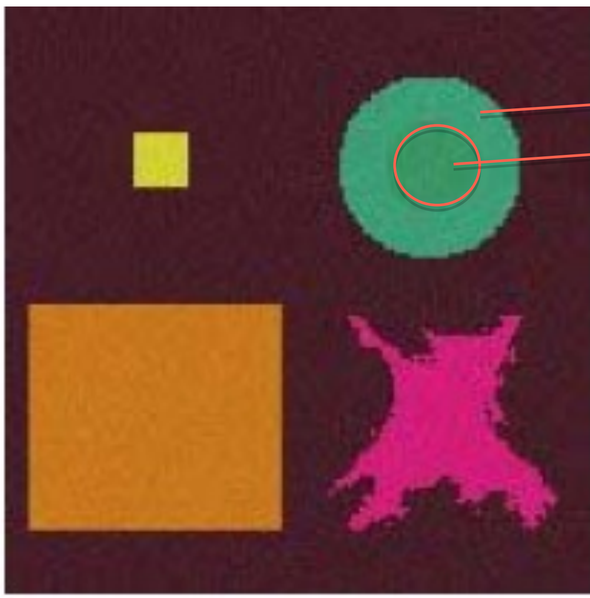
Fuzzy Logic

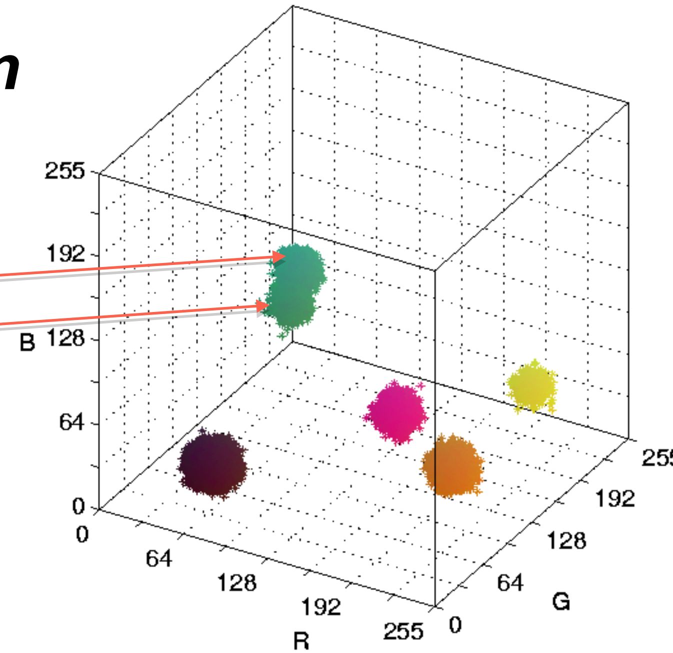
« What men really want is not knowledge but certainty. »
Bertrand Russel

Introduction

Colour image segmentation



Image



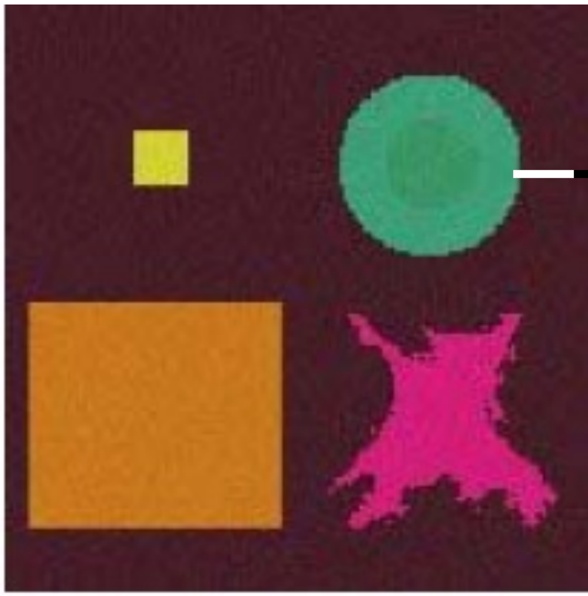
RGB color space

An image is considered as a collection of regions.

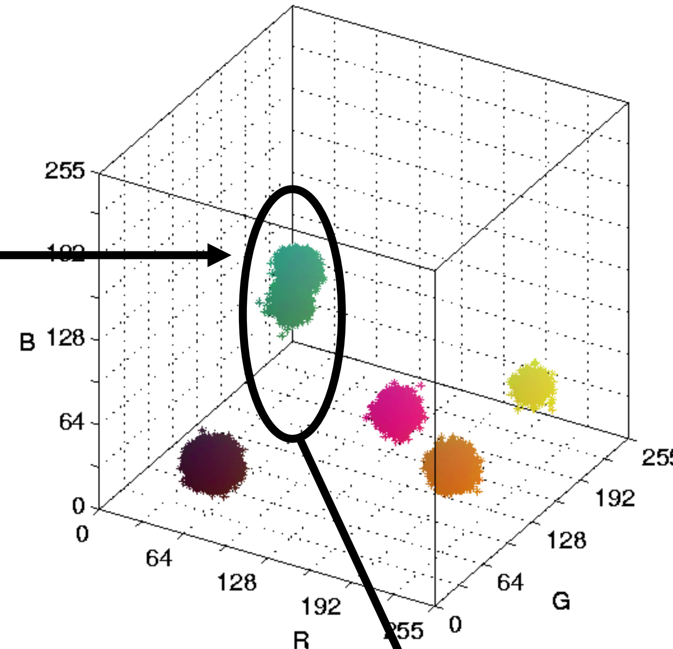
But, it is not easy (most of the time) to determine the number of regions

Introduction

Colour image segmentation based on clustering techniques

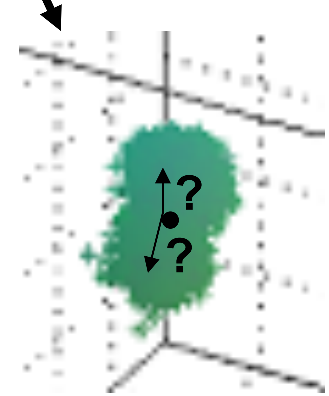


Image



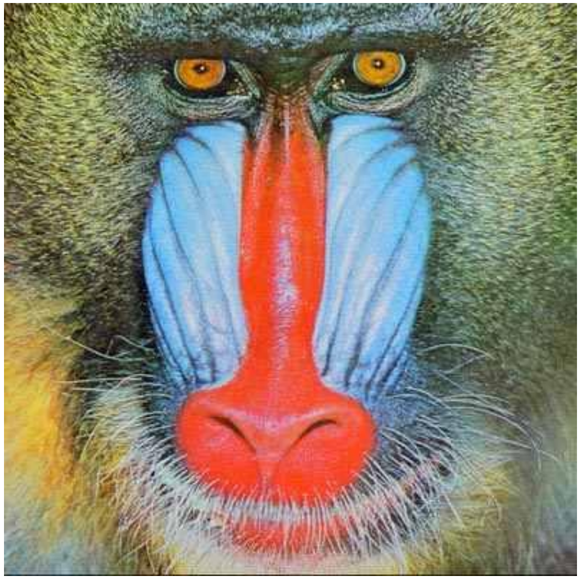
RGB color space

In a « perfect » world … a pixel belongs or does not belong to a cluster, but… is there an easy way to achieve a sure and good result ?

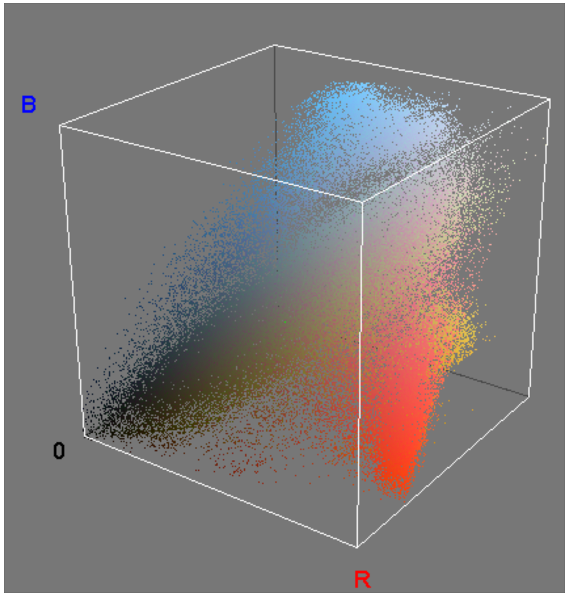


Introduction

Colour image segmentation



Image



RGB color space

An image is considered as a collection of regions.

But, it is not easy (most of the time) to determine the number of regions

Introduction

Colour image segmentation

« Crisp » approaches

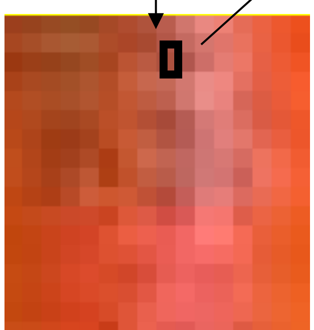


Image = a collection of regions.

A pixel $p(i,j)$ of the image is such as: $p(i,j) \in R_k$ or $p(i,j) \notin R_k$

We feel that a pixel belongs more or less to a region.

Introduction

Colour image segmentation

Fuzzy approaches

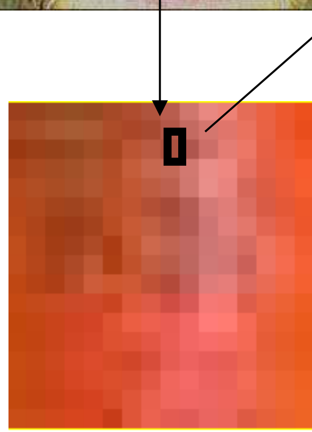


Image = a collection of regions.
Quantifying the membership degree of a pixel $p(i,j)$ to each possible region.
Then, assigning the pixel to the most plausible region (decision-making).

A possible solution

Try to take into account the partial membership to different regions

So :

- ❑ How to express and to quantify the membership degrees?
- ❑ How to manage the membership degrees in a decision-making process?

One solution: use a fuzzy logic approach

Fuzzy logic

- A way to manage uncertainty about the membership degree of an « object » to a particular class of objects among a set of classes (or clusters).

- Something which is quite « natural » in the real life…

- A step forward compared with the "classical" logic

- Some basic tools of the fuzzy logic are given hereafter

We works with fuzzy logic....

<u>Example of fuzzy rules:</u>

Rules for driving when we get closer of a crossroad controlled by traffic lights.

If the traffic light is red...	If my speed is raised...	And if the traffic light is close...	Then I brake hardly.
If the traffic light is red...	If my speed is low...	And if the traffic light is far…	Then I maintain my speed.
If the traffic light is orange...	If my speed is average	And if the traffic light is far...	Then I brake softly.
If the traffic light is green...	If my speed is low...	And if the traffic light is close …	Then I accelerate.

The fuzzy rules are expressed in natural language

…without knowing it!

Transposition of our example according to a more mathematical "less fuzzy" model:

If the light is red, if my speed exceeds 85,6 km/h and if the traffic light is unless 62,3 meters, then I press on the brake with a strength of 33,2 Newtons!!!

Our brain works in fuzzy logic

It estimates the input variables in a rough (approximative) way (low/ weak, high, far, close), and does the same for the output variables (light or strong braking) and promulgates a set of rules allowing to determine the outputs according to the inputs.

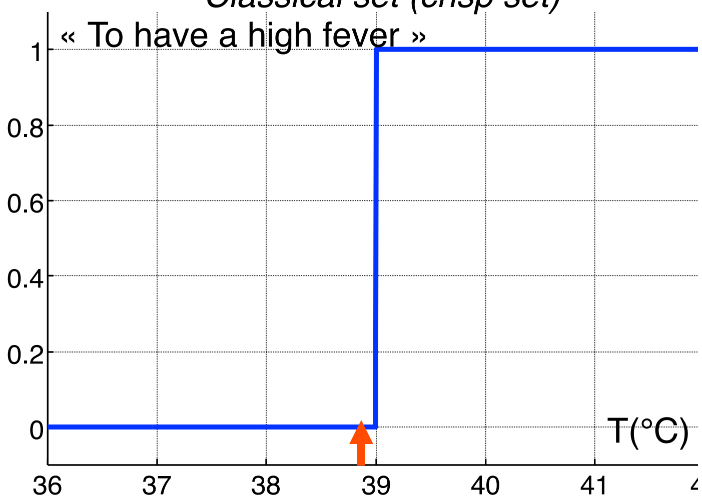
Limits of the boolean logic

A patient who is suffering from an hepatitis has generally the following symptoms:

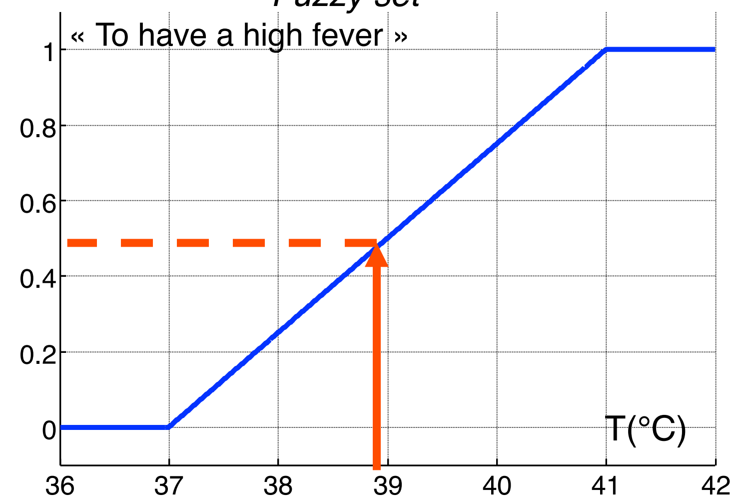
- ❑ the patient has a high fever;
- ❑ the skin presents a yellow tint;
- ❑ he has nausea.

If the temperature of the patient is 38,9°C, then:

Classical set (crisp set)



Fuzzy set



Classical logic: The patient has no high fever, then he is not suffering from hepatitis.

Fuzzy logic: The patient has high fever with a level of 48%⇒ The patient is suffering from an hepatitis with a degree of severity equal to ***x*** % (related to a fuzzy set indicating the severity not represented here).

Application fields of the fuzzy logic

- Help to decision, to diagnosis.
  (medicine, …)
- Databases
  (fuzzy objects and/or fuzzy requests)
- Pattern recognition
- Fuzzy control of systems
- **Image processing**
- …

Points of reference

- 1965: Concept introduced by Pr. Lotfi Zadeh (Berkeley):

  « Fuzzy set theory »: Definition of fuzzy sets and associated operators

- 1970: First applications: Expert systems, decision-making support in medicine, business…

- 1974: first industrial application : fuzzy control of a steam boiler done by Mamdani

- 1985: japanese people sell consumer products with « Fuzzy Logic Inside ».

Points of reference

1990: Generalization

- Household electrical appliances (washing machines, vacuum cleaners, pressure cookers, etc.),
- Audiovisual systems (autofocus, camcorder with stabilizer of images, photocopiers)
- Car : BVA, ABS, suspension, air conditioning, etc.)
- Decision-making support systems for diagnosis, recognition,
- Systems of control / command in most of the industrial domains of production
- ….

- There are dedicated processors and specific interfaces of development (*cf.* 68HC12 from Motorola)
  - Ex: processors WARP (Weight Associative Rule Processor) developed by SGS-THOMSON such as:
    - Number of rules : 256
    - Number of inputs : 16
    - Number of outputs : 16
    - Method of rule aggregation : centre of gravity
    - Processing time: 200 microseconds for 200 rules.

Warning

The approach of the problems by the fuzzy logic is different from that adopted, *a priori*, in a scientific approach (seems to be in fact).

It is much more pragmatic than determinist.

The decision in fuzzy logic is based on the notion of expertise, which allows to quantify the fuzziness (vagueness) from a priori knowledge or knowledge acquired before.

<u>Remark</u>:
It is not necessary to have a model of the inputs/outputs of a car to be able to drive it in a satisfactory way.

Main concepts

Two key points :

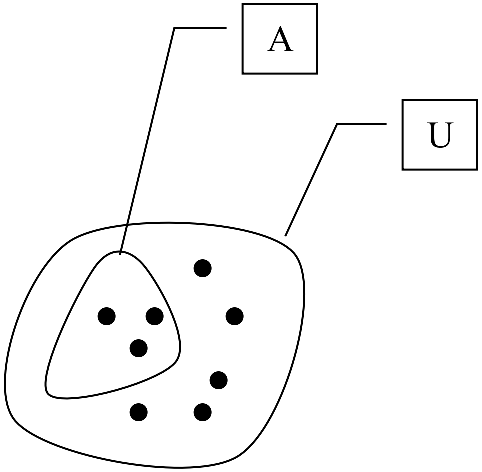
1. The fuzzy sets, the fuzzy variables and associated operators (*e.g.* for aggregation)
2. Decision-making from a basis of rules of type « IF…THEN… ».

☞ fuzzy inference.

Fuzzy set

U: the universe of discourse.

A: a fuzzy set of U



Classical set theory:

If μ_A is the membership function of the crisp set A, then:

$$\forall x \in U \quad \mu_A(x) = 0 \quad \text{if } x \notin A$$
$$\mu_A(x) = 1 \quad \text{if } x \in A$$

Fuzzy set theory:

If μ_A is the membership function of the fuzzy set A, then:

$$\forall x \in U \quad \mu_A(x) \in [0,1]$$

If $\mu_A(x) = 0.3$, it means "x belongs to A with a membership degree equal to 0.3".

Membership degree = value of truth.

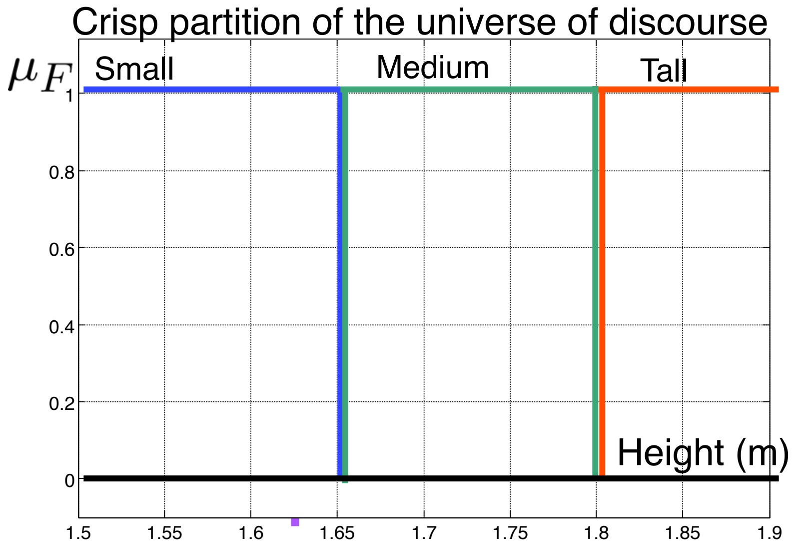
A fuzzy set is totally determined by its membership function.
The membership degree quantifies the grade of membership of the considered element to the considered fuzzy set

Fuzzy set

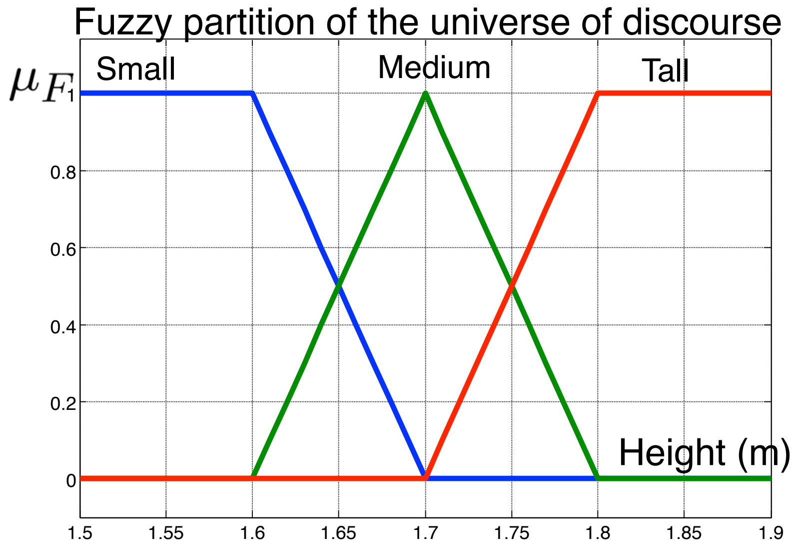
A fuzzy set *F* is defined on a set of values called « the universe of discourse » *U*. *F* is characterized by a membership function:

$$\mu_F : x \in U \longrightarrow \mu_F(x) \in [0, 1]$$

which quantifies the membership degree of each element of *U* to *F*.



Classical logic
(crisp sets)

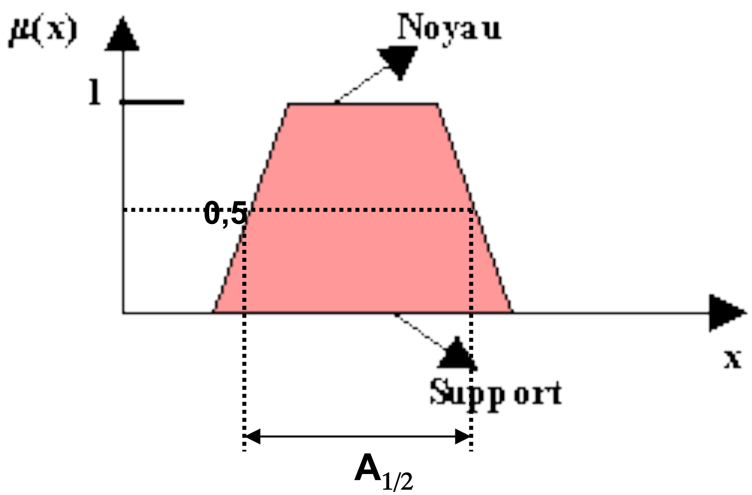


Fuzzy logic
(Gradual evolution)

Fuzzy set

Definition-Properties

$$A = \{(x, \mu_A(x))/x \in U\}$$



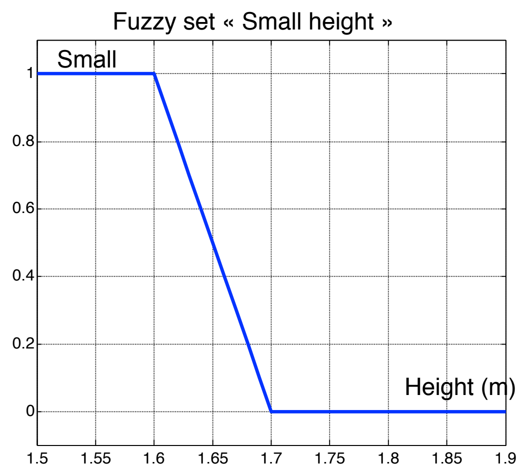
$$supp(A) = \{x \in U/\mu_A(x) \neq 0\}$$

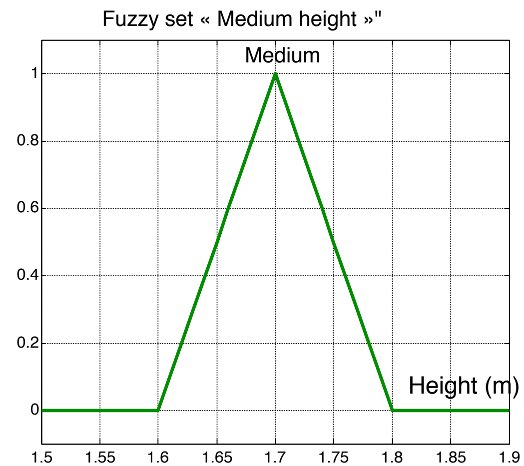
$$noy(A) = \{x \in U/\mu_A(x) = 1\}$$

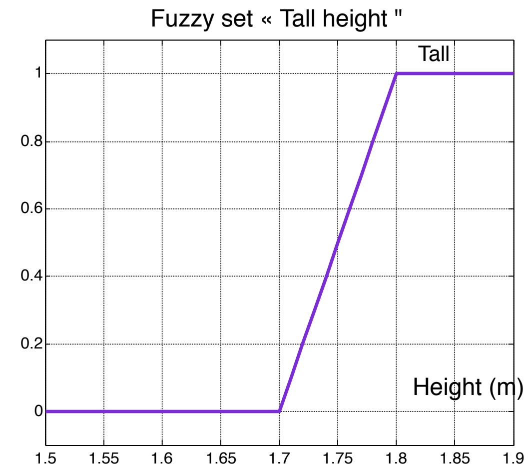
A fuzzy part *A* of *U* can be characterized by the set of its α-*cuts* too.
An **α-cut** of a fuzzy set *A* is the crisp set (classical set) of elements having a membership degree upper or equal to α.

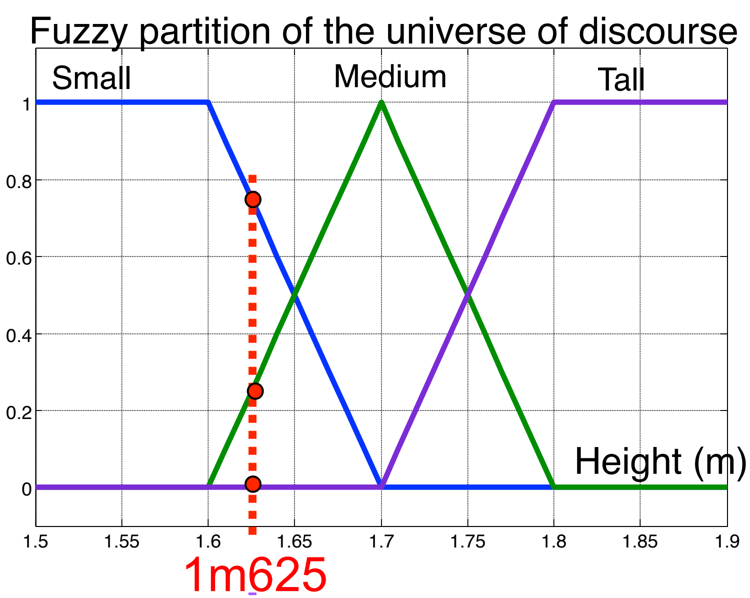
$$\alpha - cut(A) = \{x \in U/\mu_A(x) \geq \alpha\} = A_\alpha$$

Examples of fuzzy sets









Here, « Pierre's height is 1m625 » or « Pierre is 1m625 tall » is expressed by fuzzy logic by:

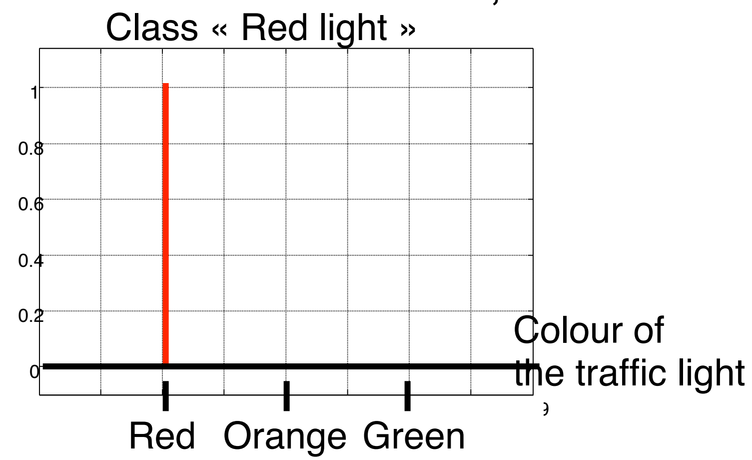
« Pierre is small » with a membership degree of 75%;
« Pierre is medium » with a membership degree of 25%;
« Pierre is tall » with a membership degree of 0%.

Particular membership functions

SIngletons:

When a certain fact corresponds to the statement of the value of a variable, we have a singleton.

$$\forall x \in U \begin{cases} \mu_{x_0}(x) = 1 \text{ iif } x = x_0 \\ \mu_{x_0}(x) = 0 \text{ iif } x \neq x_0 \end{cases}$$

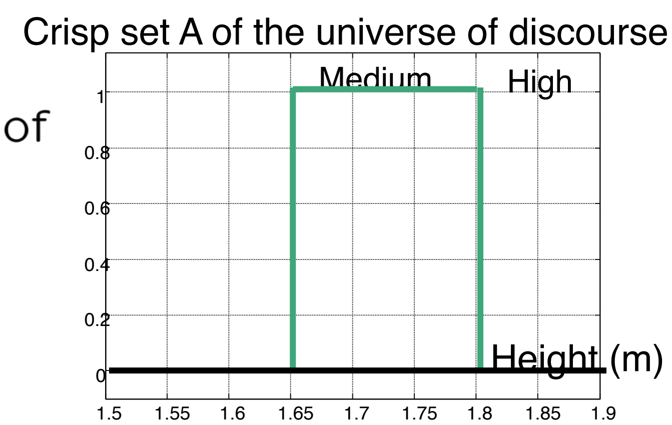


Crisp sets:

Crisp sets (classical sets) are particular cases of fuzzy sets. Their membership functions are characterictic functions.

Let be A a crisp set defined on a universe of discourse U.

$$\forall x \in U \begin{cases} \mu_A(x) = 0 \text{ iif } x \notin A \\ \mu_A(x) = 1 \text{ iif } x \in A \end{cases}$$



☞ **Fuzzy logic includes the certain (sure) data**

Operators of the fuzzy logic

As in the classical set theory, one defines the union, the intersection, the complement….of fuzzy sets.

Boolean logic = a particular case of the fuzzy logic

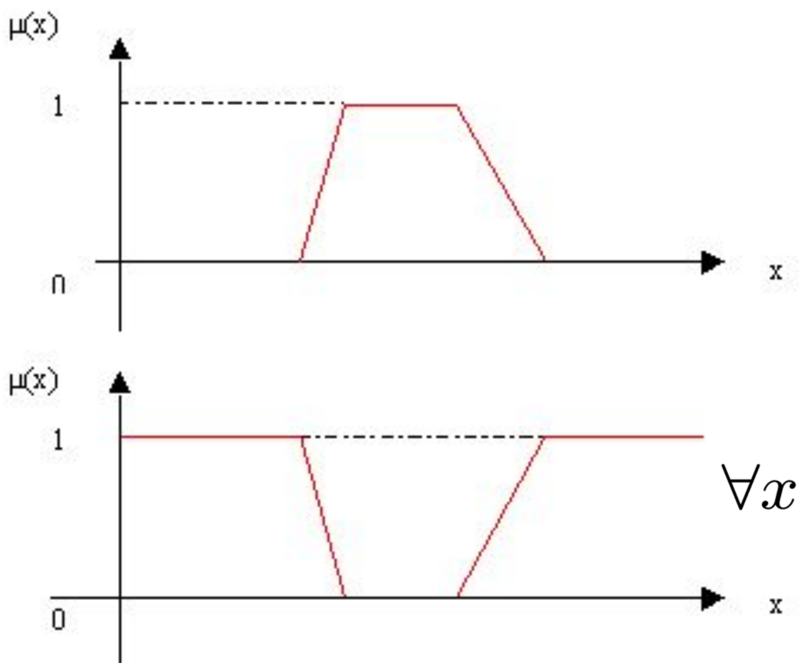
$$\Downarrow$$

All the results obtained in boolean logic have to be verified in the fuzzy logic

Complement (negation)

If A and B are two fuzzy sets and $\mu_A(x)$ and $\mu_B(x)$ their membership functions, one defines:

- **The complement** $\bar{A} = A^c$ of A



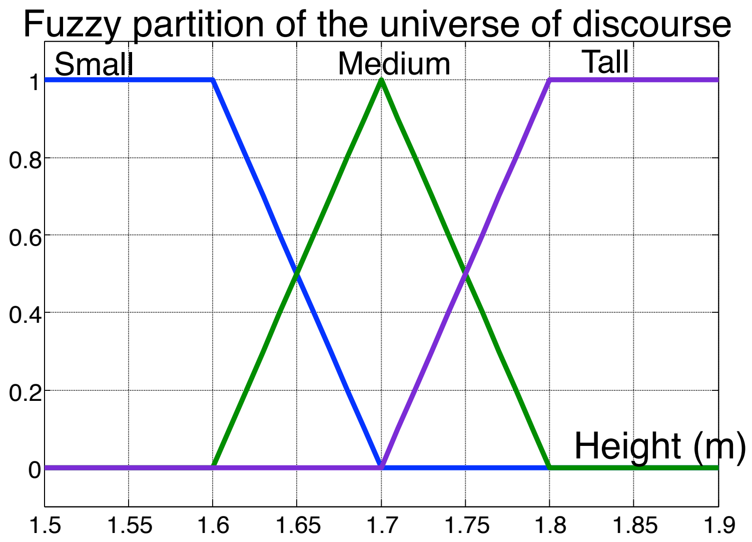
$$\forall x \in U \quad \mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

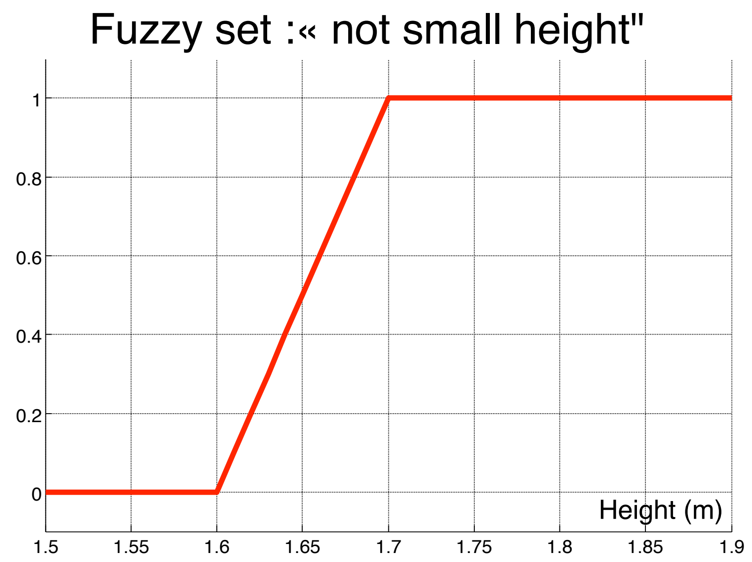
Complement (negation)

A: fuzzy set of small persons.

The set of « not small persons » (« not small height ») is a fuzzy set having the following membership function:

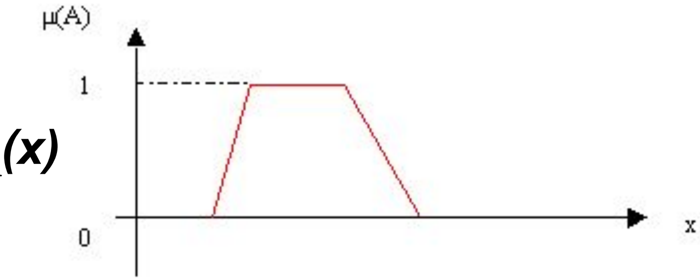
$$\forall x \in U \qquad \mu_{\bar{A}}(x) = 1 - \mu_A(x)$$



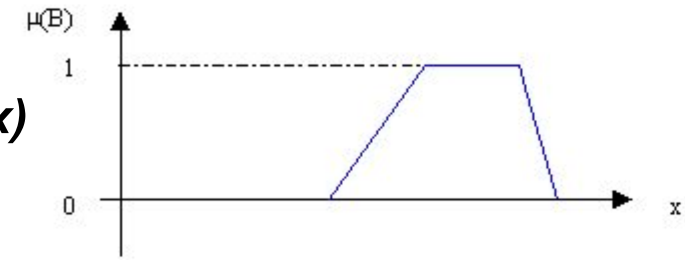


Intersection, union

$\mu_A(x)$

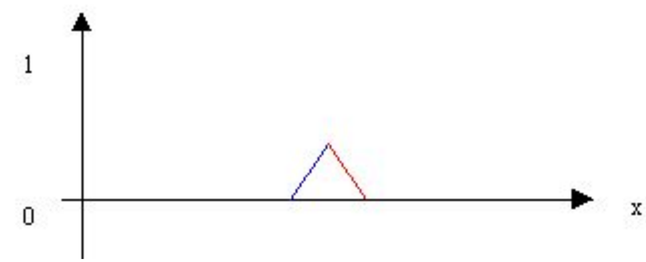


$\mu_B(x)$



- **The set « *A* AND *B* » *(A∩B)* is defined by:**

$$\forall x \in U, \mu_{A\cap B}(x) = \min_{x\in U}\left(\mu_A(x), \mu_B(x)\right)$$

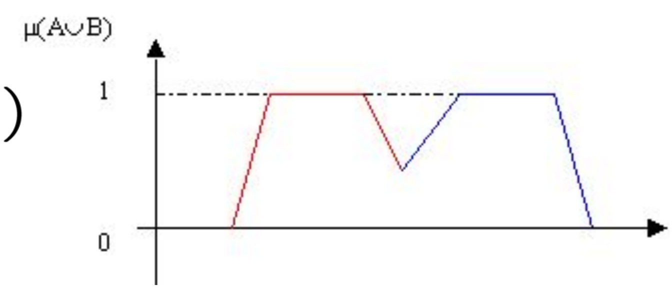


T-norme

- **The set « *A* OR *B* » *(A∪B)* is defined by:**

$$\forall x \in U, \mu_{A\cup B}(x) = \max_{x\in U}\left(\mu_A(x), \mu_B(x)\right)$$

T-conorme (⊥-norme)



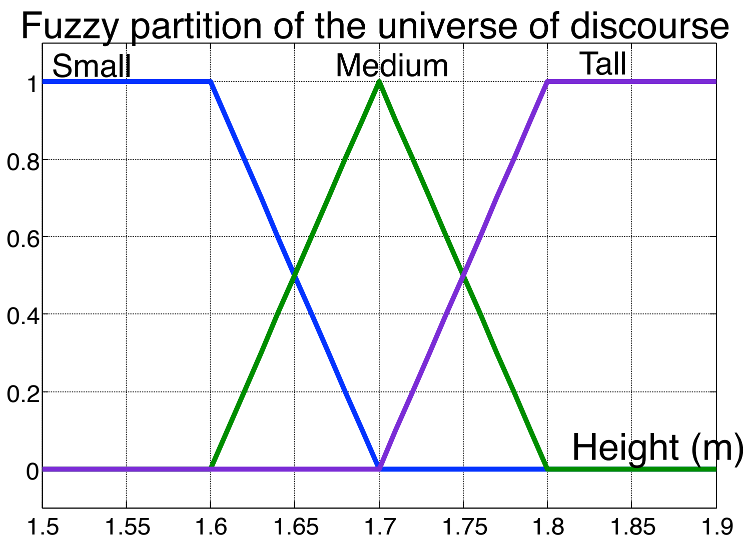
Remark: T means triangular

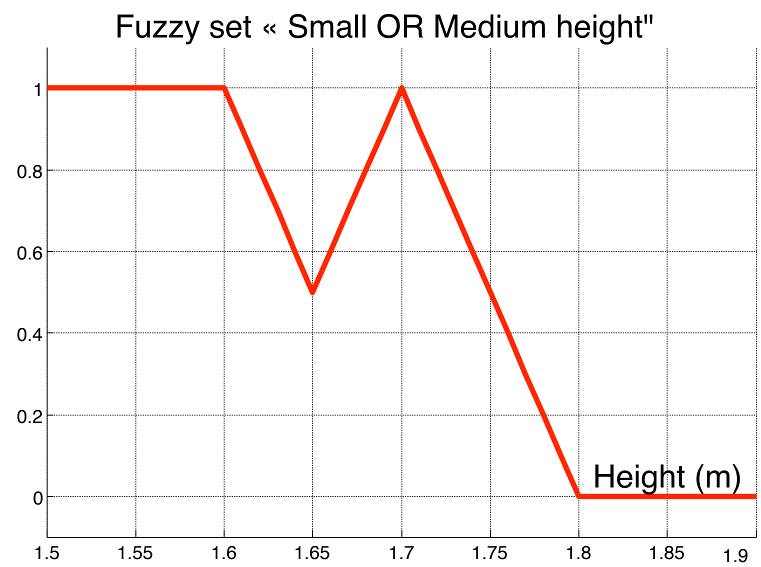
Union

A : fuzzy set describing small persons (« Small height»).
B : fuzzy set describing medium height persons (« Medium height »).

The set of « small OR average persons » is a fuzzy set having the following membership function: :

$$\forall x \in U, \mu_{A \cup B}(x) = \max_{x \in U} \left(\mu_A(x), \mu_B(x)\right)$$



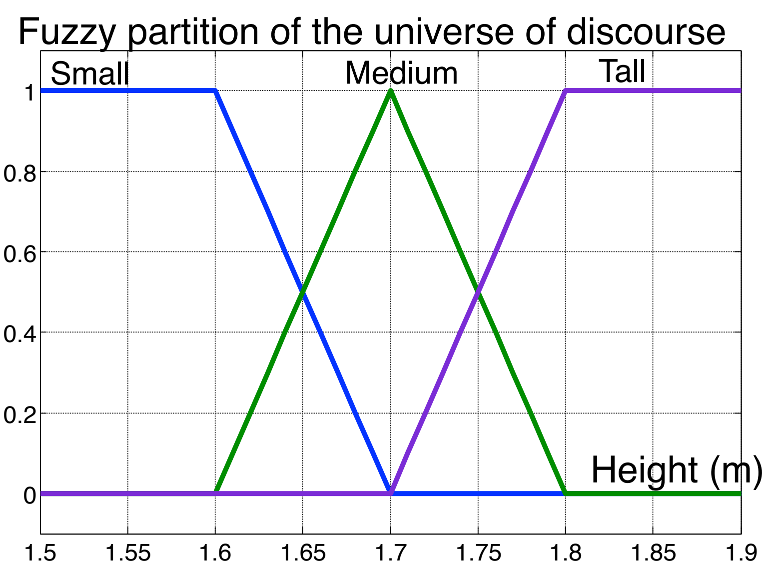


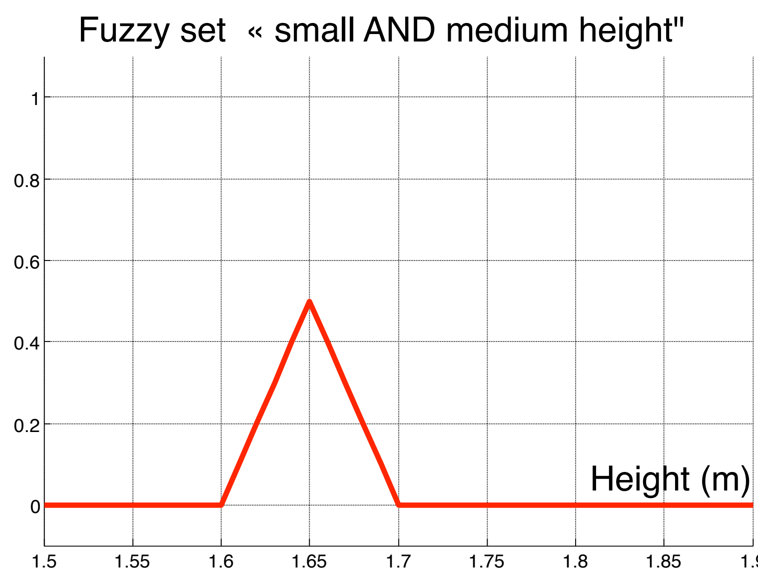
Intersection

A : fuzzy set describing small persons (« Small height »).
B : fuzzy set describing medium height persons (« Medium height »).

The set of « small AND medium height persons » is a fuzzy set having the following membership function: :

$$\forall x \in U, \mu_{A \cap B}(x) = \min_{x \in U} \left(\mu_A(x), \mu_B(x) \right)$$





Most used operators

Name	Intersection AND (t-norm)	Union OR (t-conorm)	Complement NOT
Zadeh's operators MIN/MAX	$\mu_{A\cap B}(x) = min(\mu_A(x), \mu_B(x))$	$\mu_{A\cup B}(x) = max(\mu_A(x), \mu_B(x))$	$\mu_{\overline{A}}(x) = 1 - \mu_A(x)$
Probabilist PROD/PROBOR	$\mu_{A\cap B}(x) = \mu_A(x) \times \mu_B(x)$	$\mu_A(x) + \mu_B(x) - \mu_A(x) \times \mu_B(x)$	$\mu_{\overline{A}}(x) = 1 - \mu_A(x)$

Operators

For all the definitions of operators AND and OR, we have the properties of the boolean operators.

Commutativity

Distributivity

$$\begin{cases} A\cup(B\cap C)=(A\cup B)\cap(A\cup C) \\ A\cap(B\cup C)=(A\cap B)\cup(A\cap C) \end{cases}$$

Associativity

$$\begin{cases} A\cup(B\cup C)=(A\cup B)\cup C \\ A\cap(B\cup C)=(A\cap B)\cap C \end{cases}$$

De Morgan's laws

$$\begin{cases} \overline{A\cup B}=\overline{A}\cap\overline{B} \\ \overline{A\cap B}=\overline{A}\cup\overline{B} \end{cases}$$

2 exceptions:

1. In fuzzy logic, the principle of the excluded third (or of the excluded middle) is contradicted:

$$A\cup\overline{A}\neq U \quad \text{i.e. } \mu_{A\cup\overline{A}}(x)\neq 1$$

2. In fuzzy logic, we can be *A* and not *A* at the same time:

$$A\cap\overline{A}\neq\varnothing \quad \text{i.e. } \mu_{A\cap\overline{A}}(x)\neq 0$$

Fuzzy logic-based system

Principle and mechanism steps

- ***Fuzzification***

The fuzzification consists in estimating the membership functions used in the predicates of rules.

- ***Degree of activation***

The degree of activation of a rule is the evaluation of the predicate of every rule by combination (the minimum in the case of Mamdani's mechanism) between the degrees of truth of the propositions of the predicate

- ***Implication***

The degree of activation of the rule allows to determine the conclusion of the rule, it is the implication. There are several operators of implication, but the most used is the "minimum". The fuzzy set of the conclusion is built by realizing the minimum between the degree of activation and the membership function (a kind of " clipping " of the membership function of the conclusion).

- ***Aggregation***

The global output fuzzy set is built by aggregation of the fuzzy sets obtained by each of the rules concerning this output. We consider that rules are connected by a logical "OR“, and we thus calculate the maximum between the resulting membership functions for every rule.

- ***Defuzzification***

At the end of the inference, the output fuzzy set is determined but it is not directly usable to give a precise information to the operator or actuator. It necessary to pass from the " fuzzy world " to the " real world ", it is the defuzzification step. There are several methods.

Principle of fuzzy reasoning

The fuzzy reasoning is based on the use of fuzzy rules and a mechanism of fuzzy inference.

Basic form of a rule : IF "predicate" THEN "conclusion"

Example of rule: IF *the temperature is very low* THEN *Heat strong*

The conclusion of a fuzzy rule is the membership of a fuzzy output variable "Heat" to a fuzzy class "Strong".

This membership depends on:

1. The considered fuzzy output class.
2. The validity degree of the predicate $\mu_{predicate}(x_0)$
3. The choosen implication method (how « THEN » is mathematically translated).

Fuzzification

The systems based on fuzzy logic deal with fuzzy inputs and provide fuzzy outputs.

☞ **The fuzzification is the step which consists in fuzzy quantifiying the real values of a variable.**

« Pierre is 1m625 tall » → Interface of fuzzification →

« Pierre is small » with a degree of 75%
« Pierre is average» with a degree of 25%
« Pierre is tall » with a degree of 0%

How to fuzzificate?

To fuzzificate, it is necessary to give:

1. The universe of discourse (for a variable, a parameter)
   i.e. the range of possible variations for each considered input.
2. A fuzzy partition of the universe.
3. Membership functions of each class or fuzzy set constituting the fuzzy partition.

It is necessary to fuzzificate the inputs **AND** the outputs of the fuzzy process.

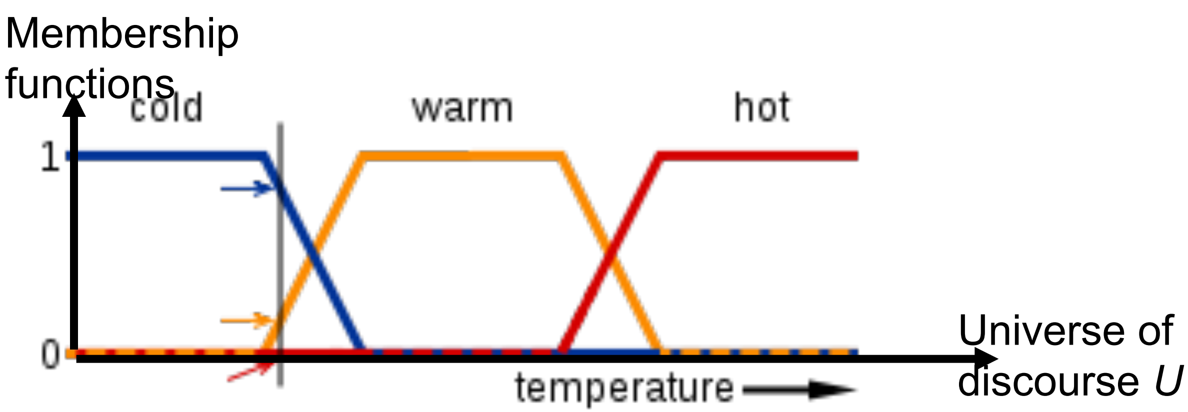
Example: According to the values of the inputs, the fuzzy system will indicate that the output (*e.g.* the heating power) will have to take the values of "low“, “medium" or “high".

The fuzzification of the variables is a delicate phase of a fuzzy logic-based process.
It is often realized in a iterative way and requires some experiments or based on expertises.

Fuzzy variables

- <u>Fuzzy logic</u> → based on fuzzy variables called *linguistic variables* **,** these non-numeric *linguistic variables* are often used to facilitate the expression of rules and facts in fuzzy logic.
- Every linguistic value constitutes then a fuzzy set of the universe of discourse.

<u>Example:</u>

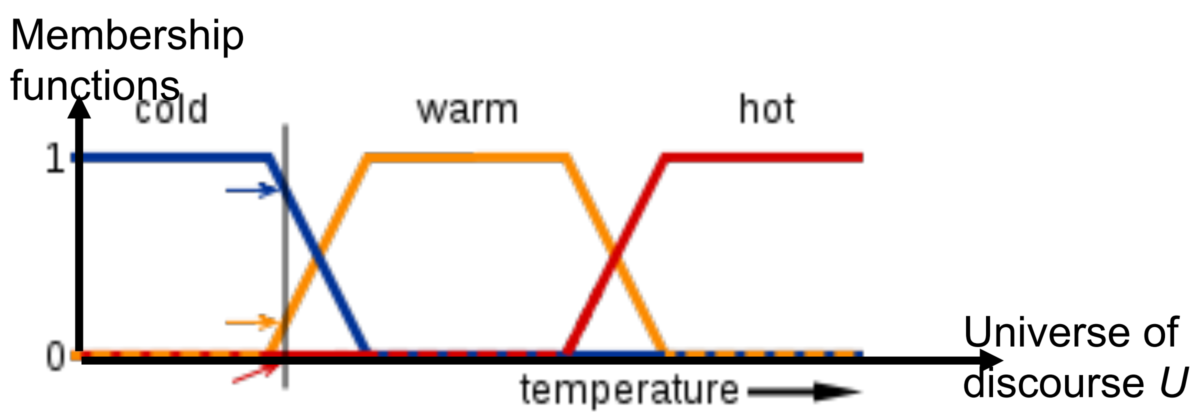


<u>Universe of discourse</u>: range of temperature from 0°C up to 200°C.
<u>Lingusitic variable</u>: the temperature.
<u>Linguistic values</u>: « cold», « warm », « hot ».

Linguistic variable

- A linguistic variable is represented by a triplet (V, U_V, T_V)
  - *V* : name of the variable (age, height, temperature, length,...)
  - U_V : range of the values taken by *V* (universe of discourse)
  - $T_V = \{A_1, A_2, \ldots\}$: set of the fuzzy sets of X_V used to characterize *V* (partition of the universe).
- For instance:

$$(V = \text{"Temperature"}, U_V = [0, 200], T_V = \{\text{"cold"}, \text{"warm"}, \text{"hot"}\})$$



Fuzzy proposition

- ***Elementary fuzzy proposition (e.f.p.)***:
  Qualification « *V* is *A* » of a linguistic variable (V, U_V, T_V)
  - Example: « Temperature is Medium »
  - "V is A" is called a ***predicate*** or ***premise***.

- ***General fuzzy proposition (g.f.p.)***: composition of elementary fuzzy propositions of linguistic variables which can be distinct.
  - Let be « *V* is *A* » an efp. of (V, U_V, T_V) , and « W is *B* » an efp of (W, U_W, T_W),
  - Examples of gfp :
    - « *V* is *A AND W is B* »
    - « *V is A OR W is B* »

Remark: epf and gpf are involved in the production of predicates.

Fuzzy rule

Having fuzzificate the input variables and output variables, it is necessary to provide the rules linking the inputs to the outputs

Aim: All the time, analyzing the state or the value of the inputs of the system to determine the state or the value of all the outputs.

Basic principle of the fuzzy logic:
More the condition on inputs is true,
More the action recommended for the outputs must be respected

Fuzzy rule

- ***Fuzzy rule principle***

Fuzzy logic–based systems use an expertise expressed in the form of a base of rules (a set of rules) of the type: IF….THEN…:

$$\text{IF "predicate" THEN "conclusion"}$$

For instance: IF (x is A) THEN (y is B)

- ***Comments***:
  - ✓ A predicate is also called « premise » or « condition ».
  - ✓ A predicate can be an elementary fuzzy proposition (e.p.f. such as « *x* is *A* »), or a combination of elementary propositions by means of logical operators AND, OR, NOT (it is called a general fuzzy proposition or g.f.p.) .

Example:

Rule R1: IF (« Weather is nice ») THEN (« Moral is good »)

Rule R2: IF (« Weather is nice » AND « Today is Sunday ») THEN (« Moral is very good »)

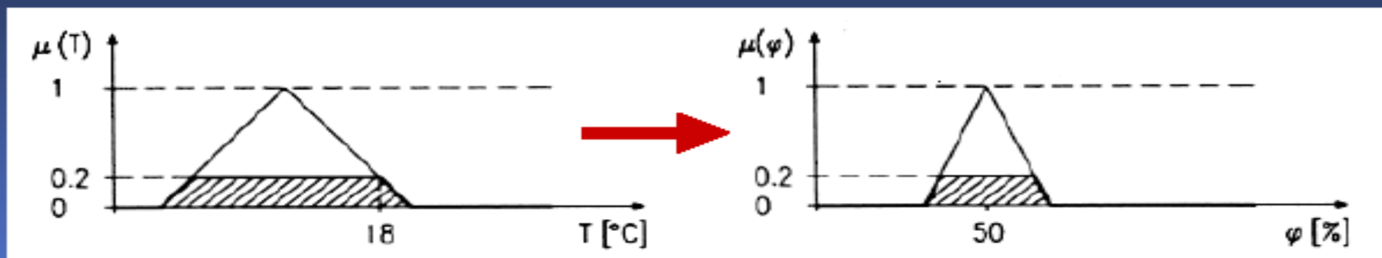
Rule R3: IF (« Weather is grey » OR « Today is not Sunday ») THEN (« Moral is low »)

- ***Key point:*** the mathematical expression of THEN (implication function)

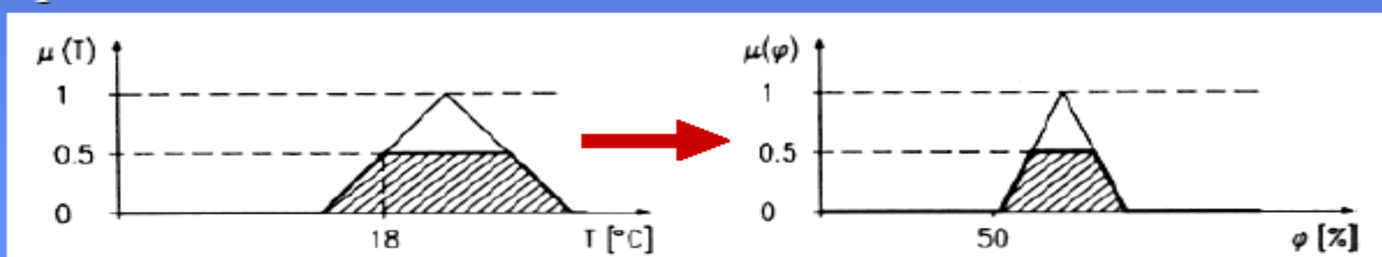
Fuzzy rule

Example of a fuzzy rule-based system

- IF *temperature* = *low* THEN *cooling valve* = *half open.*



- IF *temperature* = *medium* THEN *cooling valve* = *almost open.*



Fuzzy inference

g.f.p.

IF {« Weather is nice » AND « Today is Sunday »} THEN « Moral is very good »

Predicates | Conjunction (combination) | Implication | Conclusion

Inference :

Logical operation by which we admit a proposal by virtue of its connection with the other proposals considered as true.

In classical logic:

IF p	THEN	q
p is true	THEN	q is true

In fuzzy logic:

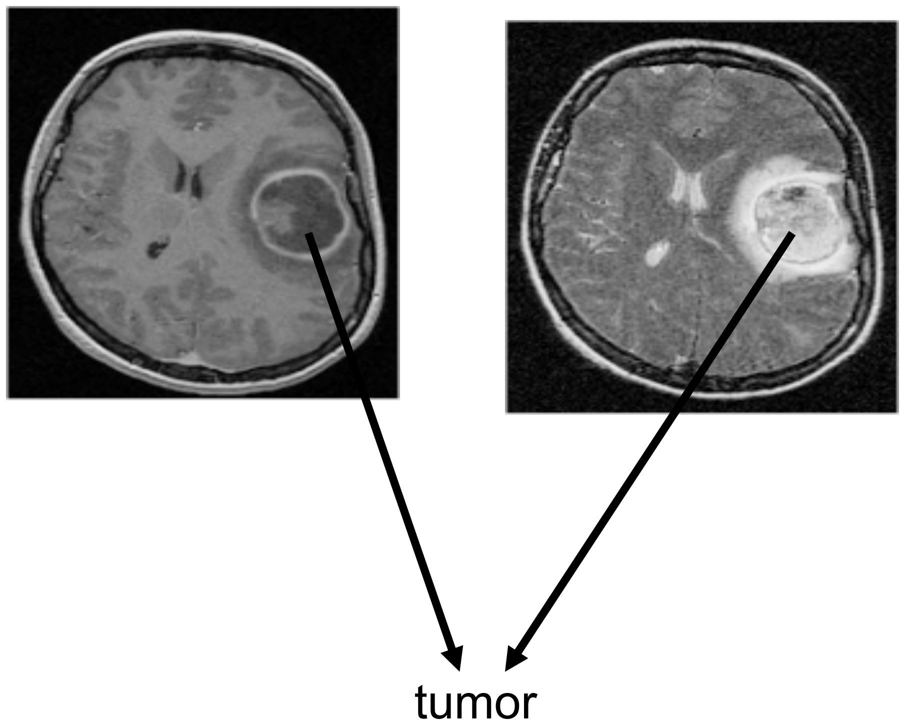
IF x is A THEN y is B

The fuzzy variable x belongs to A with a validity degree equal to $\mu_A(x_0)$.

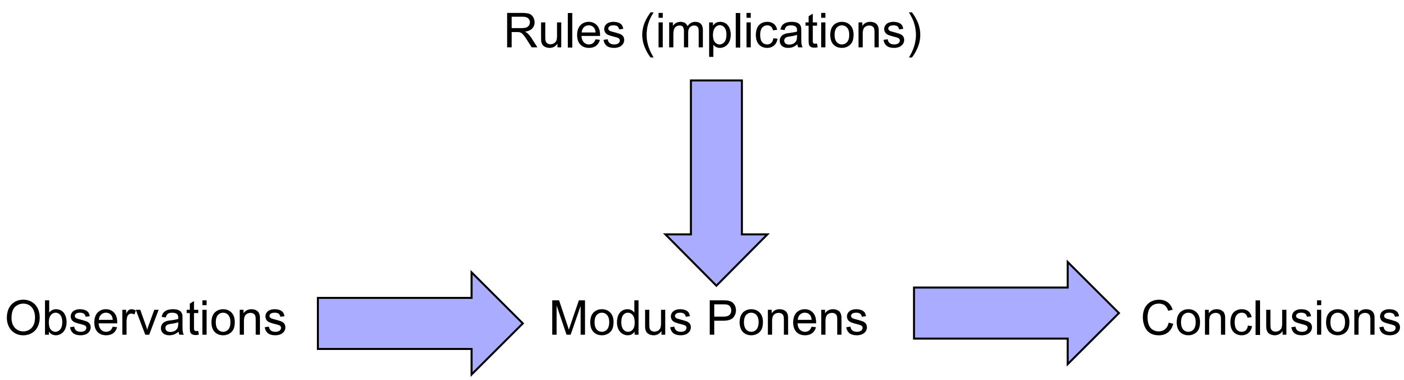
The fuzzy variable y belongs to B with a degree whici depends on the validity degree $\mu_A(x_0)$ of the predicate.

Fuzzy inference

IF {« grey level in T1 is medium » AND « grey level in T1gado is high »} THEN « presence of tumor is high »



Classical reasoning



Classical reasoning

- *Modus Ponens in the classical logic*

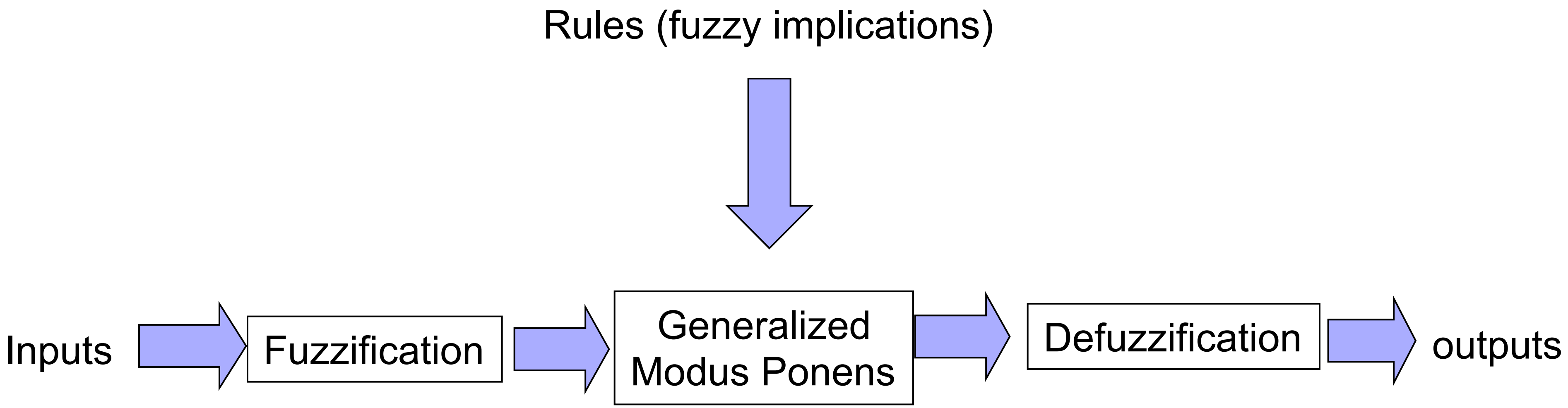
Rule:	Predicate	$\Rightarrow$	Conclusion
Observation:	Observed predicate		
Deduction:			Conclusion

$\rightarrow$ Rule of deduction to infer knowledge

- *A famous example:*

Rule:	H is a human being	$\Rightarrow$	H is mortal
Observation:	Socrate is a human being		
Deduction:			Socrate is mortal

Fuzzy reasoning



Fuzzy reasoning

❑ *Generalized Modus Ponens (GMP)*

Extension of MP to fuzzy logic (fuzzy propositions).
Let (V, U_V, T_V) and (W, U_W, T_W) be two linguistic variables

Fuzzy rule:	V is A	$\Rightarrow$	W is B
	μ_A		μ_B
Observation:	V is A'		
	$\mu_{A'}$		
Deduction:			W is B'
			$\mu_{B'}$

μ_A, μ_B and $\mu_{A'}$ are known and we look for $\mu_{B'}(y), \forall y \in U_y$

From a fuzzy rule and from an observed fact *A'* for *V*, we deduct a value *B'* for W.

Value of truth of a fuzzy proposition

Classical proposition:	truth value $\in \{0,1\}$ (FALSE or TRUE).
Fuzzy proposition:	truth value $\in [0,1]$.
Truth value p_A of proposition "V is A":	μ_A, membership function of A.
Truth value of the negation "V is not A":	$p_{\bar{A}} = 1 - \mu_A$
Truth value p of a g.f.p.:	aggregation* of the truth values p_A and p_B of each e.f.p. (in the case of 2 e.f.p.).
Conjunction "V is A AND W is B":	$p_{A \wedge B} = \min(p_A, p_B)$
Disjunction "V is A OR W is B":	$p_{A \vee B} = \max(p_A, p_B)$

*: the aggregation depends on the realized composition (AND, OR,...)

Fuzzy implication

- ***Reminder***
  - $"V \text{ is } A" \Rightarrow "W \text{ is } B"$ *is read « **if** V is A **then** W is B »*
  - *« V is A » = predicate or premise*
  - *« W is B » = conclusion*
  - *For instance: « **if** Age of a person is Young **then** Salary is Low »*

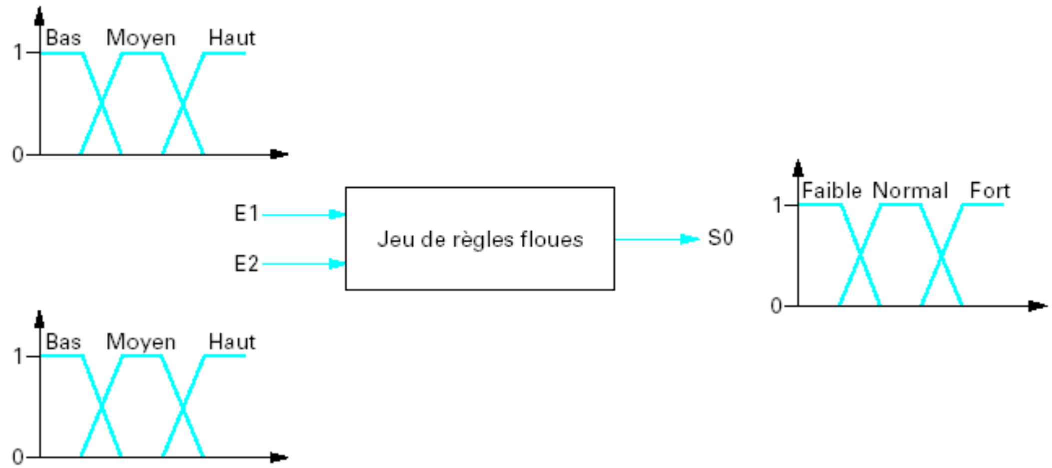
- *The degree of activation of a rule* determines the conclusion of the rule. It is what we call the implication.
- *Truth value of the implication « V is A ⇒ W is B » :* evaluated by means of an implication function f_i

$$f_i : U_x \times U_y \to [0, 1]$$
$$\forall x \in U_x, \forall y \in U_y, f_i(x, y) = \Phi\left(\mu_A(x), \mu_B(y)\right)$$

Remark: Φ is a function that maps the input space $[0; 1] \times [0; 1]$ to the output space $[0; 1]$. It is equivalent to a classical implication when the propositions are classical.

Fuzzy implication

2 main methods for implication:



<u>Mamdani's method</u>:

$$\forall y \in U_y, \mu'_{conclusion}(y) = \min_{y \in U_y} \left(\mu_{predicate}(x_0), \mu_{conclusion}(y) \right)$$

<u>Larsen's method</u>:

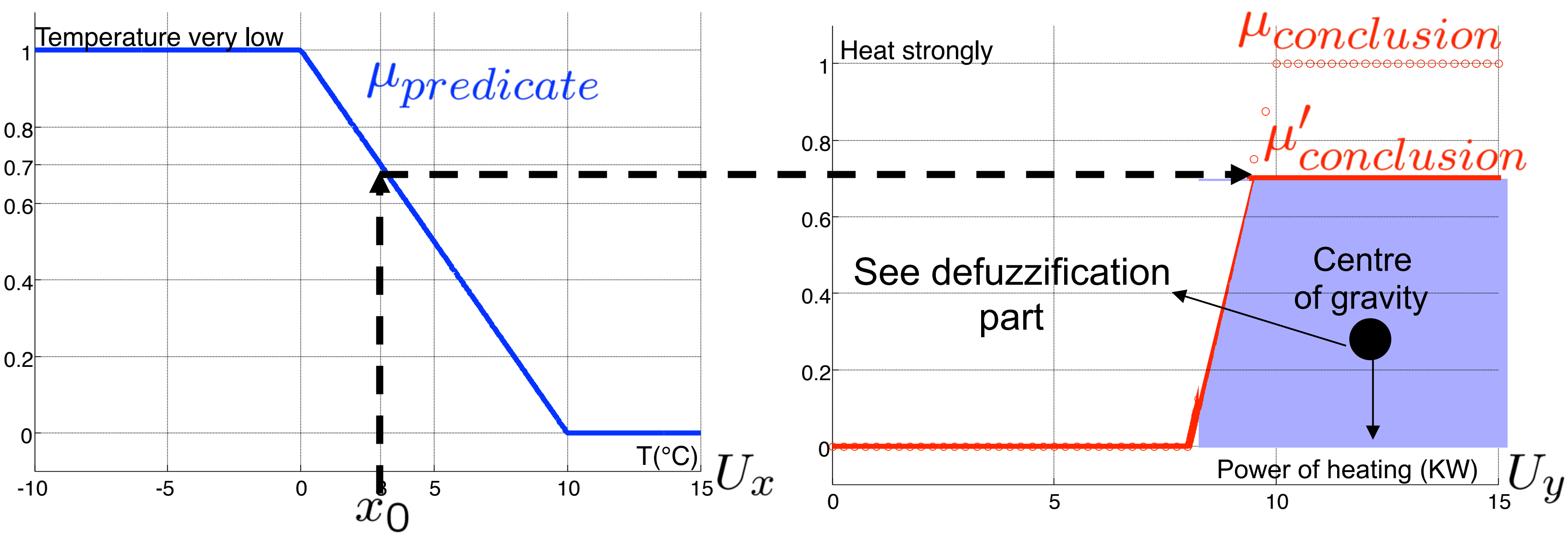
$$\forall y \in U_y, \mu'_{conclusion}(y) = \mu_{predicate}(x_0) . \mu_{conclusion}(y)$$

Methods of implication

Example (Mamdani)

Rule: <u>If</u> *the temperature is very low* <u>then</u> *Heat strongly*

$$\forall y \in U_y, \mu'_{conclusion}(y) = \min_{y \in U_y} \left(\mu_{predicate}(x_0), \mu_{conclusion}(y) \right)$$



According to the considered rule, if T=3°C then the power of heating =12KW

Activation of rules

$$
\begin{array}{ll}
R_1 : \text{IF } (X_1 \text{ is } A_{11}) \text{ AND } (X_2 \text{ is } A_{12}) & \text{THEN } (Y \text{ is } B_1) \\
R_2 : \text{IF } (X_1 \text{ is } A_{21}) \text{ OR } (X_2 \text{ is } A_{22}) & \text{THEN } (Y \text{ is } B_2) \\
R_3 : \text{IF } (X_1 \text{ is } A_{31}) \text{ AND } (X_2 \text{ is } A_{32}) \text{ AND } (X_3 \text{ is } A_{33}) & \text{THEN } (Y \text{ is } B_3) \\
\vdots &
\end{array}
$$

- A rule is activated as soon as it has a predicate having a truth value not null.
- Several rules can be simultaneously activated and can recommend actions with various degrees of validities; these actions can be contradictory.
- Remark: For every rule R_i, the implication gives: $\forall y \in U_y, \mu_{B_i}(y)$

It is advisable to aggregate rules to supply a membership of the fuzzy output variable in a strengthened fuzzy class.

Aggregation

Composition of rules (compositional rule of inference)

- The activated rules are connected with an operator OR:

$$\forall y \in U_y, \mu_B(y) = \max_i \left(\mu_{B_i}(y)\right)$$
$$i \in \{\text{subscript of the activated rules}\}$$

It gives then an output fuzzy set resulting from the aggregation of the activated rules.

Aim: to aggregate rules to supply a membership of the fuzzy output variable in a strengthened fuzzy class.

- The mechanism of inference which is the most used is the one proposed by Mamdani. It represents a simplification of the more general mechanism based on " the fuzzy implication " and the " Generalized Modus Ponens .“

Fuzzy inference

- ***Generalized Modus Ponens (GMP)***

  Extension of MP to fuzzy logic (fuzzy propositions)

  Let (V, U_V, T_V) and (W, U_W, T_W) two linguistic variables.

- ***Fuzzy rule*** $"V$ is $A" \Rightarrow "W$ is $B"$

  ✓<u>Implication</u>: $\forall x \in U_x, \forall y \in U_y, f_i(x,y) = \Phi\left(\mu_A(x), \mu_B(y)\right)$

  GMP combines the fuzzy rules with the observation « *V* is *A'* » to build the conclusion *B'*.

- ***Operator of GMP***: a function T to combine f_i and $\mu_{A'}$ which maps $[0;1] \times [0;1]$ to $[0;1]$

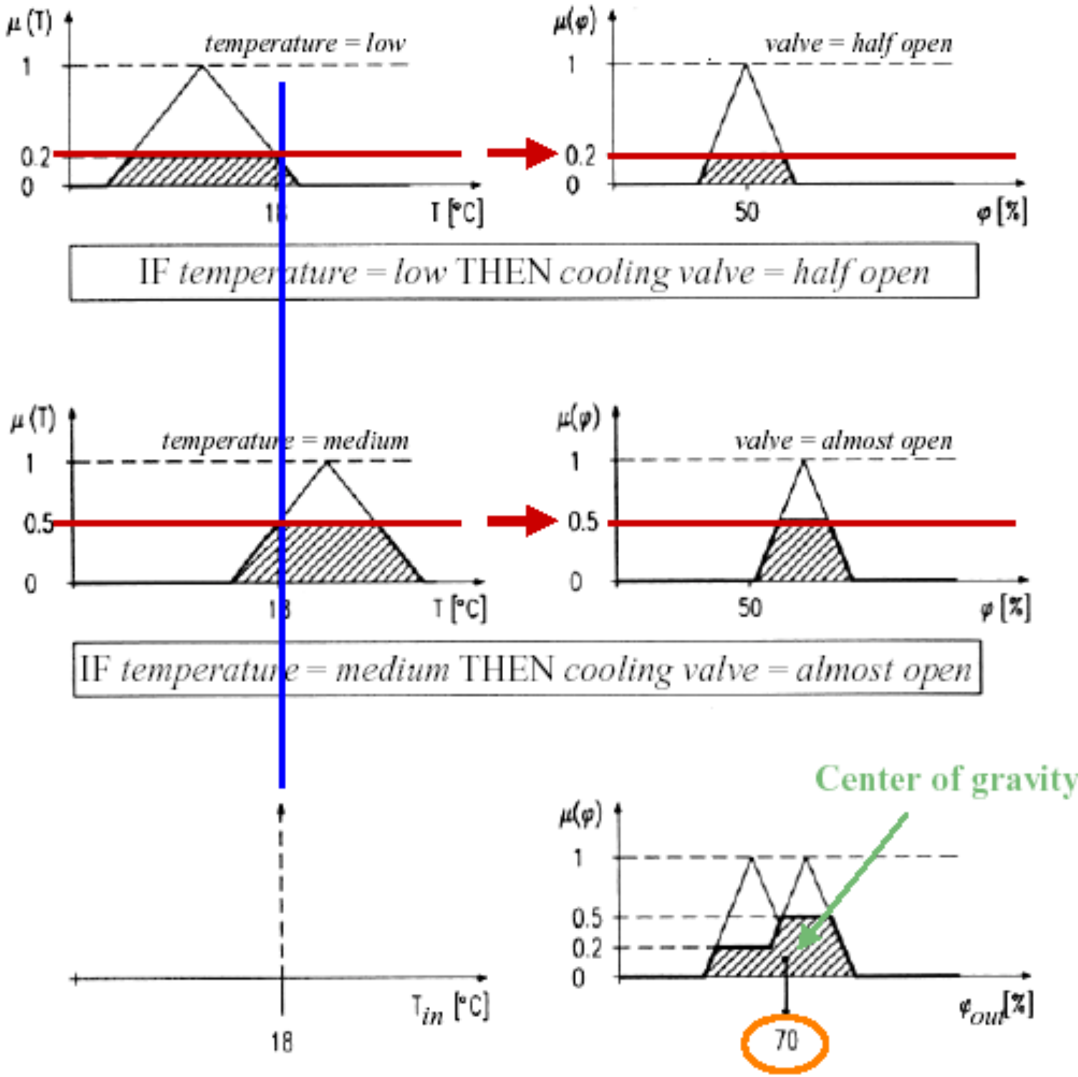
  ✓T is a T-norm.

  ✓T is linked to f_i such that GMP is compatible with MP.

- We have : $\forall y \in U_y, \mu_{B'}(y) = \sup\limits_{y \in U_y} T\left(f_i(x,y), \mu_{A'}(x)\right)$

Mamdani's inference

Max-Min inference



Defuzzification

❑ ***Principle***: At the end of the inference, an output fuzzy set is determined but it is not directly usable to give a precise information to an operator or an actuator. It necessary to pass from the " fuzzy world " to the " real world ", it is the ***defuzzification step***. There are several methods.

❑ In fuzzy control, defuzzification by COG is almost always used. It takes into account the influence of all the values proposed by the fuzzy solution.

❑ The defuzzification MM is rather used when it is a question of discriminating between values output (ex: pattern recognition).

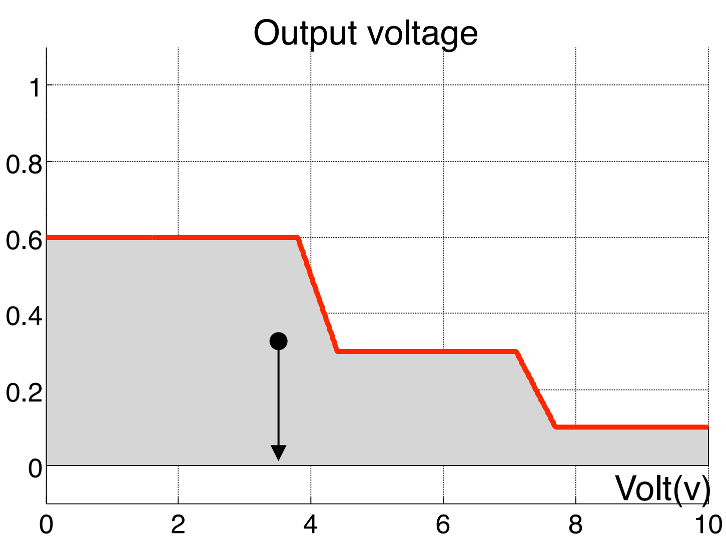
❑ Remark: other methods exist like largest of maximums, smallest of maximums.

Main methods for defuzzification

Center of gravity (COG)

It is the absciss of the center of gravity of the surface under the result curve

$$\text{Output value} = \frac{\int_{U_y} y.\mu(y).dy}{\int_{U_y} \mu(y).dy}$$



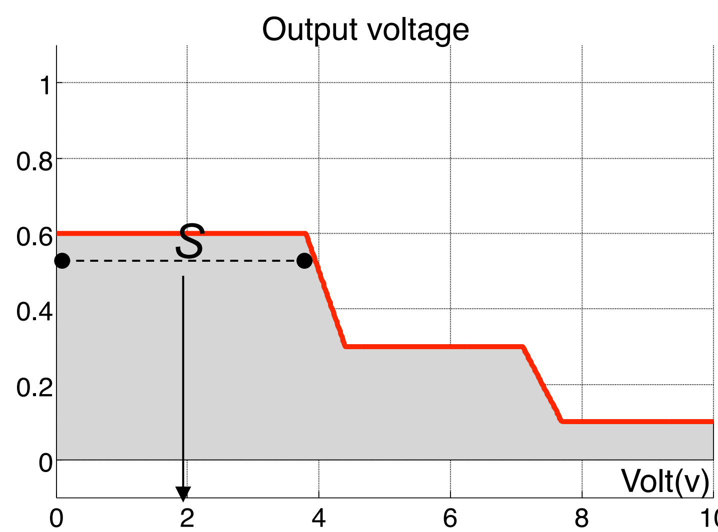
3,5V

Mean of maximums (MM)

It is the mean of the most likely values of the outputs

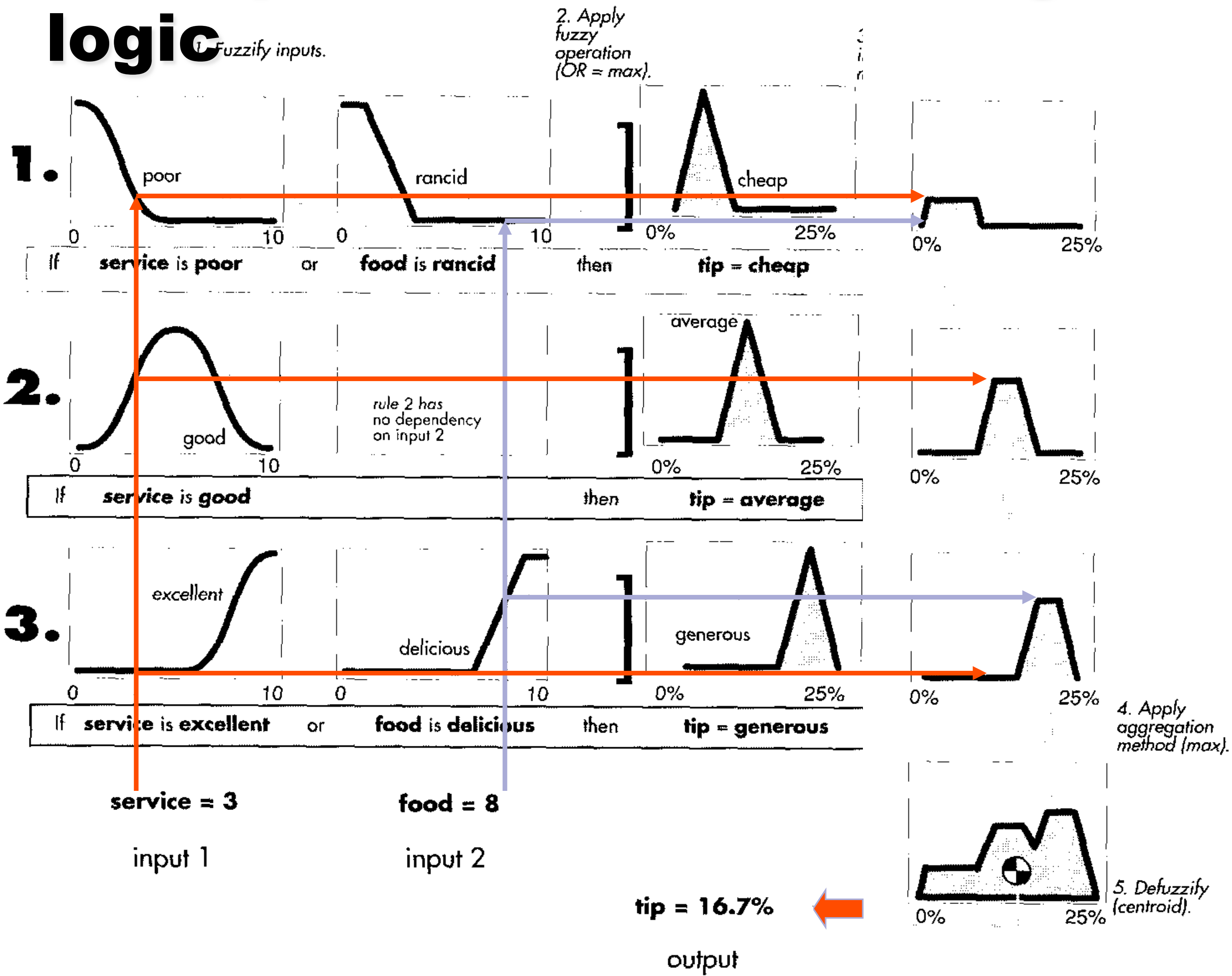
$$\text{Output value} = \frac{\int_{S} y dy}{\int_{S} dy}$$

$$\text{with: } S = \left\{ y_0 \in U_y / \mu(y_0) = \sup_{y \in U_y} (\mu(y)) \right\}$$



1,9V

Example of decision-making in fuzzy logic



Application to image processing

- Segmentation of colour images
- Based on clustering techniques
  - Partition of a population (collection of data described by a set of features)
  - Assignment of each sample (data) to a cluster
- Some classical algorithms:
  1. HCM (Hard C-Means ; not based on fuzzy logic)
  2. FCM (Fuzzy C-Means)
  3. PCM (Possibilistic C-Means)
  4. Dave's algorithm

This is the end of this part!