Simulation of deep-water waves based on JONSWAP spectrum and realization by MATLAB

Article *in* International Conference on Geoinformatics · June 2011

DOI: 10.1109/GeoInformatics.2011.5981100

2 authors:

Qiuying Guo
Shanghai Dianji University

Zunyi xu
Shandong Jianzhu (Architecture and Engineering) University

Simulation of Deep-Water Waves Based on JONSWAP Spectrum and Realization by MATLAB

Qiuying Guo[1]*, Zunyi Xu[2]
[1]School of Civil Engineering Shandong Jianzhu University, Jinan, China
[2]School of Computer Science and Technology Shandong Jianzhu University, Jinan, China
*Corresponding author, e-mail: qyguo11@sohu.com

***Abstract*—Simulating real virtual ocean environment is necessary for the research of interaction simulation of underwater gravity aided inertial navigation system. 3D modeling and simulating deep ocean waves are the key techniques to realize virtual ocean environment. JONSWAP spectrum is a deep wind wave spectrum and is suitable for simulating deep-water waves. Based on directional ocean wave spectrum composed of JONSWAP spectrum and directional spreading function, numerical simulation of 3D deep-water waves in the case of different wind velocities is realized using MATLAB. The simulation speed is fast and the effect is good. The simulation can provide some technological supports for interaction simulation of gravity aided navigation system for underwater vehicles.**

Keywords-ocean wave simulation; spectrum of ocean waves; JONSWAP spectrum; directional spectrum

I. INTRODUCTION

Countries all over the world pay more and more attention to the exploration & utilize of ocean resources because ocean contains abundant resources. So many countries now put premium on the research of basic marine science. Underwater gravity aided inertial navigation system is one of the important research fields [1]. Due to restriction of experiment conditions of underwater surveying, currently the research of underwater aided navigation is mainly based on computer simulation, which is to set up virtual environment for running underwater aided navigation system. The research plays an important role in testing feasibility and correctness of schemes and optimizing parameters of the system.

Research of underwater aided navigation interactive simulation firstly needs to set up vivid virtual ocean environment, which can provide virtual reality environment for operating underwater vehicles and lay the foundation for research of naval vessel simulation and ocean weaponry. Ocean waves are the most evident natural phenomenon on the sea surface, so it is one of key techniques of realizing virtual ocean environment to model and simulate 3D deep-water waves.

Simulation of ocean waves firstly needs modeling ocean waves. Modeling ocean waves mainly has several methods, such as method based on computational fluid mechanics, method based on ocean wave spectrum and method based on geometric modeling [2]. Among which method based on ocean wave spectrum is the most widely adopted method because it needs relatively small computational load and its parameters are obtained by long-term ocean observations and has some authenticity. The commonly used ocean wave spectrums are Pierson-Moscowitz (P-M), JONSWAP and Neumann etc. For the deep water area, the first factor to influence the growth and development of wind waves is wind speed, and the other factor is wind fetch length.

II. JONSWAP SPECTRUM

JONSWAP spectrum is a deep-water wind wave spectrum, which was developed by some institutes of England, Netherland, America and Germany after analyzing and fitting data collected during the "Joint North Sea Wave Observation Project" and is used extensively in the ocean wave research and engineering practice. Its spectrum function is [3]:

$$S(\omega) = \frac{ag^2}{\omega^5}\exp[-\frac{5}{4}(\frac{\omega_m}{\omega})^4]\bullet\gamma^{\exp\left(-\frac{(\omega-\omega_m)^2}{2\sigma^2\omega_m^2}\right)} \tag{1}$$

Where, g is acceleration of gravity, ω is the wave frequency. a is the intensity of the spectrum that relates to the wind speed and fetch length and has the following experience formula:

$$a = 0.076\bar{X}^{-0.22} \tag{2}$$

Where, $\bar{X} = gx/U^2$, U is the wind speed at 10m above the sea surface, x is the fetch length. Typical values of a in the northern sea are in the range of 0.0081 to 0.01. $\omega_m$is the peak wave-frequency, which is the maximum value appeared in the frequency spectrum. The peak of the JONSWAP spectrum is empirically defined by:

$$\omega_m = 22(g/U)(\bar{X})^{-0.33} \tag{3}$$

γ is peak enhancement factor, which is used to represent wind-wave growth state, its values are in the range of 1.5~6,typical value is 3.3.σ is peak shape factor, its values are defined by:

Supported by the National Natural Science Foundation of China (No. 60972052).

978-1-61284-848-8/11/$26.00 ©2011 IEEE

$$\sigma = \begin{cases} 0.07, & \omega \le \omega_m \\ 0.09, & \omega > \omega_m \end{cases} \tag{4}$$

Fig. 1 shows JONSWAP spectrum energy distribution curve with different wind speed. From Fig. 1 we can see that JONSWAP spectrum is narrow band spectrum, and its energy is mainly focused on some frequency band. For example, spectrum energy is mainly focused on 1.4～5.0 rad/s for a wind speed of 10 m/s, and spectrum energy is mainly focused on 1.2～4.0 rad/s for a wind speed of 16 m/s, which can save simulation time and improve simulation speed.

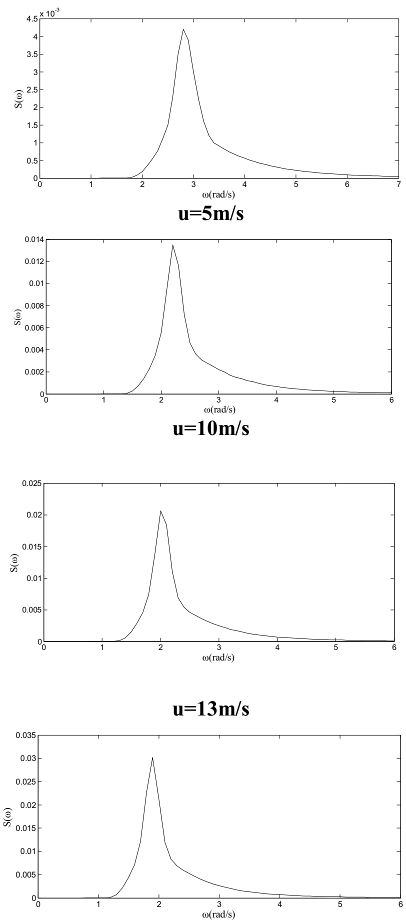


Figure 1. JONSWAP spectrum energy distribution curves with different wind speeds

III. THE DIRECTIONAL SPECTRUM OF OCEAN WAVES

Sea surface fluctuation of a fixed point depends on not only wave frequency but also direction of propagation. So, the frequency spectrum of ocean waves is still not enough to fully describe the characteristic of wind waves. Assuming linear wave theory, the field of ocean waves is characterized by the directional wave spectrum. The directional spectrum of ocean waves which considers both wave energy's frequency distribution and direction distribution can fully describe the characteristic of wind waves. According to small amplitude wave theory, the total energy provided by all the composed waves is [3]:

$$E = \int_0^{\infty}\int_0^{\infty} S(\omega,\theta) d\omega = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \frac{1}{2} a_{ij}^2 \tag{5}$$

Where, E is wave energy, ω is the wave frequency, θ is the direction angle, $S(\omega,\theta)$ is the directional spectrum function of random waves. The directional spectrum is expressed as the following equation:

$$S(\omega,\theta) = s(\omega) \bullet D(\omega,\theta) \tag{6}$$

Where, $S(\omega)$ is the frequency spectrum, $D(\omega,\theta)$ is called direction spreading function, which satisfies the following condition:

$$\int_{-\pi}^{\pi} D(\omega,\theta) = 1 \tag{7}$$

Usually it is considered that frequency distribution of wave energy is independent of direction distribution, therefore directional spectrum can be expressed as the following equation:

$$S(\omega,\theta) = s(\omega) \bullet D(\theta) \tag{8}$$

The commonly used direction spreading functions are [2]:

$$D(\theta) = \frac{2}{\pi}\cos^2\theta \quad (|\theta| \le \frac{\pi}{2}) \tag{9}$$

$$D(\theta) = \frac{8}{3\pi}\cos^4\theta \quad (|\theta| \le \frac{\pi}{2}) \tag{10}$$

Therefore, there are varied forms of directional spectrum functions to represent wind waves if we combine different frequency spectrum functions with direction spreading function. If JONSWAP spectrum is chosen as frequency spectrum of ocean waves, the directional spectrums of ocean waves can be expressed as:

$$S(\omega,\theta) = \frac{\alpha g^2}{\omega^5}\exp[-\frac{5}{4}(\frac{\omega_m}{\omega})^4] \bullet \gamma^{\exp\left(-\frac{(\omega-\omega_m)^2}{2\sigma^2\omega_m^2}\right)} \bullet \frac{2}{\pi}\cos^2\theta \tag{11}$$

$$S(\omega,\theta) = \frac{\alpha g^2}{\omega^5}\exp[-\frac{5}{4}(\frac{\omega_m}{\omega})^4] \bullet \gamma^{\exp\left(-\frac{(\omega-\omega_m)^2}{2\sigma^2\omega_m^2}\right)} \bullet \frac{8}{3\pi}\cos^2\theta \tag{12}$$

IV. MATHEMATICAL MODEL OF SIMULATING 3D OCEAN WAVES BASED ON JONSWAP SPECTRUM

Sea waves are of stochastic nature in the stable sea conditions and, mathematically, are represented as Gaussian stationary and ergodic processes. So sea waves can be viewed as wave superposition of infinite simple Cosine waves spreading in the direction of θ angle relatively with x axis in *(x,y)* plane, and those Cosine waves are with different amplitudes, different frequencies and different initial phases. The sea surface elevation $H(x,y,t)$ can be represented by the Double Summation Model [3, 4]:

$$H(x,y,t)=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}a_{ij}\bullet\cos(k_i x\cos\theta_j+k_i y\sin\theta_j-\omega_i t-\varepsilon_{ij}) \tag{13}$$

Where, a_{ij} is the wave amplitude of frequency i and directional angle j, θ_j is spreading directional angle of a single wave($0<\theta\leqslant 2\pi$), ω_i is representative frequency in the range of frequency division. k_i is wave number($k_i=\omega_i^2/g$), ε_{ij} is initial phase angle distributed at random($0\leqslant\varepsilon_{ij}<2\pi$).

According to (5):

$$a_{ij}\approx\sqrt{2S(\omega_i,\theta_i)\bullet\Delta\omega_i\bullet\Delta\theta_j} \tag{14}$$

Where, $S(\omega,\theta)$ is directional spectrum function of ocean waves, which can be represented by (11) and (12). ω_i is representative frequency, θ_j is representative directional angle, $\Delta\omega$ and $\Delta\theta$ are increments of ω and θ respectively. The sea surface elevation of 3D ocean waves can be represented by:

$$H(x,y,t)=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sqrt{2S(\omega_i,\theta_j)\bullet\Delta\omega_i\bullet\Delta\theta_j}\bullet\cos(k_i x\cos\theta_j+k_i y\sin\theta_j-\omega_i t-\varepsilon_{ij}) \tag{15}$$

V. REALIZATION OF SIMULATING 3D OCEAN WAVES BASED ON MATLAB

The simulation of 3D ocean waves can be realized by MATLAB because it has powerful functions of calculation and provides many functions of 3D graphic processing. Firstly the wind speed and the fetch length are determined while simulating 3D ocean waves. Then discretization of the wave frequency and the direction angle are performed according to certain principles. Finally characteristic essential factors of each composition wave such as wave amplitude, angular frequency and phase are calculated according to the sea surface elevation of (15). And further the sea surface elevation is calculated at every point (x, y) and 3D sea level wave field can be plotted.

According to JONSWAP spectrum energy distribution curves and the general method (interval equal division method) of dividing frequency interval and direction interval, the adopted frequency band, wave frequency interval and direction angle interval at different wind speeds are summarized in table 1, and the fetch length is assumed as 100km in this paper.

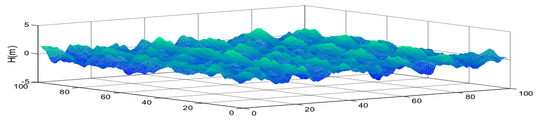
The simulation experiment of 3D ocean waves is realized by dell notebook computer based on MATLAB2008 software, the computer hardware configuration is Intel Core i3, 2GB memory, ATI Mobility Radeon HD 565 chip card. The model of ocean waves is set up based on 100×100 grid. In the grid the data of every point include the information of (x, y, H), where H is wave elevation and (x, y) is horizontal coordinates of grid point.

TABLE I. SIMULATION FREQUENCY BANDS, EWAVE FREQUENCY INTERVAL, DIRECTION ANGLE INTERVAL WITH DIFFERENT WIND SPEEDS AND SIMULATION TIME

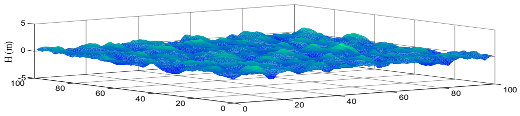
Wind speed (m/s)	Simulation frequency band (rad/s)	Interval of wave frequency Δω (rad/s)	Interval of direction angle Δθ (rad)	Simulation time (s)
5	1.8-6.0	0.4 0.2 0.1	π/5 π/10 π/10	3 12 22
10	1.4-5.0	0.4 0.2 0.1	π/5 π/10 π/10	2 11 22
13	1.3-4.5	0.2 0.1	π/10 π/10	9 18
16	1.2-4.0	0.1 0.05	π/10 π/20	15 60

The 3D sea level wave field can be plotted using 3-d graphics function of surf(), the colour of ocean waves can be defined by the colour mapping function of colormap(). MATLAB software provides many commonly used colour mappings, and the function of colormap(winter) may be selected when plotting ocean waves. The function of shading interp can be used to color image points by interpolation form, thus the lifelike wave picture can be plotted. The simulation calculation time at different conditions is listed in TABLE I. Fig. 2 shows the simulation scene of 3D ocean waves at different speeds.

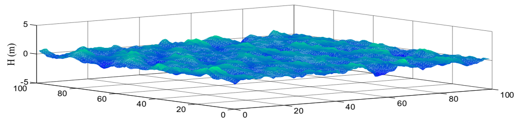
From TABLE I. and Fig. 2, we can see that sea waves have close relation with wind speed, and the bigger the wind, the greater the sea waves. The sea waves also have relation with the fetch length. The time needed for simulation has relations with simulation frequency band, interval of wave frequency ($\Delta\omega$) and interval of direction. angle($\Delta\theta$). The narrower the frequency band and the bigger $\Delta\omega$ and $\Delta\theta$, the less time is needed for simulation. On the contrary, the wider the frequency band and the smaller $\Delta\omega$ and $\Delta\theta$, the more time is needed for simulation, and the simulated sea waves are more detailed and vivid. For example, when u=5m/s, frequency band between 1.8-6.0 rad/s, $\Delta\omega$=0.4 rad/s and $\Delta\theta$=π/5, it only needs 3 seconds to simulate sea waves within the region of 100×100 grid, and when $\Delta\omega$=0.2rad/s, $\Delta\theta$=π/10 and other conditions are same with above it needs 12 seconds. when u=10m/s, frequency band between 1.4-5.0 rad/s, $\Delta\omega$=0.4 rad/s and $\Delta\theta$=π/5, it only needs 2 seconds to simulate sea waves within the region of 100×100 grid, and when $\Delta\omega$=0.1rad/s,$\Delta\theta$=π/10 and other conditions are same with above it needs 22 seconds. Therefore, the sea waves can be simulated quickly and vividly using MATLAB if suitable simulation frequency band, interval of wave frequency and interval of direction angle are selected.



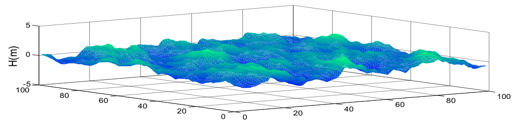
(a) u=5m/s, △ω=0.4, $\Delta\theta = \pi/5$



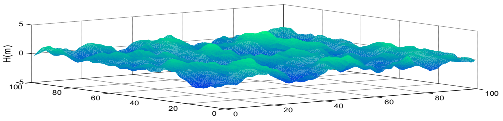
(b) u=5m/s, △ω=0.2, $\Delta\theta = \pi/10$



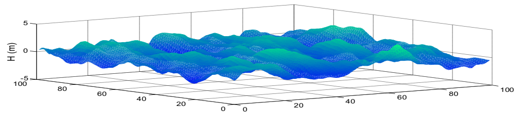
(c) u=5m/s, △ω=0.1, $\Delta\theta = \pi/10$



(d) u=10m/s,△ω=0.2, $\Delta\theta = \pi/10$



(e) u=13m/s,△ω=0.1, $\Delta\theta = \pi/10$



(d) u=16m/s, △ω=0.1, $\Delta\theta = \pi/10$

Figure 2. 3D ocean waves at different conditions based on JON SWAP spectrum (wind direction $\vec{W} = \pi/4$)

VI. CONCLUSIONS

Simulation of deep-water waves is realized using MATLAB based on JONSWAP spectrum in this paper, which can provide some technical supports for the research of interaction simulation of underwater gravity aided inertial navigation system. Through simulation experiments we can draw the following conclusions:

- The sea waves have close relation with wind speed, and also have relation with the fetch length. The function of JONSWAP spectrum has considered the wind speed and the fetch length, so better simulation results can be obtained.
- If suitable simulation frequency band, interval of wave frequency and interval of direction angle are selected, the ocean waves can be simulated quickly and vividly using MATLAB.
- The time needed for simulation has relations with simulation frequency band, interval of wave frequency ($\triangle\omega$) and interval of direction angle($\triangle\theta$). The wider the frequency band and the smaller $\triangle\omega$ and $\triangle\theta$, the simulated sea waves are more detailed and vivid, but the more time is needed for simulation. So the best simulation results may be obtained when comprehensively determine the factors of the needed realistic level for simulated sea waves, the needed real-time degree, wind speed and calculating speed of computer.

ACKNOWLEDGMENT

The authors would like to express appreciations to colleagues in our laboratory for their valuable comments and other helps.

REFERENCES

[1] Z. Xu, "Research on the theory, models, matching algorithms and simulation for the underwater gravity aided inertial navigation system," Ph. D. dissertation, China University of Mining and Technology (Beijing), China, 2007.

[2] J. Liu, "Research on algorithm for simulating ocean waves based on spectrum of ocean waves," M. A. thesis, Hunan University, China, 2005.

[3] J. Tao, Numerical Simulation of Water Waves, 1rd ed.,Tianjin: Tianjin University Press, 2005, pp. 43-48.

[4] F. Zeng, "Real-time wave simulation based on texture-mapping," Computer Simulation, vol. 24(10), 2007, pp. 195-198.

[5] H. Yang and J. Sun, "Wave simulation based on ocean wave spectrums," Journal of System Simulation, vol. 14(9), 2002, pp. 1175-1178.

[6] W. Jin and P. Pu, "Sea wave real-time simulation based on spectrum," Journal of Chongqing University of Posts and Telecommunications (Natural Science Edition), vol. 21(3), 2009, pp. 435-438.

[7] X. Hou, P. Shen, and W. Wang, "Method of ocean wave synthesis based on directional spectrum," Journal of System Simulation, vol. 22(1), 2010, pp. 130-134.

[8] C. Xie, Y. Jin, and X. Liu, "Real-time ocean wave in multi-channel marine simulator", Proc.Conf. Virtual Reality Continuum and Its Applications in Industry, ACM Press New York, NY, USA, Singapore, pp. 332–335, 2004.

[9] J. Schneider and R. Westermann, "Towards real-time visual simulation of water surfaces," Proc. IEEE Conf. Vision Modelling and Visualization, Germany: IEEE Computer Society press, pp. 211-218, 2001.

[10] M. Wang, MATLAB and Science Computing, 2rd ed., Beijing: Electronics Industry Press, 2004, pp. 107-128.