Applied Numerical Methods
(MATH 151A)
Assignment 1

Note:

- **Due day**: Jan 24th 10:00 a.m. Assignments submitted after the due date will not be accepted. Print your homework and submit it in discussion section.

1. Consider the following non-linear equation:

$$f(x) := x^2 - 0.7x = 0 \quad \text{on } [0.5, 1]$$

   (a) Show that $f(x)$ has **exactly** one root on [0.5,1] without solving the equation.

   (b) Consider the bisection algorithm starting with the interval $[0.5, 1]$, i.e. consider $[a_1, b_1] = [0.5, 1]$ and $p_1 = 0.75$. Find the minimum number of iterations required to approximate the solution with an absolute error of less than 10^{-5}.

2. Suppose $f(x)$ is continuous on $[a, b]$, and $f(x) \in [a, b]$ for any $x \in [a, b]$. Show that f has at least one fixed point on $[a, b]$. (Hint: use Intermediate Value Theorem.)

3. Given the following sequence $\{p_n\}_{n=0}^{\infty}$:

$$\begin{cases} p_{n+1} = \frac{p_n^2+3}{2p_n} \\ p_0 \text{ is given.} \end{cases}$$

   (a) Calculate p_1 and p_2 with $p_0 = 3$.

   (b) Find all the possible limits of sequence $\{p_n\}_{n=0}^{\infty}$ (for all possible p_0).

   (c) Show by definition that the given sequence is actually a sequence generated by Newton's method to find a solution of the equation $x^2 - 3 = 0$.

4. Consider the following non-linear equation:

$$f(x) := x^2 - 3 = 0 \quad \text{on } [0, 4]$$

   (a) Write the formula for the next term in the sequence in terms of the two previous terms for the secant method p_n to solve the above equation, with the starting point at $p_0 = \frac{1}{2}$ and $p_1 = 3$, and calculate p_2 and p_3.

   (b) Calculate p_2 and p_3 in the sequence of the method of false position p_n to solve the above equation, with the starting point at $p_0 = \frac{1}{2}$ and $p_1 = 3$.

5. (Programming problem)

   Solve the following equation by the following methods:

$$\sin(x) - x = 0 \quad \text{on } [0, \frac{\pi}{2}]$$

   (a) Write a complete program to implement the secant method to solve the equation within an accuracy of at least 10^{-5} (using $|p_n - p_{n-1}|$) with initial values p_0 and p_1 as $\frac{\pi}{4}$ and $\frac{3\pi}{8}$ respectively. Plot the graph of the sequence $\{p_n\}$ generated by the secant method.

   (b) Write a complete programme to implement the Newton's Method to solve the equation within an accuracy of at least 10^{-5} (using $|p_n - p_{n-1}|$) with initial value p_0 as $\frac{\pi}{4}$. Plot the graph of the sequence $\{p_n\}$ generated by Newton's method.