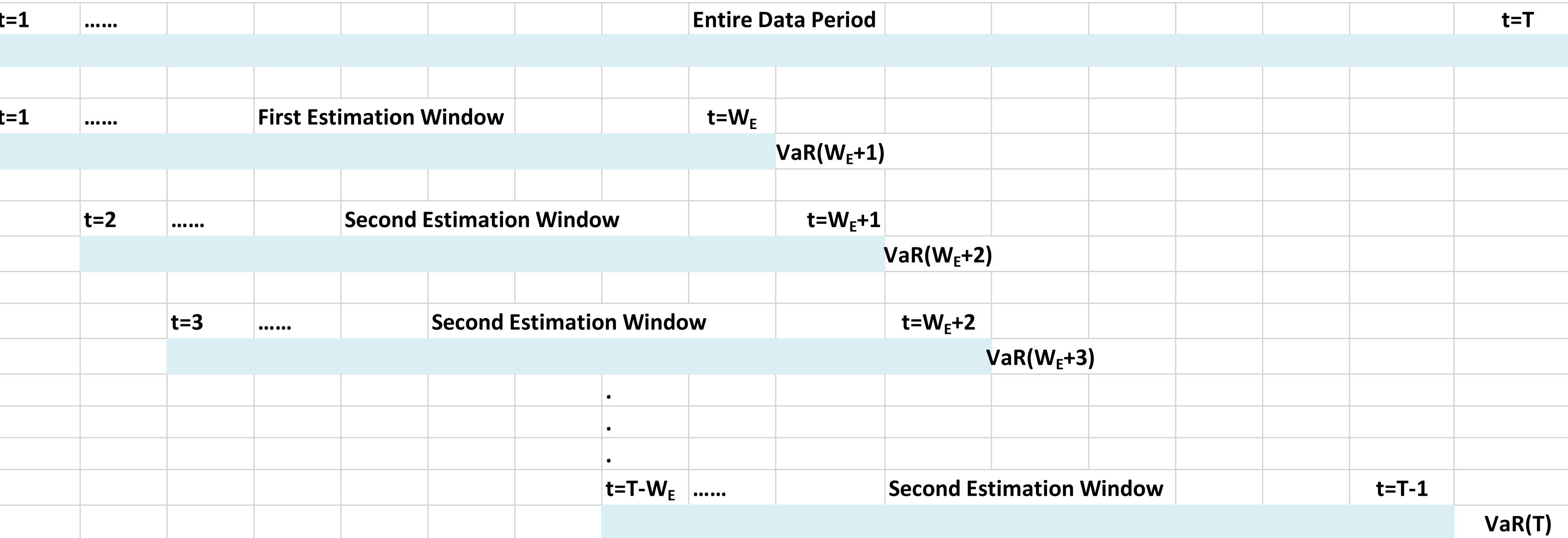
# Back Testing Basics

- Back Testing is a procedure that compares ex-ante calculated risk (e.g., Volatility, VaR or ES) with ex-post (probability of) realized risks
- Good idea to test any risk metrics/models. Useful in identifying the weaknesses of risk-forecasting models and providing ideas for improvement
- Regulation required (Basel II and III)

# Back Testing Basics

- Estimation window and testing window (Training vs. Testing data sets in Machine Learning speak)
  - Estimation window: the time period data from which is used to forecast risk
  - Testing window: the time period data from which is used to test the forecasted risk
- Forecast the next period's risk using data in the estimation window
- Estimation window length: long enough to have 10 violations for reliable statistical testing

# Back Testing Basics




**Estimation window in Dates**

| Start | End | VaR forecast (date) |
| --- | --- | --- |
| 1/1/1999 | 12/31/2000 | VaR(1/1/2001) |
| 1/2/1999 | 1/1/2001 | VaR(1/2/2001) |
| . | . | . |
| . | . | . |
| . | . | . |
| 12/31/2007 | 12/30/2009 | VaR(12/31/2009) |

# Back Testing Basics

**Definition** **(VaR violation)** *An event such that:*

$$\eta_t = \begin{cases} 1 & \text{if } y_t \le -VaR_t \\ 0 & \text{if } y_t > -VaR_t. \end{cases}$$

*$v_1$ is the count of $\eta_t = 1$ and $v_0$ is the count of $\eta_t = 0$, which are simply obtained by:*

$$v_1 = \sum \eta_t$$

$$v_0 = W_T - v_1.$$

**Definition** **(Violation ratio)** *The violation ratio is:*

$$VR = \frac{\text{Observed number of violations}}{\text{Expected number of violations}} = \frac{v_1}{p \times W_T}.$$

where p is the VaR threshold and $W_T$ is the size of the test sample

- Rule of thumb for using violation ratio
  - If VR is between [0.8, 1.2] then consider the risk model is precise
  - If VR<0.5 or VR>1.5 then not precise

# Back Testing Basics

**Estimation window in Dates**

| **t** | **$t+W_E-1$** | **VaR forecast (day)** | **Realized Return<VaR($t+W_E-1$)?** |
|---|---|---|---|
| 1 | 500 | VaR(501) | 1 |
| 2 | 501 | VaR(502) | 0 |
| . | . | . | . |
| . | . | . | . |
| . | . | . | . |
| 1999 | 2499 | VaR(2500) | 0 |

| | |
|---|---|
| Number of observed violations | sum(total number of 1) |
| Expected violations | $pxW_T$ |
| Violation Ratio | sum(total number of 1)/($pxW_T$) |

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# Regulation and Back Testing

- Regulation required (Basel II and III) VaR testing
- Example
  - $Capital_t = C_t = A_t \max(VaR_t(1\%), \overline{VaR_t}(1\%)) + \text{Constant}$

  where $\overline{VaR_t}(1\%)$ - Average VaR over previous 60 days

$$A_t \text{ is the scaling factor} = \begin{cases} 3 & \text{if } v_1 \le 4 \text{ or green} \\ 3 + 0.2 * (v_1 - 4) & \text{if } 5 \le v_1 \le 9 \text{ or Yellow} \\ 4 & \text{if } 10 \le v_1 \text{ or Red} \end{cases}$$

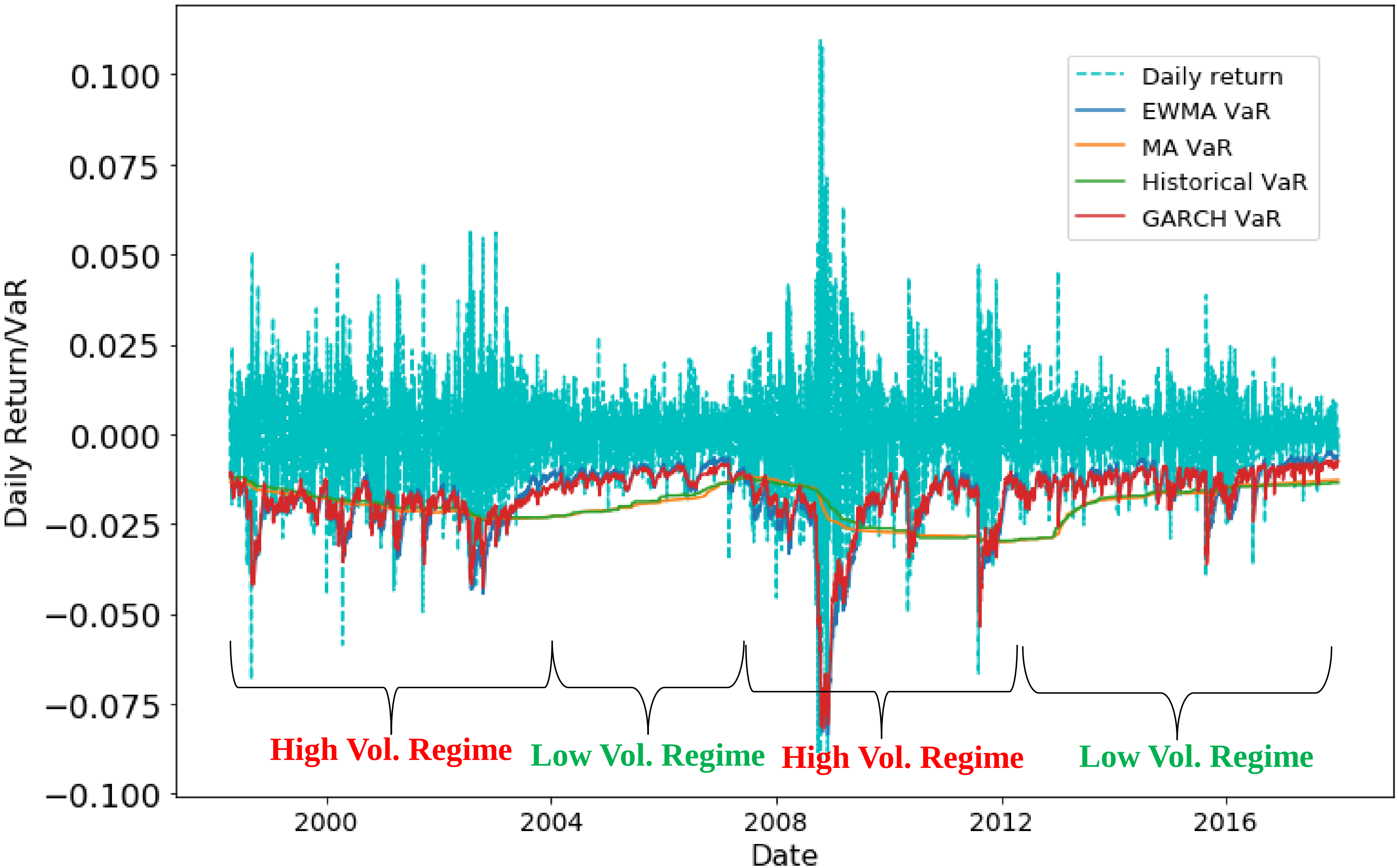
  - Define $v_t$= # of VaR(1%) violations over the past 250 days
  - How this works?
    - If volatility goes up, $v_t$ goes up, then need to reduce risk and possibly raise cash

# Example: S&P500 Risk Forecast

- VR ratio test failed for two methods for the 2/11/1994-12/31/2017 Period
- MA has the lowest Violation ratio, EWMA and GARCH very similar to each other with ratios slightly above the threshold of 1.2 (acceptable), while Historical method VR is fully within the acceptable range
- But is violation ratio the only thing we should focus on?

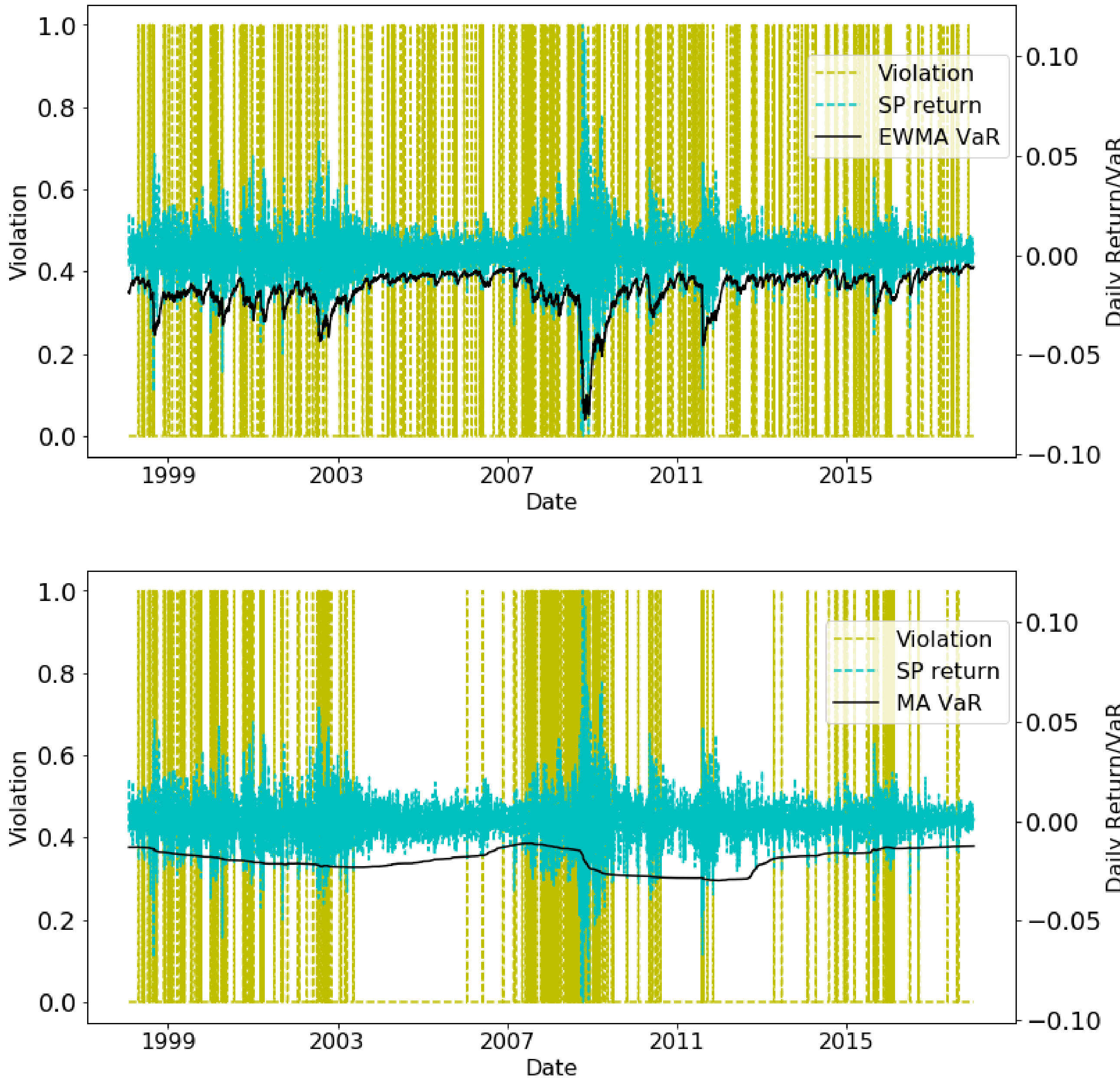
| | Violation Ratio | Std of VaR |
|---|---|---|
| EWMA | 1.220826 | 0.010556 |
| MA | 1.057251 | 0.005723 |
| Historical | 1.121085 | 0.005444 |
| GARCH | 1.248753 | 0.009449 |

# Example: S&P500 Risk Forecast



# S&P500 Risk Forecast – EWMA vs. MA VaR

- MA method produces a lot of violations during the financial crisis – Not good
- EWMA produces less violations during financial crisis but more during 'normal' market condition
- We want less violations during crisis

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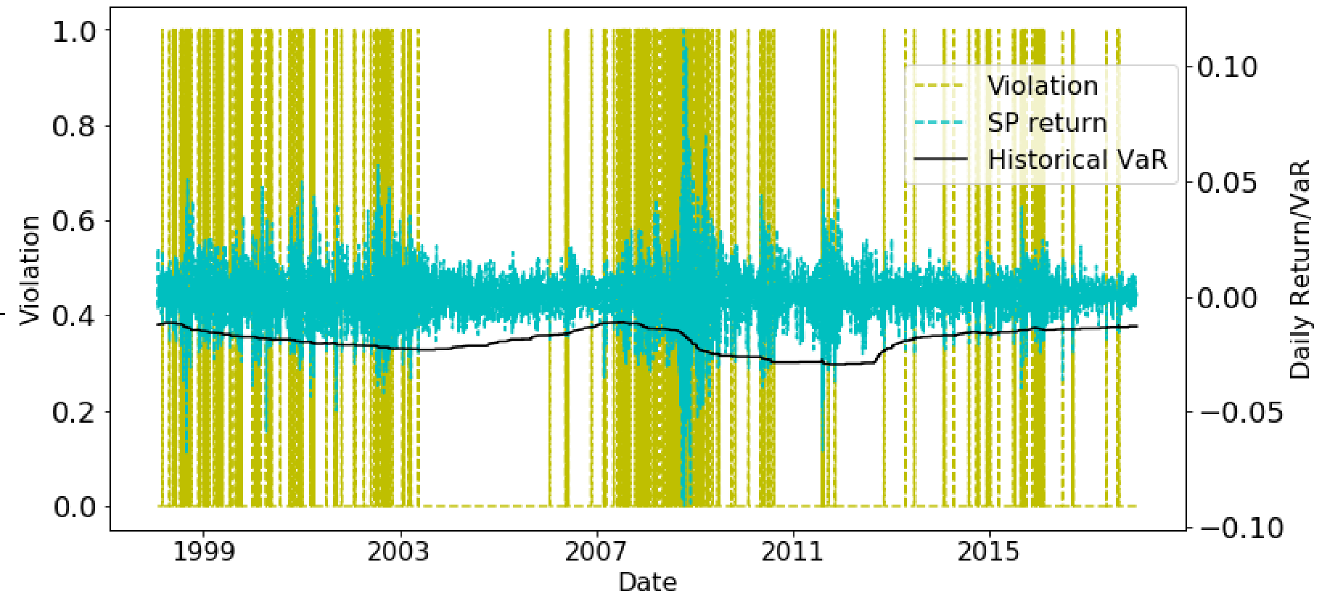
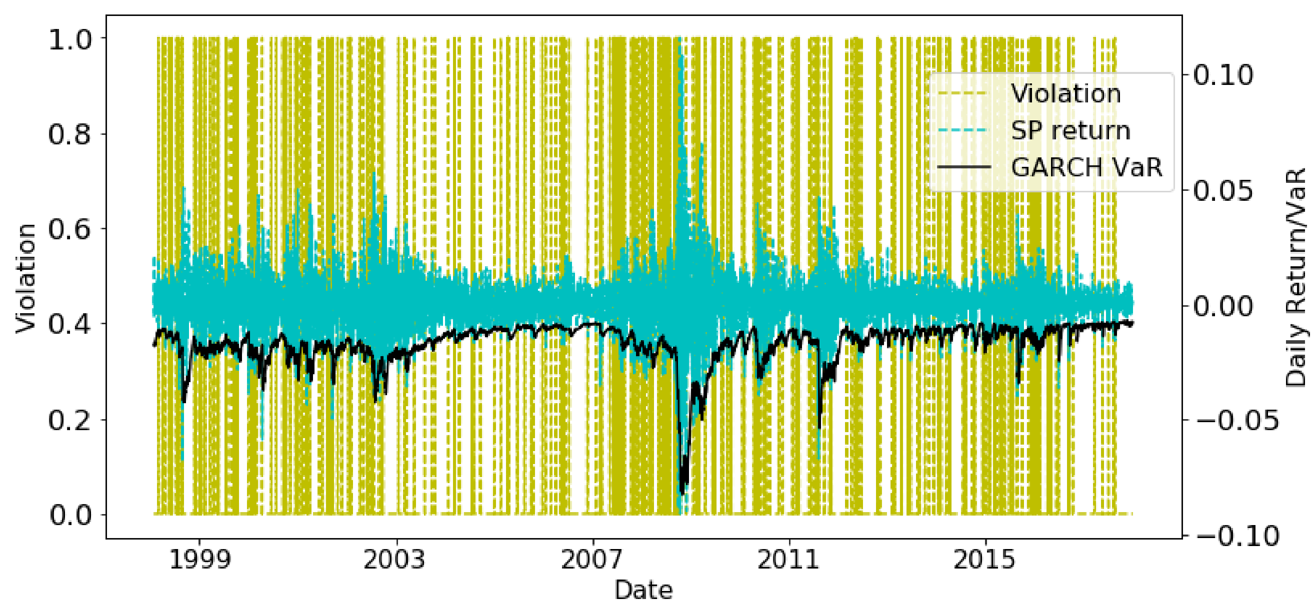
# S&P500 Risk Forecast – Historical vs. GARCH VaR

- Historical method produces a lot of violations during the financial crisis – Not good
- GARCH produces less violations during financial crisis but more during 'normal' market condition
- We want less violations during crisis

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**Conditional Risk**

- **Simple way to identify risk regimes and create conditional flags**
- **Conditional covariance**

# From Lecture 3: Conditional Analytics

- Conditional expected returns for Risk-on and Risk-off regimes:

| *Mean Monthly Regime Returns* | | | | | |
|---|---|---|---|---|---|
| | **Mkt-RF** | **SMB** | **HML** | **RF** | **MoM** |
| **RiskOn** | 1.46 | 0.18 | 0.18 | 0.23 | 0.34 |
| **RiskOff** | -2.19 | 0.05 | 0.25 | 0.22 | 1.28 |

- Conditional covariance matrix for Risk-on and Risk-off regimes:

| *RiskOn Monthly Covariance Matrix* | | | | | |
|---|---|---|---|---|---|
| | **Mkt-RF** | **SMB** | **HML** | **RF** | **MoM** |
| **Mkt-RF** | 12.15 | 2.08 | -2.02 | -0.02 | -2.44 |
| **SMB** | 2.08 | 10.82 | -2.81 | -0.03 | 2.16 |
| **HML** | -2.02 | -2.81 | 7.22 | -0.05 | -4.82 |
| **RF** | -0.02 | -0.03 | -0.05 | 0.04 | 0.12 |
| **MoM** | -2.44 | 2.16 | -4.82 | 0.12 | 20.44 |

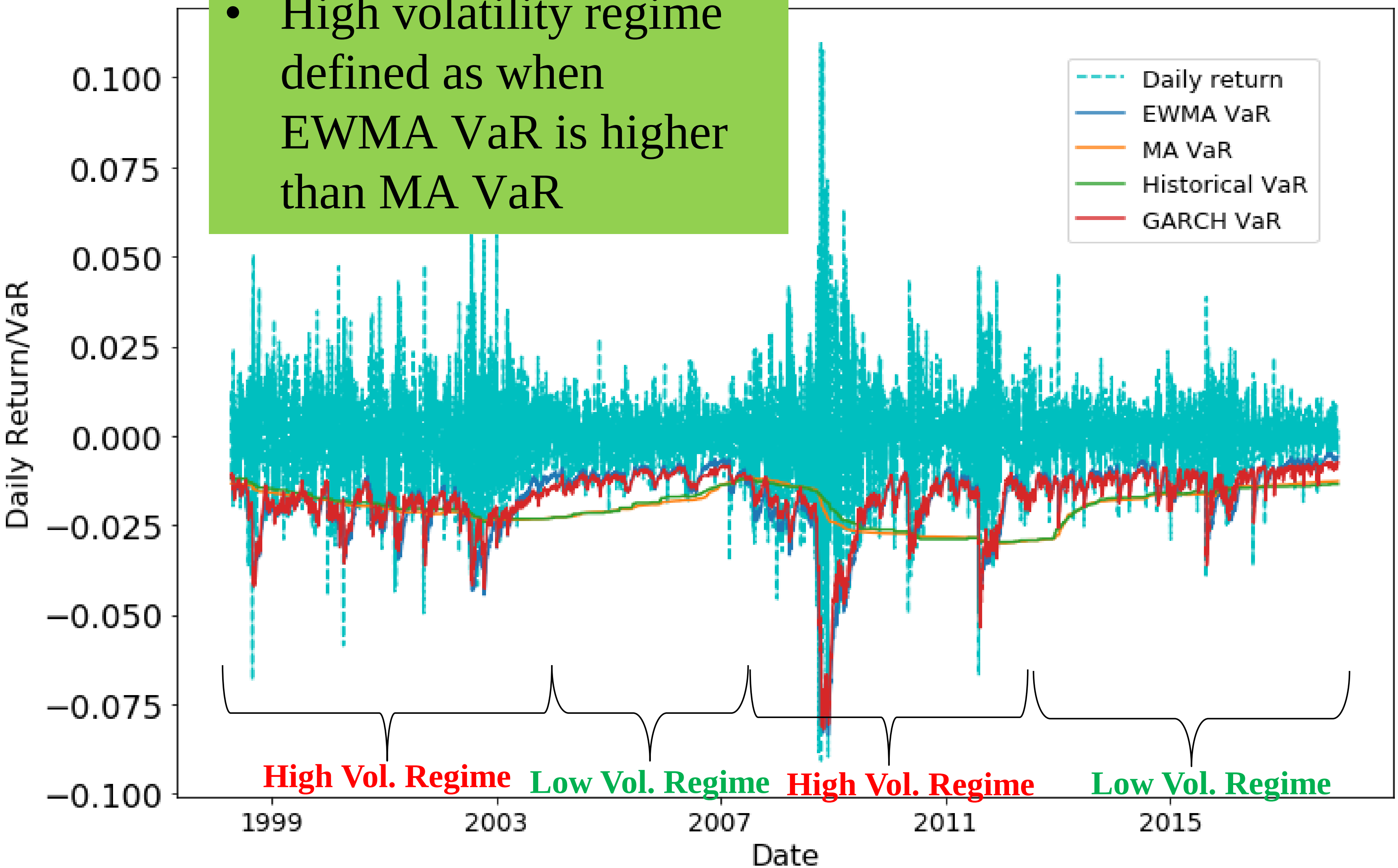
| *RiskOff Monthly Covariance Matrix* | | | | | |
|---|---|---|---|---|---|
| | **Mkt-RF** | **SMB** | **HML** | **RF** | **MoM** |
| **Mkt-RF** | 27.53 | 5.55 | -2.39 | -0.10 | -12.31 |
| **SMB** | 5.55 | 8.79 | -1.76 | -0.10 | -4.29 |
| **HML** | -2.39 | -1.76 | 15.52 | 0.26 | 4.93 |
| **RF** | -0.10 | -0.10 | 0.26 | 0.04 | 0.01 |
| **MoM** | -12.31 | -4.29 | 4.93 | 0.01 | 30.71 |

# How Does Conditional Risk Work?

- Two steps
  1. Develop methodology to flag historical time periods, which allows the calculation of risk (e.g. covariance matrix) conditional on the flag being one way or the other
  2. Forecast the flag for the coming time period. Then use the flag and the corresponding conditional risk from step 1 (e.g., High vol. regime) as forecast for risk

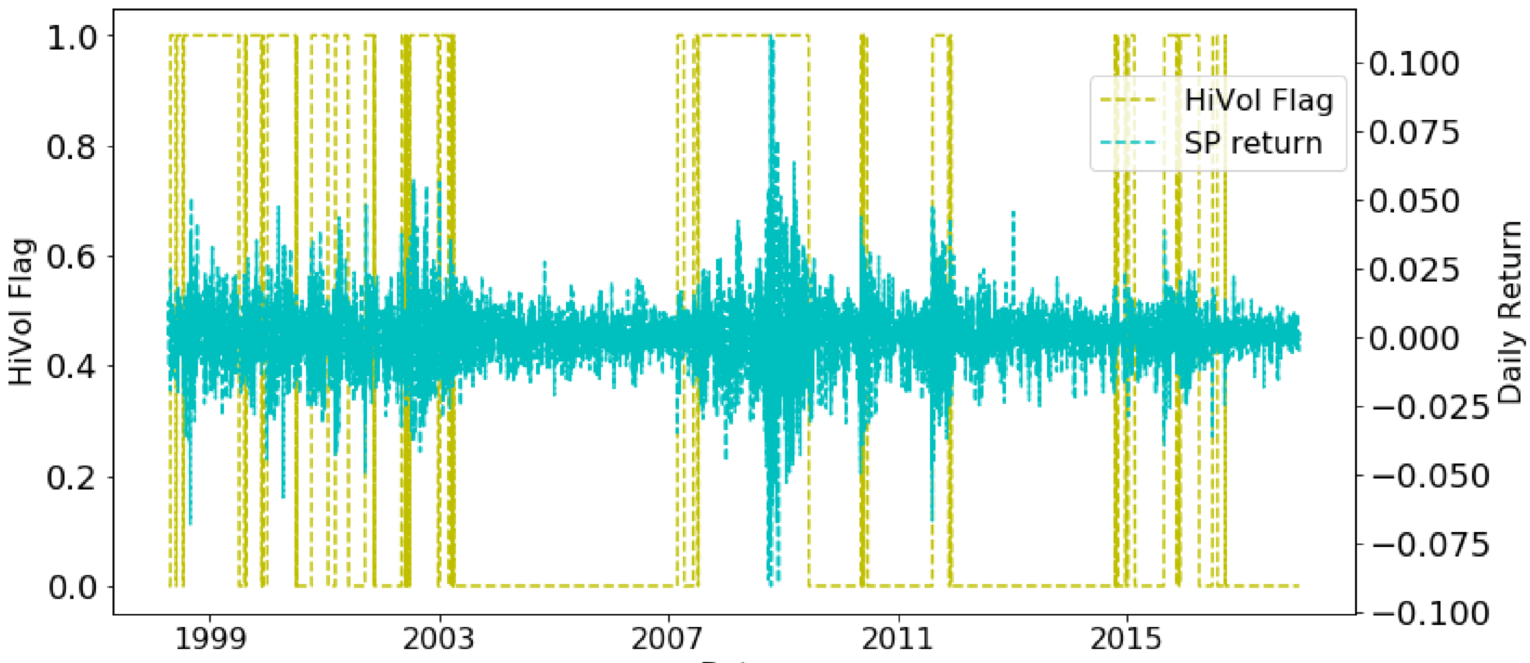
# Step 1: How to create historical flags

- High volatility regime defined as when EWMA VaR is higher than MA VaR



# How Does Conditional Risk Work?

- An example with two simple steps
  1. Develop methodology to flag historical time periods, which allows the calculation of risk (e.g. covariance matrix) conditional on the flag being one way or the other
  2. Forecast the flag for the coming time period. Then use the flag and the corresponding conditional risk from step 1 (e.g., High vol. regime) as forecast for risk

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# Step 2: Conditional Analytics

- Forecast next period's risk regime flag:
- Conditional covariance matrix for low vol. regime is hence being used to forecast next time period volatility:

cov_ret_LowVol - DataFrame

| Index | SP500 | Nikkei |
|---|---|---|
| SP500 | 7.54302e-05 | 2.30101e-05 |
| Nikkei | 2.30101e-05 | 0.000167843 |

**Copula**

- **Theoretical background and process**
- **Properties of different types of Copulas using an example**

# Copulas

- Copulas provide the means to create a multivariate distribution with certain type of dependence
- The central idea behind copulas is to separate out the distribution of individual assets (marginal distribution) from the joint-distribution (copula) that links them together
- For example, each assets can have a normally distributed returns, but the joint-distribution is Clayton so it produces higher correlation in down markets

# The Theory of Copulas

- Suppose X and Y are two random variables representing the returns of two different assets

$$X \sim f;\ Y \sim g$$

Where f and g are the marginal distributions

- Together the joint distribution and the marginal distributions are represented by the joint density

$$(X, Y) \sim h$$

- The idea behind copulas is that we focus separately on marginal distributions *(f, g)* and the function that combined them together into the joint distribution, *h*, *which is the copula*

# The process to create Copulas

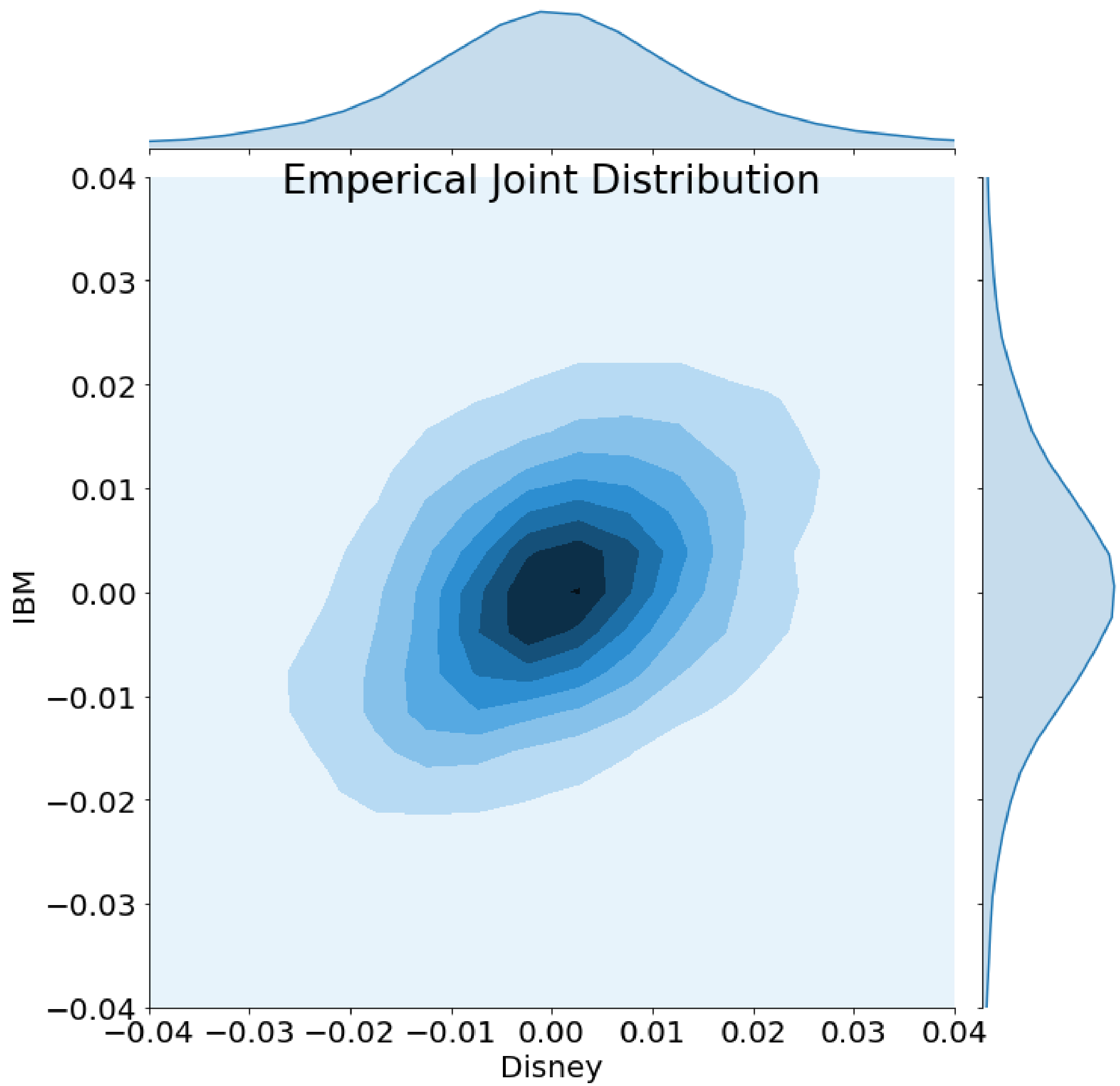
1. Create joint distribution function *h*, through either what's naturally embedded in return data, or by using random number generators
2. Create uniform distributions for the each of the asset returns
$$U=F(X);\ V=G(Y)$$
where F and G are the cdf functions of returns X and Y
3. Transform the uniform distributions into marginal distribution for each of the assets by applying the inverse cdf function. E.g., inverse student-t function creates new X, Y as t distributions
$$X=t^{-1}(U),\ Y=t^{-1}(V)$$

# An Example

- Two stocks: IBS and Disney
- *Data between 1986 and 2017*

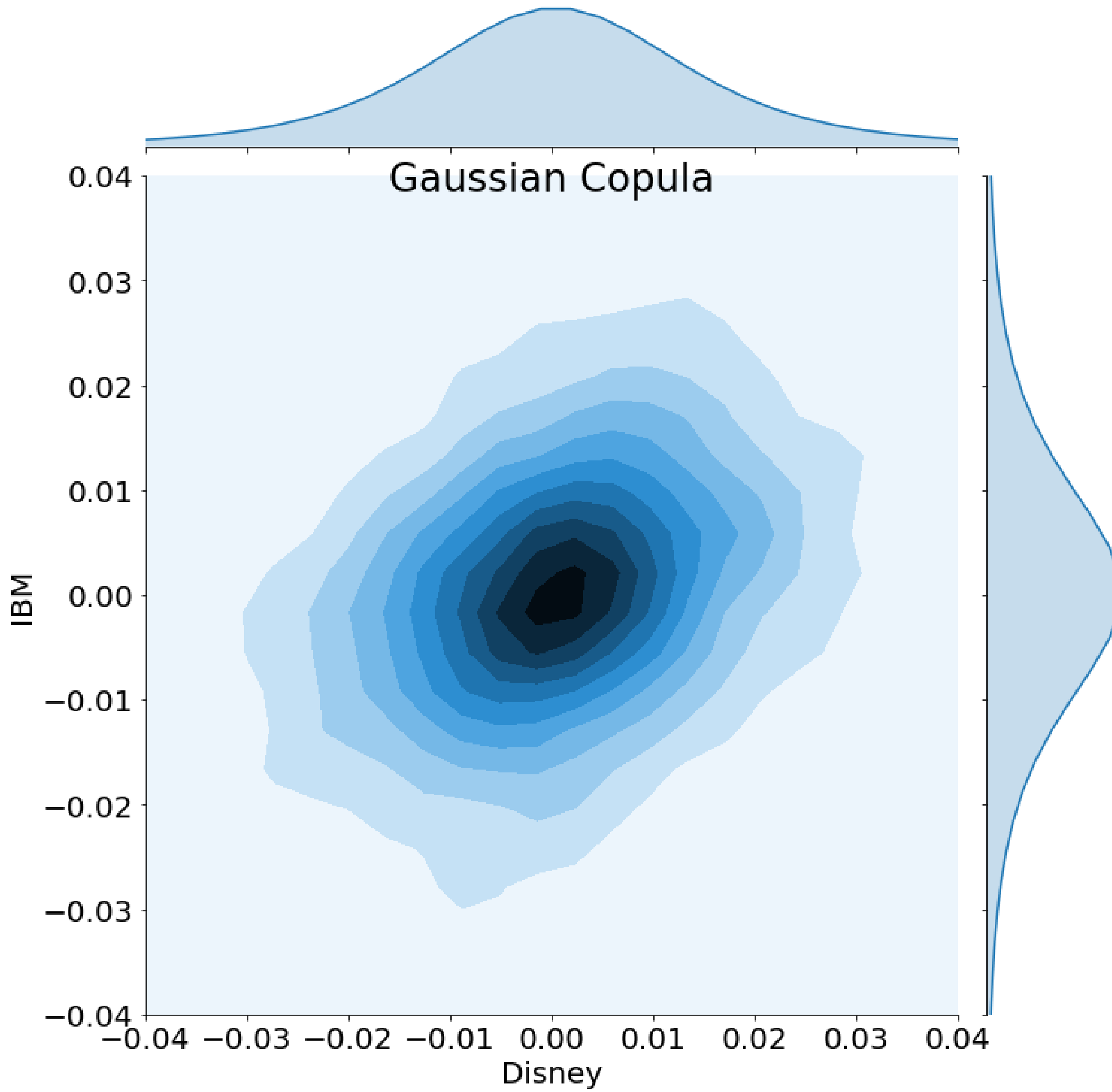
# Different types of Copulas – Empirical (IBM vs. Dis)

- Empirical data distribution: slightly higher correlation along left tail (exceedance correlation)



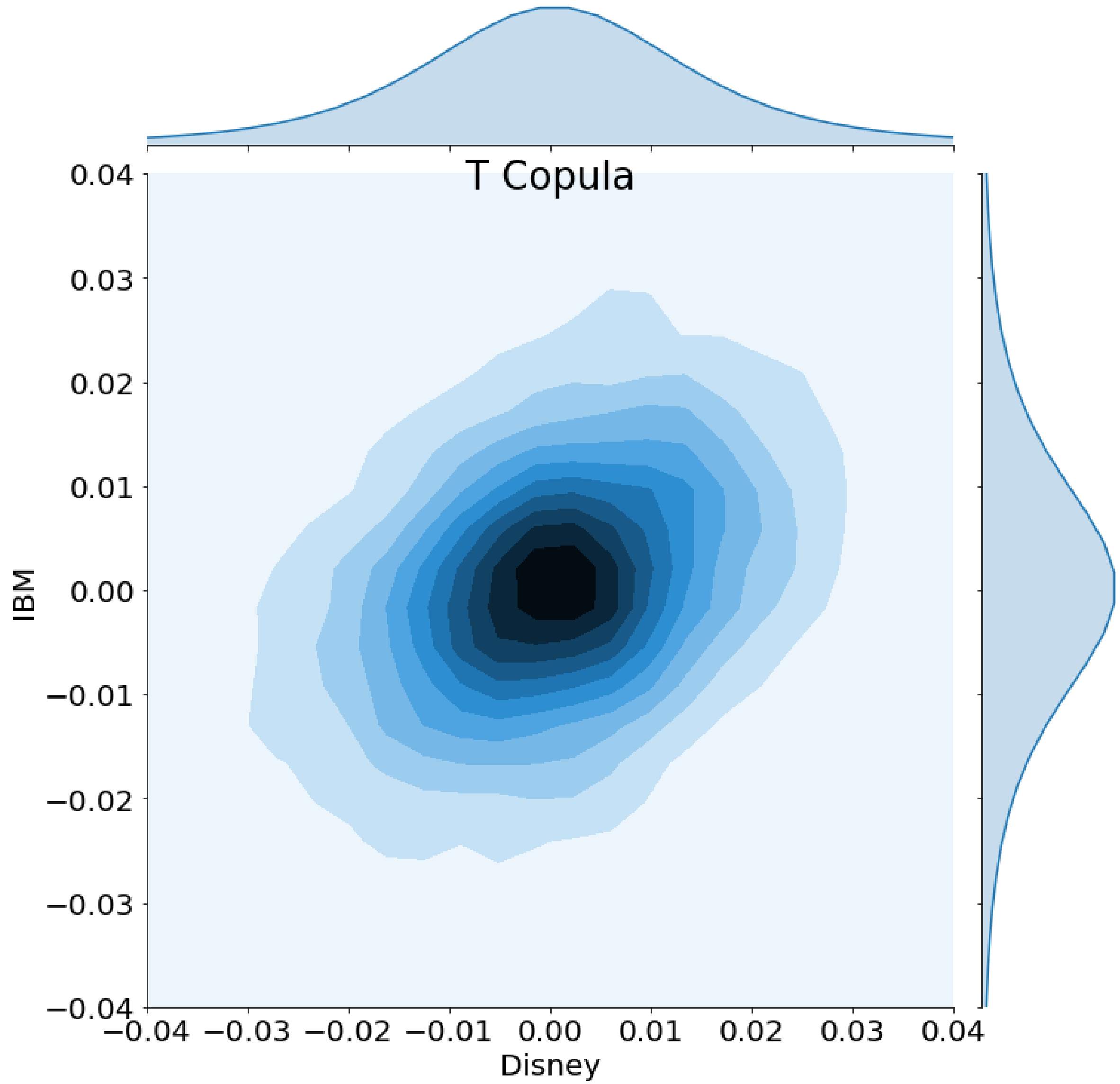
# Different types of Copulas - Gaussian

- Gaussian Copula + student t marginals: similar correlations along tails
- Not capturing the higher tail correlation from the realized data



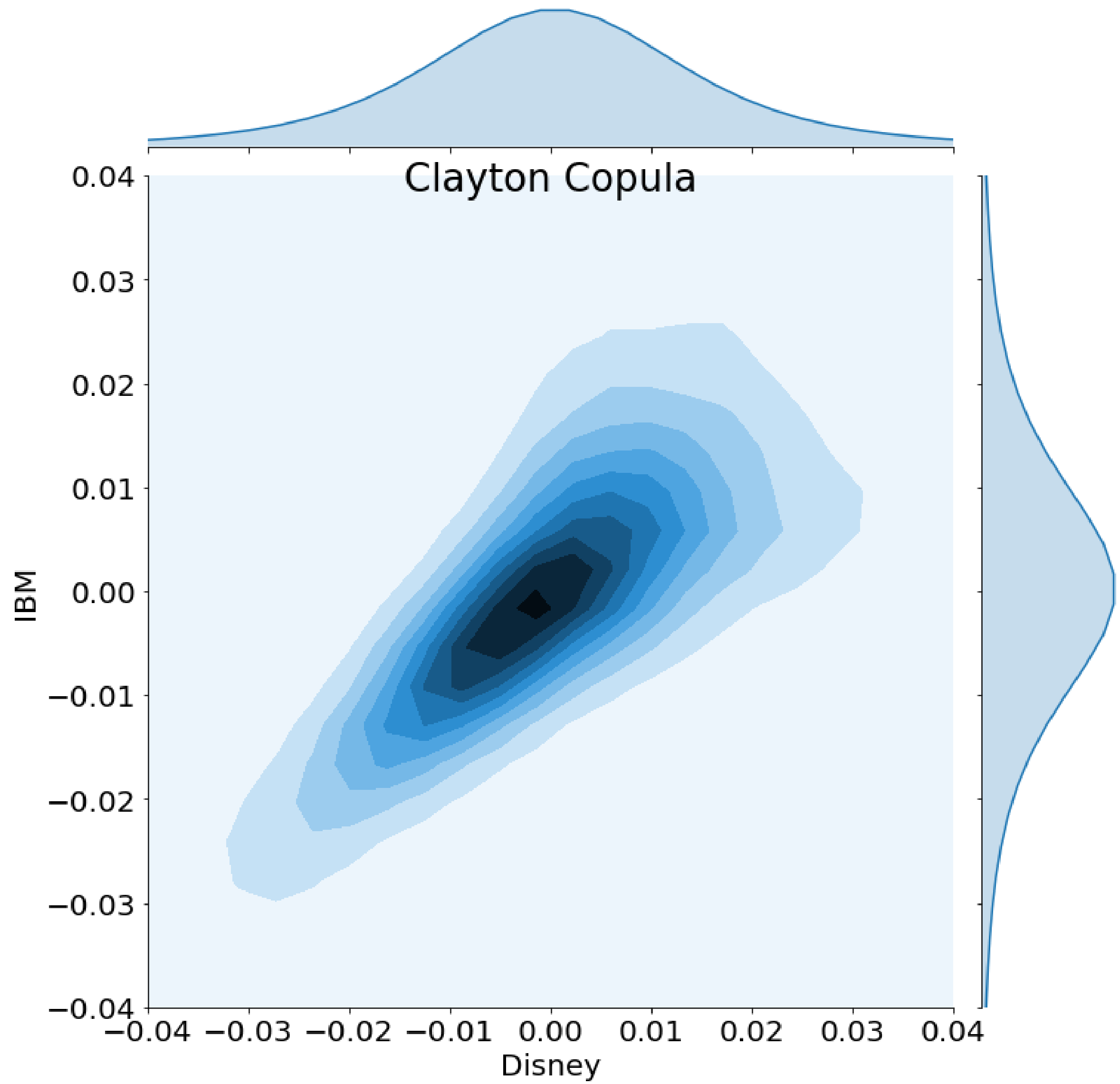
# Different types of Copulas – Student t (IBM vs. Dis)

- t Copula + student t marginals : no exceedance correlation (t Copula is symmetric)



# Different types of Copulas - Clayton

- Clayton Copula + student t marginals: higher correlation along left tail (exceedance correlation)
- Clayton parameter $\theta = 3$

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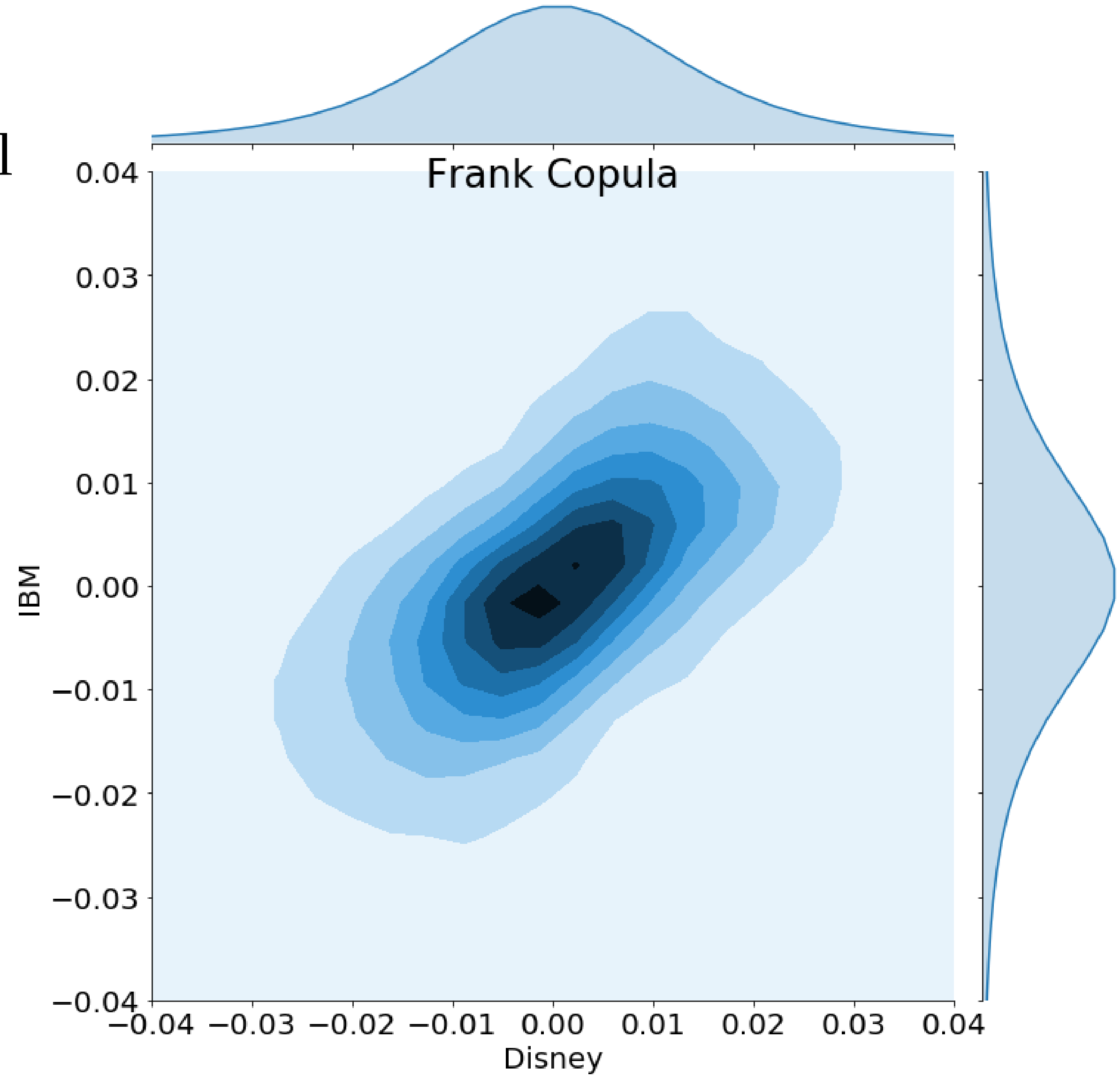
# Different types of Copulas - Frank

- Frank Copula + student t marginals: tighter but still symmetric correlation vs. Gaussian



# Different types of Copulas - Gumbel

- Gumbel Copula + student t marginals: higher correlation along right tail (exceedance correlation)