

## **Team Members**



Jinyi Li Statistics



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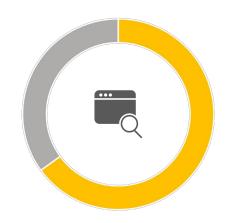
Zheng Wang Statistics, Computational Math

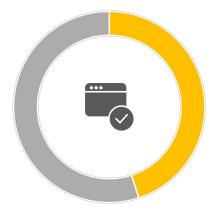


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## **Problem Statement**

- Can we use features to predict whether an eye is Persian?
- Can being Persian predict certain measurements about eye features?





# **Data Cleaning**



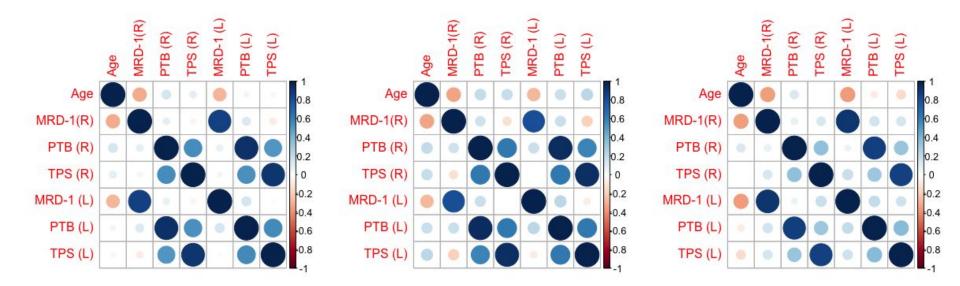
Build model without the online datasheet

Built model separately for male and female



All

- The predictors we used are MRD-1 (L),MRD-1(R),PTB (R), TPS (R), TPS (L), PTB (L) and we want to find the correlation between these features.
- Correlation plots for Caucasian, Persian and All people:

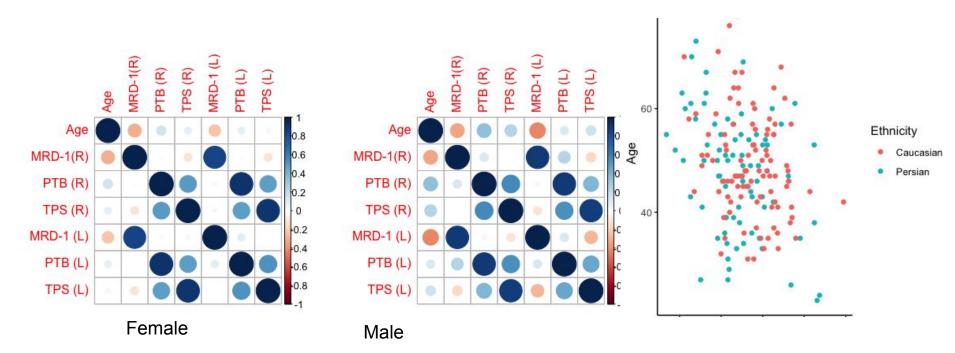


Caucasian

Persian

Correlation plots for Female and Male

• Investigate linear relation between age and MRD



**MRD** 

#### **Correlation Analysis:**

#### Observation:

- 1. MRD, PTB and TPS all show fairly high correlation between left and right eyes ( $\sim$ 0.8).
- 2. TPS and PTB correlate with each other ( $\sim$ 0.6). Such correlation is slightly higher among Caucasians and females.
- 3. Although not very strong, age negatively correlates with MRD, particularly among males (~0.4).

#### Insight:

- 1. We will only use features of one side of eyes in models to avoid multicollinearity.
- 2. We should consider adding age to the model as a covariate.

Note: The gender distribution of the dataset is very skewed, with 145 observations of females and only 34 observations of males. Therefore, the conclusion we got from the male samples could be biased.

#### **Distribution Analysis**

#### Observation

- 1. MRD shows Gaussian distribution, while TPS seems lognormal. PTB is ambiguous and may be either.
- 2. MRD is apparently differentiated by ethnicity.
- 3. After splitting the data by gender, both PTB and TPS show obvious differences between the ethnic groups.
- 4. 2. The distribution of PTB(L) for Male looks abnormal. Since PTB(R), which is highly correlated to PTB(L), shows a normal Gaussian distribution, we assume that the exotic shape arises due to the small sample size (only 21 Caucasians and 13 Persians in this sample).

#### Insight

- 1. All of the three eye features, MRD, TPS and PTB, could be significant to predict ethnicity.
- 2. The models might perform better if we log-transform TPS.
- 3. We should create separate models for both genders instead of neglecting the covariate

#### **Investigation of Derived Variables**

#### Observation

1. According to the distribution plots, the derived variables, MRD/TPS and MRD/PTB are significantly differentiated by gender and negatively correlate with age.

#### Insight

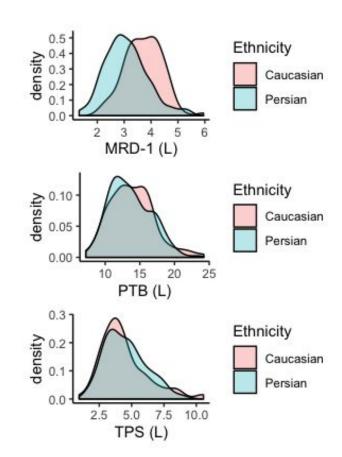
1. The above two observations suggest that we can potentially predict the two derived variables, MRD/TPS and MRD/PTB, with gender and age.

Note: Since MRD is a significant feature itself, it is probable that the significance of MRD/TPS and MRD/PTB comes from it, which undermines the value of this observation.

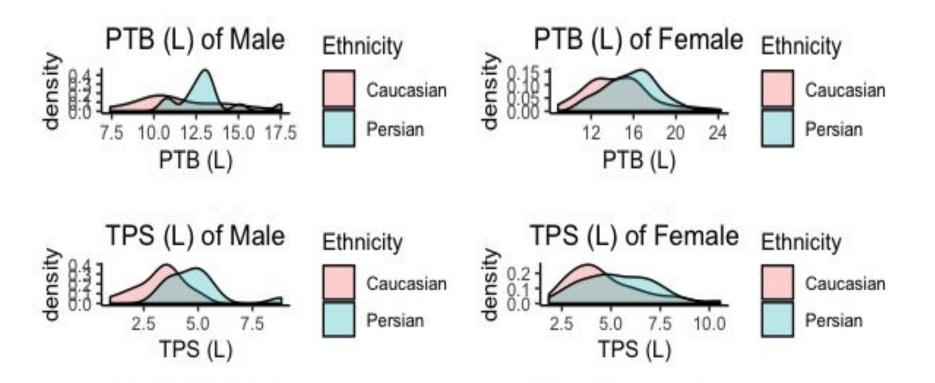
## **Distribution**

Based on the density distribution of each eye features we can find:

- MRD shows Gaussian distribution, while TPS seems lognormal. PTB is ambiguous and may be either.
- 2. Caucasian and Persian have obviously different MRD on both left and right eyes.



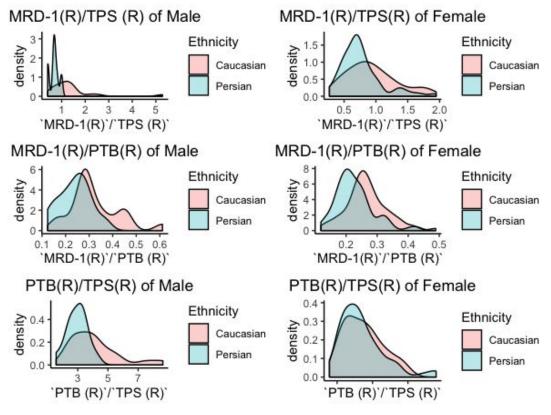
# Distribution, partitioned by gender



# Distribution of Variables Derived from Eye Features, partitioned by gender

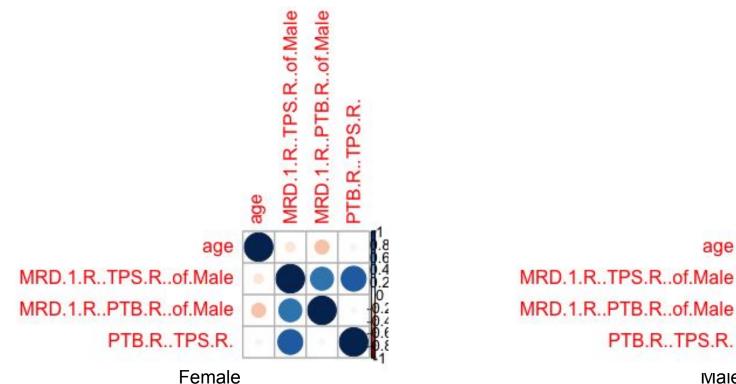
Based on the density distribution of variables derived from eye features we can find:

The derived variables, MRD/TPS and MRD/PTB are significantly differentiated by gender.



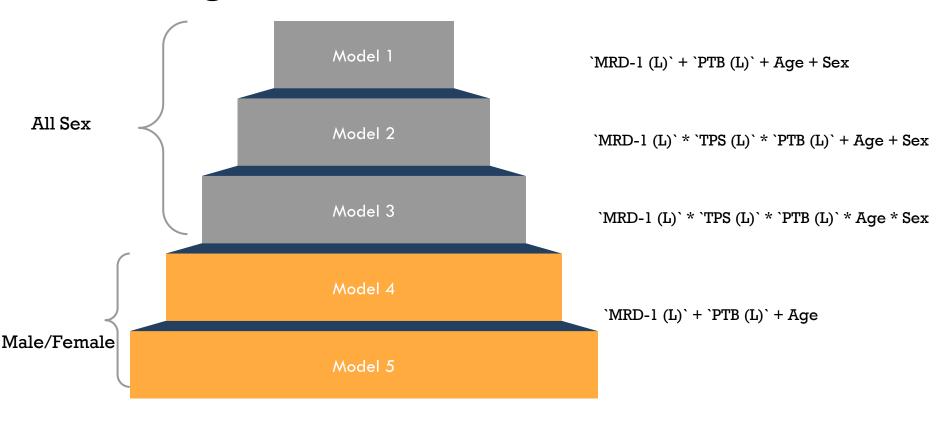
# **Correlation plots**

Males' ages correlate with their MRD/TPS and MRD/PTB, with coefficients of 0.34 and 0.5 respectively. Such correlations are weaker on females, but they still exist.



Male

# **Modeling**



 $Model \ l: `MRD-l \ (L)` * `TPS \ (L)` * `PTB \ (L)` + Age + Sex$ 

#### Sample Training data Summary Table

#### Deviance Residuals:

Min 1Q Median 3Q Max -1.8925 -0.9555 -0.5393 1.0273 2.0754

#### Coefficients:

Estimate Std. Error z value Pr(>|z|) (Intercept) 2.65652 1.62835 1.631 0.102803 `MRD-1 (L)` -1.15581 0.29726 -3.888 0.000101 \*\*\* `PTB (L)` 0.25014 0.07315 3.420 0.000627 \*\*\* -0.05239 0.01909 -2.744 0.006061 \*\* Age 0.53622 SexM 0.52480 1.022 0.306893

# of Training observations: 143 # of Testing observations: 36

#### CV 1

	FALSE	TRUE
FALSE	13	8
TRUE	6	9

#### CV 2

	FALSE	TRUE
FALSE	17	7
TRUE	1	11

#### CV3

	FALSE	TRUE
FALSE	13	9
TRUE	6	8

	FALSE	TRUE
FALSE	17	6
TRUE	2	11

#### Model 2: `MRD-1 (L)` + `PTB (L)` + Age + Sex

#### Sample Training data Summary Table

#### Deviance Residuals:

Min 1Q Median 3Q Max -1.8356 -1.0323 -0.3919 1.0112 2.3766

#### Coefficients:

Estimate Std. Error z value Pr(>|z|) (Intercept) -2.020116 15.159539 -0.133 0.8940 `MRD-1 (L)` -2.112333 4.434989 -0.4760.6339 `TPS (L)` -1.413245 2.827919 -0.500 0.6173 `PTB (L)` 0.898603 1.099691 0.817 0.4138 -0.032128 0.017878 -1.7970.0723 . Age 0.4141 SexM 0.436470 0.534445 0.817 `MRD-1 (L)`: TPS (L)` 0.795761 0.842974 0.944 0.3452 `MRD-1 (L)`: `PTB (L)` -0.040935 0.314959 -0.1300.8966 `TPS (L)`: `PTB (L)` 0.007423 0.186018 0.9682 0.040 `MRD-1 (L)`:`TPS (L)`:`PTB (L)` -0.028620 0.054818 -0.522 0.6016

# of Training observations: 143 # of Testing observations: 36

#### CV 1

	FALSE	TRUE
FALSE	16	7
TRUE	2	11

#### CV 2

	FALSE	TRUE
FALSE	14	7
TRUE	8	7

#### CV3

	FALSE	TRUE
FALSE	14	5
TRUE	4	13

	FALSE	TRUE
FALSE	11	2
TRUE	13	10

#### Model 3: 'MRD-1 (L)' \* 'TPS (L)' \* 'PTB (L)' \* Age \* Sex

#### Sample Training data Summary Table

```
glm(formula = Ethnicity ~ `MRD-1 (L)` * `TPS (L)` * `PTB (L)` *
    Age * Sex, family = "binomial", data = train)
Deviance Residuals:
    Min
               10
                     Median
                                   3Q
                                           Max
-2.01522 -0.62528 -0.00001
                             0.44251
                                       2.31912
Coefficients:
                                         Estimate Std. Error z value Pr(>|z|)
(Intercept)
                                        -1.983e+01 2.440e+02 -0.081
                                                                       0.935
`MRD-1 (L)`
                                        1.846e+01 7.055e+01
                                                              0.262
                                                                       0.794
`TPS (L)`
                                        -9.180e+00 4.481e+01
                                                              -0.205
                                                                       0.838
`PTB (L)`
                                        4.446e+00 1.687e+01
                                                              0.264
                                                                       0.792
                                        1.080e+00 4.630e+00
                                                               0.233
                                                                       0.816
Age
SexM
                                        2.583e+02 9.622e+06
                                                              0.000
                                                                       1.000
`MRD-1 (L)`: TPS (L)`
                                        -1.797e-01 1.284e+01
                                                                       0.989
                                                              -0.014
`MRD-1 (L)`: `PTB (L)`
                                        -2.181e+00 4.866e+00
                                                              -0.448
                                                                       0.654
`TPS (L)`: `PTB (L)`
                                        4.748e-01 2.976e+00
                                                               0.160
                                                                       0.873
`MRD-1 (L)`:Age
                                        -5.405e-01 1.344e+00
                                                             -0.402
                                                                       0.688
`TPS (L)`:Age
                                        -1.562e-01 8.430e-01
                                                              -0.185
                                                                       0.853
`PTB (L)`:Age
                                        -1.118e-01 3.130e-01
                                                              -0.357
                                                                       0.721
`MRD-1 (L)`:SexM
                                        -1.764e+02 2.360e+06
                                                               0.000
                                                                       1.000
`TPS (L)`:SexM
                                       -4.423e+02 2.517e+06
                                                               0.000
                                                                       1.000
`PTB (L)`:SexM
                                        -2.755e+02 6.961e+05
                                                               0.000
                                                                       1.000
Age:SexM
                                        -1.214e+02 2.169e+05
                                                              -0.001
                                                                       1.000
`MRD-1 (L)`:`TPS (L)`:`PTB (L)`
                                        7.667e-02 8.534e-01
                                                               0.090
                                                                       0.928
`MRD-1 (L)`: `TPS (L)`:Age
                                        8.324e-02 2.431e-01
                                                               0.342
                                                                       0.732
`MRD-1 (L)`: `PTB (L)`:Age
                                        4.884e-02 9.080e-02
                                                               0.538
                                                                       0.591
`TPS (L)`: `PTB (L)`:Age
                                        1.010e-02 5.439e-02
                                                               0.186
                                                                       0.853
`MRD-1 (L)`:`TPS (L)`:SexM
                                        1.777e+02 6.343e+05
                                                               0.000
                                                                       1.000
`MRD-1 (L)`:`PTB (L)`:SexM
                                        7.965e+01 1.666e+05
                                                               0.000
                                                                       1.000
`TPS (L)`: PTB (L)`:SexM
                                        1.009e+02 1.858e+05
                                                               0.001
                                                                       1.000
`MRD-1 (L)`:Age:SexM
                                        3.521e+01 5.307e+04
                                                               0.001
                                                                       0.999
`TPS (L)`:Age:SexM
                                        3.887e+01 5.494e+04
                                                               0.001
                                                                       0.999
`PTB (L)`:Age:SexM
                                        1.678e+01 1.578e+04
                                                              0.001
                                                                       0.999
`MRD-1 (L)`:`TPS (L)`:`PTB (L)`:Age
                                        -6.027e-03 1.575e-02
                                                             -0.383
                                                                       0.702
`MRD-1 (L)`:`TPS (L)`:`PTB (L)`:SexM
                                        -3.180e+01 4.595e+04
                                                                       0.999
`MRD-1 (L)`:`TPS (L)`:Age:SexM
                                        -1.186e+01 1.374e+04
                                                             -0.001
                                                                       0.999
`MRD-1 (L)`:`PTB (L)`:Age:SexM
                                        -4.741e+00 3.767e+03
                                                                       0.999
`TPS (L)`: PTB (L)`:Age:SexM
                                        -4.948e+00 4.070e+03
                                                              -0.001
                                                                       0.999
0.999
```

# of Training observations: 143 # of Testing observations: 36

#### CV 1

	FALSE	TRUE
FALSE	12	11
TRUE	7	6

#### CV 2

	FALSE	TRUE
FALSE	13	6
TRUE	6	11

#### CV3

	FALSE	TRUE
FALSE	16	4
TRUE	6	10

	FALSE	TRUE
FALSE	14	6
TRUE	4	12

#### Model 4: 'MRD-1 (L)' + 'PTB (L)' + Age (Female)

#### Sample Training data Summary Table

glm(formula = Ethnicity ~ `MRD-1 (L)` + `PTB (L)` + Age, family = "binomial",
 data = trainFemale)

#### Deviance Residuals:

Min 1Q Median 3Q Max -1.8788 -0.9213 -0.4312 0.9600 2.0432

#### Coefficients:

# of Training observations: 116 # of Testing observations: 29

#### CV 1

	FALSE	TRUE
FALSE	13	4
TRUE	4	8

#### CV 2

	FALSE	TRUE	
FALSE	14	4	
TRUE	2	9	

#### CV3

	FALSE	TRUE	
FALSE	11	4	
TRUE	4	10	

	FALSE	TRUE
FALSE	12	6
TRUE	4	7

#### Model 5: 'MRD-1 (L)' + 'PTB (L)' + Age (Male)

#### Sample Training data Summary Table

glm(formula = Ethnicity ~ `MRD-1 (L)` + `PTB (L)` + Age, family = "binomial",
 data = trainMale)

#### Deviance Residuals:

Min 1Q Median 3Q Max -1.5348 -0.8073 -0.4046 0.9015 1.9734

#### Coefficients:

Estimate Std. Error z value Pr(>|z|) (Intercept) -3.64960 3.45440 -1.057 0.291 `MRD-1 (L)` -0.91818 0.64764 -1.418 0.156 `PTB (L)` 0.45805 0.22187 2.064 0.039 \* Age 0.01087 0.04227 0.257 0.797

# of Training observations: 27 # of Testing observations: 7

#### CV 1

	FALSE	TRUE
FALSE	2	1
TRUE	2	2

#### CV 2

	FALSE	TRUE	
FALSE	3	2	
TRUE	0	2	

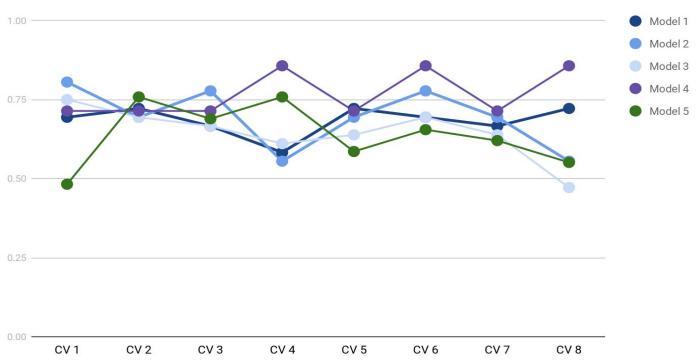
#### CV3

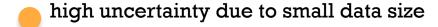
	FALSE	TRUE
FALSE	4	2
TRUE	0	1

	FALSE	TRUE	
FALSE	5	0	
TRUE	1	1	

# Result



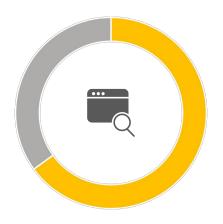






Add validation dataset

## **Artificial Neural Network**



#### **ANN Performance**

Training Accuracy: 0.65 Validation Accuracy: 0.70



### Compare to other Models

ANN model has similar performance as our simpler models

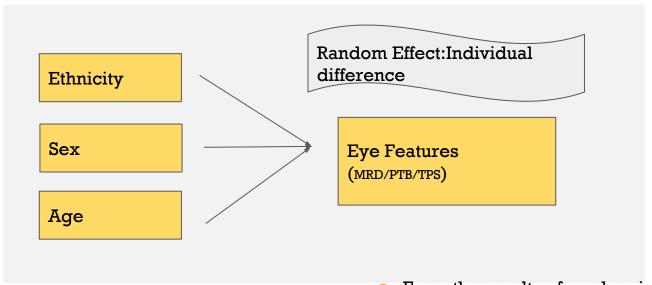
#### **ANN** part

To serve as a comparison to the logistic regression model, we decided to sacrifice all interpretability and run an artificial neural network model to learn a mapping from age, sex, and all metrics of eyes to the ethnicity of the patient.

In order to do this, we use an ANN with 1 input layer, 4 hidden layer, and 1 output layer. For each of the hidden layer, we choose to use 64, 128, 32, and 16 dimensions. Overall, this would give us a complex model with thousands of trainable parameters. Although the model learned from the dataset, we have is not interpretable (since the ANN is like a black box, and there are just too many parameters to track), ANN usually provides us with the ultimate flexibility and accuracy. Any derived parameters that would work can be automatically learned by a neural network.

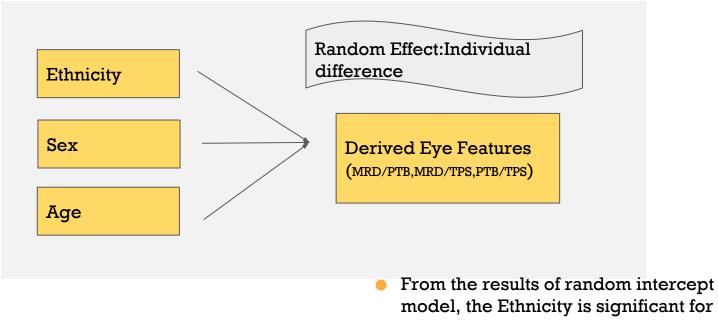
In terms of the result we are getting, it, however, turns out that the performance of ANN is similar to our simple logistic models. This indicates that there are significant noises in the dataset, and the best we could do in predicting the ethnicity is about 70%. Moreover, the fact that the simple logistic model actually performs similarly or even better than more complex models such as ANN indicates that the true relationship between ethnicity and metrics of the eyes should be simple, and an extremely naïve model is likely the solution to the problem.

# Can being Persian Affect the eye features?



- From the results of random intercept model, the p-value of three predictors are significant
- Sex and Age plays an important role to decide eye features

# Can being Persian Affect the eye features?



- model, the Ethnicity is significant for MRD/PTB and MRD/TPS.
- Sex and Age plays an important role to decide eye features

#### **Random Effect Model Part**

In this part of project, we are solving the question "how the ethnicity of a patient affects the metrics of their eyes". In this part, we explore how all of the three metrics (MRD, PTB, TPS) and three derived metrics (MRD/PTB, MRD/TPB, PTB/TPS). Based on our result, we found that in fact that the ethnicity is a significant predictor for metrics MRD, PTB, TPS, MRD/PTB, and MRD/TPB.

#### Fitted models are the following:

• Notice that effect of Age is small, since it is expected that patients who are just 1 year older are not going to have big difference in their eyes

```
MRD = 4.8021 - 0.3448 * Ethnicity is Persian - 0.2629 * Sex is Male - 0.0210 * Age
PTB = 12.4018 - 1.4596 * Ethnicity is Persian - 3.0753 * Sex is Male - 0.0432 * Age
TPS = 3.7385 - 0.7382 * Ethnicity is Persian - 1.0937 * Sex is Male - 0.0174 * Age
MRD/TPS = 1.4241 - 0.2900 * Ethnicity is Persian + 0.2358 * Sex is Male - 0.0091 * Age
MRD/PTB = 0.3773 - 0.0549 * Ethnicity is Persian + 0.0420 * Sex is Male - 0.0022 * Age
PTB/TPS = 3.6112 - 0.2544 * Ethnicity is Persian + 0.1317 * Sex is Male - 0.0017 * Age
```

	Metrics	Ethnicity's Coefficient	P-value	Significant	Interpretation
We summarize the interpretation (of Ethnicity) we draw from our models in this	MRD	-0.3448	0.0005	Yes	At a particular age and with a particular gender, a patient who is Persian is expected to have an MRD that is 0.3448 unit lower than patient who is Caucasian
form:	РТВ	1.4596	0.0004	Yes	At a particular age and with a particular gender, a patient who is Persian is expected to have a PTB that is 1.4596 unit higher than patient who is Caucasian
	TPS	0.7382	0.0046	Yes	At a particular age and with a particular gender, a patient who is Persian is expected to have a TPS that is 0.7382 unit higher than patient who is Caucasian
	MRD/TPS	-0.2900	0.0000	Yes	At a particular age and with a particular gender, a patient who is Persian is expected to have an MRD/TPS ratio that is 0.29 lower than patient who is Caucasian
	MRD/PTB	-0.0549	0.0000	Yes	At a particular age and with a particular gender, a patient who is Persian is expected to have an MRD/PTB ratio that is 0.0549 lower than patient who is Caucasian
	PTB/TPS	-0.2544	0.1427	No	At a particular age and with a particular gender, a patient who is Persian is expected to have a PTB/TPS ratio that is 0.2544 lower than patient who is Caucasian

We notice that from the result of this part, it seems contradictory with respect to our logistic modeling sections, as a lot of the metrics are not significant in the logistic regression model. The reasons are the following:

- 1. In logistic models, the parameters can be affected by multicollinearity, which would inflate the variances of coefficient and cause some of them to appear as insignificant.
- 2. In logistic models, we did not exclude the noise (i.e. the random effect of patient). Thus, the variances of the coefficients will be a combination of variances due to random effect as well as variances due to the predictors themselves. This cause some of the predictors to become insignificant. Notice, we did not exclude the random effect in our logistic regression model part since the question we are attempting to answer require a predictive model. Thus, we do not seek to exclude those random effect as they won't impact the coefficients in the model and won't make any improvement to model accuracies.

## **Conclusion**



Both ANN and logistic models failed to achieve a great validation score in predicting the Ethnicity, indicating there could be high noises.

Significant split observed with random intercept model (patient as random effect) indicates Ethnicity makes a differences in many metrics

Although ethnicity will split various metrics of a eye, this differences in the metrics will vary with each patient

