MATH 151A Homework 3

Zheng Wang

March 8, 2019

Question 1

(a) The Lagrange interpolation is the following:

$$\begin{aligned} h(x) &= f(x_0)\frac{x(x-1)}{x_0(x_0-1)} + f(x_1)\frac{(x+1)(x-1)}{(x_1+1)(x_1-1)} + f(x_2)\frac{x(x+1)}{(x_2+1)x_2} \\ &= \frac{f(-1)}{2}\cdot(x^2-x) - f(0)\cdot(x^2-1) + \frac{f(1)}{2}\cdot(x^2+x) \end{aligned}$$

(b) The error term is given by the following:

$$E(x) = \frac{f'''(\xi(x))}{3!}(x-x_0)(x-x_1)(x-x_2) = \frac{f'''(\xi(x))}{6}x(x+1)(x-1)$$

(c) The result of the integral are as follows:

$$\begin{aligned} \int_{-1}^{1} h(x)dx &= \int_{-1}^{1} \frac{f(-1)}{2}\cdot(x^2-x) - f(0)\cdot(x^2-1) + \frac{f(1)}{2}\cdot(x^2+x) \\ &= \frac{f(-1)}{2}\cdot\frac{2}{3} + f(0)\cdot\frac{4}{3} + \frac{f(1)}{2}\cdot\frac{2}{3} \\ &= \frac{1}{3}\cdot f(-1) + \frac{4}{3}\cdot f(0) + \frac{1}{3}\cdot f(1) \end{aligned}$$

(d) The statement is true and the result will be exact. This is because if $f(x)$ is a polynomial of degree less than or equal to 2, then we have $f'''(x) = 0 \quad \forall x \in \mathbb{R}$. Thus $E(x) = 0 \quad \forall x \in \mathbb{R}$ Then we have the following:

$$\int_{-1}^{1} f(x)dx = \int_{-1}^{1} [h(x) + E(x)]dx = \int_{-1}^{1} h(x)dx$$

Thus, the result will be exact. ■

(e) The error term is given by:

$$\text{Error} = \int_{-1}^{1} f(x) - h(x)dx = \int_{-1}^{1} E(x)dx = \int_{-1}^{1} \frac{f'''(\xi(x))}{6}x(x+1)(x-1)dx$$

NOTE: here the function $g(x) = x(x+1)(x-1)$ changes sign in the interval $[-1,1]$, so Weighted Mean Value Theorem for Integrals cannot be used to further simplify.

Question 2

(a) Here we take $x_0 = 0$, $x_1 = 2$, and $x_2 = 4$; $h = 2$. By simply apply the Simpson's Rule, we have:

$$\int_0^4 f(x) \approx \frac{2}{3}[f(0) + 4f(2) + f(4)] = 4$$

Thus, the estimated integral is $\boxed{\int_0^4 f(x)dx \approx 4}$.

(b) Here we are using the composite Simpson's Rule, we have the following:

$$\begin{aligned}
\int_0^4 f(x)dx &= \int_0^2 f(x)dx + \int_2^4 f(x)dx \\
&\approx \frac{1}{3}[f(0) + 4 \cdot f(1) + f(2)] + \frac{1}{3}[f(2) + 4 \cdot f(3) + f(4)] \\
&= \frac{10}{3} + \frac{10}{3} \\
&= \frac{20}{3}
\end{aligned}$$

Thus, the estimated integral is $\boxed{\int_0^4 f(x)dx \approx \frac{20}{3}}$.

Question 3 (Coding)

(a) The program can be find in file "**hw4_5_trape.m**".

(b) The program can be find in file "**hw4_5_Simps.m**".

The code for part (a) is the following:

```
% Math 151A
% Homework 4
% Question 3(a)
% Wang, Zheng

%% input N
N = input('Please input the number of subintervals, N:');

%% calculate the output
fprintf('Estimating by Composite Trapezoidal Rule...\n')
fprintf('Integral of cos(x) from 0 to pi is%12.8f\n',compos_Trape(N))

%% useful functions
function fx = f(x)
    fx = cos(x);
end
```

```
function res=compos_Trape(N)
    a = 0;
    b = pi;
     h = (b-a)/N;
    fa = f(a);
    fb = f(b);
    fo = 0;
    for j=1:(N-1)
       fo = fo + 2*f(a+j*h);
    end
    res = h*(fo+fa+fb)/2;
end
```

The code for part (b) is the following:

```
% Math 151A
% Homework 4
% Question 3(b)
% Wang, Zheng

%% input N
N = input('Please input the number of subintervals, N:');
if mod(N,2) ~= 0
error('To use Composite Simpson''s Rule, N must be even.')
end

%% calculate the output
fprintf('Estimating by Composite Simpson''s Rule...\n')
fprintf('Integral of cos(x) from 0 to pi is %12.12f\n',compos_Simps(N))

%% useful functions

function fx = f(x)
    fx = cos(x);
end

function res=compos_Simps(N)
    a = 0;
    b = pi;
    h = (b-a)/N;
    f_t = 0;
    for j=1:(N/2)
        f_t = f_t + ( f(a+(2*j-2)*h) + 4*f(a+(2*j-1)*h) + f(a+(2*j)*h) );
    end
    res = h*f_t/3;
```

```
end
```