project-Copy1

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1 Rutherford Scattering (short project for PHYS 1113)

1.0.1 Coded by WANG, Xuechi

1.0.2 Analysed by LUO, Yueyang

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
import tqdm
```

[2]:
```
from scipy import constants
```

1.1 Initialize some constants

[3]:
```
Z1 = 2  # Atomic number of alpha particle
Z2 = 79  # Atomic number of gold nucleus
e = constants.e  # Elementary charge (C)
epsilon_0 = constants.epsilon_0  # Permittivity of free space (F/m)
(m, _, _) = constants.physical_constants['alpha particle mass']  # Mass of␣
 ↪alpha particle (kg)
k = 10  # Initial distance in X direction (sufficiently large)
pi = constants.pi
```

1.2 Set initial condition

[4]:
```
b = 2  # Impact parameter (can be varied)
X_0 = -k
Y_0 = b
V_X0 = 1.0
V_Y0 = 0.0
delta_tau = 0.001
```

[5]:
```
V_X = np.array([V_X0])
V_Y = np.array([V_Y0])
X = np.array([X_0])
Y = np.array([Y_0])
```

1.3 Calculate motion by iteration

1.3.1 The following code may not use in the real experiment due to performance issue (np.append) but it's ok for small stimulation

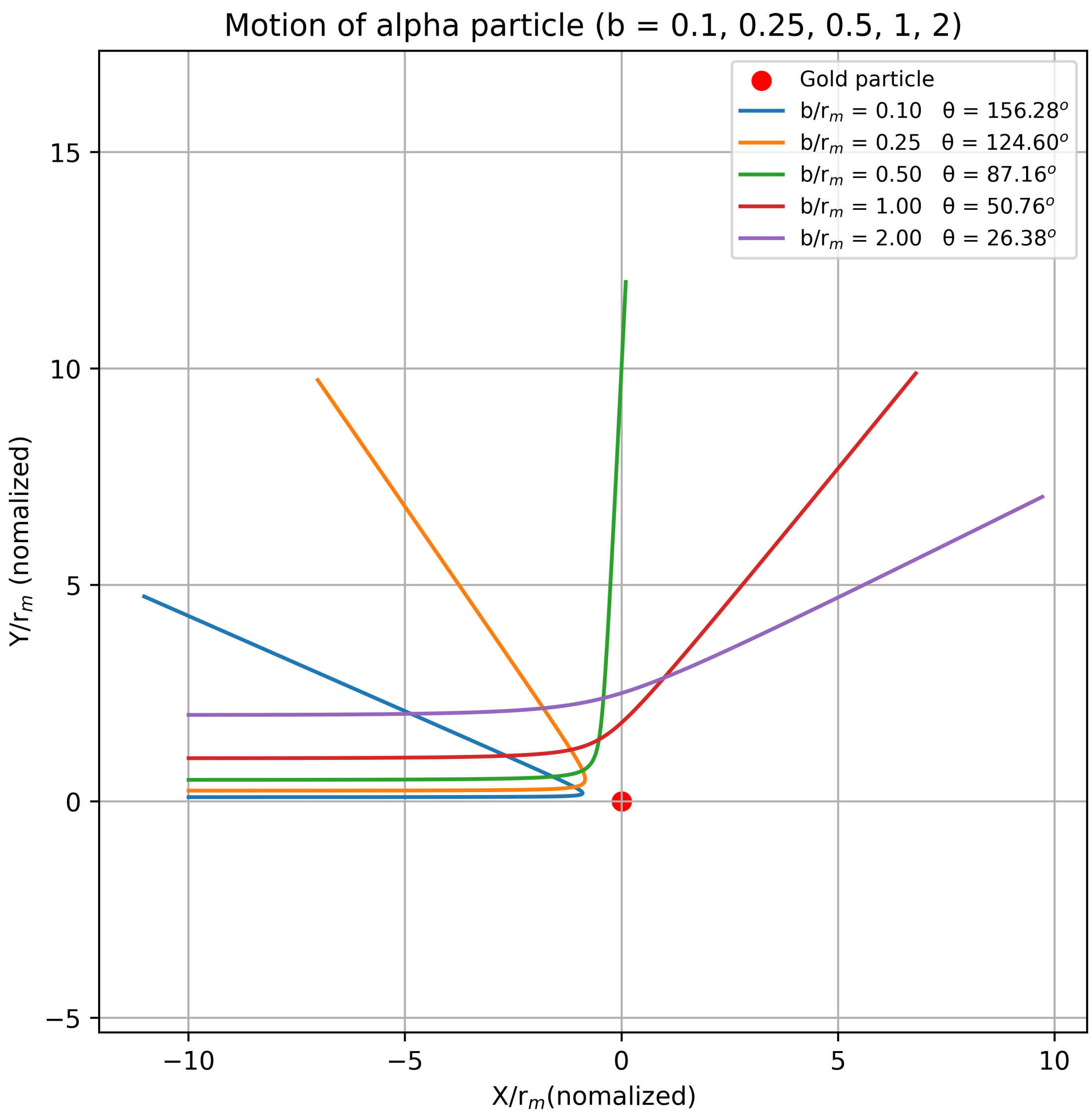
[6]:
```
def cal_X_Y(b):
    X_0 = -k
    Y_0 = b
    V_X0 = 1.0
    V_Y0 = 0.0
    delta_tau = 0.001
    V_X = np.array([V_X0])
    V_Y = np.array([V_Y0])
    X = np.array([X_0])
    Y = np.array([Y_0])
    while (np.sqrt(X[-1]**2 + Y[-1]**2) < 12): # R < 15
        X = np.append(X, X[-1] + delta_tau * V_X[-1])
        Y = np.append(Y, Y[-1] + delta_tau * V_Y[-1])
        A_X = X[-1]/(2*(np.sqrt(X[-1]**2 + Y[-1]**2)**3))
        A_Y = Y[-1]/(2*(np.sqrt(X[-1]**2 + Y[-1]**2)**3))
        V_X = np.append(V_X, V_X[-1] + delta_tau * A_X)
        V_Y = np.append(V_Y, V_Y[-1] + delta_tau * A_Y)
    return (X, Y, V_X, V_Y)
```

1.4 Plot figures of particle motion

[7]:
```
plt.figure(figsize=(38.4/5.5, 38.4/5.5), dpi=550)
plt.scatter([0], [0], marker='o', linewidths=2, label='Gold particle', c='r')
plt.axis('equal')
plt.grid()
plt.title(f"Motion of alpha particle (b = 0.1, 0.25, 0.5, 1, 2)")
for i in [0.1,0.25,0.5,1,2]:
    (X, Y, V_X, V_Y) = cal_X_Y(i)
    theta = np.arctan(V_Y[-1] / V_X[-1])
    if theta < 0:
        theta = np.pi - np.abs(theta)
    plt.plot(X, Y, label='b/r$_m$ = {:.2f}   '.format(i) + '  = {:.2f}$^o$'.
 ↪format(theta*180/pi))
    print(i)
plt.xlabel("X/r$_m$(nomalized)")
plt.ylabel("Y/r$_m$ (nomalized)")
plt.legend(fontsize="small")
```

```
0.1
0.25
0.5
1
2
```

[7]: <matplotlib.legend.Legend at 0x2ad4891a110>



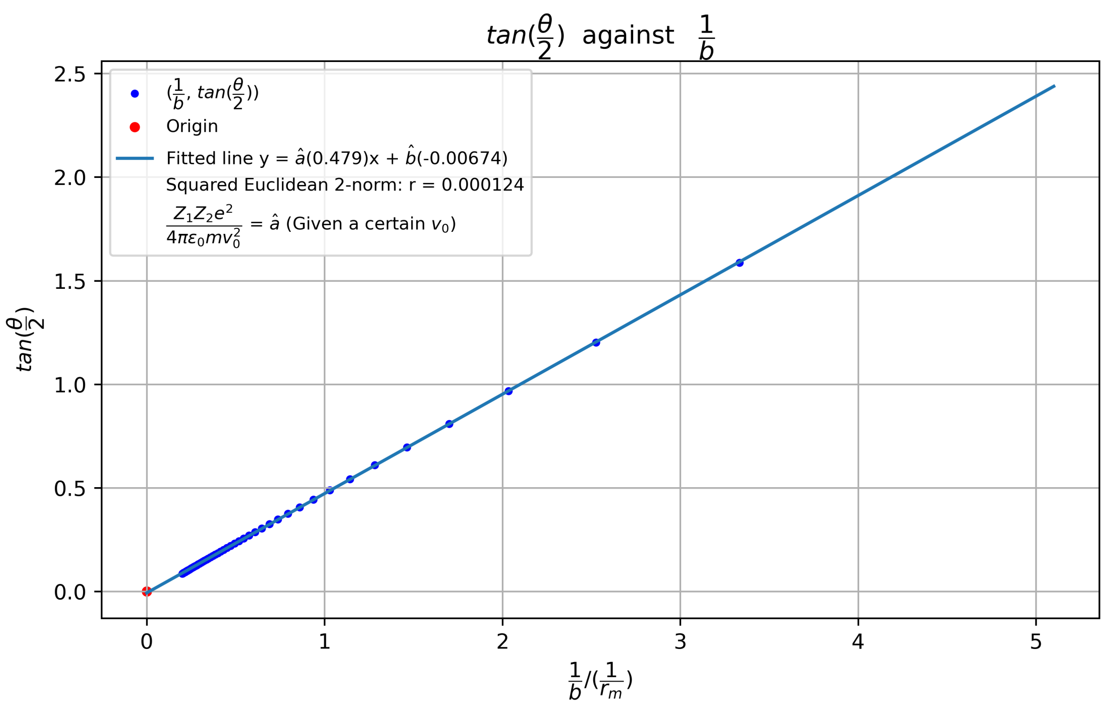
$Plot\ \ tan\dfrac{\theta}{2}\ \ against\ \ \dfrac{1}{b}/(\dfrac{1}{r_m})$

[8]:
```
plt.figure(figsize=(38.4/4.5, 21.6/4.5), dpi=450)
plt.xlabel(r"$\dfrac{1}{b} /(\dfrac{1}{r_m})$",)
plt.title("$tan(\\dfrac{ }{2})$  against   $\\dfrac{1}{b}$")
plt.ylabel(r'$tan(\dfrac{ }{2})$')
tan_theta_de_2 = np.array([])
b_devided_1 = np.array([])
for i in tqdm.tqdm(np.linspace(0.3,5,50), ascii=True):
    (X, Y, V_X, V_Y) = cal_X_Y(i)
```

```
    theta = np.arctan(V_Y[-1] / V_X[-1])
    if theta < 0:
        theta = np.pi - np.abs(theta)
    tan_theta_de_2 = np.append(tan_theta_de_2, np.tan(theta/2))
    b_devided_1 = np.append(b_devided_1, 1/i)
    #plt.scatter(1/i, np.tan(theta/2 - 1e-8), c='b', s=10, label='($\\dfrac{1}{b}$, $tan(\\dfrac{ }{2})$')
[a, b] = np.linalg.lstsq(np.vstack([b_devided_1 , np.ones(len(b_devided_1))]).T, tan_theta_de_2)[0]
plt.grid()
plt.scatter(b_devided_1, tan_theta_de_2, c='b', s=8, label='($\\dfrac{1}{b}$, $tan(\\dfrac{ }{2}$))')
plt.scatter(0,0, s=15, c='r', label="Origin")
plt.plot(np.linspace(0,5.1, 40), a * np.linspace(0,5.1, 40) + b, label="Fitted line y = $\\^a$(0.479)x + $\\^b$(-0.00674)")
plt.scatter(0,0, label="Squared Euclidean 2-norm: r = 0.000124",c='black',s=0)
plt.scatter(0,0, label="$\\dfrac{Z_1 Z_2 e^2}{4 \\pi \\epsilon_0 m v_0^2}$ = $\\^a$ (Given a certain $v_0$)",c='black',s=0)
plt.legend(fontsize='small')
```

```
100%|##########################################| 50/50 [00:29<00:00,  1.67it/s]
```

```
[8]: <matplotlib.legend.Legend at 0x2ad4e3d5dd0>
```



1.5 Least square calculation (to find linear relationship)

$mx + c = y$

1.5.1 return m (Slope), c (intercept), r (Residual)

[9]:
```
np.linalg.lstsq(np.vstack([b_devided_1 , np.ones(len(b_devided_1))]).T,␣
↪tan_theta_de_2)
```

[9]:
```
(array([ 0.47931386, -0.0067472 ]),
 array([0.00012417]),
 np.int32(2),
 array([8.7231768 , 3.56777856]))
```

1.6

1.7 Verify the conservation of total energy and angular momentum

$E = \frac{1}{2}(V_X^2 + V_Y^2) + \frac{1}{2R}$

[10]:
```
# Total energy
E = 0.5*(V_X**2 + V_Y**2)+0.5/np.sqrt(X**2+Y**2)
```

[11]:
```
# Glance at data of energy
E[0:20]
```

[11]:
```
array([0.54472136, 0.54472136, 0.54472136, 0.54472136, 0.54472136,
       0.54472136, 0.54472136, 0.54472136, 0.54472136, 0.54472136,
       0.54472136, 0.54472136, 0.54472136, 0.54472136, 0.54472136,
       0.54472136, 0.54472136, 0.54472136, 0.54472136, 0.54472135])
```

$L = XV_Y - YV_X = (\vec{R} \times \frac{\vec{P}}{m})z$

[12]:
```
# Angular momentum
L = X*V_Y-Y*V_X
```

[13]:
```
# Glance at data of angular momentum
L[0:20]
```

[13]:
```
array([-5., -5., -5., -5., -5., -5., -5., -5., -5., -5., -5., -5., -5.,
       -5., -5., -5., -5., -5., -5., -5.])
```

1.8 Further evaluate the Range, Mean Deviation, Standard Deviation, Medium of E and L

1.8.1 Range

[14]:
```python
def cal_range(A):
    return np.max(A) - np.min(A)
```

[15]:
```python
print(f"{' ':^4}"+f"{'b':^6}"+"       "+f"{'Range of E':^12}"+"       "+f"{'Range of L':^12}")
count = 1
for i in np.linspace(0.1,5,25):
    (X, Y, V_X, V_Y) = cal_X_Y(i)
    print(f"{count:>2}{'':<2}",end="")
    count+=1
    E = 0.5*(V_X**2 + V_Y**2)+0.5/np.sqrt(X**2+Y**2)
    L = X*V_Y-Y*V_X
    e_range = cal_range(E)
    l_range = cal_range(L)
    print(f'{i:.4f}       {e_range:e}       {l_range:e}')
```

```
      b          Range of E         Range of L
 1  0.1000       1.784878e-04       4.241052e-14
 2  0.3042       1.570807e-04       4.085621e-14
 3  0.5083       1.285386e-04       6.883383e-15
 4  0.7125       1.028133e-04       4.218847e-14
 5  0.9167       8.239112e-05       6.128431e-14
 6  1.1208       6.677031e-05       3.463896e-14
 7  1.3250       5.486576e-05       2.997602e-14
 8  1.5292       4.571204e-05       4.618528e-14
 9  1.7333       3.857931e-05       1.083578e-13
10  1.9375       3.294126e-05       6.528111e-14
11  2.1417       2.842202e-05       3.330669e-14
12  2.3458       2.475193e-05       4.973799e-14
13  2.5500       2.173541e-05       5.240253e-14
14  2.7542       1.922875e-05       5.906386e-14
15  2.9583       1.712492e-05       5.062617e-14
16  3.1625       1.534312e-05       5.107026e-14
17  3.3667       1.382158e-05       3.597123e-14
18  3.5708       1.251247e-05       5.728751e-14
19  3.7750       1.137838e-05       5.062617e-14
20  3.9792       1.038970e-05       4.884981e-14
21  4.1833       9.522801e-06       1.101341e-13
22  4.3875       8.758609e-06       6.750156e-14
23  4.5917       8.081635e-06       8.348877e-14
24  4.7958       7.479185e-06       6.661338e-14
25  5.0000       6.940780e-06       8.704149e-14
```

1.8.2 Mean Deviation

[16]:
```python
def cal_mean_deviation(A):
    return np.sum(np.abs(A - np.mean(A)))
```

[17]:
```python
print(f"{' ':^4}"+f"{'b':^6}"+"          "+f"{'Mean Deviation of E':^0}"+"     "+f"{'Mean Deviation of L'}")
count = 1
for i in np.linspace(0.1,5,25):
    (X, Y, V_X, V_Y) = cal_X_Y(i)
    print(f"{count:>2}{'':<2}",end="")
    count+=1
    E = 0.5*(V_X**2 + V_Y**2)+0.5/np.sqrt(X**2+Y**2)
    L = X*V_Y-Y*V_X
    e_mean_deviation = cal_mean_deviation(E)
    l_mean_deviation = cal_mean_deviation(L)
    print(f'{i:.4f}            {e_mean_deviation:e}            {l_mean_deviation:e}')
```

```
      b          Mean Deviation of E     Mean Deviation of L
 1  0.1000          4.978914e-01            2.389738e-10
 2  0.3042          4.569606e-01            1.371497e-10
 3  0.5083          4.006224e-01            2.777334e-11
 4  0.7125          3.469121e-01            8.983947e-11
 5  0.9167          3.011266e-01            1.551100e-10
 6  1.1208          2.632905e-01            1.009788e-10
 7  1.3250          2.321179e-01            9.522916e-11
 8  1.5292          2.062585e-01            2.056466e-10
 9  1.7333          1.845882e-01            4.028620e-10
10  1.9375          1.662367e-01            3.088014e-10
11  2.1417          1.505349e-01            1.008265e-10
12  2.3458          1.369735e-01            1.952327e-10
13  2.5500          1.251587e-01            1.929070e-10
14  2.7542          1.147883e-01            2.146714e-10
15  2.9583          1.056208e-01            1.877849e-10
16  3.1625          9.746669e-02            2.218705e-10
17  3.3667          9.017099e-02            1.300369e-10
18  3.5708          8.361313e-02            1.777560e-10
19  3.7750          7.768778e-02            2.882787e-10
20  3.9792          7.231439e-02            1.960654e-10
21  4.1833          6.742057e-02            4.840572e-10
22  4.3875          6.294803e-02            2.823439e-10
23  4.5917          5.884750e-02            4.195675e-10
24  4.7958          5.507561e-02            2.517915e-10
25  5.0000          5.159796e-02            3.247038e-10
```

1.8.3 Standard Deviation

[18]:
```
def cal_standard_deviation(A):
    return np.std(A)
```

[19]:
```
print(f"{' ':^4}"+f"{'b':^6}"+"       "+f"{'STD of E':^12}"+"       "+f"{'STD␣
↪of L':^12}")
count = 1
for i in np.linspace(0.1,5,25):
    (X, Y, V_X, V_Y) = cal_X_Y(i)
    print(f"{count:>2}{'':<2}",end="")
    count+=1
    E = 0.5*(V_X**2 + V_Y**2)+0.5/np.sqrt(X**2+Y**2)
    L = X*V_Y-Y*V_X
    e_std = cal_standard_deviation(E)
    l_std = cal_standard_deviation(L)
    print(f'{i:.4f}       {e_std:e}       {l_std:e}')
```

```
      b            STD of E           STD of L
 1  0.1000       3.320740e-05       1.194140e-14
 2  0.3042       2.996694e-05       7.959646e-15
 3  0.5083       2.556366e-05       1.515659e-15
 4  0.7125       2.146605e-05       5.906410e-15
 5  0.9167       1.808092e-05       1.183860e-14
 6  1.1208       1.537878e-05       6.196091e-15
 7  1.3250       1.322903e-05       5.641177e-15
 8  1.5292       1.150501e-05       1.175675e-14
 9  1.7333       1.010644e-05       2.438029e-14
10  1.9375       8.957156e-06       1.613814e-14
11  2.1417       8.001361e-06       5.976566e-15
12  2.3458       7.197313e-06       1.043728e-14
13  2.5500       6.513935e-06       1.154586e-14
14  2.7542       5.927313e-06       1.223040e-14
15  2.9583       5.419515e-06       1.120235e-14
16  3.1625       4.976414e-06       1.202829e-14
17  3.3667       4.587130e-06       7.435204e-15
18  3.5708       4.242674e-06       1.124232e-14
19  3.7750       3.936271e-06       1.507648e-14
20  3.9792       3.662050e-06       1.103614e-14
21  4.1833       3.415511e-06       2.693135e-14
22  4.3875       3.192811e-06       1.621267e-14
23  4.5917       2.990764e-06       2.264887e-14
24  4.7958       2.806762e-06       1.432514e-14
25  5.0000       2.638512e-06       2.016989e-14
```

1.8.4 Medium

[20]:
```
def cal_medium(A):
    return np.median(A)
```

[22]:
```
print(f"{' ':^4}"+f"{'b':^6}"+"        "+f"{'Medium of E':^11}"+"       ␣
↪"+f"{'Medium of L':^11}")
count = 1
for i in np.linspace(0.1,5,25):
    (X, Y, V_X, V_Y) = cal_X_Y(i)
    print(f"{count:>2}{'':<2}",end="")
    count+=1
    E = 0.5*(V_X**2 + V_Y**2)+0.5/np.sqrt(X**2+Y**2)
    L = X*V_Y-Y*V_X
    e_mid = cal_medium(E)
    l_mid = cal_medium(L)
    print(f'{i:.4f}       {e_mid:e}       {l_mid:e}')
```

```
      b          Medium of E        Medium of L
 1  0.1000      5.500021e-01      -1.000000e-01
 2  0.3042      5.499815e-01      -3.041667e-01
 3  0.5083      5.499401e-01      -5.083333e-01
 4  0.7125      5.498781e-01      -7.125000e-01
 5  0.9167      5.497958e-01      -9.166667e-01
 6  1.1208      5.496934e-01      -1.120833e+00
 7  1.3250      5.495712e-01      -1.325000e+00
 8  1.5292      5.494299e-01      -1.529167e+00
 9  1.7333      5.492698e-01      -1.733333e+00
10  1.9375      5.490915e-01      -1.937500e+00
11  2.1417      5.488956e-01      -2.141667e+00
12  2.3458      5.486828e-01      -2.345833e+00
13  2.5500      5.484538e-01      -2.550000e+00
14  2.7542      5.482092e-01      -2.754167e+00
15  2.9583      5.479500e-01      -2.958333e+00
16  3.1625      5.476768e-01      -3.162500e+00
17  3.3667      5.473905e-01      -3.366667e+00
18  3.5708      5.470918e-01      -3.570833e+00
19  3.7750      5.467817e-01      -3.775000e+00
20  3.9792      5.464608e-01      -3.979167e+00
21  4.1833      5.461301e-01      -4.183333e+00
22  4.3875      5.457901e-01      -4.387500e+00
23  4.5917      5.454419e-01      -4.591667e+00
24  4.7958      5.450862e-01      -4.795833e+00
25  5.0000      5.447239e-01      -5.000000e+00
```

2 According Equations

$$\text{m}\ \frac{d^2\vec{r}}{dt^2} = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 r^2}\hat{e_r}$$

$$\hat{e_r} = \frac{\vec{r}}{r} = \frac{x\hat{i} + y\hat{j}}{r}$$

$$\frac{1}{2}m{v^2}_0 = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 r_m}\hat{e_r}$$

$$\text{v}_0 = \frac{r_m}{T}$$

$$\text{T} = \sqrt{\frac{2\pi m\epsilon_0 r_m^3}{Z_1 Z_2 e^2}}$$

$$\text{t=T} \qquad \vec{r} = r_m\vec{R}$$

$$\frac{d^2\vec{R}}{d\tau^2} = \frac{1}{2R^2}\hat{e_r}$$

$$\text{A}_X = \frac{X}{2R^3} \quad A_Y = \frac{Y}{2R^3}$$

$$\text{R} = \sqrt{X^2 + Y^2}$$

$$\text{X(n+1) = X(n)+V}_X(n)\Delta\tau \quad V(n+1) = V(n) + A_X(n)\Delta\tau$$

$$\text{E} = \frac{1}{2}(V_X^2 + V_Y^2) + \frac{1}{2R} \quad L = XV_Y - YV_X = (\vec{R}\times\frac{\vec{P}}{m})z$$

$$\tan\ \frac{\theta}{2} = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 m v_0^2}\frac{1}{b}$$