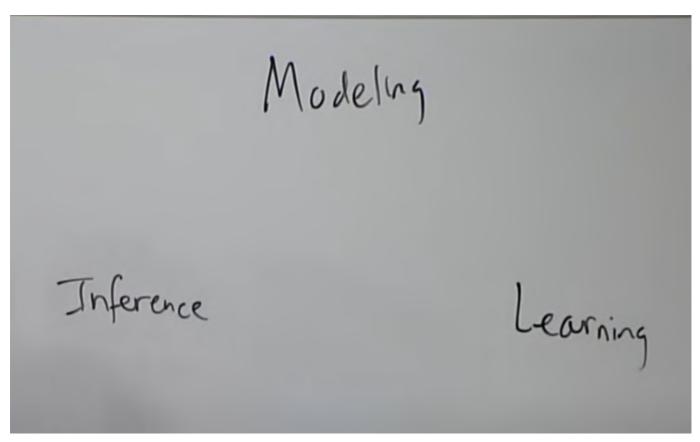
### Intro



Models: MDP Inference Algorithms: be able to use model to predict and answer queries

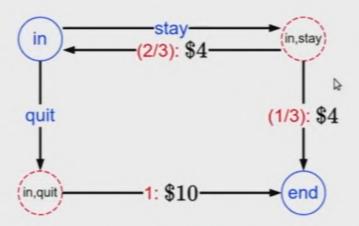
- Value iteration: Compute the optimal policy.
- Policy evaluation: Estmate the value for a particular policy.

Learning: How you actually learn these models

• And this lecture is going to be about learning - Reinforcement learning

#### Review

## Review: MDPs





### **Definition: Markov decision process**

States: the set of states

 $s_{\mathrm{start}} \in \mathrm{States}$ : starting state

Actions(s): possible actions from state s

 $T(s,a,s^\prime)$ : probability of  $s^\prime$  if take action a in state s

 $\operatorname{Reward}(s,a,s')$ : reward for the transition (s,a,s')

 $\operatorname{IsEnd}(s)$ : whether at end of game

 $0 \le \gamma \le 1$ : discount factor (default: 1)

Policy A mapping from states to action A policy tells u where to go/which action to take

When we run a policy, we get a path unti you hit the end state(In RL it's called an episode)

What happend out of the episode: You can look at the utility: The discounted sum of rewards along the way.

### Review: MDPs

- Following a **policy**  $\pi$  produces a path (**episode**)  $s_0; a_1, r_1, s_1; a_2, r_2, s_2; a_3, r_3, s_3; \ldots; a_n, r_n, s_n$
- Value function  $V_{\pi}(s)$ : expected utility if follow  $\pi$  from state s

$$V_{\pi}(s) =_{\triangleright} egin{cases} 0 & ext{if IsEnd}(s) \ Q_{\pi}(s,\pi(s)) & ext{otherwise.} \end{cases}$$

• **Q-value** function  $Q_{\pi}(s, a)$ : expected utility if first take action a from state s and then follow  $\pi$ 

$$Q_{\pi}(s, a) = \sum_{s'} T(s, a, s') [\text{Reward}(s, a, s') + \gamma V_{\pi}(s')]$$

In last lecture we didn't really worked with utility, because we can use iterative method to compute/approximate the expected utility(The value)

## Unknown transitions and rewards



### Definition: Markov decision process-

States: the set of states

 $s_{ ext{start}} \in ext{States}$ : starting state

Actions(s): possible actions from state s

T(s, a, s'): probability of s' if take action a in state s

 $\operatorname{Reward}(s,a,s')$ : reward for the transition (s,a,s')

 $\operatorname{IsEnd}(s)$ : whether at end of game

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## Unknown transitions and rewards



### Definition: Markov decision process-

States: the set of states

 $s_{ ext{start}} \in ext{States}$ : starting state

 $\operatorname{Actions}(s)$ : possible actions from state s

 $\operatorname{IsEnd}(s)$ : whether at end of game

 $0 \leq \gamma \leq 1$ : discount factor (default: 1)

### reinforcement learning!

IF you say goodbye to transition and rewards, that is called reinforcement learning

For MDP: I give you everything there, and u just need to find the optimal policy For ML: We don't know what rewards and transition(probability) that we are gonna get

### Reinforcement Learning Intro

MDP and RL

# From MDPs to reinforcement learning



### -Markov decision process (offline)-

- Have mental model of how the world works.
- Find policy to collect maximum rewards.



### Reinforcement learning (online)-

- · Don't know how the world works.
- Perform actions in the world to find out and collect rewards.

#### MDP is a offline thing

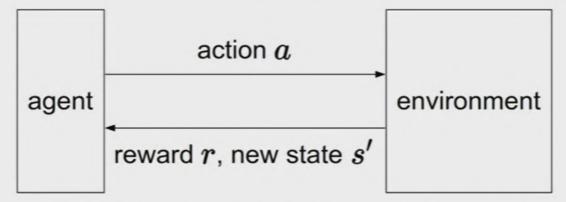
- You already have a exact model of how the "world" works
- All you need to do is just sit down and think(Compute) the best policy that collects the best rewards.

#### RL is a online thing

- We don't know how the "real world" works
- So we cannot just sit there and think(Compute) because it won't help at all
- We have to go out and perform action in the real worlds, by doing so, hopefully we'll learn something and we'll get some rewards.

#### General Framework

# Reinforcement learning framework



### Algorithm: reinforcement learning template-

For 
$$t=1,2,3,\ldots$$
 Choose action  $a_t=\pi_{\mathrm{act}}(s_{t-1})$  (how?) Receive reward  $r_t$  and observe new state  $s_t$  Update parameters (how?)

You can treat yourself as an agent You take action a to the environment, the environment returns you a new state s' and reward r Two main quesions here:

- What am I gonna act(Which action should I choose)
- Upon recived reward and new state, How/what should I do to update my model of "the world".

### Monte Carlo methods

#### Model-based Monte Carlo

Monte Carlo: means using simples

In the beginning of RL, we have nth but data Let's try to build MDP from that data (Estimate MDP)

## Model-based Monte Carlo

Data:  $s_0; a_1, r_1, s_1; a_2, r_2, s_2; a_3, r_3, s_3; \ldots; a_n, r_n, s_n$ 

-``

Key idea: model-based learning—

Estimate the MDP:  $T(s,a,s^\prime)$  and  $\mathrm{Reward}(s,a,s^\prime)$ 

#### Transitions:

$$\hat{T}(s,a,s') = rac{\# ext{ times } (s,a,s') ext{ occurs}}{\# ext{ times } (s,a) ext{ occurs}}$$

#### Rewards:

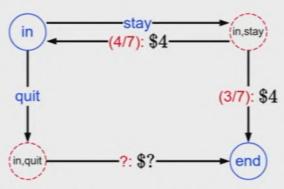
$$\widehat{\operatorname{Reward}}(s,a,s') = r \text{ in } (s,a,r,s')$$

In simple, we just need to figure out what the transitions and rewards are.

- Transition: Collect the data and check the ratio
- Reward: Just observe the reward that the env returns.

Q/A: Do we always know all states and actions? For states we can just observe/collect as they come For actions we have to know becasue we are an agent and we have to play the game

## Model-based Monte Carlo



Data (following policy  $\pi(s) = \mathrm{stay}$ ):

[in; stay, 4, end]

- Estimates converge to true values (under certain conditions)
- With estimated MDP  $(\hat{T}, \widehat{\operatorname{Reward}})$ , compute policy using value iteration

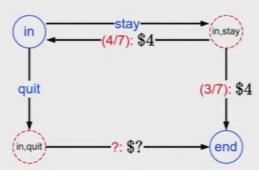
So it works like: You gather a bunch of data (Episodes), estimate them, build MDP, then compute

#### Potential Problem

You might not explored all the states/rewards

Unless you have a policy let u goes and covers all the states

### **Problem**



Problem: won't even see (s,a) if  $a 
eq \pi(s)$   $(a = \operatorname{quit})$ 



## Key idea: exploration—

To do reinforcement learning, need to explore the state

Solution: need  $\pi$  to **explore** explicitly (more on this later)

So we need to figure out how to get the data, to explore the state space, which is one of the key challenge of RL Solution: We need a random policy

#### Model-free Monte Carlo

### From model-based to model-free

$$\hat{Q}_{\mathrm{opt}}(s,a) = \sum_{s'} \hat{T}(s,a,s') [\widehat{\mathrm{Reward}}(s,a,s') + \gamma \hat{V}_{\mathrm{opt}}(s')]$$

All that matters for prediction is (estimate of)  $Q_{\mathrm{opt}}(s,a)$ .



Key idea: model-free learning

Try to estimate  $Q_{
m opt}(s,a)$  directly.

For MDP, at the end of the day, all we need is Qopt(s,a) Qopt(s,a) is: The maximun possible utility I could get if I'm in chance node (s,a) and I followed the optimal policy

So if we know Qopt(s,a), we would just need to follow the optimal policy and it's done, we don't need to know about the rewards/transitions.

So why don't we estimate Qopt directly, without building up the whole MDP model

### Model-free Monte Carlo

Data (following policy  $\pi$ ):

$$s_0; a_1, r_1, s_1; a_2, r_2, s_2; a_3, r_3, s_3; \ldots; a_n, r_n, s_n$$

Recall:

 $Q_{\pi}(s,a)$  is expected utility starting at s, first taking action a, and then following policy  $\pi$ 

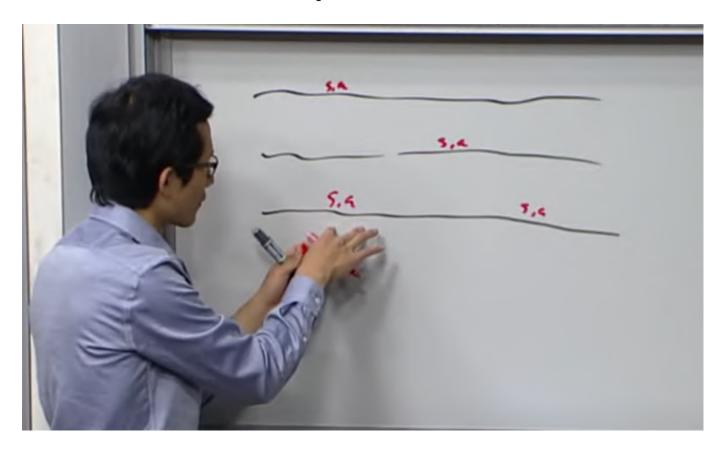
**Utility:** 

$$u_t = r_t + \gamma \cdot r_{t+1} + \gamma^2 \cdot r_{t+2} + \cdots$$

Estimate:

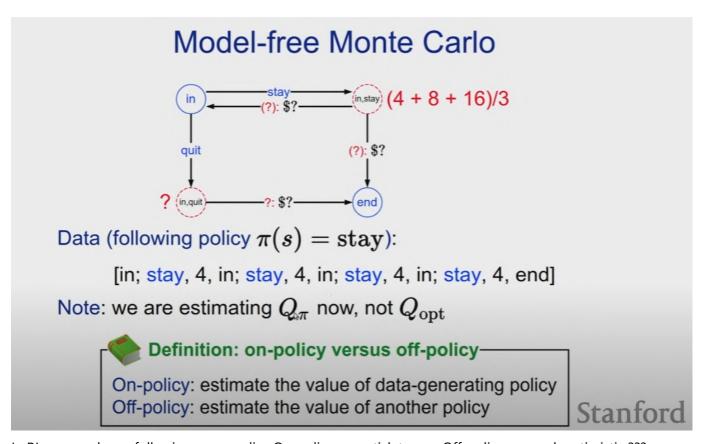
$$\hat{Q}_{\pi}(s,a) = ext{average of } u_t ext{ where } s_{t-1} = s, a_t = a_t$$
 (and  $s,a$  doesn't occur in  $s_t$ , Stanford

SO this means, you only Average the utility that you get, only on a timestamp (t) that i am in a particular state s and I (keep)took action a.



We just avarage whenever we see a utility of s,a but no double counting

The small hat ^ on each notion means that: this comes form some quantity(estimate) and differentiate it from the true value



In RL we are always following some policy On-policy: you stick to one Off-policy: general, optimistic ???

Q/A: Is monte free always follow the same policy? A: It is just for this particular example, it is a on policy In other examples we can also try optimize the policy

Q/A: Which one is better, model free or model based? Interesting question. If u have a exact/correct model for the "world", model-based is kind of the way to go, becasue you need less data points and less computation(u already know the model!)

However in real world it is really hard to get the model 100% correct. Now with deep RL many ppl go for model free cuz u can solve diffcult problem without constructing the MDP

## Model-free Monte Carlo (equivalences)

Data (following policy  $\pi$ ):

$$s_0; a_1, r_1, s_1; a_2, r_2, s_2; a_3, r_3, s_3; \ldots; a_n, r_n, s_n$$

Coriginal formulation-

$$\hat{Q}_{\pi}(s,a) = ext{average of } u_t ext{ where } s_{t-1} = s, a_t = a$$

Equivalent formulation (convex combination)

On each (s, a, u):

$$\eta = \frac{1}{1 + (\# \text{ updates to } (s,a))}$$

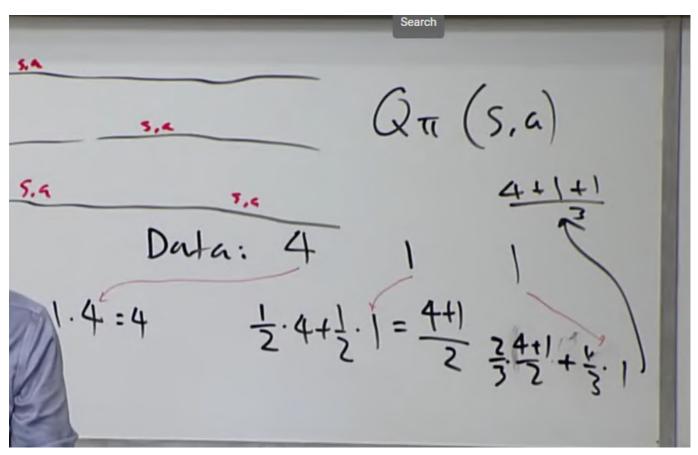
$$\hat{Q}_{\pi}(s,a) \leftarrow (1-\eta)\hat{Q}_{\pi}(s,a) + \eta u$$

Interpretations

#### **Avarange**

We've talked about it

#### **Convex Combination**



Theme variations N: number of occurences It is balancing between the old utilities/values that I had and the new utility that I saw

The old is with weight 1-n, the new one is weight n = 1/number of occurences

#### **Stochastic Gradient Decent**

# Model-free Monte Carlo (equivalences)

-Equivalent formulation (convex combination)

On each (s, a, u):

$$\hat{Q}_{\pi}(s,a) \leftarrow (1-\eta)\hat{oldsymbol{Q}}_{\pi}(s,a) + \eta u$$

Equivalent formulation (stochastic gradient)

On each (s, a, u):

$$\hat{Q}_{\pi}(s,a) \leftarrow \hat{Q}_{\pi}(s,a) - \eta [\underbrace{\hat{Q}_{\pi}(s,a)}_{ ext{prediction}} - \underbrace{u}_{ ext{target}}]$$

Implied objective: least squares regression

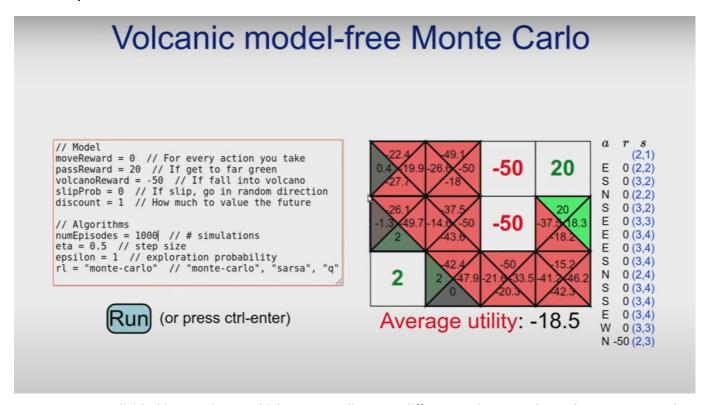
$$(\hat{Q}_{\pi}(s,a) = u)^2$$

u: the new data u've got All the updates are like:

Residual = Prediction - Target

This is implicitly doing SGD on the square loss of the Residual, so u want them to close to each other

#### An Exapmle



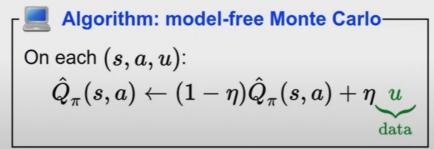
A state (square) divided into 4 pieces, which corrsponding to 4 different actions(North south east west), and the number there is the Qpi value if take that action from that state The policy is: move comletely

### Bootstrapping

## Using the utility

## Data (following policy $\pi(s) = \mathrm{stay}$ ):

$$\begin{array}{ll} \text{[in; stay, 4, end]} & u=4 \\ \text{[in; stay, 4, in; stay, 4, end]} & u=8 \\ \text{[in; stay, 4, in; stay, 4, in; stay, 4, end]} & u=12 \\ \text{[in; stay, 4, in; stay, 4, in; stay, 4, in; stay, 4, end]} & u=\frac{1}{8}6 \end{array}$$



Each episode has a utility asscioated with it

The idea was to use avarange, with large amount of data to compute, u can get close to true value But with limited data the variance is big

The key idea is?

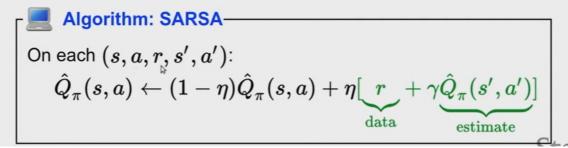
#### **SARSA**

## Using the reward + Q-value

Current estimate:  $\hat{Q}_{\pi}(s, ext{stay}) = 11$ 

Data (following policy  $\pi(s) = \text{stay}$ ):

$$\begin{array}{ll} \hbox{[in; stay, 4, end]} & 4+0 \\ \hbox{[in; stay, 4, in; stay, 4, end]} & 4+11 \\ \hbox{[in; stay, 4, in; stay, 4, in; stay, 4, end]} & 4+11 \\ \hbox{[in; stay, 4, in; stay, 4, in; stay, 4, in; stay, 4, end]} & 4+11 \\ \end{array}$$



Why called SARSA: You are in state s, you took action a, you got a reward r, and you end up in state s', and

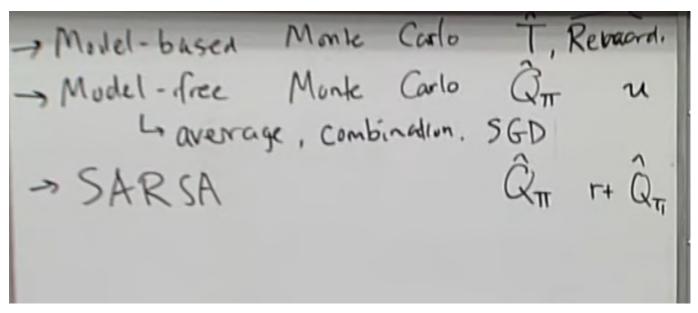
you took another action a'.

So for every quintuple that you see, you are gonna perform an update. And this is the convex combination, where you take part of the old value and try to merge them with the new value.

The new value is: the **immidiate reward** + the discount of your estimate.

Remember what is the estimate: It is trying to be the expectation of rewards that you will get in the future.

So if Qpi estimate could be close to the true value then this will be a lot better cuz it will reduce the variance.



SO Model-free monte carlo is updating based on u SARSA is updating based on immidiate reward r + the discount of your estimate.

## Model-free Monte Carlo versus SARSA



Key idea: bootstrapping-

SARSA uses estimate  $\hat{Q}_{\pi}(s,a)$  instead of just raw data u.

u

based on one path

unbiased

large variance

wait until end to update

 $r+\hat{Q}_{\pi}(s',a')$ 

based on estimate

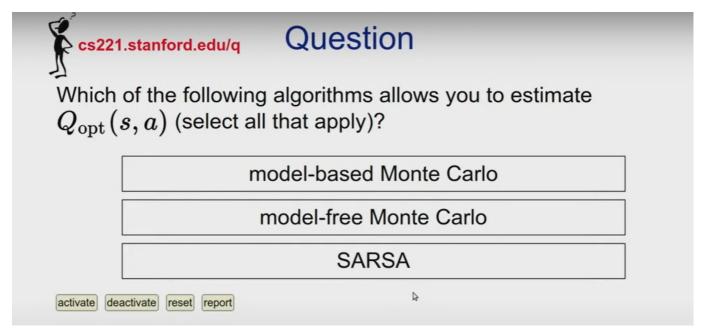
biased

small variance

can update immediately

SARSA is biased because it's based on its previous expirences Monte Carlo - wait until end to update: Cuz you

need to reach the end state of your model so that u will know the utility But for SARSA you can immmidiately update because all u need to see is just a local windows of (s,a,r,s',a')



A. With the exact model u can compute everything B & C. Model-free and SARSA are estimating the Qpi(the value of a policy) but not Qopt, which is a problem, because all we want is the Oopt That's when we need Q-learning which allows you to get Qopt

## Q-learning

Problem: model-free Monte Carlo and SARSA only estimate  $Q_{\pi}$ , but want  $Q_{\mathrm{opt}}$  to act optimally

Output	MDP	reinforcement learning
$Q_{\pi}$	policy evaluation	model-free Monte Carlo, SARSA
$Q_{ m opt}$	value iteration	Q-learning

Q-learning

## Q-learning

#### MDP recurrence:

$$Q_{ ext{opt}}(s, a) = \sum_{s'} T(s, a, s') [ ext{Reward}(s, a, s') + \gamma V_{ ext{opt}}(s')]$$

### Algorithm: Q-learning [Watkins/Dayan, 1992]-

On each 
$$(s, a, r, s')$$
:  $\hat{Q}_{\mathrm{opt}}(s, a) \leftarrow (1 - \eta) \underbrace{\hat{Q}_{\mathrm{opt}}(s, a)}_{\mathrm{prediction}} + \eta \underbrace{(r + \gamma \hat{V}_{\mathrm{opt}}(s'))}_{\mathrm{target}}$  Recall:  $\hat{V}_{\mathrm{opt}}(s') = \max_{a' \in \mathrm{Actions}(s')} \hat{Q}_{\mathrm{opt}}(s', a')$ 

## SARSA versus Q-learning



### Algorithm: SARSA-

On each (s, a, r, s', a'):

$$\hat{Q}_{\pi}(s,a) \leftarrow (1-\eta) \hat{Q}_{\pi}(s,a) + \eta(r+\gamma \hat{Q}_{\pi}(s',a'))$$

### Algorithm: Q-learning [Watkins/Dayan, 1992]-

$$\begin{aligned} &\text{On each } (s, a, r, s') : \\ &\hat{Q}_{\text{opt}}(s, a) \leftarrow (1 - \eta) \hat{Q}_{\text{opt}}(s, a) + \eta (r + \gamma \max_{a' \in \text{Actions}(s \nmid s)} \hat{Q}_{\text{opt}}(s', a')) ] \end{aligned}$$

For Q-learning it doesn't metter what action a' you gonna took for the next step, cuz we'll just take the one that maximizes(optimal one).

This is why SARSA is estimating the value of a policy

Cuz what a' shows up there is a function of policy

and Q-learning is estimating the optimal policy cuz i don't care the next action, i'll just take the maximum(optimal) one

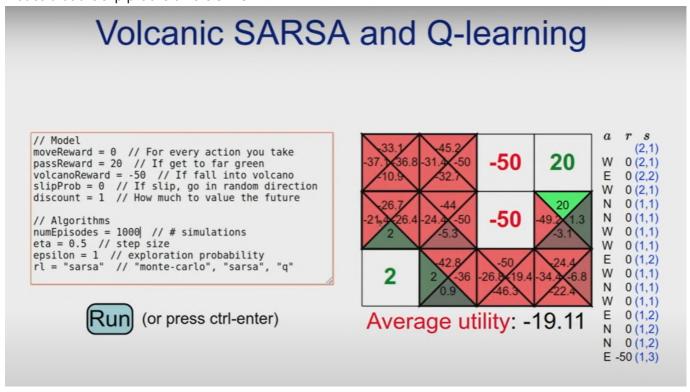
This is same ituiation of policy evaluaiton and value iteration.

Q/A: Is q-learning on-policy or off-policy A: off-policy, cuz i am following whatever policy i can follow and estimate the value of the optimal policy, which is probably not the original policy at the beginning

While SARSA is a on-policy because i am estimating the value of Qpi

#### Volcano example

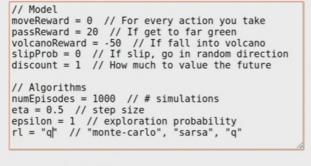
Notice that the slip prob is 0 here SARSA



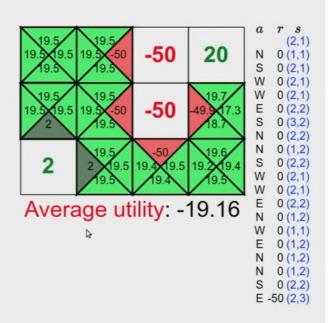
The value here is Qpi, which is the value of the policy i'm following(the random policy)

#### Q-learning

## Volcanic SARSA and Q-learning







The value of optimal policy Learning how to hehave optimally

### **Encountering the Unknown**

So we've just mentioned the algothrims. If i hand u some data and told u there's a fixed policy, you can estimate all those values



## **Exploration**



Algorithm: reinforcement learning template-

For 
$$t=1,2,3,\ldots$$

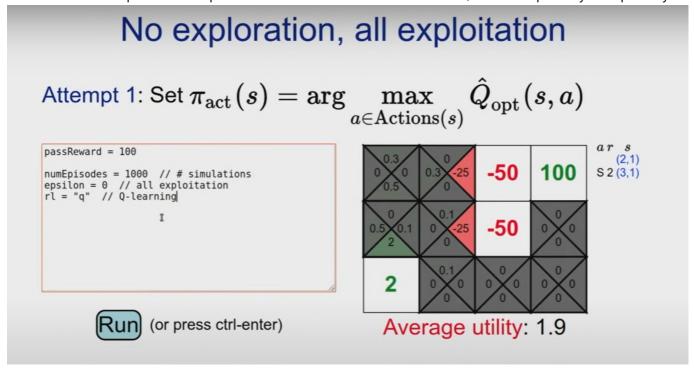
Choose action  $a_t = \pi_{\mathrm{act}}(s_{t-1})$  (how?)

Receive reward  $r_t$  and observe new state  $s_t$  Update parameters (how?)

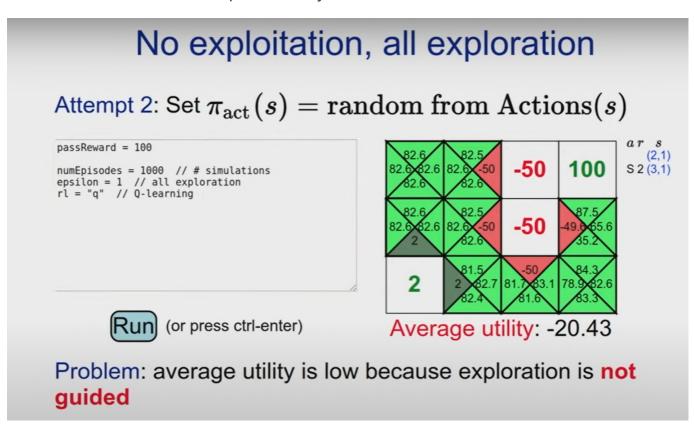
$$s_0; a_1, r_1, s_1; a_2, r_2, s_2; a_3, r_3, s_3; \ldots; a_n, r_n, s_n$$

Which **exploration policy**  $\pi_{\mathrm{act}}$  to use?

However there's a question of exploration: We don't even see all the states, how do u possibly act optimally



In this case, once we found a local optimium, we'll just stick to it lol



I just try differently way and learn from it, however the average utility is still bad cuz the policy was just random policy The exploration is **not guided** 

## Exploration/exploitation tradeoff



Key idea: balance-

Need to balance exploration and exploitation.



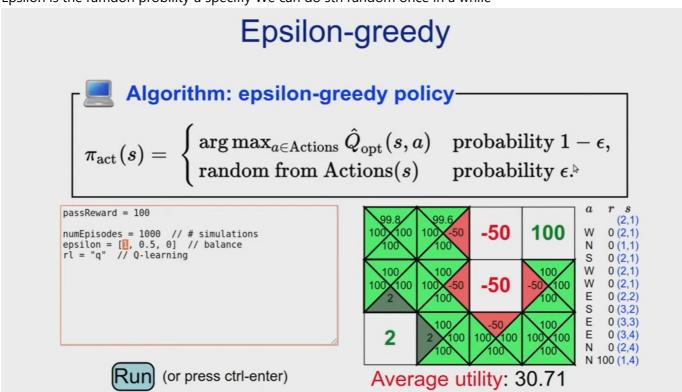
### Examples from life: restaurants, routes, research

RL kind of capture the life You are aiming for a best reward, you have to learn how the world works and improves ur policy

#### **Epsilon-greedy**

Balancing the exploration and exploitation This assumes that ur doing Q-learning

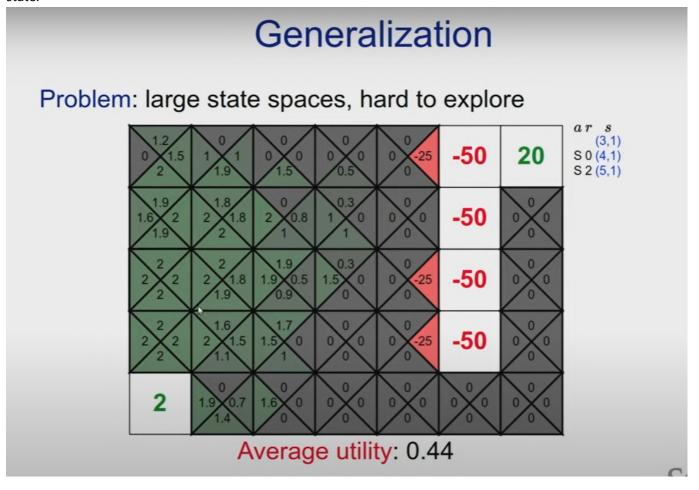
Epsilon is the ramdon probility u specifiy We can do sth random once in a while



Epsilon When u turns older, you start to explore less and exploit more lol

#### Generalization

The problem is that the state space is huge and there's no way you can go down and track every possible state.



Large state space

## Q-learning

### Stochastic gradient update:

$$\hat{Q}_{\mathrm{opt}}(s, a) \leftarrow \hat{Q}_{\mathrm{opt}}(s, a) - \eta [\underbrace{\hat{Q}_{\mathrm{opt}}(s, a)}_{\mathrm{prediction}} - \underbrace{(r + \gamma \hat{V}_{\mathrm{opt}}(s'))}_{\mathrm{target}}]$$

This is **rote learning**: every  $\hat{Q}_{ ext{opt}}(s,a)$  has a different value

Problem: doesn't generalize to unseen states/actions

## I can't generalize between states and actions. Um, okay

I we follow the current way, we cannot generalize between states and actions

#### **Function approximation**

Tell you new states that you haven't see before

## **Function approximation**



Key idea: linear regression model-

Define **features**  $\phi(s,a)$  and **weights**  ${f w}$ :

$$\hat{Q}_{ ext{opt}}(s,a;\mathbf{w}) = \mathbf{w} \cdot \phi(s,a)$$

## Example: features for volcano crossing-

$$egin{aligned} \phi_1(s,a) &= \mathbf{1}[a=\mathrm{W}] & \phi_7(s,a) &= \mathbf{1}[s=(5,*)] \ \phi_2(s,a) &= \mathbf{1}[a=\mathrm{E}] & \phi_8(s,a) &= \mathbf{1}[s=(*,6)] \ & ... \end{aligned}$$

Featrues: eg: it's better to go to the West/East It's better to be at the 5th row/ 6th col

So we have a smaller set of features that we can try to use it, to generalize different states you might see.



Algorithm: Q-learning with function approximation-

On each 
$$(s, a, r, s')$$
:  $\mathbf{w} \leftarrow \mathbf{w} - \eta [\hat{Q}_{\mathrm{opt}}(s, a; \mathbf{w}) - \underbrace{(r + \gamma \hat{V}_{\mathrm{opt}}(s'))}_{\mathrm{target}}] \phi(s, a)$ 

Implied objective function:

$$(\underbrace{\hat{Q}_{ ext{opt}}(s, a; \mathbf{w})}_{ ext{prediction}} - \underbrace{(r + \gamma \hat{V}_{ ext{opt}}(s'))}_{ ext{target}})^2$$

You take ur weight vector, take an update of redidual \* the feature vector. eta: like step size

## Covering the unknown



Epsilon-greedy: balance the exploration/exploitation tradeoff

Function approximation: can generalize to unseen states

### Summary



## Summary so far

- Online setting: learn and take actions in the real world!
- Exploration/exploitation tradeoff
- Monte Carlo: estimate transitions, rewards, Q-values from data
- Bootstrapping: update towards target that depends on estimate rather than just raw data

## Challenges in reinforcement learning

Binary classification (sentiment classification, SVMs):

· Stateless, full feedback

Reinforcement learning (flying helicopters, Q-learning):

Stateful, partial feedback



Key idea: partial feedback—Only learn about actions you take.



Key idea: state-

Rewards depend on previous actions ⇒ can have delayed rewards.

Stanford

## States and information

stateless state

full feedback

supervised learning (binary classification)

supervised learning (structured prediction)

partial feedback

multi-armed bandits

reinforcement learning

## Deep reinforcement learning

just use a neural network for  $\hat{Q}_{ ext{opt}}(s,a)$ 

Playing Atari [Google DeepMind, 2013]:



- last 4 frames (images) ⇒ 3-layer NN ⇒ keystroke
- $\epsilon$ -greedy, train over 10M frames with 1M replay memory
- Human-level performance on some games (breakout), less good on others (space invaders)

Reinforcement learning is generally really hard, because of enumorous states and delayed feedback.

## **Applications**



Autonomous helicopters: control helicopter to do maneuvers in the air



Backgammon: TD-Gammon plays 1-2 million games against itself, human-level performance

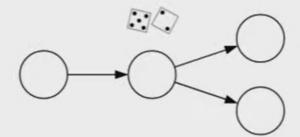


Elevator scheduling; send which elevators to which floors to maximize throughput of building



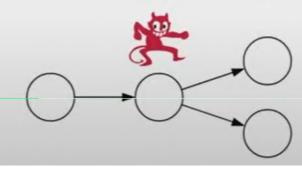
Managing datacenters; actions: bring up and shut down machine to minimize time/cost

## Markov decision processes: against nature (e



## Next time...

Adversarial games: against opponent (e.g., cl



For RL we are playing against nature Next time we'll play against an opponent who are trying to get us