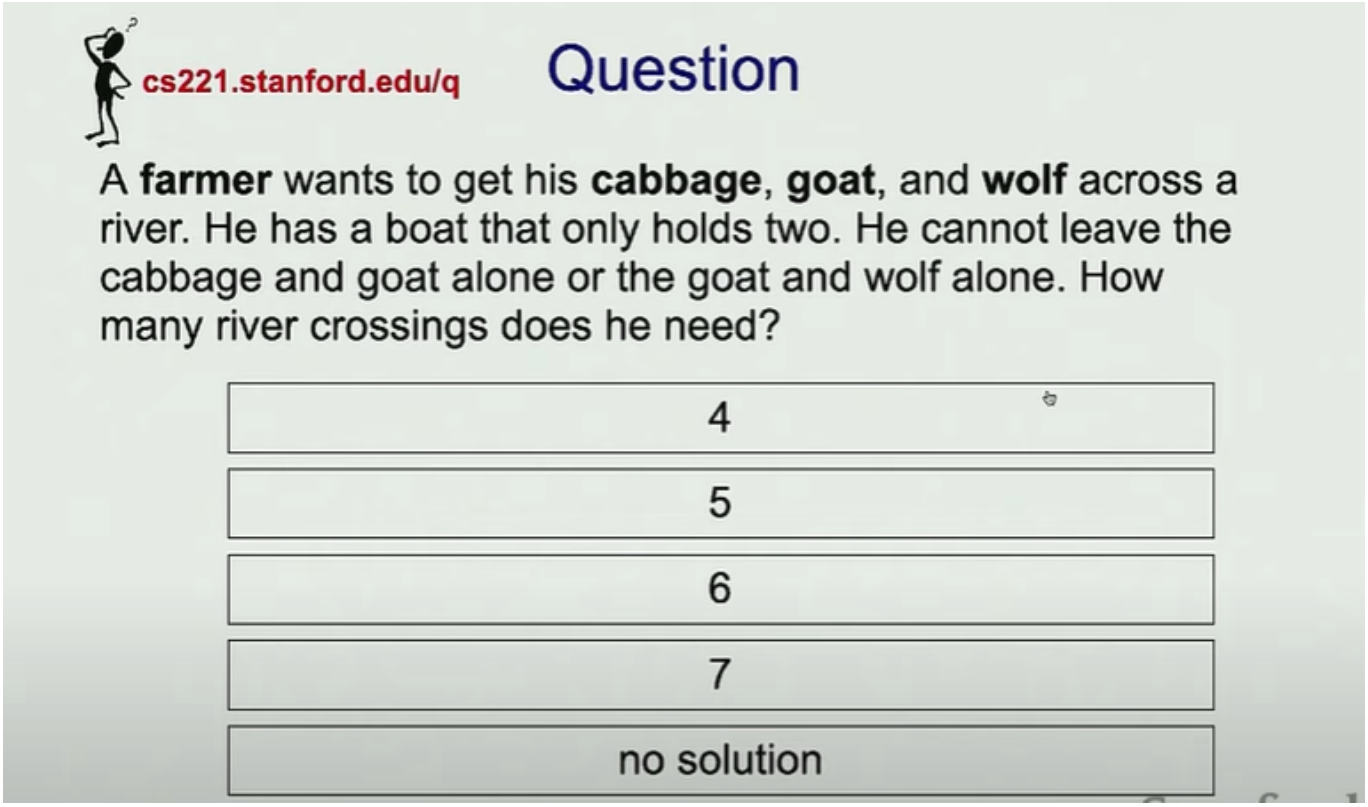
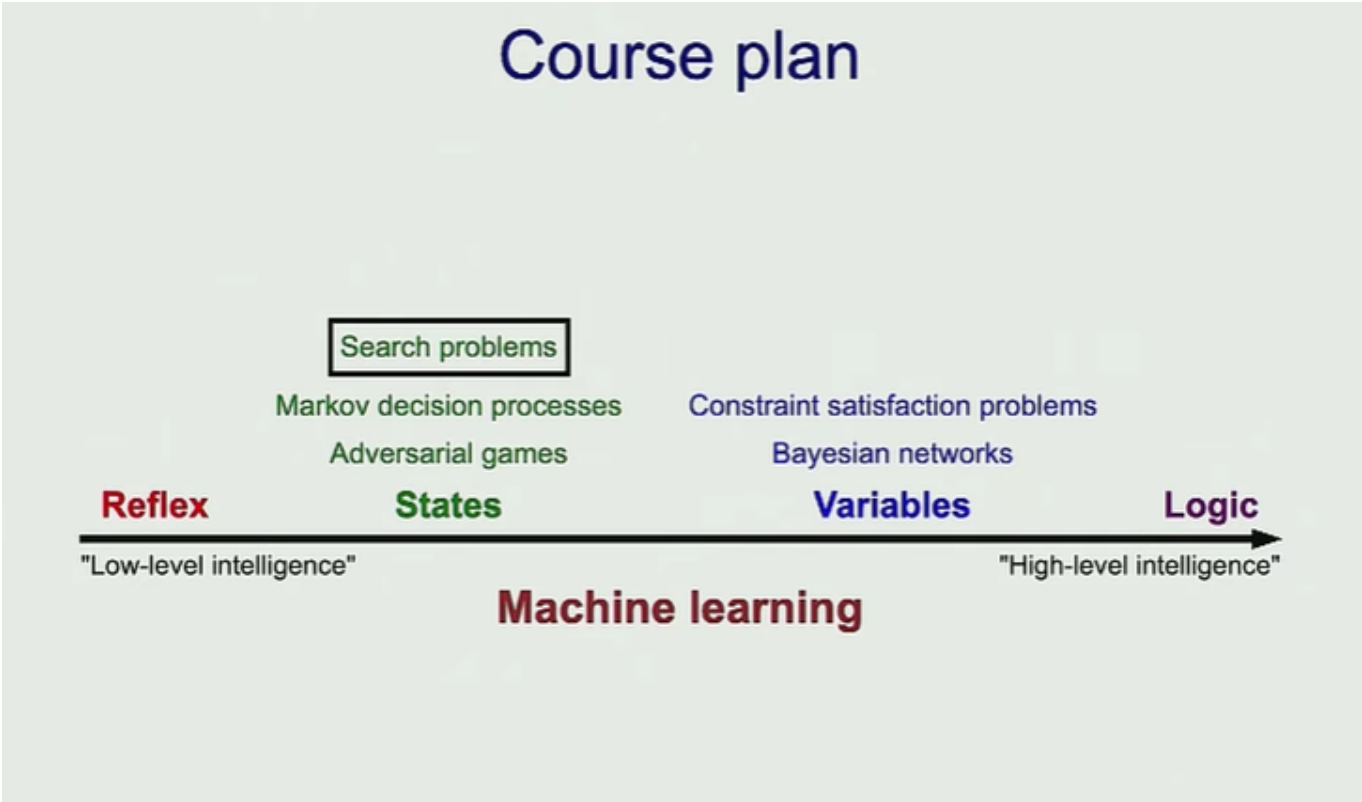
4. Search 1 - Dynamic Programming, Uniform Cost Search

State-based models and Search

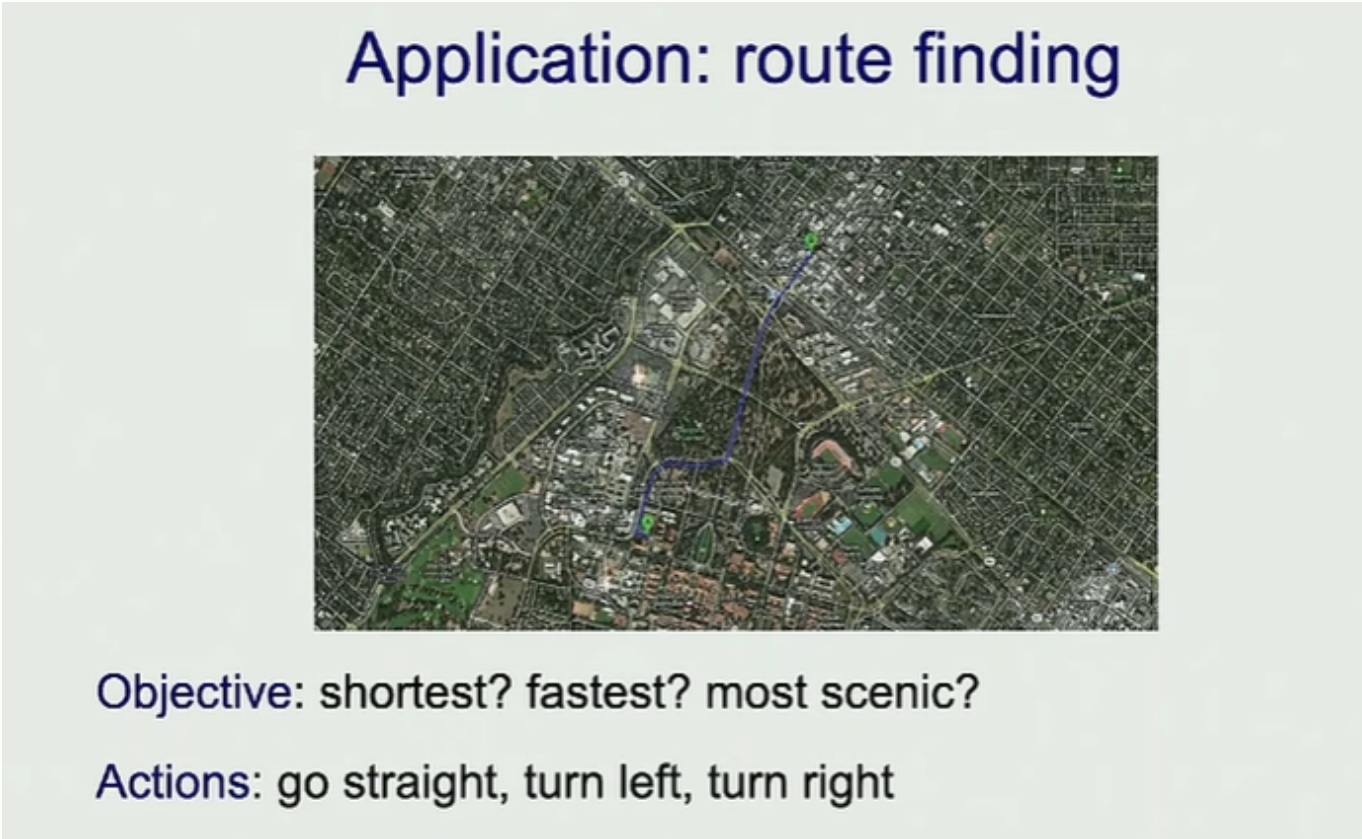
Intro



Open question Dynamic Programming Try all the possibilities



What are search problems?



Application: robot motion planning

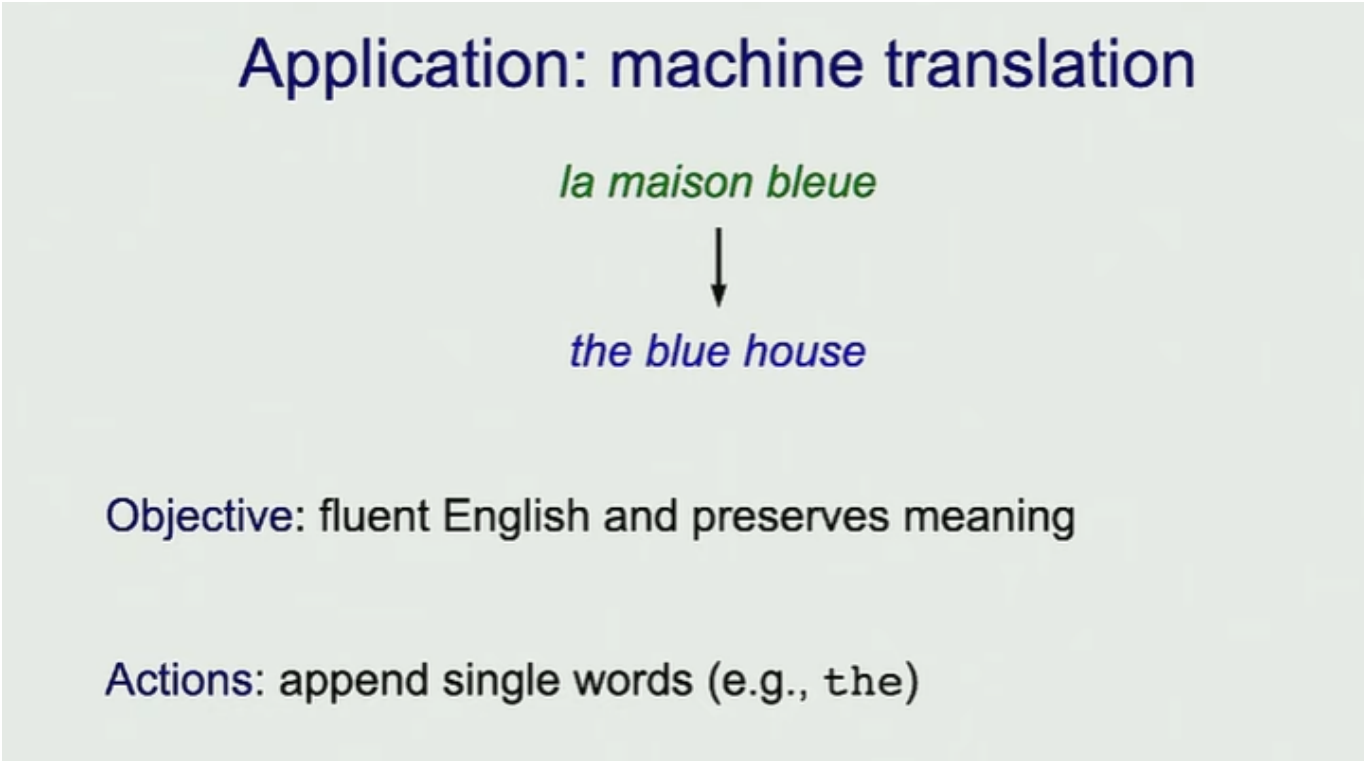
Objective: fastest? most energy efficient? safest? most expressive?

Actions: translate and rotate joints

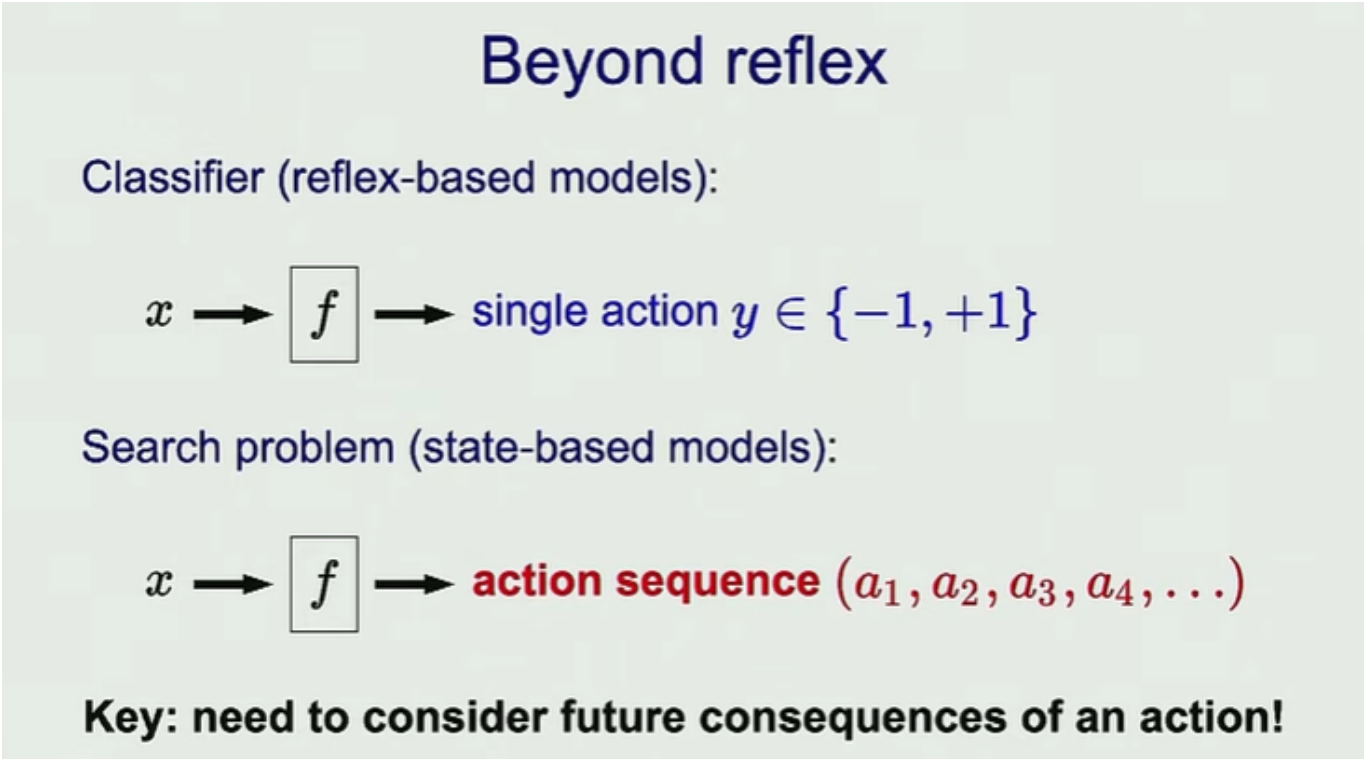
Application: solving puzzles

Objective: reach a certain configuration

Actions: move pieces (e.g., `Move12Down`)



What's different from reflex-based models?



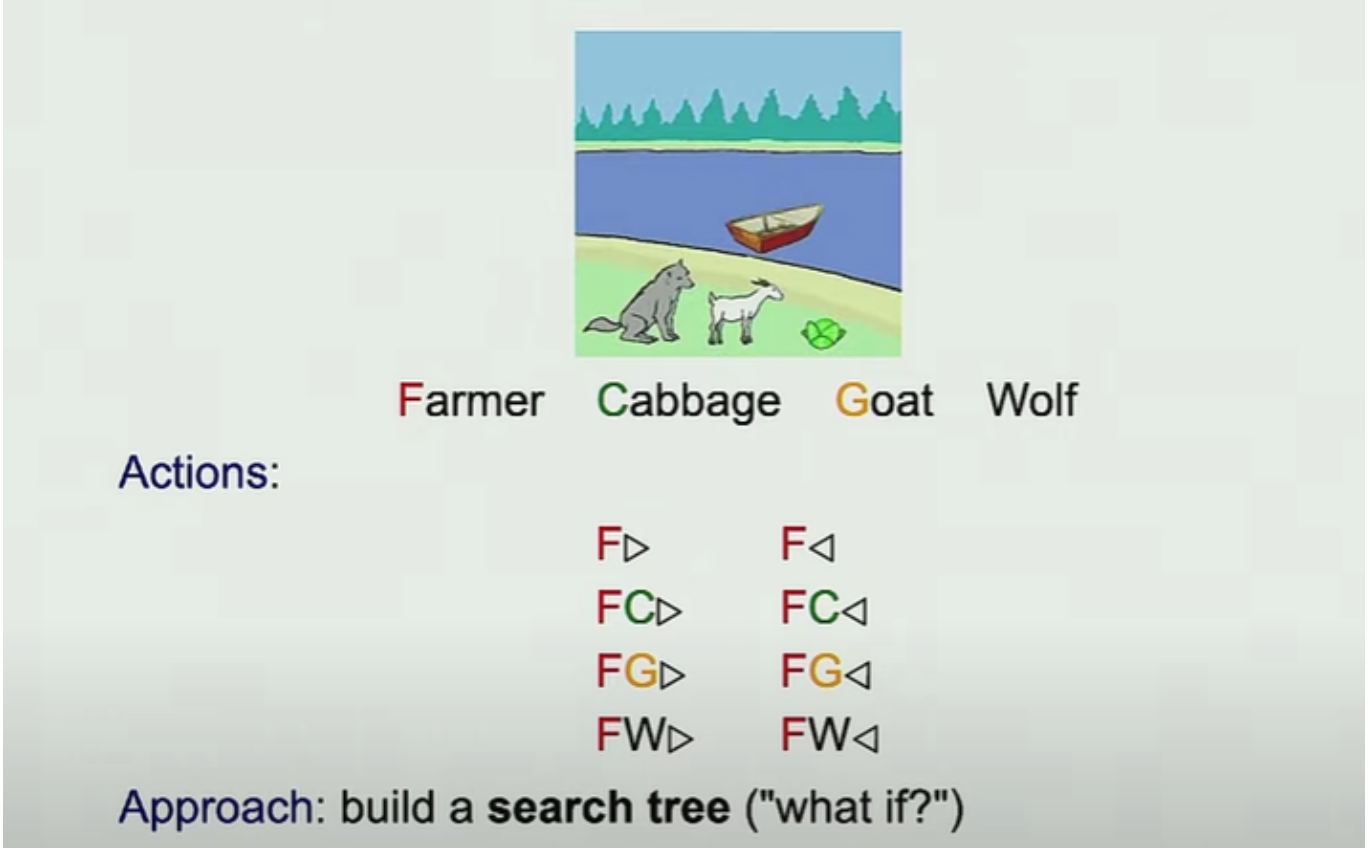
For state based models,You need to think about future Cuz for each step, it's gonna change ur state

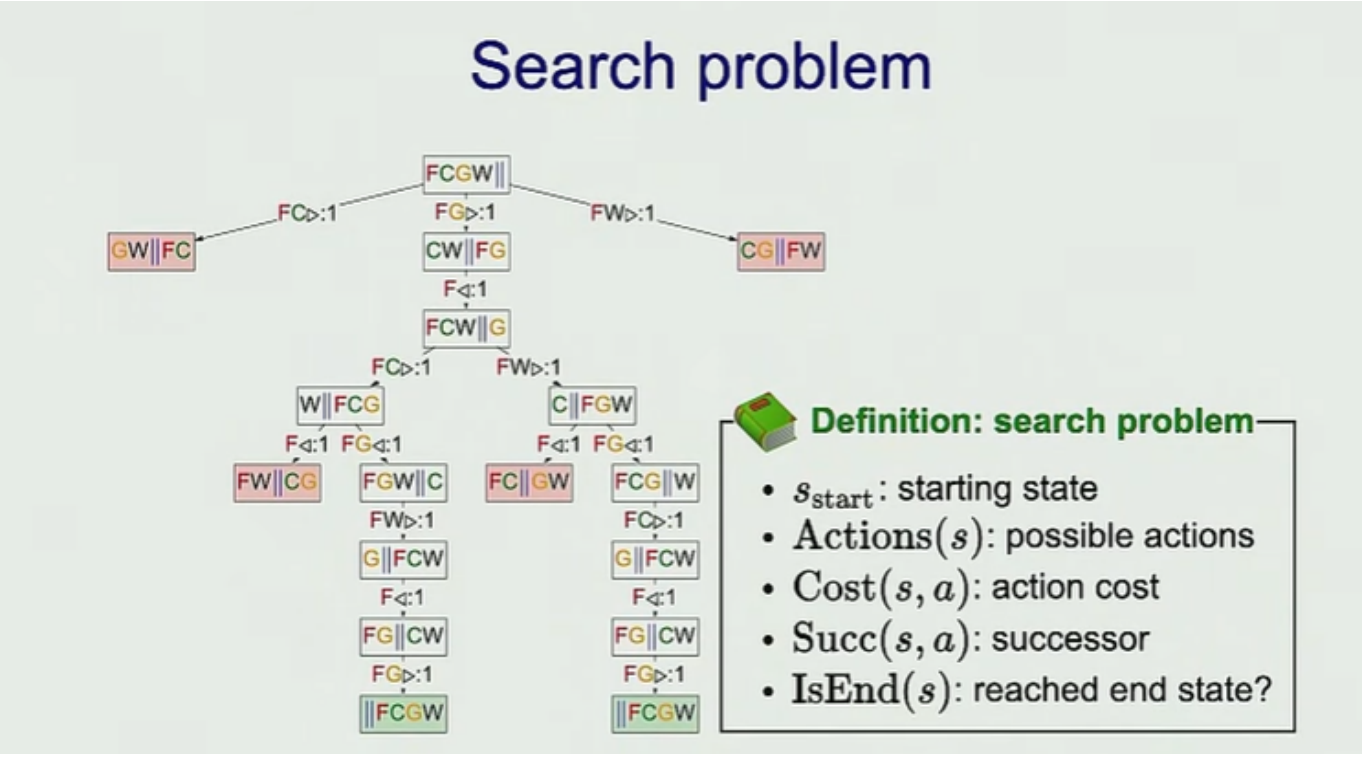
Road map

We are going to talk about three different algo for doing inference, for searching problems

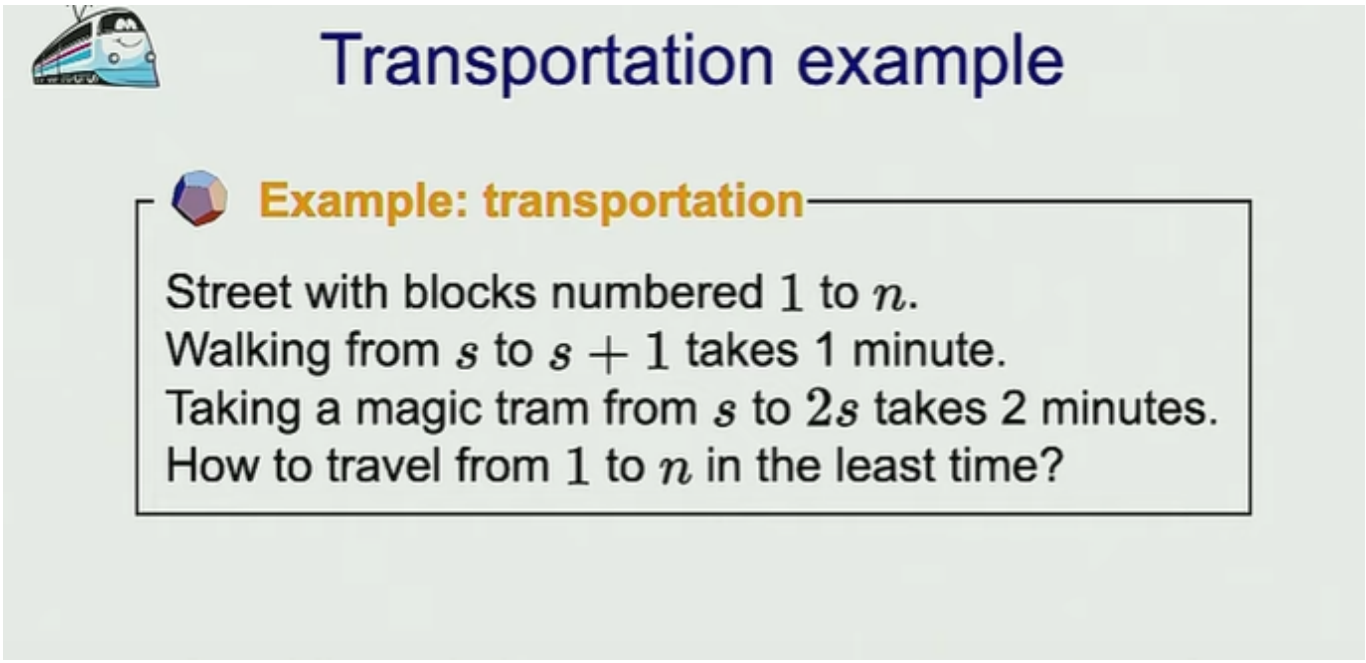
Tree Search

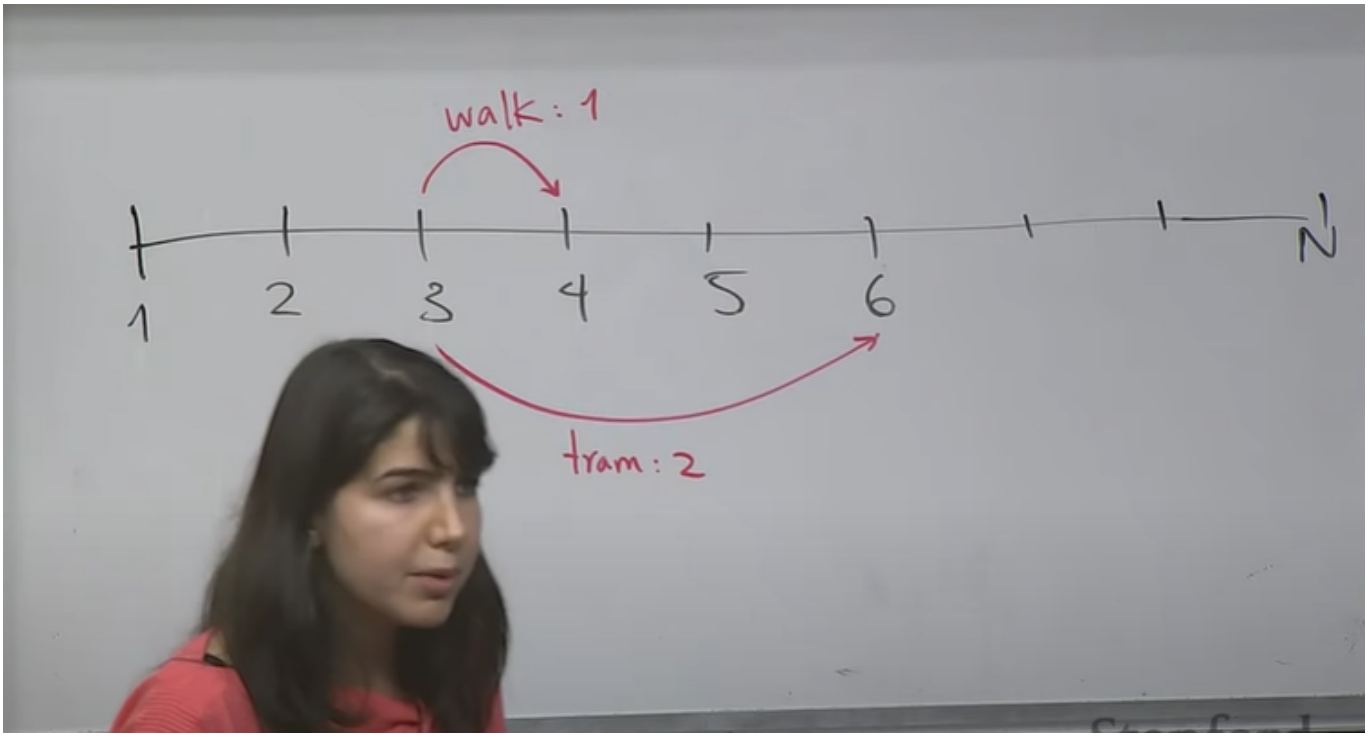
Enumerate all the actions we can take



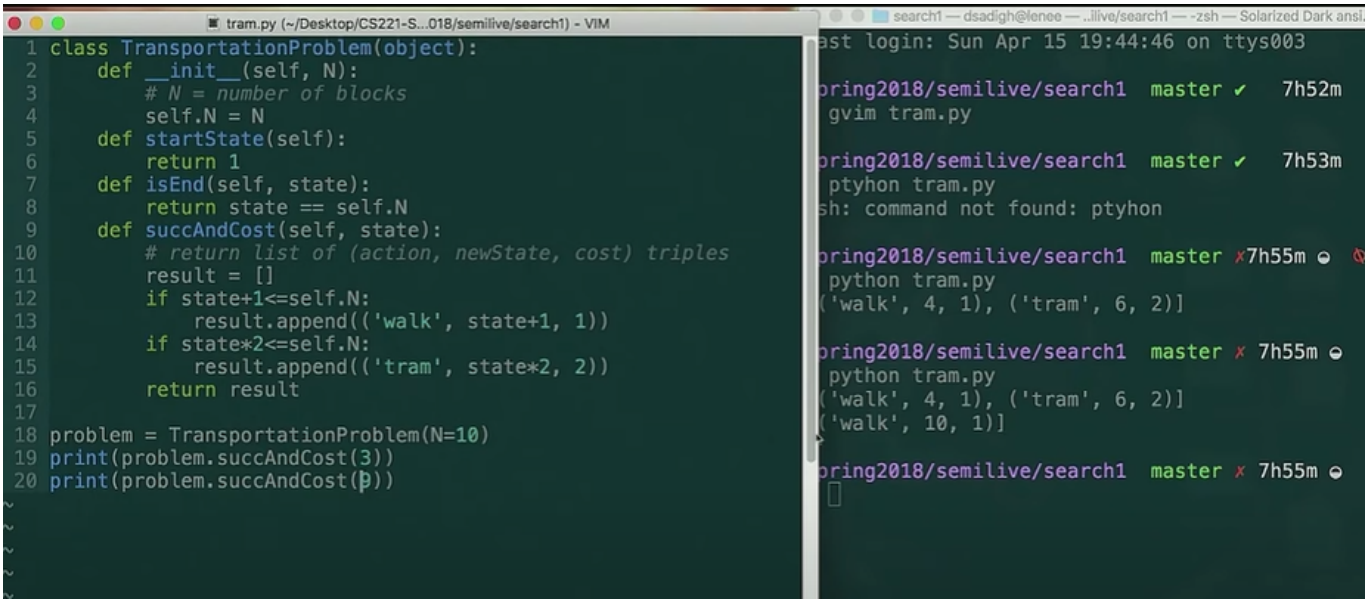


Walk or tram


Transportation example
Example: transportation
Street with blocks numbered 1 to n.
Walking from s to $s+1$ takes 1 minute.
Taking a magic tram from s to $2s$ takes 2 minutes.
How to travel from 1 to n in the least time?

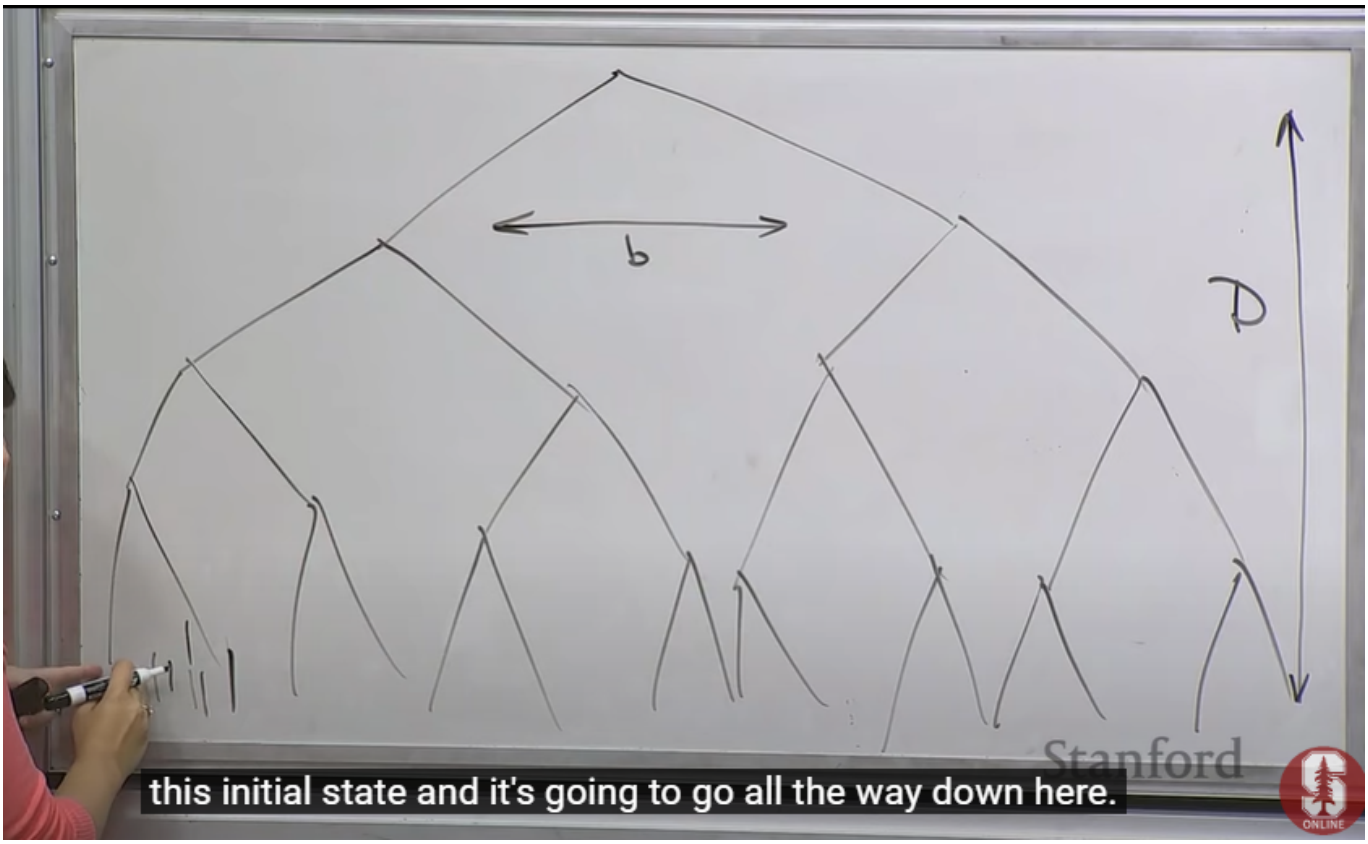

walk: 1
1
2
3
4
5
6
N
tram: 2

Defining the search problem model

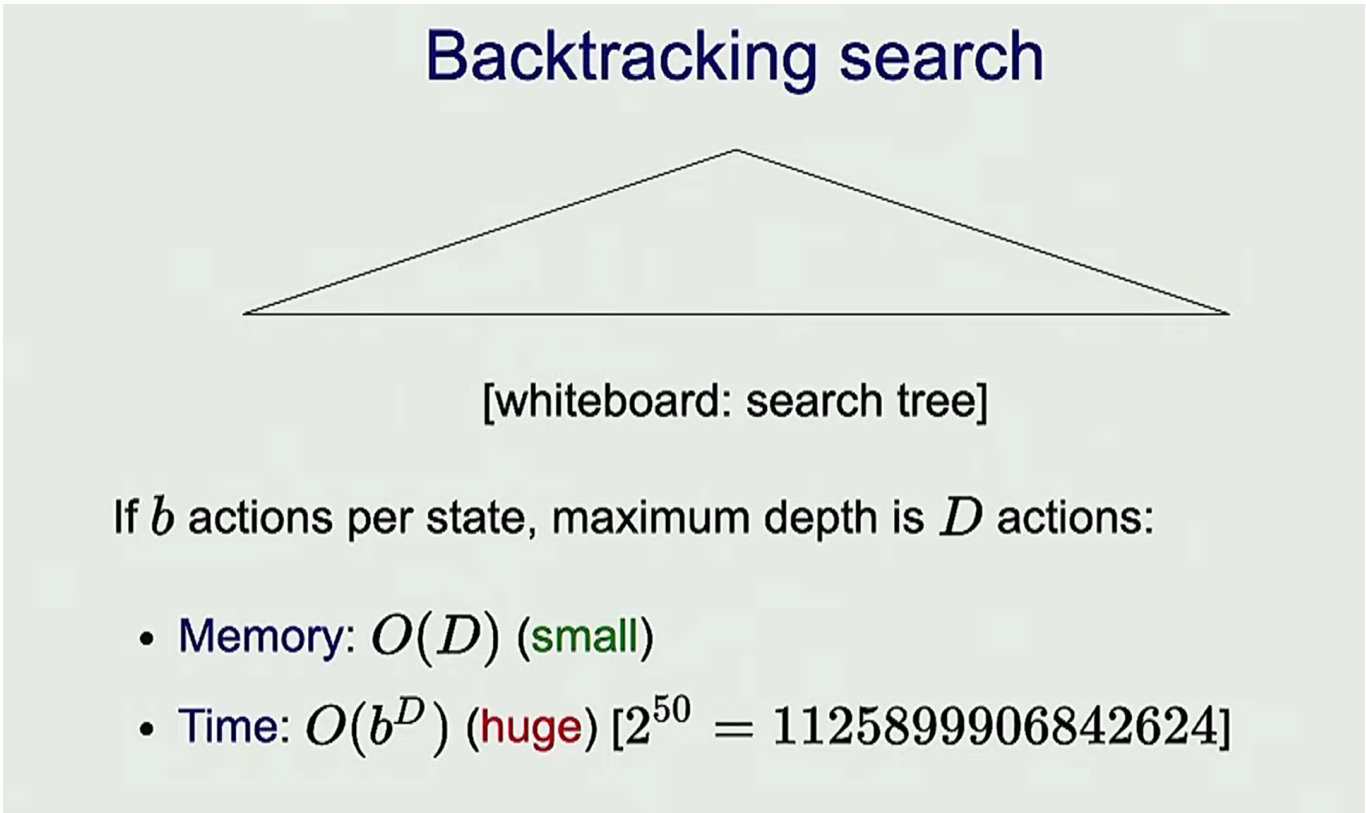


Algo	Cost	Time	Space
Backtracking Search	Any	O(b^D)	O(D)
DFS	0	Worst case O(b^D)	O(D)
BFS	cost >= 0 (assuming all the cost are the same)	Worst Case O(b^D)	Worst case O(b^D) ???
DFS - ID	cost >= 0 (assuming all the cost are the same)	Worst case O(b^D)	O(D)

Backtracking Search



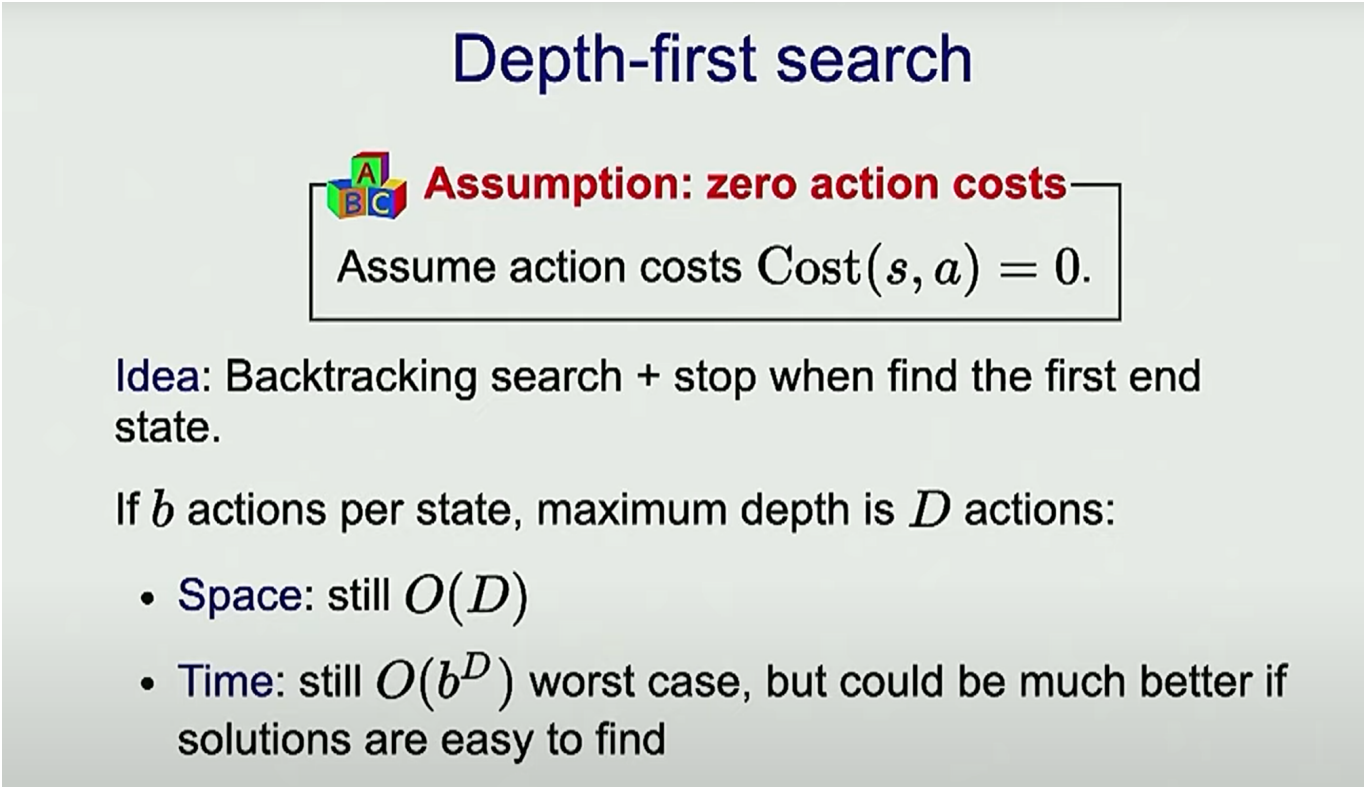
b : branching factor (How many branches it gonna hava from one node) D : Depth of the tree



The time complexity is quite bad though lol

DFS

When you don't care the cost of going back and forth

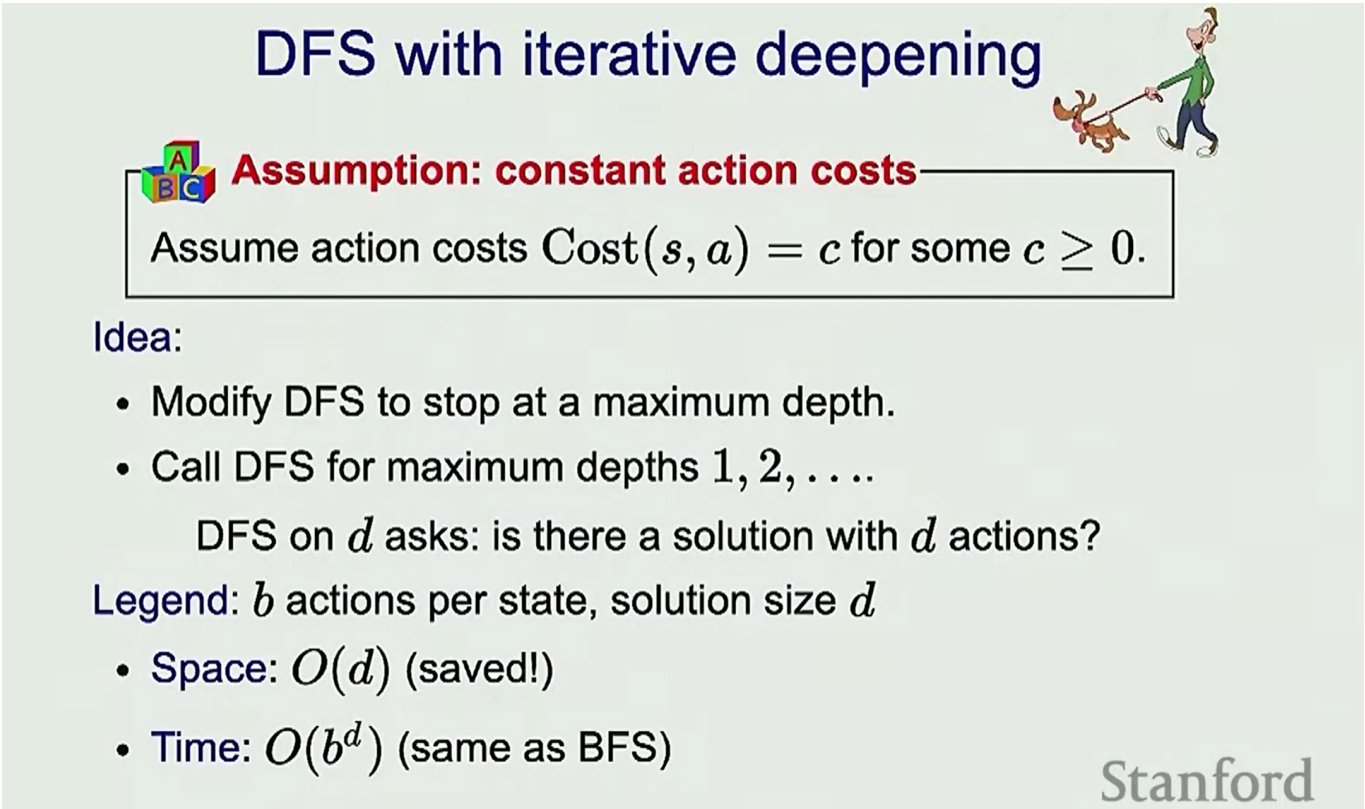


BFS

This is useful when the costs are similar

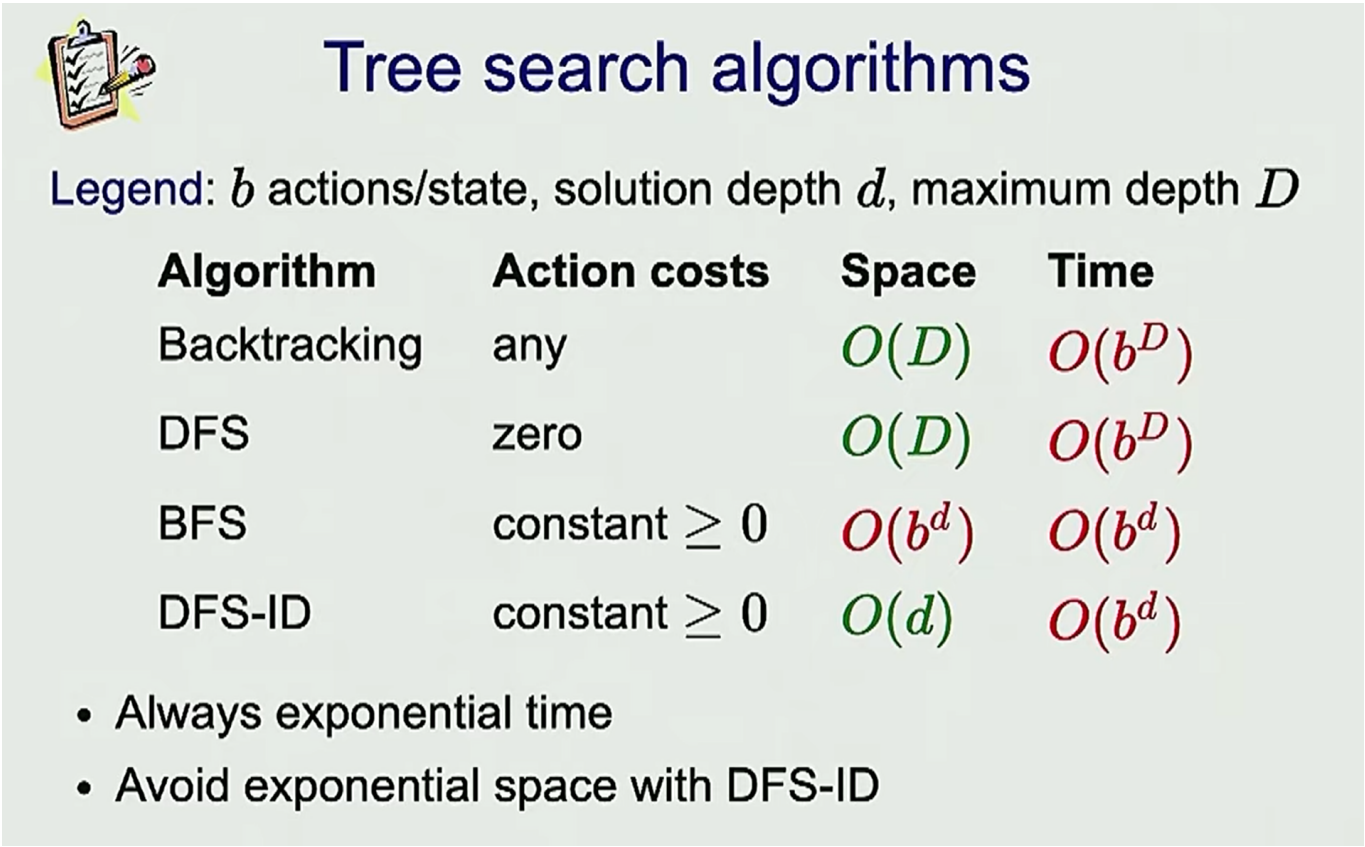
DFS - Itrative deepening

A combination of BFS and DFS



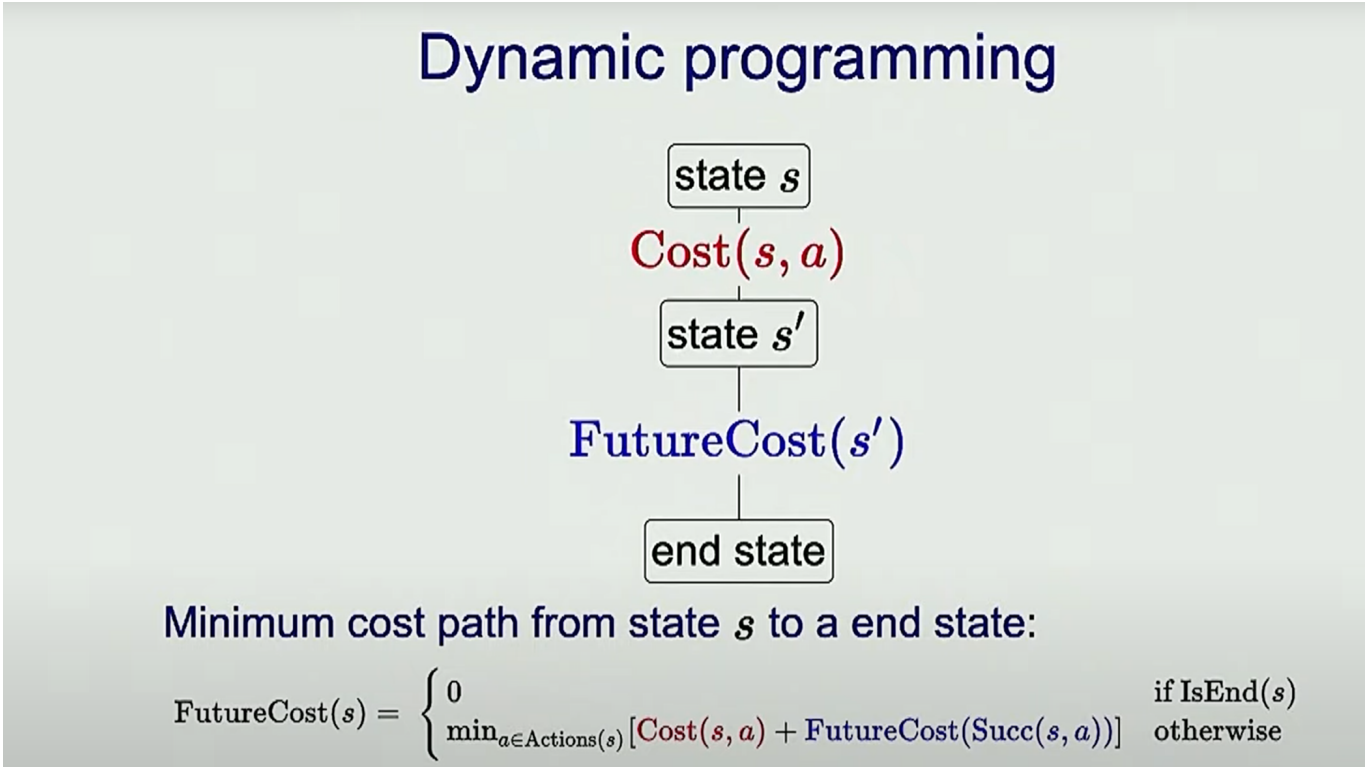
You can visit the same node multiple times, because u are runnning DFS each time u proceed

SO in the worst case, when u have to search through the whole tree, the actual time complexity is O(b^d * b^d) => o((b^d)^2) => b^d*d However, in time complexity O(), there's no big difference between b^ddd and b^d, cuz we care about the 数量级 only



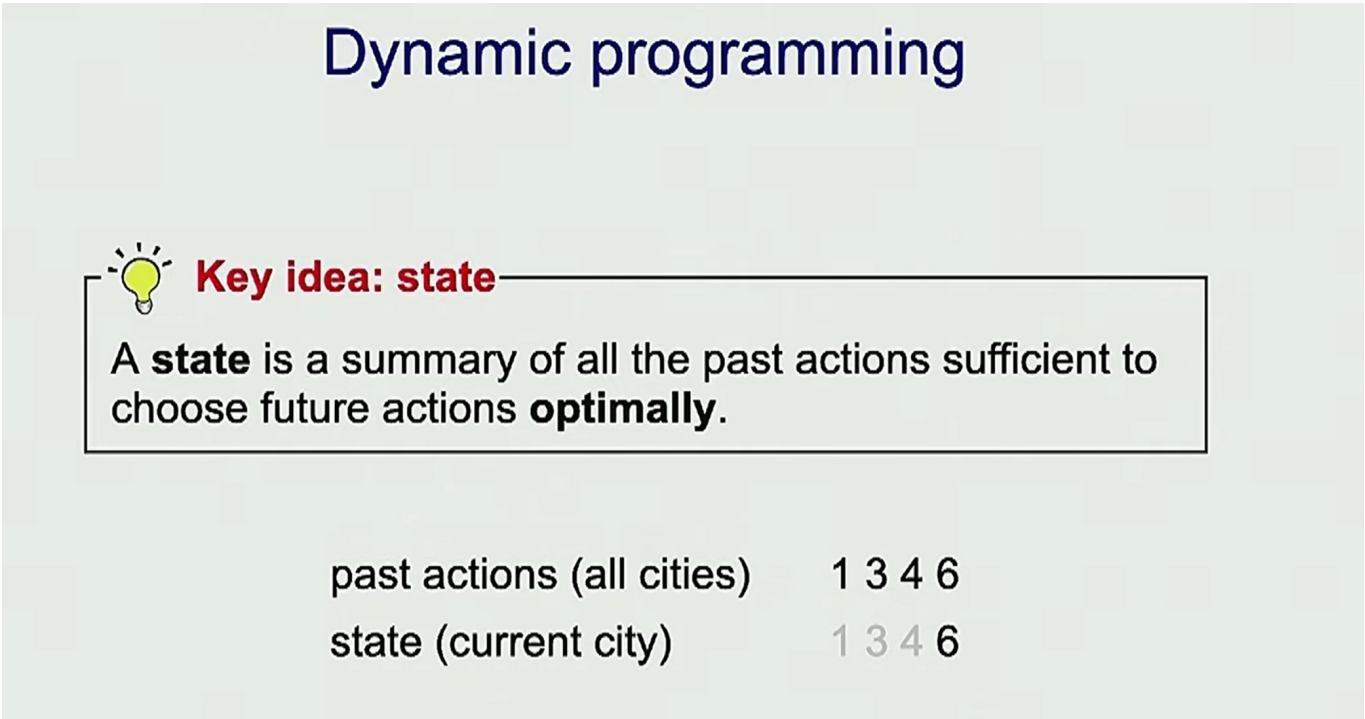
The exponential time is not good, that's when dynamic programming comes to paly 😃

Dynamic Programming



Reduce the recomputation

State: a summary of all past actions



Limitation

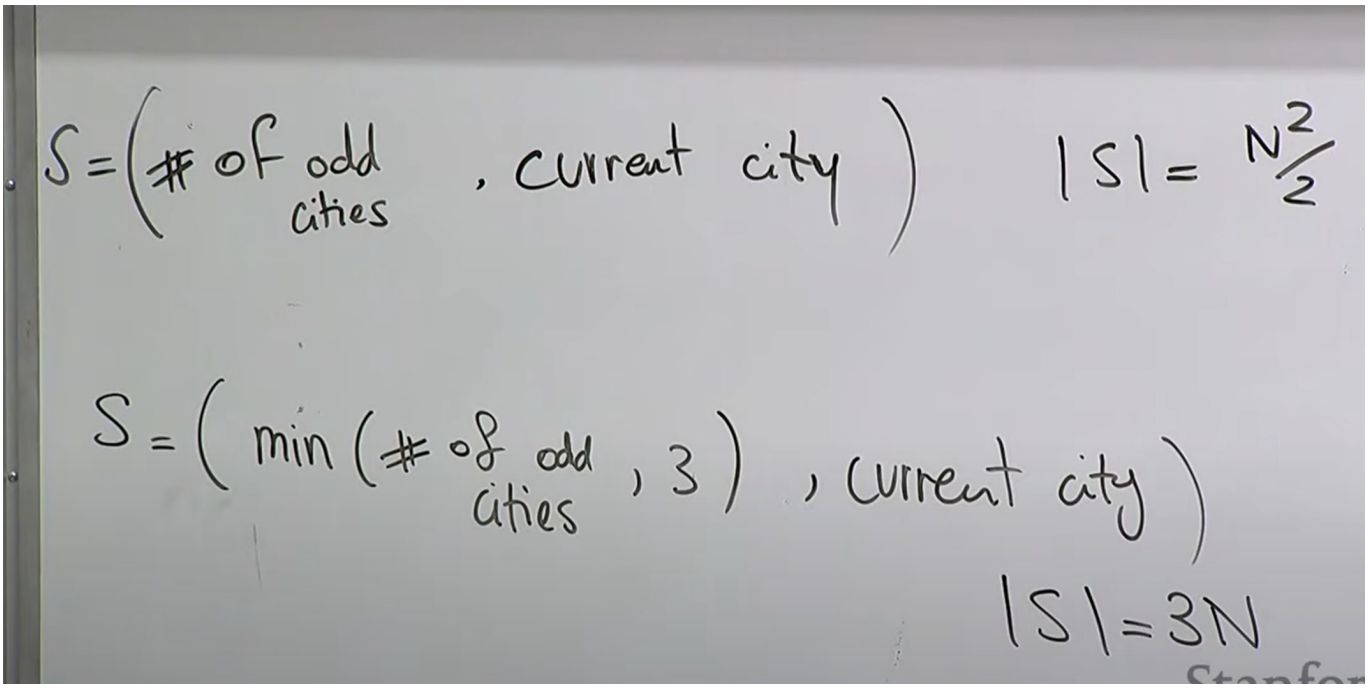
It does not run well with grapth with cycles

Define the state

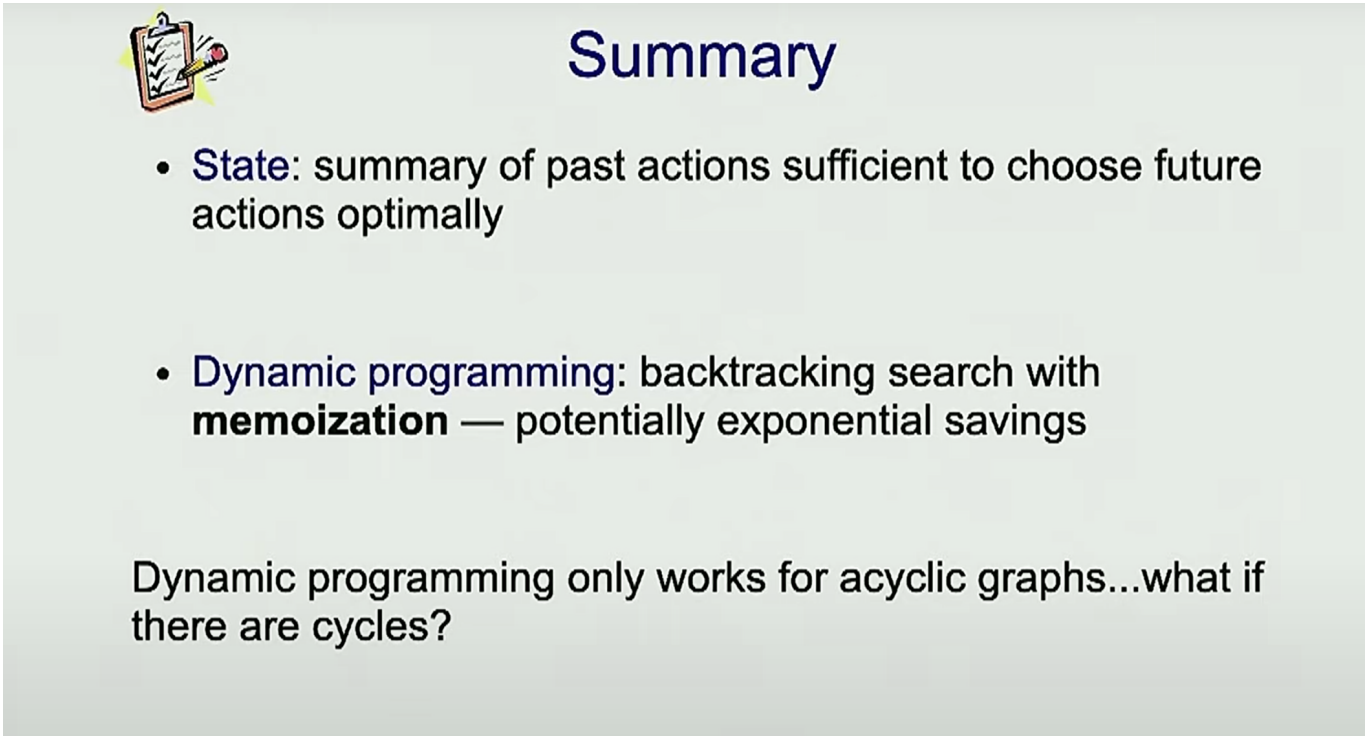
What if we have a rule saying u cannot go 3 odd cities in a row We have to have a context of our past actions

State [prev city, cur city] state space: n^2 The state space is too big, thus the program will be complax =>

State [bool(if prev was odd), cur city] state space: 2*n



For example #means number This can bring the complexity of state space from n^2 => 3n, which is linear



Bring the time O() from expential => polinomial

Uniform Cost Search

Uniform cost search (UCS)

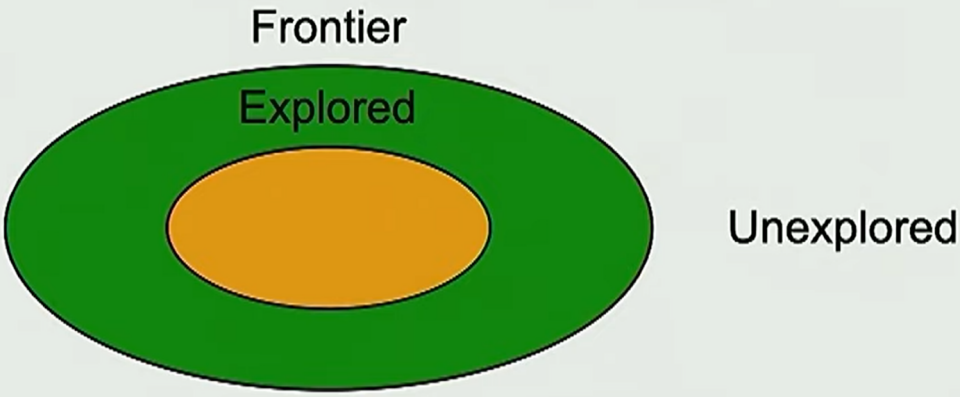
Key idea: state ordering

UCS enumerates states in order of increasing past cost.

Assumption: non-negativity

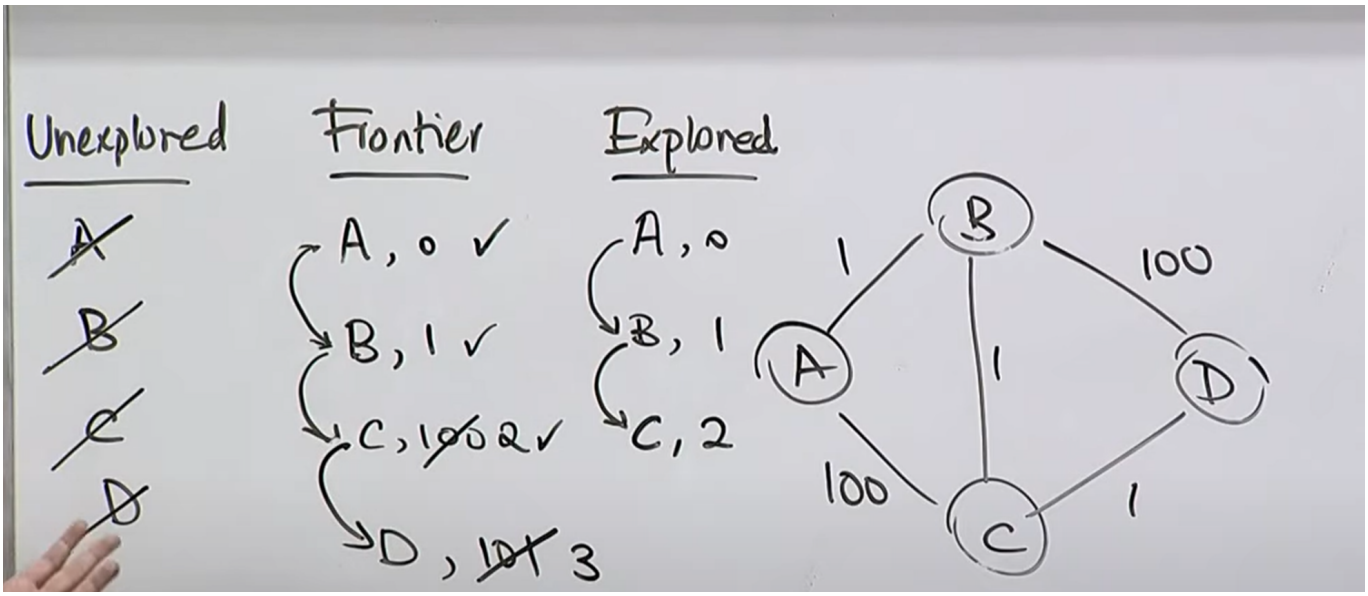
All action costs are non-negative: $\text{Cost}(s, a) \geq 0$.

High-level strategy



- Explored: states we've found the optimal path to
- Frontier: states we've seen, still figuring out how to get there cheaply
- Unexplored: states we haven't seen

Forntier: Explored by not sure about the optimal path to get there YET



In runtime, pop out the best one from forntier

What is the difference from A*??

Code

```
from enum import Enum
import sys
from queue import PriorityQueue

sys.setrecursionlimit(100000)

Destination = 10

### Model (Search Problem)
class TransportationProblem(object):

    WALK_COST = 1
    TRAM_COST = 2

    WALK = "walk"
    TRAM = "tram"

    def __init__(self, destination):
        # N number of blocks
        self.destination = destination

    def startState(self) -> int:
        return 1

    def isEnd(self, state):
        return state == self.destination

    def succAndCost(self, state : int):
        '''
```

```
        Return a list of (action, newState, cost) triples
        Meaning return the a list of: actions we can take, what new state we gonna
endup at, and what the cost gonna be
        '''
        result = []
        if(state + 1 <= self.destination):
            result.append((self.WALK, state + 1, self.WALK_COST))

        if(state * 2 <= self.destination):
            result.append((self.TRAM, state * 2, self.TRAM_COST))

        return result


### Algorithms
def backtrackingSearch(self, problem):

    best = {
        "totalCost" : sys.maxsize,
        "history": None
    }

    memo = {}

    def recurse(currentState, history, totalCost):

        if(currentState in memo):
            for totalCost, history in memo[currentState]:
                best["totalCost"] = totalCost
                best["history"] = history
            return

        if(problem.isEnd(currentState)):
            #Update the best cost if we find a better solution
            if(totalCost < best["totalCost"]):
                best["totalCost"] = totalCost
                best["history"] = history
                memo[currentState] = (totalCost, history)

        for action, newState, cost in problem.succAndCost(currentState):
            recurse(newState, history + [(action, newState, cost)], totalCost +
cost)

    recurse(problem.startState(), history=[], totalCost=0)

    return best

def printSolution(solution):
    totalCost = solution["totalCost"]
    history = solution["history"]
    print("minimum cost is {}".format(totalCost))

    for h in history:
        print(h)
```

```python
# You just need to know the current state
def dynamicProgramming(problem):

    memo = {} # state -> futureCost(state) action, newState, cost
    def futureCost(state):

        if problem.isEnd(state):
            return 0

        if state in memo:
            return memo[state][0]

        minFutureCostWithAction = min(
            (curCost + futureCost(newState), action, newState, curCost)
            for action, newState, curCost in problem.succAndCost(state)
        )

        memo[state] = minFutureCostWithAction
        minFutureCost = minFutureCostWithAction[0]
        return minFutureCost


    state = problem.startState()
    minCost = futureCost(state)

    # Recover History
    history = []
    while not problem.isEnd(state):
        _, action, newState, cost= memo[state]
        history.append((action, newState, cost))
        state = newState

    return {
        "totalCost" : minCost,
        "history": history
    }



# searching method in a graph that visits nodes in order of their path cost from
the start node.
# It delves into the graph, visiting the node with the smallest cumulative path
cost first.
def UniformCostSearch(problem):
    frontier = PriorityQueue()
    frontier.put((0, problem.startState()))

    while(True):
        # Move minumun cost from frontier to explored
        pastCost, state = frontier.get()
```

```
        # If the end state popped up, meaning we have already have the minumum
cost to the end state
        # Everything else left in the queue would have a larger past cost.
        # If it's not (The end state may or may not in the queue yet), meaning
there's other possible path
        # That may have a lower cost
        if(problem.isEnd(state)):
            return {
                "totalCost" : pastCost,
                "history": []
            }

        # Add all the successors of current state to frontier
        for action, newState, cost in problem.succAndCost(state):
            frontier.put((pastCost + cost, newState))




### Inference
problem = TransportationProblem(destination=Destination)
# solution = problem.backtrackingSearch(problem)
solution = dynamicProgramming(problem)
printSolution(solution)

# solution = UniformCostSearch(problem)
# problem.printSolution(solution)
# print(problem.succAndCost(9))
```