

## Invited Review

## A survey for the quadratic assignment problem ☆

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Received 13 September 2004; accepted 2 September 2005

Available online 27 December 2005

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**Abstract**

The **quadratic assignment problem** (QAP), one of the most difficult problems in the NP-hard class, models many real-life problems in several areas such as **facilities location**, parallel and **distributed computing**, and **combinatorial data analysis**. Combinatorial optimization problems, such as the **traveling salesman problem**, maximal clique and graph partitioning can be formulated as a QAP. In this paper, we present some of the most important QAP formulations and classify them according to their mathematical sources. We also present a discussion on the theoretical resources used to define lower bounds for exact and heuristic algorithms. We then give a **detailed discussion** of the progress made in both **exact** and **heuristic solution methods**, including those formulated according to **metaheuristic** strategies. Finally, we analyze the contributions brought about by the study of different approaches.

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**Keywords:** Assignment; Integer programming; Combinatorial optimization; Facilities planning and design; Metaheuristics; Branch and bound

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**1. Introduction**

Let us consider the problem of **assigning facilities to locations** in such a way that **each facility is** designated to **exactly one location** and vice-versa. The **distances between locations**, the **demand flows**

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☆ A preliminary version of this survey, with 278 references, is Loiola et al. (2004).

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among the facilities and, in the general case, the *facility versus location* assignment costs are known. The international literature identifies the quadratic assignment problem (QAP) as the problem of finding a minimum cost allocation of facilities into locations, taking the costs as the sum of all possible distance-flow products.

The main motivation for this survey is the continuous interest in QAP, shown by a number of researchers worldwide, for the theory, applications and solution techniques for this problem. Among the many references listed at the end of this paper, we found over 100 that were published since 1999. The last surveys, books and review articles in the literature are Burkard (1991), Malucelli (1993), Pardalos et al. (1994), Burkard and Çela (1996), Çela (1998) and Burkard et al. (1998a). The article of Anstreicher (2003) reviews only the recent advances on algorithms. An article by Drezner et al. (in press) surveys the state-of-the-art in both heuristic and exact methods.

Koopmans and Beckmann (1957) first proposed the QAP as a mathematical model related to economic activities. Since then, it has appeared in several practical applications: Steinberg (1961) used the QAP to minimize the number of connections between components in a backboard wiring; Heffley (1972, 1980) applied it to economic problems; Francis and White (1974) developed a decision framework for assigning a new facility (police posts, supermarkets, schools) in order to serve a given set of clients; Geoffrion and Graves (1976) focused on scheduling problems; Pollatschek et al. (1976) invoked the QAP to define the best design for typewriter keyboards and control panels; Krarup and Pruzan (1978) applied it to archeology; Hubert (1987) in statistical analysis; Forsberg et al. (1994) used it in the analysis of reaction chemistry and Brusco and Stahl (2000) in numerical analysis. Nevertheless, the facilities layout problem is the most popular application for the QAP: Dickey and Hopkins (1972) applied the QAP to the assignment of buildings in a University campus, Elshafei (1977) in a hospital planning and Bos (1993) in a problem related to forest parks. Benjaafar (2002) introduced a formulation of the facility layout design problem in order to minimize work-in-process (WIP). In his work, he shows that layouts obtained using a WIP-based formulation can be very different from those obtained using the conventional QAP-formulation. For example, a QAP-optimal layout can be WIP-infeasible. Rabak and Sichman (2003), Miranda et al. (2005) and Duman and Ilhan (in press) studied the placement of electronic components.

The index assignment problem (Ben-David and Malah, 2005) has to do with error control in communications and can be shown to be a special case of the QAP. Wess and Zeitlhofer (2004) studied the problem of memory layout optimization in signal processors. Other applications can be found in Scriabin and Vergin (1975), Hubert and Schulz (1976), Heffley (1977), Los (1978), Khare et al. (1988a,b), Krackhardt (1988), Bland and Dawson (1991), Balakrishnan et al. (1992), Lacksonen and Ensore (1993), Medova (1994), Phillips and Rosen (1994), Gouveia and Voß (1995), Bozer and Suk-Chul (1996), Talbot and Cawley (1996), White (1996), Mason and Rönnqvist (1997), Ostrowski and Ruoppila (1997), Ball et al. (1998), Haghani and Chen (1998), Kochhar et al. (1998), Martin (1998), Sarker et al. (1998), Spiliopoulos and Sofianopoulou (1998), Tansel and Bilen (1998), Tavakkoli-Moghaddain and Shayan (1998), Urban (1998), Gong et al. (1999), Rossin et al. (1999), Bartolomei-Suarez and Egbelu (2000), Ho and Moodie (2000), Urban et al. (2000), Hahn and Krarup (2001), Pitsoulis et al. (2001), Takagi (2001), Siu and Chang (2002), Wang and Sarker (2002), Youssef et al. (2003), Yu and Sarker (2003), Ciriani et al. (2004), Solimanpur et al. (2004) and Abbiw-Jackson et al. (in press).

Since its first formulation, the QAP has been drawing researchers' attention worldwide, not only because of its practical and theoretical importance, but also because of its complexity. The QAP is one of the most difficult combinatorial optimization problems. In general, instances of size  $n > 30$  cannot be solved in reasonable time. Sahni and Gonzales (1976) had shown that QAP is NP-hard and that, unless  $P = NP$ , it is not possible to find an  $f$ -approximation algorithm, for a constant  $f$ . Such results are valid even when flows and distances appear as symmetric coefficient matrices. Due to its high computation complexity, the QAP has been chosen as the first major test application for the GRIBB project (great international branch-and-bound search). This project is seeking to establish a software library for solving a large class of parallel

search problems by the use of numerous computers around the world accessed by Internet. Preliminary results from test runs are presented in Moe (2003).

Several NP-hard combinatorial optimization problems, such as the traveling salesman problem, the bin-packing problem and the max clique problem, can be modeled as QAPs. The search for local optima in classical internet-available instances is a tendency that allows for the comparison of technique performances, even when the optimum is unknown, or when the use of exact algorithms in these instances is possible, Burkard et al. (1996a, 1998b) and Çela (1998)). In the QAP case, we can mention as examples, instances with recently proved optimal solutions: Bur26 (b to h), (2004) and Tai25a (2003) by Hahn; Ste36a (2001) by Brixius and Anstreicher; Bur26a (2001) by Hahn; Kra30a by Hahn; Kra30b, Kra32 and Tho30 (2000) by Anstreicher, Brixius, Goux and Linderöth; Nug30 (one of the most renowned and challenging instances) (2000) by Anstreicher, Brixius, Goux and Linderöth; Ste36b and Ste36c (1999) by Nyström. In 2003, Misevicius enhanced the best-known solution for Tai50a, Tai80a and Tai100a using a modified tabu search. Those results motivated the article of Anstreicher (2003) that registers the recent advances in QAP-solutions and describes the new algorithms and computational structures used. Besides, the new instances are available for tests in Burkard et al. (1991, 1997), Li and Pardalos (1992) and QAPLIB (2004). Also, there are instance generators with known optimum values that are currently used for testing algorithms, Çela (1998). Finally, Palubeckis (1999, 2000), Drezner et al. (in press) and Stützle and Fernandes (2004) present new instance sets that are reported to be difficult for metaheuristics.

Experts in combinatorial optimization tend to search for particular polynomially solvable versions of NP-hard problems and research mechanisms to measure the difficulty of problem instances. In the case of the QAP, Christofides and Gerrard (1981) studied some special instances of the QAP; Sylla and Babu (1987) developed a methodology for an orderly quadratic assignment problem; Chen (1995) presented other QAP-cases, followed by Çela (1998), who presented several polynomially solvable instances; Herroelen and Vangils (1985), Cyganski et al. (1994), Mautor and Roucairol (1994b) showed that Palubetski's QAP instances are degenerate; Angel and Zissimopoulos (1998, 2000, 2001, 2002) discussed the difficulty of other QAP instances based on the variance of their flow and distance sets; Abreu et al. (2002) derived a polynomial expression for the variance of the solution costs and defined a measure of the difficulty instances and Barvinok and Stephen (2003) constructed a distribution of QAP solution-values.

In the challenge of identifying new structural properties for QAP instances, many formulations have appeared, based on various points of view. Here we propose to collect these formulations, highlighting their most important features, and to classify them according to the technique used, such as integer programming, positive semi-definite programming, discrete and combinatorial mathematics, graph and group theory, or linear algebra via spectral theory. Most of these formulations are equivalent, except for those that characterize more general problems. Considering the difficulty of the QAP, these formulations encourage addition of mathematical resources for the development of new solution techniques. By the end of this article, we discuss the contributions obtained from these formulations, building several tables and charts from the extensive bibliography concerning the elaboration of exact and heuristic algorithms, lower bound calculation, instance class characteristics and recording the development of QAP since 1957 to the present.

The following surveys are essential references for those who want to have a more complete understanding of this problem: Hanan and Kurtzberg (1972), Burkard (1984, 1991, 2002), Pardalos et al. (1994) and Burkard et al. (1998a), as well as, the books by Pardalos and Wolkowicz (1994), Padberg and Rijal (1996), Dell'Amico et al. (1997) and Çela (1998).

## 2. Formulations of QAP and related problems

In this section, we present the most important QAP formulations and describe the type of solution approach adopted for each formulation.

## 2.1. Selected QAP formulations

### 2.1.1. Integer linear programming formulations (IP)

First, we present the QAP as a Boolean program followed by a linear programming problem, where the binary constraints are relaxed. The Boolean formulation was initially proposed by Koopmans and Beckmann (1957) and later used in several works such as Steinberg (1961), Lawler (1963), Gavett and Plyter (1966), Elshafei (1977), Bazaraa and Sherali (1979), Bazaraa and Kirca (1983), Christofides and Benavent (1989), Bos (1993), Mans et al. (1995), Jünger and Kaibel (2000, 2001a,b), Liang (1996), Torki et al. (1996), Tsuchiya et al. (1996, 2001), Ball et al. (1998), Ishii and Sato (1998), Kaibel (1998), Kochhar et al. (1998), Martin (1998), Spiliopoulos and Sofianopoulou (1998), and most recently, Siu and Chang (2002), Yu and Sarker (2003) and, finally, Fedjki and Duffuaa (2004).

We consider  $f_{ij}$  the flow between facilities  $i$  and  $j$ , and  $d_{kp}$  the distance between locations  $k$  and  $p$ . It is our goal to calculate:

$$\min \sum_{i,j=1}^n \sum_{k,p=1}^n f_{ij} d_{kp} x_{ik} x_{jp} \quad (2.1)$$

$$\text{s.t.} \quad \sum_{i=1}^n x_{ij} = 1 \quad 1 \leq j \leq n, \quad (2.2)$$

$$\sum_{j=1}^n x_{ij} = 1 \quad 1 \leq i \leq n, \quad (2.3)$$

$$x_{ij} \in \{0, 1\} \quad 1 \leq i, j \leq n. \quad (2.4)$$

If we consider the cost of assignment of activities to locations, a general form for a QAP instance of order  $n$  is given by three matrices  $F = [f_{ij}]$ ,  $D = [d_{kp}]$  and  $B = [b_{ik}]$ , the first two matrices defining the flows between facilities and the distances between locations,  $b_{ik}$  being the allocation costs of facilities to locations. This problem can be defined as:

$$\min \sum_{i,j=1}^n \sum_{k,p=1}^n f_{ij} d_{kp} x_{ik} x_{jp} + \sum_{i,k=1}^n b_{ik} x_{ik} \quad (2.5)$$

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$$\text{s.t.} \quad (2.2), (2.3) \text{ and } (2.4).$$

Since the linear term of (2.5) is easy to solve, most authors discarded it.

A more general QAP version was proposed by Lawler (1963) and involves costs  $c_{ijkp}$  that do not necessarily correspond to products of flows and distances. The Lawler formulation is as follows:

$$\min \sum_{i,j=1}^n \sum_{k,p=1}^n c_{ijkp} x_{ik} x_{jp} \quad (2.6)$$

$$\text{s.t.} \quad (2.2), (2.3) \text{ and } (2.4).$$

This model was also used in Bazaraa and Elshafei (1979), Drezner (1995), Sarker et al. (1995, 1998), Brüngger et al. (1997, 1998), Chiang and Chiang (1998), Hahn and Grant (1998), Hahn et al. (1998), Gong et al. (1999) and Rossin et al. (1999).

### 2.1.2. Mixed integer linear programming (MILP) formulations

The QAP, as a mixed integer programming formulation, is found in the literature in different forms. All of them replace the quadratic terms by linear terms. For example, Lawler (1963) used  $n^4$  variables,

$$c_{ijkp} = f_{ij} d_{kp} \quad \text{and} \quad y_{ijkp} = x_{ik} x_{jp}, \quad 1 \leq i, j, k, p \leq n.$$

$i$  来自运输流矩阵,  $k$  来自不同地点之间的距离矩阵。如果把  $i$  分配给  $k$ , 那么  $x_{ik}$  等于 1。一个机器只能分配给一个位置,  $i$  对应的其他位置都为 0

Other formulations use relaxations of the original problem. In this category, one can find the papers by Love and Wong (1976a,b), Kaufman and Broeckx (1978), Bazaraa and Sherali (1980), Christofides et al. (1980), Burkard and Bonniger (1983), Frieze and Yadegar (1983), Assad and Xu (1985), Adams and Sherali (1986), Christofides and Benavent (1989) and the works of Adams and Johnson (1994), Drezner (1995), Gouveia and Voß (1995), Milis and Magirou (1995), Padberg and Rijal (1996), White (1996), Ramachandran and Pekny (1998), Karisch et al. (1999) and of Ramakrishnan et al. (2002).

In general, QAP linearizations based on MILP models present a huge number of variables and constraints, a fact that makes this approach unpopular. However, these linearizations, together with some constraint relaxations, lead to the achievement of much improved lower bounds for the optimal solution. In this regard, we have the works of Kaufman and Broeckx (1978), Bazaraa and Sherali (1980), Frieze and Yadegar (1983), Adams and Sherali (1986), Adams and Johnson (1994) and Padberg and Rijal (1996). Çela (1998) mentions three QAP linearizations: Kaufman and Broeckx (1978), which has the advantage of a smaller number of restrictions; Frieze and Yadegar (1983), for achieving the best lower bounds via Lagrangian relaxation and Padberg and Rijal (1996) owing to its polytope description. The formulation presented by Frieze and Yadegar (1983) describes the QAP in a linear form, using  $n^4$  real variables,  $n^2$  Boolean variables and  $n^4 + 4n^3 + n^2 + 2n$  constraints. The authors show that the formulation given in (2.7)–(2.16) below is equivalent to Eqs. (2.1)–(2.4), Çela (1998).

$$\min \sum_{i,j=1}^n \sum_{k,p=1}^n f_{ij} d_{kp} \cdot y_{ijkp} \quad (2.7)$$

$$\text{s.t.} \quad \sum_{i=1}^n x_{ik} = 1 \quad 1 \leq k \leq n, \quad \text{当 } k=1 \text{ 时, } i \text{ 集合中, 能分配给第一个位置的机器有且仅有一个。} \quad (2.8)$$

$$\sum_{k=1}^n x_{ik} = 1 \quad 1 \leq i \leq n, \quad (2.9)$$

$$\sum_{i=1}^n y_{ijkp} = x_{jp} \quad 1 \leq j, k, p \leq n, \quad (2.10)$$

$$\sum_{j=1}^n y_{ijkp} = x_{ik} \quad 1 \leq i, k, p \leq n, \quad (2.11)$$

$$\sum_{k=1}^n y_{ijkp} = x_{jp} \quad 1 \leq i, j, p \leq n, \quad (2.12)$$

$$\sum_{p=1}^n y_{ijkp} = x_{ik} \quad 1 \leq i, j, k \leq n, \quad (2.13)$$

$$y_{iikk} = x_{ik} \quad 1 \leq i, k \leq n, \quad (2.14)$$

$$x_{ik} \in \{0, 1\} \quad 1 \leq i, k \leq n, \quad (2.15)$$

$$0 \leq y_{ijkp} \leq 1 \quad 1 \leq i, j, k, p \leq n. \quad (2.16)$$

### 2.1.3. Formulations by permutations

Taking a simple approach, the pairwise allocation of facility costs to adjacent locations is proportional to flows and to distances between them. The QAP formulation that arises from this proportionality and uses the permutation concept can be found in Hillier and Michael (1966), Graves and Whinston (1970), Pierce and Crowston (1971), Burkard and Stratman (1978), Roucairol (1979, 1987), Burkard (1984), Frenk et al. (1985), Bland and Dawson (1991, 1994), Battiti and Tecchiolli (1994), Bui and Moon (1994), Chakrapani and Skorin-Kapov (1994), Fleurent and Ferland (1994), Li et al. (1994b), Mautor and Roucairol

(1994a,b), Li and Smith (1995), Taillard (1995), Bozer and Suk-Chul (1996), Colorni et al. (1996), Huntley and Brown (1996), Peng et al. (1996), Tian et al. (1996, 1999), Cung et al. (1997), Mavridou and Pardalos (1997), Merz and Freisleben (1997), Nissen (1997), Pardalos et al. (1997), Angel and Zissimopoulos (1998), Deineko and Woeginger (1998), Talbi et al. (1998a,b, 2001), Tansel and Bilen (1998), Abreu et al. (1999), Fleurent and Glover (1999), Gambardella et al. (1999) and Maniezzo and Colorni (1999). More recently, the following articles were released: Ahuja et al. (2000), Angel and Zissimopoulos (2000, 2001, 2002), Stützle and Holger (2000), Arkin et al. (2001), Pitsoulis et al. (2001), Abreu et al. (2002), Gutin and Yeo (2002), Hasegawa et al. (2002), Boaventura-Netto (2003) and Rangel and Abreu (2003). Costa and Boaventura-Netto (1994) studied the non-symmetrical QAP through a directed graph formulation.

Let  $S_n$  be the set of all permutations with  $n$  elements and  $\pi \in S_n$ . Consider  $f_{ij}$  the flows between facilities  $i$  and  $j$  and  $d_{\pi(i)\pi(j)}$  the distances between locations  $\pi(i)$  and  $\pi(j)$ . If each permutation  $\pi$  represents an allocation of facilities to locations, the problem expression becomes:

$$\min_{\pi \in S_n} \sum_{i,j=1}^n f_{ij} d_{\pi(i)\pi(j)}. \quad \text{与bel eg类似，讲究机器i和j被安排的地址的排列。} \quad (2.17)$$

This formulation is equivalent to the first one presented in (2.1)–(2.4), since the constraints (2.2) and (2.3) define permutation matrices  $X = [x_{ij}]$  related to  $S_n$  elements, as in (2.17), where, for all  $1 \leq i, j \leq n$ ,

$$x_{ij} = \begin{cases} 1, & \text{if } \pi(i) = j; \\ 0, & \text{if } \pi(i) \neq j. \end{cases} \quad (2.18)$$

#### 2.1.4. Trace formulation

This formulation is supported by linear algebra and exploits the trace function (the sum of the matrix main diagonal elements) in order to determine QAP lower bounds for the cost. This approach allows for the application of spectral theory, which makes possible the use of semi-definite programming to the QAP. The trace formulation, by Edwards (1980), can be stated as:

$$\min_{X \in S_n} \text{tr}(F \cdot X \cdot D \cdot X^t). \quad (2.19)$$

Afterwards, this approach was used in several works: Finke et al. (1987), Hadley et al. (1990, 1992a,b,c), Hadley (1994), Karisch et al. (1994), Karisch and Rendl (1995), Zhao et al. (1998), Anstreicher et al. (1999), Wolkowicz (2000a,b) and Anstreicher and Brixius (2001).

The following formulations define QAP relaxations through the dual of the Lagrangean dual, as we find in Karisch et al. (1994), Zhao et al. (1998), Wolkowicz (2000a,b). Let  $e$  be a vector with each coordinate equal to 1. If  $X$  is a permutation matrix and  $B$  is a cost matrix, then the SDP formulation is:

$$\min \quad \text{tr}(F \cdot X \cdot D - 2B)X^t \quad (2.20)$$

$$\text{s.t.} \quad Xe = e, \quad (2.21)$$

$$X^t e = e, \quad (2.22)$$

$$X_{ij} \in \{0, 1\} \quad \forall i, j. \quad (2.23)$$

Another formulation that follows this approach, Zhao et al. (1998), is

$$\min \quad \text{tr} F \cdot X \cdot D \cdot X^t - 2BX^t \quad (2.24)$$

$$\text{s.t.} \quad XX^t = X^t X = I, \quad (2.25)$$

$$Xe = X^t e = e, \quad (2.26)$$

$$X_{ij}^2 - X_{ij} = 0 \quad \forall i, j. \quad (2.27)$$



From the formulations (2.20)–(2.23) and (2.24)–(2.27) one can derive semi-definite relaxations for the QAP. Roupin (2004) presents a simple algorithm to obtain SDP relaxations for any quadratic or linear program with bivalent variables, starting from an existing linear relaxation of the considered combinatorial problem. This algorithm is applied to obtain semi-definite relaxations for three classical combinatorial problems: the  $k$ -Cluster Problem, the Quadratic Assignment Problem, and the Constrained-Memory Allocation Problem.

### 2.1.5. Graph formulation

Let us consider two undirected weighted complete graphs, the first one having its edges associated to flows and the second one, to distances. The QAP can be thought as the problem of finding an optimal allocation of the vertices of one graph on those of the other. In this formulation the solution costs are given as the sum of products of corresponding edge weights (see Fig. 1).

The algebraic and combinatorial approach adopted by Abreu et al. (1999) was the impetus for Marins et al. (2004) to define a new algebraic graph-theoretical approach involving line-graph automorphisms. The line-graph of a given graph  $G$ , denoted  $L(G)$ , is determined by taking each edges of  $G$  as a vertex of  $L(G)$ , while an edge of  $L(G)$  is defined as a pair of edges that are adjacent in  $G$ . A graph automorphism is a permutation of its vertices that preserves the edges. The set of all automorphisms of  $G$  together with the permutations composition is a group denoted by  $\text{Aut}(L(G))$  (Kreher and Stinson (1998)).

From a theorem by Whitney (1932), if  $G = K_n$ ,  $n \neq 2$  and  $4$ , then  $\text{Aut}(G)$  and  $\text{Aut}(L(G))$  constitute isomorphic groups. Based on this result, Marins et al. (2004) noticed that solving the QAP means either finding a permutation  $\pi \in S_n$  or finding a  $L(K_n)$  automorphism, which is a  $C_{n,2}$  permutation, that minimizes the following expression:

$$\min_{\pi \in \text{Aut}(L(K_n))} \sum_{i=1}^N f_i d_{\pi(i)}. \quad (2.28)$$

It is appropriate to mention White (1995), wherein a number of QAP representations are discussed with respect to their convexity and concavity and also Yamada (1992), where a formulation is presented for the QAP on an  $n$ -dimensional grid.

### 2.2. QAP related problems

The most classical QAP-related problem is, obviously, the Linear Assignment Problem (LAP), which is polynomial and easily solved by the Hungarian method. As several presentations of this problem can be found in the literature (for example, Burkard, 2002), we do not discuss the LAP here. We prefer to begin with another linear problem also used in QAP studies.

The three-index assignment problem (3-dimensional AP or 3AP), first suggested by Pierskalla (1967a,b, 1968), searches for two permutations  $\pi$  and  $\varphi \in S_n$ , so that the following expression is minimized:

$$\min_{\pi, \varphi \in S_n} \sum_{i=1}^n c_{i\pi(i)\varphi(i)}. \quad (2.29)$$

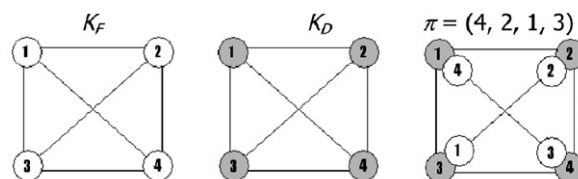


Fig. 1. Allocation of cliques  $K_F$  over  $K_D$  concerning permutation  $\pi = (4, 2, 1, 3)$ .

Burkard and Fröhlich (1980) proposed a branch-and-bound algorithm to solve the 3AP. Emelichev et al. (1984) described transportation models with multiple indices, with details, based on this formulation. The 3-dimensional AP has been studied by several QAP experts: Vlach (1967), Frieze (1974, 1983), Frieze and Yadegar (1981), Burkard et al. (1986, 1996a,b), Euler (1987), Balas and Saltzman (1989, 1991), Bandelt et al. (1991), Crama and Spieksma (1992), Balas and Qi (1993), Burkard and Rudolf (1993), Qi et al. (1994), Magos and Miliotis (1994), Poore (1994a,b, 1995), Burkard and Çela (1996), Magos (1996), Poore and Robertson (1997) and Burkard (2002).

A wide range of QAP theoretical studies involve several related quadratic problems, such as the **quadratic bottleneck assignment problem**, the **biquadratic assignment problem**, the **3-dimensional QAP**, the **quadratic semi-assignment problem** and the **multiobjective QAP**. Almost all of these problems were reported in Burkard (2002).

### 2.2.1. The quadratic bottleneck assignment problem (QBAP)

Steinberg (1961) considered QBAP a variation of QAP with applications to backboard wiring. In that work, a placement algorithm was presented for the optimal connection of  $n$  elements in individual locations so that the length wire needed to connect two elements is minimized. The basic claim of the paper is: *the optimal weighted-wire-length equals the least among the maximum-wire-length norms*. This concept arises from the principle that it may be better to minimize the largest cost in a problem, than to minimize the overall cost.

The QBAP general program is obtained from the QAP formulation by substituting the maximum operation in the objective function for the sums, which suggests the term *bottleneck function*:

$$\min_{\pi \in S_n} \max \{f_{ij} d_{\pi(i)\pi(j)} : 1 \leq i, j \leq n\}. \quad (2.30)$$

A general formulation related to (2.30) was studied in Burkard and Finke (1982), Burkard and Zimmermann (1982), Kellerer and Wirsching (1998) and Burkard (2002).

### 2.2.2. The biquadratic assignment problem (BiQAP)

Proposed by Burkard et al. (1994), this problem can be found in other works, such as Burkard and Çela (1995), Mavridou et al. (1998) and Burkard (2002). The flow and distance matrices are order  $n^4$  and the BiQAP-formulation is:

$$\min_{\pi \in S_n} \sum_{i,j,k,l=1}^n f_{ijkl} d_{\pi(i)\pi(j)\pi(k)\pi(l)}. \quad (2.31)$$

### 2.2.3. The quadratic 3-dimensional assignment problem (Q3AP)

Pierskalla (1967b) introduced it in a technical memorandum. The work was never published in the open literature. Since then, nothing on the subject has been found in the publication databases. Hahn et al. (2004) re-discovered the Q3AP while working with others on a problem arising in data transmission system design. The purpose of the work is to jointly optimize pairs of mappings for multiple transmissions using higher order signal constellations. The resulting problem formulation is:

$$\min \left\{ \sum_{i=1}^N \sum_{j=1}^N \sum_{p=1}^N b_{ijp} u_{ij} w_{ip} + \sum_{i=1}^N \sum_{j=1}^N \sum_{p=1}^N \sum_{k=1}^N \sum_{n=1}^N \sum_{q=1}^N \right. \\ \left. \times C_{ijpknq} u_{ij} u_{kn} w_{ip} w_{kq} : \mathbf{u} \in \mathbf{X}, \mathbf{w} \in \mathbf{X}; \mathbf{u}, \mathbf{w} \text{ binary} \right\}, \quad (2.32)$$

where

$$\mathbf{x} \in \mathbf{X} \equiv \left\{ \mathbf{x} \geq 0 : \sum_{i=1}^N x_{ij} = 1 \text{ for } j = 1, \dots, N; \sum_{j=1}^N x_{ij} = 1 \text{ for } i = 1, \dots, N \right\}. \quad (2.33)$$



#### 2.2.4. The quadratic semi-assignment problem (QSAP)

This is a special case used to model clustering and partitioning problems by Hansen and Lih (1992). It can be written as:

$$\min \sum_{k=1}^m \sum_{i,j=1}^n c_{ij} x_{ik} x_{jk} \quad (2.34)$$

$$\text{s.t.} \quad \sum_{k=1}^m x_{ik} = 1 \quad 1 \leq i \leq n, \quad (2.35)$$

$$x_{ij} \in \{0, 1\} \quad 1 \leq i, j \leq n. \quad (2.36)$$

Other applications can be found in Simeone (1986a,b) and Bullnheimer (1998). References for polynomial heuristics and lower bounds are found in Freeman et al. (1966), Magirou and Milis (1989), Carraresi and Malucelli (1994) and Billionnet and Elloumi (2001). Samra et al. (2005) presented a simple, but effective method of enhancing and exploiting diversity from multiple packet transmissions in systems that employ non-binary linear modulations such as phase-shift keying (PSK) and quadrature amplitude modulation (QAM). The optimal adaptation scheme reduces to solving the general form of the QAP, wherein the problem costs cannot be expressed as products of flows and distances (see Eq. (2.6), above).

#### 2.2.5. The multiobjective QAP (mQAP)

Knowles and Corne (2002) presented another QAP variation considering several flow and distance matrices. This problem is a *benchmark case* for multiobjective metaheuristics or multiobjective evolutionary algorithms. According to the authors, this model is **more suitable for some layout problems**, such as the allocation of facilities in hospitals, where it is desired to minimize the products of the flows by the distances between doctors and patients, and between nurses and medical equipment simultaneously. The mathematical expression is then,

$$\begin{aligned} \min_{\pi \in \mathcal{S}_n} \vec{C}(\pi) &= \{C^1(\pi), C^2(\pi), \dots, C^m(\pi)\}, \\ \text{where } C^k(\pi) &= \sum_{i,j=1}^n f_{ij}^k d_{\pi(i)\pi(j)}, \quad 1 \leq k \leq m. \end{aligned} \quad (2.37)$$

In this last constraint,  $f_{ij}^k$  denotes the  $k$ th flow between  $i$ - and  $j$ -facilities. More recently, Knowles and Corne (2003) presented instance generators for the multiobjective version of QAP. Lopez-Ibanez et al. (2004) discuss the design of ACO algorithms. Paquete and Stützle (2004) developed a study of stochastic local search algorithms for the biobjective QAP with different degrees of correlation between the flow matrices. Klemen et al. (2004) analyze a parallel technique and Day and Lamont (2005) present a specific algorithm.

A combination QAP queuing model is discussed by Smith and Li (2001), in the context of traffic networks.

### 3. Lower bounds

The study of **lower bounds** is very important for the development of algorithms to solve mathematical programming and combinatorial **optimization problems**. Generally, the exact methods employ implicit enumeration, in an attempt to guarantee the optimum and, at the same time, to avoid the total enumeration of the feasible solutions. The performance of these methods depends on the computational quality and efficiency of the lower bounds.

Lower bounds are fundamental tools for branch-and-bound techniques and for the evaluation of the quality of the solutions obtained from some heuristic algorithms. One can measure the quality of a lower bound by the gap between its value and the optimal solution. Good lower bounds should be close to the optimum. Lower bounds are useful in exact solution methods, only when they can be calculated quickly. When used in heuristic methods, their quality is more important than speed of calculation.

The QAP lower bound presented by Gilmore (1962) and Lawler (1963) is one of the best known. Its importance is due to its simplicity and its low computational cost. However, it shows an important drawback as its gap grows very quickly with the size of the problem, making it a weak bound for bigger instances. The most recent and promising trends of research are based on semi-definite programming, reformulation–linearization and lift-and-project techniques, although they usually need an extra computational effort. Anstreicher and Brixius (2001) reported a new QAP bound using semi-definite and convex quadratic programming with good relation between cost and quality. White (1994b) used a data decomposition method, linking the actual data to the data of a special class of assignment problems for which bounds are computationally tractable.

The Gilmore and Lawler lower bound (GLB) is given by the solution of the following linear assignment problem (LAP):

$$\min \sum_{i,j=1}^n (b_{ij} + l_{ij}) \cdot x_{ij} \quad (3.1)$$

$$\text{s.t.} \quad \sum_{i=1}^n x_{ij} = 1 \quad 1 \leq j \leq n; \quad (3.2)$$

$$\sum_{j=1}^n x_{ij} = 1 \quad 1 \leq i \leq n; \quad (3.3)$$

$$x_{ij} \in \{0, 1\} \quad 1 \leq i, j \leq n. \quad (3.4)$$

In order to solve (3.1)–(3.4), it is necessary to find the coefficients  $l_{ij}$ , as below:

$$l_{ij} = \min \sum_{k,p=1}^n c_{ijkp} \cdot y_{ijkp} \quad k \neq i, p \neq j \quad (3.5)$$

$$\text{s.t.} \quad \sum_{k=1}^n y_{ijkp} = 1 \quad 1 \leq i, j, p \leq n, \quad (3.6)$$

$$\sum_{p=1}^n y_{ijkp} = 1 \quad 1 \leq i, j, k \leq n, \quad (3.7)$$

$$y_{ijkp} \in \{0, 1\} \quad 1 \leq i, j, k, p \leq n. \quad (3.8)$$

Roucairol (1979, 1987), Edwards (1980), Frieze and Yadegar (1983), Finke et al. (1987), White (1994a), Burkard (1991), Brügger et al. (1997, 1998), and Spiliopoulos and Sofianopoulou (1998) present improvement methods for the GLB and its application to algorithms used to solve QAP.

### 3.1. Bounds based on MILP relaxations

The optimal solution for a MILP-formulation is a lower bound for the corresponding QAP and each dual solution of the linear programming is also a lower bound for the QAP. Several researchers as in Frieze and Yadegar (1983), Assad and Xu (1985), Adams and Johnson (1994), Ramachandran and Pekny (1998) and Karisch et al. (1999) used this principle. Lagrangean relaxation has been applied to the QAP (Michelon and Maculan, 1991). Drezner (1995) also proved that the linear programming relaxation is equal or better than the GLB bound. Adams et al. (in press) calculate bounds using a level-2 reformulation linearization technique (2-RLT) due to Hahn et al. (2001b). The RLT is a general theory for reformulating mixed 0–1

linear and polynomial programs in higher-variable spaces in such a manner that tight polyhedral outer-approximations of the convex hull of solutions are obtained (Adams and Sherali, 1986, 1990). In Sherali and Adams (1999a,b) both first and second-level constructs for the QAP were presented as an illustration of the general methodology.

### 3.2. Bounds based on GLB reformulations

These bounds were adopted by several authors including Frieze and Yadegar (1983), Assad and Xu (1985), Carraresi and Malucelli (1992, 1994) and Adams and Johnson (1994). A bound based on a dual formulation was proposed in Hahn and Grant (1998) and Hahn et al. (1998). The bounds given by Assad and Xu (1985) and by Carraresi and Malucelli (1992, 1994) are comparable to the ones obtained by Frieze and Yadegar (1983) in terms of quality, with the advantage that they demand less computational time. However, there is no theoretical proof concerning its convergence. Those bounds characterize a finite sequence of problems related to the original one, producing a non-decreasing GLB sequence. The computational results in Hahn and Grant (1998) have shown that these bounds are competitive in terms of quality when compared to some of the best bounds and still better in computational time. Sergeev (2004) works with a continuous relaxation of the Adams–Johnson technique.

### 3.3. Bounds based on interior points methods

Resende et al. (1995) used Drezner (1995) theory and solved a MILP linear relaxation using an interior points algorithm (Karmarkar and Ramakrishnan, 1991). This technique gives better quality lower bounds than those obtained by Adams and Johnson (1994). However, these bounds require much computational effort, and they are not recommended for branch-and-bound algorithms. In this case, it is better to use the Hahn and Grant (1998) dual ascent lower bound.

### 3.4. Variance reduction bounds

Initially proposed by Li et al. (1994a), these bounds are based on reduction schemes and are defined from the variance of  $F$  and  $D$  matrices. These bounds, when used in a branch-and-bound algorithm, take less computational time and generally obtain better performance than GLB. They show more efficiency when the flow and distance matrices have high variances.

### 3.5. Bounds based on graph formulation

As we discussed previously, a pair of  $n \times n$  matrices  $F$  and  $D$  associated to a given QAP instance can be seen as the adjacency matrices of two weighted complete graphs  $K_F$  and  $K_D$ . It is known that  $\pi \in S_n$  defines an isomorphism between  $K_F$  and  $K_D$ , then solving the QAP means finding an isomorphism  $\pi \in S_n$  such that  $Z_\pi$  is minimum. Gavett and Plyter (1966) and Christofides and Gerrard (1981) used this concept, decomposing  $K_F$  and  $K_D$  in isomorphic spanning subgraphs to find lower bounds through an LAP relaxation.

### 3.6. Spectral bounds

We consider here the bounds derived from the trace formulation, using the calculation of data matrix eigenvalues. For some time, the quality of the results compensated the computational hardness of the calculations, but recently some of these bounds have been superseded by reformulation–linearization and semi-definite programming bounds. Some references on spectral bounds are Finke et al. (1987), Rendl (1985), Hadley et al. (1990, 1992a,b), Rendl and Wolkowicz (1992) and Karisch et al. (1994).

### 3.7. Semi-definite programming and reformulation–linearization bounds

This new trend uses a number of theoretical tools to obtain linear programming representations of QAP. Zhao et al. (1998) study semi-definite programming (SDP) relaxations; Anstreicher (2001) compares SDP relaxations and eigenvalue bounds; Anstreicher and Brixius (2001) propose a SDP representation of a basic eigenvalue bound; Burer and Vandenbussche (2006) applied Lagrangean relaxation on a lift-and-project QAP relaxation, following the ideas in Lovász and Schrijver (1991), thus obtaining very tight SDP bounds.

A report, by Rendl and Sotirov (2003), discusses a very good semi-definite programming (SDP) lower bound for the QAP. In 2003, when the report was written, it reported the tightest lower bounds for a large number of QAPLIB instances. The Rendl and Sotirov bound is also found in Fischer et al. (2006). In an as yet unfinished technical report, Povh and Rendl (2006) point out that the strongest relaxation (R3) from Rendl and Sotirov (2003) and the relaxation from Burer and Vandenbussche (2006) are actually the same. The differing lower bound values in the two papers are due to the fact that Rendl and Sotirov use the bundle method, which gives only underestimates of the true bound, while Burer and Vandenbussche are able to compute this bound more accurately.

### 3.8. A comparison of QAP lower bounds

Table 1 presents a comparison of the different QAP lower bounds discussed above. In the table, GLB62 is the Gilmore-Lawler bound from Gilmore (1962); RRD95 is the interior-point bound from Resende et al. (1995); HG98 is the 1-RLT dual ascent bound from Hahn and Grant (1998); KCCEB99 is the dual-based bound from Karisch et al. (1999); AB01 is the quadratic programming bound from Anstreicher and Brixius (2001); RRRP02 is the 2-RLT interior point bound from Ramakrishnan et al. (2002); RS03 is the SDP bound from Rendl and Sotirov (2003), BV04 is the lift-and-project SDP bound from Burer and Vandenbussche (2006) and HH01 is the Hahn-Hightower 2-RLT dual ascent bound from Adams et al. (to appear). The best lower bounds are in the shaded cells of the table. Lower bounds are listed in the table in order of date they were published, with the exception of HH01. The HH01 bound was first published in 2001, but the HH01 bound calculations in Table 1 were recently calculated and thus were placed in the rightmost

Table 1  
Comparison of lower bounds

	Optimum	GLB62	RRD95	HG98	KCCEB99	AB01	RRRP02	RS03	BV04	HH01
Had16	3720	3358		3558	3553	3595		3699	3672	3720*
Had18	5358	4776		5083	5078	5143		5317	5299	5358*
Had20	6922	6166		6571	6567	6677		6885	6811	6922*
Kra30a	88900	68360		75853	75566	68572		77647	86678	86247
Kra30b	91420	69065		76562	76235	69021		81156	87699	87107
Nug12	578	493	523	523	521	498	578*	557	568	578*
Nug15	1150	963	1041	1039	1033	1001		1122	1141	1150*
Nug20	2570	2057	2182	2179	2173	2290		2451	2506	2508
Nug30	6124	4539		4793	4785	5365		5803	5934	5750
Rou15	354210	298548		323943	323589	303777		333287	350207	354210*
Rou20	725520	559948		642058	641425	607822		663833	695123	699390
Tai20a	703482	580674		617206	616644	585139		637300	671685	675870
Tai25a	1167256	962417		1006749	1005978	983456		1041337	1112862	1091653
Tai30a	1818146	1504688		1566309	1565313	1518059		1652186	1706875	1686290
Tho30	149936	90578		99995	99855	124684		136059	142814	136708

\* Problem solved exactly by lower bound calculation

column. It should be noted that HH01 is a dual ascent procedure that underestimates the RLT-2 lower bound described in [Adams et al. \(to appear\)](#) and in [Ramakrishnan et al. \(2002\)](#).

#### 4. Resolution methods

The methods used in combinatorial optimization problems can be either exact or heuristic. In the first case, the most frequent used strategies are branch-and-bound or dynamic programming general methods. On the other hand, there are a number of heuristic techniques using different conceptions. In what follows, we discuss both approaches and we quote their most important references.

##### 4.1. Exact algorithms

The different methods used to achieve a global optimum for the QAP include branch-and-bound, cutting planes or combinations of these methods, like branch-and-cut, and dynamic programming. Branch-and-bound are the most known and used algorithms and are defined from allocation and cutting rules, which define lower bounds for the problem. We can find the first enumerative schemes that use lower bounds to eliminate undesired solutions: [Gilmore \(1962\)](#), [Land \(1963\)](#) and [Lawler \(1963\)](#). Several references concerning QAP branch-and-bound algorithms are available, as [Gavett and Plyter \(1966\)](#), [Nugent et al. \(1968\)](#), [Graves and Whinston \(1970\)](#), [Pierce and Crowston \(1971\)](#), [Burkard and Stratman \(1978\)](#), [Bazaraa and Elshafei \(1979\)](#), [Mirchandani and Obata \(1979\)](#), [Roucairol \(1979\)](#), [Burkard and Derigs \(1980\)](#), [Edwards \(1980\)](#), [Bazaraa and Kirca \(1983\)](#), [Kaku and Thompson \(1986\)](#), [Pardalos and Crouse \(1989\)](#), [Burkard \(1991\)](#), [Laursen \(1993\)](#), [Mans et al. \(1995\)](#), [Bozer and Suk-Chul \(1996\)](#), [Pardalos et al. \(1997\)](#), [Brünger et al. \(1998\)](#), [Ball et al. \(1998\)](#), [Spiliopoulos and Sofianopoulou \(1998\)](#), [Brixius and Anstreicher \(2001\)](#) and [Hahn et al. \(2001a,b\)](#). In recent years, procedures that combine branch-and-bound techniques with parallel implementation are being widely used. Due to them, the best results for the QAP are being achieved. Yet, it is important to observe that the success for the instances of bigger sizes is also related to the hardware technological improvements, [Roucairol \(1987\)](#), [Pardalos and Crouse \(1989\)](#), [Mautor and Roucairol \(1994a\)](#), [Brünger et al. \(1997\)](#), [Clausen and Perregaard \(1997\)](#).

Dynamic programming is a technique used for QAP special cases where the flow matrix is the adjacency matrix of a tree. [Christofides and Benavent \(1989\)](#) studied this case using a MILP approach to the relaxed problem. It was then solved with a dynamic programming algorithm, taking advantage of the polynomial complexity of the instances. This technique was also used by [Urban \(1998\)](#).

Cutting plane methods introduced by [Bazaraa and Sherali \(1980\)](#), initially, did not present satisfactory results. However, they contributed in the formulation of some heuristics that use MILP and Benders decomposition. The employed technique is not widely used so far, but good quality solutions for QAP cases are being presented. The slow convergence of this method makes it proper only for small instances ([Kaufman and Broeckx \(1978\)](#), [Bazaraa and Sherali \(1980, 1982\)](#) and [Burkard and Bonniger \(1983\)](#)). Recently, [Miranda et al. \(2005\)](#) use Benders decomposition algorithm to deal with a motherboard design problem, including linear costs in the formulation.

The branch-and-cut technique, a variation proposed by [Padberg and Rinaldi \(1991\)](#), appears to be an alternative cutting strategy that exploits the polytope defined by the feasible solutions of the problem. Its main advantage over cutting planes is that the cuts are associated with the polytope's facets. Cuts associated with facets are more effective than the ones produced by cutting planes, so the convergence to an optimal solution is accelerated. The dearth of knowledge about the QAP polytope is the reason why polyhedral cutting planes are not widely used for this problem. In this context, some researchers have been describing basic properties of the polytope that can contribute to future algorithm development: [Jünger and Kaibel \(2000, 2001a,b\)](#), [Padberg and Rijal \(1996\)](#), [Kaibel \(1998\)](#) and [Blanchard et al. \(2003\)](#).

#### 4.1.1. The effects of methodology and computer speed improvements

In Table 2, we present a summary of what was achieved by QAP research in the last 35 years, using as an example the progress made on solving exactly the classical Nugent instances, Nugent et al. (1968). The data came from Hahn (2000) and Brixius and Anstreicher (2001). The first result (for Nug08) was obtained by complete enumeration; all the others have been obtained by several branch-and-bound variations. Owing to lack of space, the references within the table are quoted by their number in the reference list. The column “Single CPU seconds” allows for some comparison of results obtained on different machines.

A number of important problem instances have been solved to optimality in recent years. The details can be found on the QAPLIB homepage (QAPLIB (2004)): Ste36b-c (by Nyström in 1999); Bur26a (by Hahn in October 2001); Ste36a (by Brixius and Anstreicher in October 2001); Kra30b, Kra32 and Tho30 (by Anstreicher et al. in November 2000); Kra30a (by Hahn in December 2000); Tai25a (by Hahn in 2003); and Bur26b-h (by Hahn in 2004).

Table 2  
Progress made on solving the Nugent instances exactly

Size	Bound	Year	Machine	CPU speed	Single CPU seconds	No. nodes	Who [Ref.]	Mins (Norm)
8	–	1968	GE 265 <sup>a</sup>		3374	40,320	Palubeckis (2000)	
8	GL	1975	CDC CYBER-76		<1		Burkard (1975)	
12	GL	1978	CDC CYBER-76		29		Burkard and Stratman (1978)	
15	GL	1980	CDC CYBER-76		2947		Burkard and Derigs (1980)	
15	GL	1994	Cray 2		121		Merz and Freisleben (2000)	
16	GL	1994	Cray 2		969		Merz and Freisleben (2000)	
20	GL	1995	i860	40 MHz	811,440	360,148,026	Colorni et al. (1996)	845
20	RLT1	1995	SPARC10	75 MHz	159,900	724,289	Hahn et al. (1998)	333
20	QP	1999	HP-C3000	300 MHz	8748	1,040,308	Anstreicher and Brixius (2001)	146
20	RLT1	1999	UltraSPARC10	360 MHz	5087	181,073	Hahn et al. (2001a)	42
22	C-M	1995	16 i860	40 MHz	48,308,400	48,538,844,413	Colorni et al. (1996)	50,321
22	RLT1	1995	DEC Alpha 500	300 MHz	1,812,420	10,768,366	Hahn et al. (1998)	10,270
22	QP	1999	HP-C3000	300 MHz	8058	1,225,892	Anstreicher and Brixius (2001)	134
22	RLT1	1999	UltraSPARC10	360 MHz	48,917	1,354,837	Hahn et al. (2001a)	408
24	GL	1997	32 Motorola 604		82,252,800	Unknown	Brünger et al. (1997)	466,099
24	RLT1	1997	DEC Alpha 500	300 MHz	4,859,940	49,542,338	Hahn (2000)	27,540
24	QP	2000	HP-C3000	300 MHz	349,794	31,865,440	Anstreicher and Brixius (2001)	5830
24	RLT2	2000	DEC Alpha 500	300 MHz	1,487,724	16,710,701	Hahn et al. (2001b)	8430
25	RLT1	1998	UltraSPARC10	360 MHz	5,698,818	108,738,131	Hahn (2000)	64,207
25	QP	2000	HP-C3000 (1) <sup>a</sup>	300 MHz	715,020	71,770,751	Anstreicher et al. (2002)	11,917
25	RLT1	2000	HP-J5000	440 MHz	1,393,117	27,409,486	Hahn et al. (2001a)	31,879
25	RLT2	2000	Dell 7150	733 MHz	254,179	11,796	Hahn et al. (2001b)	5816
27	QP	2000	HP-C3000 (2) <sup>a</sup>	300 MHz	5,676,480	~402,000,000	Anstreicher et al. (2002)	94,608
27	RLT2	2001	IBM S80	450 MHz	1,579,956.31	46,315	Hahn et al. (2001b)	37,639
28	QP	2000	HP-C3000 (3) <sup>a</sup>	300 MHz	27,751,680	~2,230,000,000	Anstreicher et al. (2002)	462,528
28	RLT2	2001	IBM S80	450 MHz	8,682,044	202,295	Hahn et al. (2001b)	206,922
30	QP	2000	HP-C3000 (4) <sup>a</sup>	300 MHz	218,859,840	11,892,208,412	Anstreicher et al. (2002)	3,647,664
30	RLT2	2004	Dell 7150	733 MHz	59,986,514	543,061	Adams et al. (in press)	1,369,692

<sup>a</sup> Means equivalent single CPU seconds in HP-C3000, for time on computational pools with active machines (average): (1) 185; (2) 185; (3) 224; (4) 653.



#### 4.2. Heuristic algorithms

Heuristic algorithms do not give a guarantee of optimality for the best solution obtained. Approximate methods are included in this category. These have the additional property that worst-case solutions are known. As a matter of fact, it is not unusual to find approximate algorithms called heuristic algorithms in the Combinatorial Optimization literature, as in [Osman and Laporte \(1996\)](#). In this context, we consider heuristic techniques as a procedure dedicated to the search of good quality solutions. Heuristic procedures include the following categories: constructive, limited enumeration and improvement methods. The most recent techniques that can be adapted to a wide range of optimization problems are called metaheuristics and will be discussed in the next section.

[Gilmore \(1962\)](#) introduced a *constructive method* that completes a permutation (i.e., feasible solution) at each iteration of the algorithm. In this method, sets  $A$  and  $L$  are introduced, the first concerning the allocated facilities and the second the occupied locations, both initially empty. The construction of a permutation  $\pi$  is made by means of a heuristic and, in each step, a new allocation  $(i, j)$  is chosen, so that  $i \notin A, j \notin L$  and making  $\pi(i) = j$ . For an instance of size  $n$ , the process is repeated until a complete permutation on the problem order is achieved. Constructive methods were used in [Armour and Buffa \(1963\)](#), [Buffa et al. \(1964\)](#), [Sarker et al. \(1995, 1998\)](#), [Tansel and Bilen \(1998\)](#), [Burkard \(1991\)](#), [Arkin et al. \(2001\)](#), [Gutin and Yeo \(2002\)](#) and [Yu and Sarker \(2003\)](#). At the end of 1990, multistart techniques are used to begin heuristic or metaheuristic methods. In this category, we cite [Misevicius \(1997\)](#), [Fleurent and Glover \(1999\)](#) and [Misevicius and Riskus \(1999\)](#).

*Enumerative methods*: Enumeration can guarantee that the obtained solution is optimum only if they can go to the end of the enumerative process. However, it is possible that a good solution, or even an optimal solution, is found by the beginning of Enumeration. It can be observed that the better the information used to guide the enumeration, the greater the chances of finding good quality solutions early. However, it usually takes much longer to guarantee optimality. In order to bound the runtime of this enumeration, stopping conditions are defined: maximum number of loops for the whole execution, or between two successive improvements; a limit for the execution time and so on. It becomes clear that any one of these stopping criteria can eliminate the optimum solution, a fact that requires some attention when using bounded enumeration methods ([Burkard and Bonniger, 1983](#); [West, 1983](#)). [Nissen and Paul \(1995\)](#) applied the *threshold accepting* technique to the QAP.

*Improvement methods* correspond to local search algorithms. Most of the QAP heuristics are in this category. An improvement method begins with a feasible solution and tries to improve it, searching for other solutions in its neighborhood. The process is repeated until no improvement can be found. The basic elements for this method are the neighborhood and the selection criterion that defines the order in which the neighbors are analyzed ([Heider, 1973](#); [Mirchandani and Obata, 1979](#); [Bruijs, 1984](#); [Pardalos et al., 1993](#); [Burkard and Çela, 1995](#); [Li and Smith, 1995](#); [Anderson, 1996](#); [Talbi et al., 1998a](#); [Deineko and Woeginger, 2000](#); [Misevicius, 2000a](#); [Mills et al., 2003](#)). Improvement methods are frequently used in metaheuristics.

It is worthwhile to mention that up to this date, approximate algorithms with performance guaranteed for one constant were only obtained for special cases of QAP. Examples are the cases where the distance matrix satisfies the triangular inequality ([Queyranne, 1986](#)) or when the problem is treated as a maximal clique problem with given maximum bound ([Arkin et al., 2001](#)). [White \(1993\)](#) proposed a new approach, where the actual data are relaxed by embedding them in a data space that satisfies an extension of the metric triangle property. The computations become simpler and bounds are given for the loss of optimality.

More recently, [Arora et al. \(2002\)](#) proposed a randomized procedure for rounding fractional perfect assignments to integral assignments. This extends the well-known LP rounding procedure of [Raghavan and Thompson](#), which is usually used to round fractional solutions of linear programs. This procedure may be used to design an additive approximation algorithm for solving the QAP.

### 4.3. Metaheuristics

Before the end of the 1980s, most of the proposed heuristic methods for combinatorial optimization problems were specific and dedicated to a given problem. After that period, this paradigm has changed. More general techniques have appeared, known as metaheuristics. They are characterized by the definition of a priori strategies adapted to the problem structure. Several of these techniques are based on some form of simulation of a natural process studied within another field of knowledge (metaphors). With the advent of metaheuristics, QAP research received new and increased interest. Recall that the QAP is considered a classical challenge or “benchmark” as we mentioned earlier, Moe (2003).

#### 4.3.1. The following metaheuristics are based on natural process metaphors

*Simulated annealing* is a local search algorithm that exploits the analogy between combinatorial optimization algorithms and statistical mechanics (Kirkpatrick et al., 1983). This analogy is made by associating the feasible solutions of the combinatorial optimization problem to states of the physical system, having costs associated to these states energies. Let  $E_i$  and  $E_{i+1}$  be two energy successive states, corresponding to two neighbor solutions and let  $\Delta E = E_{i+1} - E_i$ . The following situations can occur: if  $\Delta E < 0$ , there is an energy reduction and the process continues. In other words, there is a reduction on the problem cost function and the new allocation may be accepted; if  $\Delta E = 0$ , there is a stability situation and there is no change in the energy state. This means that the problem cost function was not changed; if  $\Delta E > 0$ , an increase on the energy is characterized and it is useful for the physical process to permit particle accommodation, i.e., the problem cost function is increased. Instead of eliminating this allocation, its use is subjected to the values of a probability function, to avoid convergence into poor local minima. Burkard and Rendl (1984) proposed one of the first applications of simulated annealing to the QAP. After that, Wilhelm and Ward (1987) presented the new equilibrium components for it. Connolly (1990) introduced an *optimal temperature* concept that gave valuable results. Later, Abreu et al. (1999) applied the technique by trying to reduce the number of inversions associated to the problem solution, together with the cost reduction. Other approaches for the simulated annealing applied to the QAP are Bos (1993), Yip and Pao (1994), Burkard and Çela (1995), Peng et al. (1996), Tian et al. (1996, 1999), Mavridou and Pardalos (1997), Chiang and Chiang (1998), Misevicius (2000b, 2003c), Tsuchiya et al. (2001), Siu and Chang (2002) and Baykasoglu (2004).

*Genetic algorithms* are techniques that simulate the natural selection and adaptation found in nature. These algorithms keep a *population* formed by a subset of *individuals* that correspond, in the QAP case, to the feasible permutations, with fitness values associated to their permutation cost. By means of the so-called *genetic operators*, and of *selection criteria*, the algorithm replaces one population by another with best available fitness values. The scheme is based on the idea that the best individuals survive and generate descendants that carry their genetic characteristics, in the same way that biological species propagate in nature.

Genetic algorithms generally begin with a population of randomly generated initial individuals (i.e., solutions). Their costs are evaluated and a subset with the lowest cost solutions is selected. Genetic operations are applied to this subset, thus generating a new solution set (a new population). The process continues until some stopping criterion is met. See Davis (1987) and Goldberg (1989). Various ideas for the use of genetic algorithms on the QAP can be found in Bui and Moon (1994), Tate and Smith (1995), Mavridou and Pardalos (1997), Kochhar et al. (1998), Tavakkoli-Moghaddain and Shayan (1998), Gong et al. (1999), Drezner and Marcoulides (2003), El-Baz (2004), and Wang and Okazaki (2005). Drezner (2005a) presented a two-phase genetic algorithm. The first one is a quick genetic strategy that runs several times, keeping the best solution at each iteration to build a starting population for Phase 2. The use of these algorithms in the QAP context presents some difficulties in getting the optimal solution, even for small instances. However, some hybrid ideas using genetic algorithms have shown to be more efficient, as discussed later in this paper.

*Scatter search* is a technique introduced by Glover (1977) in a heuristic study of integer linear programming problems. It is an evolutionary method that takes linear combinations of solution vectors in order to produce new solution vectors in successive generations. This metaheuristic is composed of initial and evolutionary phases. In the initial phase, a reference set of good solutions is made. In each subsequent (evolutionary) phase, new solutions are generated using strategically selected combinations of the reference subset. Then, a set of the best newly generated solutions is moved into the reference set. The evolutionary phase procedure is repeated until a stop criterion is satisfied. Application of scatter search to the QAP can be found in Cung et al. (1997).

*Ant colony optimization (ACO)* refers to a class of distributed algorithms that has as its most important feature the definition of properties in the interaction of several simple agents (the ants). Its principle is the way in which ants are able to find a path from the colony to a food source. The set of ants, cooperating in an ordinary activity to solve a problem, constitute the ant system. The main characteristic of this method is the fact that the interaction of these agents generates a synergetic effect, because the quality of the obtained solutions increases when these agents work together, interacting among themselves. Numerical results for the QAP are presented in Maniezzo and Colorni (1995, 1999), Colorni et al. (1996) and Dorigo et al. (1996). Gambardella et al. (1999) show ant colony as a competitive metaheuristic, mainly for instances that have few good solutions close to each other. Other references are in Stützle and Dorigo (1999), Stützle and Holger (2000), Talbi et al. (2001), Middendorf et al. (2002), Solimanpur et al. (2004), Randall (2004) and Ying and Liao (2004). A novel external memory implementation is introduced, based on the use of partially complete sequences of solution components from above-average quality individuals (i.e., ants), over a number of previous iterations. Elements of such variable-size partial permutation sequences are taken from randomly selected locations of parental individuals and stored in an external memory called the partial permutation memory, Acan (2005).

Although *neural networks* and *Markov chains* are structurally different from metaheuristics, they are also based on a nature metaphor and they have been applied to the QAP, Bousonocalzon and Manning (1995), Liang (1996), Obuchi et al. (1996), Tsuchiya et al. (1996), Ishii and Sato (1998, 2001), Rossin et al. (1999), Shin and Niitsuma (2000), Nishiyama et al. (2001), Hasegawa et al. (2002) and Uwate et al. (2004).

#### 4.3.2. The following metaheuristics are based directly on theoretical and experimental considerations

*Tabu search* is a local search algorithm that was introduced by Glover (1989a,b) to find good quality solutions for integer programming problems. Its main feature is an updated list of the best solutions that were found in the search process. Each solution receives a priority value or an aspiration criterion. Their basic ingredients are: a *tabu list*, used to keep the history of the search process evolution; a mechanism that allows the acceptance or rejection of a new allocation in the neighborhood, based on the tabu list information and on their priorities; and a mechanism that allows the alternation between neighborhood diversification and intensification strategies. Adaptations for the QAP can be found in Skorin-Kapov (1990, 1994), Taillard (1991), Bland and Dawson (1991), Rogger et al. (1992), Chakrapani and Skorin-Kapov (1993), Misevicius (2003a, 2005) and Drezner (2005b). Despite the inconvenience of depending on the size of the tabu list and the way this list is managed, the performances of those algorithms show them as being very efficient strategies for the QAP, as analyzed by Taillard (1991) and Battiti and Tecchiolli (1994). Taillard (1995) presents a comparison between the uses of tabu search and genetic algorithm, when applied to the QAP.

*Greedy randomized adaptive search procedure (GRASP)* is a random and iterative technique where, at each step, an approximate solution for the problem is obtained. The final solution is the best resulting one among all iterations. At each step, the first solution is constructed through a random greedy function and the following solutions are obtained by applying on the previous solution a local search algorithm that gives a new best solution regarding to the previous one. At the end of all iterations, the resulting solution is the best generated one. It is not guaranteed that GRASP solutions do not stick to a local optimum, so it is

important to apply the local search phase to try to improve them. The use of suitable data structures and careful implementations allow an efficient local search. This technique was applied to the QAP by several researchers, as follows: Li et al. (1994b), Feo and Resende (1995), Resende et al. (1996), Fleurent and Glover (1999), Ahuja et al. (2000), Pitsoulis et al. (2001) and Rangel et al. (2000). Oliveira et al. (2004) built a GRASP using the path-relinking strategy, which looks for improvements along the paths joining pairs of good solutions. GRASP was also applied to a QAP variation: BiQAP, Mavridou et al. (1998).

*Variable neighborhood search (VNS)* was introduced by Mladenovic and Hansen (1997). It is based on systematic movement within a set of neighborhoods, conveniently defined. A number of change rules can be invoked. A change is applied when the exploring the current neighborhood does not produce a better solution. VNS has been applied to large combinatorial problem instances. In Taillard and Gambardella (1999), three VNS strategies are proposed for the QAP. One of them is a search over variable neighborhood, according to the basic paradigm. The other two are hybrids in combination with some of the previously described methods.

Stützle (in press) applied an iterated local search (ILS), a simple and powerful stochastic local search, wherein, to avoid stagnation behavior, using acceptance criteria that allow moves to poorer local optima enhances the algorithm. The paper proposes population-based ILS extensions.

There are several other hybrid algorithms for the QAP. In Bölte and Thonemann (1996), a combination of simulated annealing and genetic algorithm is presented. Battiti and Tecchiolli (1994), Bland and Dawson (1994), Chiang and Chiang (1998) and Misevicius (2001, 2004a) use tabu search with simulated annealing, while Talbi et al. (1998b) and Hasegawa et al. (2002) use tabu search with a neural network, and Youssef et al. (2003) use tabu search, simulated annealing and fuzzy logic together. Some hybrid algorithms combine a genetic algorithm with tabu search, Fleurent and Ferland (1994), Drezner (2003), or with a greedy algorithm, Ahuja et al. (2000). All were proved to be more promising than the genetic algorithm alone.

More recently, there are more complex procedures in this class, such as Lim et al. (2000, 2002), who work with hybrid genetic algorithms based on k-gene exchange local search and Misevicius (2004b), who introduced new results for the quadratic assignment problem used an improved hybrid genetic procedure. Dunker et al. (2004) combined dynamic programming with evolutionary computation for solving a dynamic facility layout problem. Balakrishnan et al. (2003) used GATS, a hybrid algorithm that considers a possible planning horizon, which combines genetic with tabu search and is designed to obtain all global optima.

Some categories of hybrid genetic algorithms are known as mimetic algorithms or evolutionary algorithms and some works can be found in this context: Brown and Huntley (1991), Nissen (1994), Huntley and Brown (1996), Merz and Freisleben (1997, 1999, 2000), Nissen (1997), Ostrowski and Ruoppila (1997). Misevicius et al. (2002) presented an algorithm based on the reconstruct and improve principle. The main components of this meta-heuristics are a reconstruction (mutation) procedure and an improvement (local search) procedure. Misevicius (2003b,d) presented a new heuristic, based on the run and recreate cause. The main components are ruin (mutation) and a recreate (improvement) procedure.

There is also a technique introduced by Goldbarg and Goldbarg (2002) that uses a variation of the genetic algorithms, known as transgenetic heuristics. In the QAP case, the results presented are comparable to others, with virtually no improvement in computational times. The use of several metaheuristics and hybrid proposals on QAP is discussed and their results are compared and analyzed in Maniezzo and Colnani (1995) and Taillard et al. (2001). Kelly et al. (1994) studied diversification strategies for the QAP; Fedjki and Duffuaa (2004) developed a work using extreme points in a search algorithm to solve the QAP. Finally, several techniques use parallel and massive computation, Roucairol (1987), Pardalos and Crouse (1989), Brown and Huntley (1991, 1996), Taillard (1991), Chakrapani and Skorin-Kapov (1993), Laursen (1993), Mans et al. (1995), Obuchi et al. (1996), Brüngger et al. (1997, 1998), Clausen et al. (1998), Talbi et al. (1998a,b, 2001), Anstreicher et al. (2002) and Moe (2003). Most of these references were cited in other procedure classes.

## 5. The main research trends and tendencies

In this section, we seek to identify the research trends with time, during the almost 50 years after the QAP first appeared in the literature. This study raises a number of questions concerning researcher preferences and also the needs for formulations, techniques and theoretical developments. We also consider the influence of hardware development throughout different periods and the possibilities brought about by the most recent conquests represented by parallel processing and metacomputing.

The bibliography presented in this work lists 365 publications, of which *about 95%* deal directly with QAP. The curve in Fig. 2 shows a steady increase in interest in the QAP in recent years. The 2005 data collected from January through July have been extrapolated in Figs. 2, 8 and 9 in order to estimate what they would have been for the entire year. Apparently, there is a moderate drop in QAP research interest in 2005.

In the following figures, the publications are grouped by the approach strategy, determined by the formulation classification given in Section 2; the kinds of lower bounds adopted according to classification of Section 3; the solution techniques or procedures given in Section 4; the reference distribution concerning algorithmic, theoretical or applied work along the time (periods of 5 years). To finish this section, we point out new research tendencies based on recent advances.

Fig. 3 presents the number of publications related to the different QAP formulations classified in this work as graphs (GR), trace (TR), mixed integer linear programming (MILP), integer linear programming (ILP) and permutations (PM). We observed that the QAP approach that identifies solutions with permutations is the most used, followed by ILP, MILP and TR formulations. The formulations using exclusively graphs are not encountered as often, perhaps because they are more recent.

Fig. 4 presents the number of publications whose primary subject is lower bounds, categorized by the classifications that are adopted in this article: graph formulation, variance reduction and interior points based bounds (OTHERS), semi-definite programming (SDP), spectral bounds (SB), MILP relaxation based bounds, (MILP) and, finally, Gilmore and Lawler (GLB) bounds.

Observing Fig. 4, we conclude that most works use lower bounds derived from the GLB and MILP bounds, followed by SB relaxation based bounds. However, GLB is the most traditional and frequently the quickest to produce results.

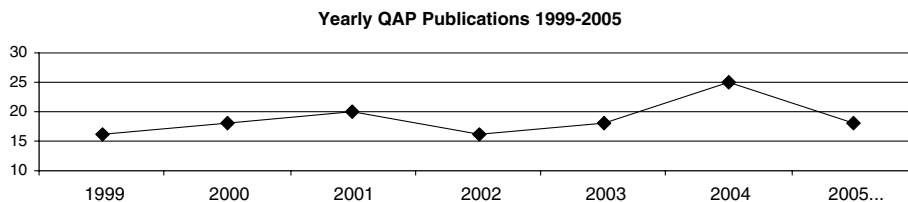


Fig. 2. Number of QAP papers published in recent years.

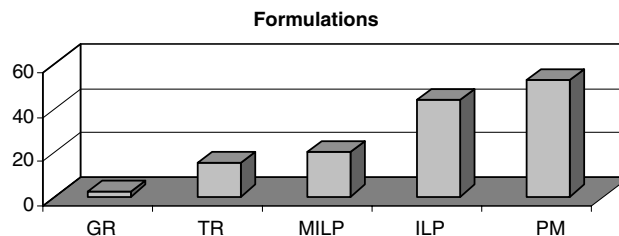


Fig. 3. Publications on different QAP formulations.



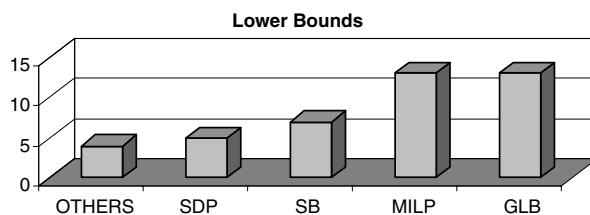


Fig. 4. Publications on QAP lower bounds.

Fig. 5 shows the distribution of publications, categorized by solution techniques that were classified in this work as Heuristic Methods, Exact Methods and Metaheuristics.

About 30 papers deal with exact methods, while more than one hundred are dedicated to heuristic or metaheuristic methods, a natural consequence of the NP-hardness of the problem. In writing this survey, the space dedicated to these three general techniques follows this trend.

Fig. 6 shows the distribution of references by metaheuristic resolution methods. In this figure, we have scatter search (SS), variable neighborhood search (VNS), greedy randomized adaptive search procedure (GRASP), genetic algorithm (GA), neural networks and others (NNO), tabu search (TS), ant colony (AC), simulated annealing (SA) and hybrid algorithms (HA).

Hybrid procedures that result from different metaheuristic compositions are by far the most utilized solution procedure. However, when we look for a comparison among pure metaheuristics, the procedures based on more traditional simulated annealing and more recently defined ant colony are the most popular.

Fig. 7 shows the distribution of QAP publications with respect to the categories: applications, theory (i.e., formulations, complexity studies and lower bounding techniques) and algorithms.

Fig. 8 distributes the number of articles by 5-year periods since 1957, when the QAP was first proposed. For each period, the work is also classified according to the same categories of Fig. 7. We observe that an explosion of interest in theory and algorithm development occurred in the period from 1992 to the present. The last period is only partially over, but level of interest and trends remain the same.

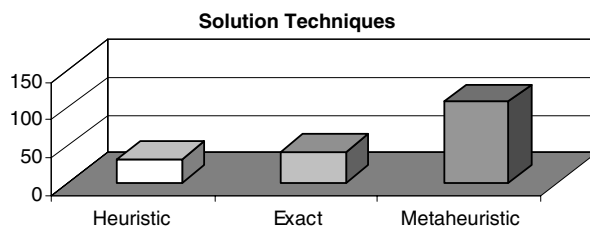


Fig. 5. Publications: solution techniques.

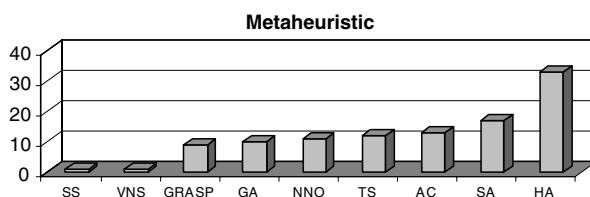


Fig. 6. Publications: metaheuristics used to the QAP.



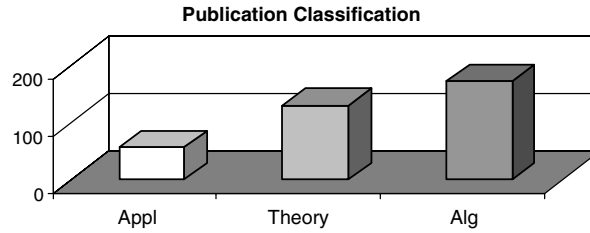


Fig. 7. Publications classification according to their contents.

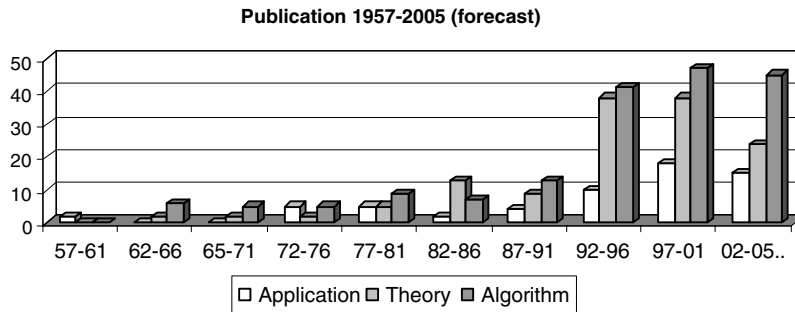


Fig. 8. Number of publications in a 5-year basis.

The QAP seems to have attracted little interest until the middle of the 70s. The 80s have seen a number of theoretical developments, followed in the late 80s by a growing interest in algorithms to which the theory naturally pointed. By the end of the 80s, with the emerging of metaheuristics, the problem received more attention, partly as a benchmark: a metaheuristic would be considered competitive if, when applied to the QAP, could achieve better results than those already known. The research by the end of the 90s profited from the development of computer technology, both in hardware and in software, including larger random access memories and available capacity management alternatives, such as parallel computing and metacomputing. This, combined with the available exact techniques, made it possible to find optimal solutions for larger instances (over  $n = 30$ ) and also to obtain better solutions for some bigger instances, [QAPLIB \(2004\)](#).

Fig. 9, which covers the recent years, shows that the interest in algorithms continues to be very strong, with a cyclical trend in theoretical developments. Applications continue to be of interest, but to a lesser extent. It is noteworthy that there is recently a growing interest in applications to communication link design and to the optimization of communications networks.

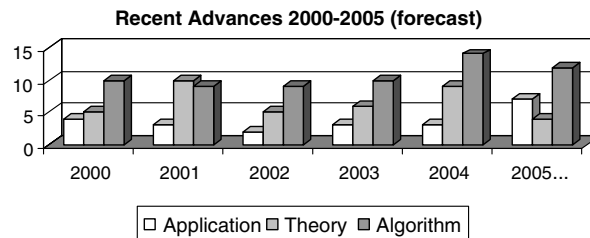


Fig. 9. Recent advances.

## Acknowledgments

We are indebted to CNPq (Council for the Scientific and Technological Development, of the Brazilian government) for the support we received both through scholarships and financial support for research. We are also indebted to Kurt Anstreicher and Zvi Drezner for their criticism and suggestions and to the anonymous referees. The reporting of results in [Tables 1 and 2](#) was partially supported by the US National Science Foundation Grant Number DMI-0400155.

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