The botanist keeps tasting tea

A gentle introduction to e-values and sequential statistical inference

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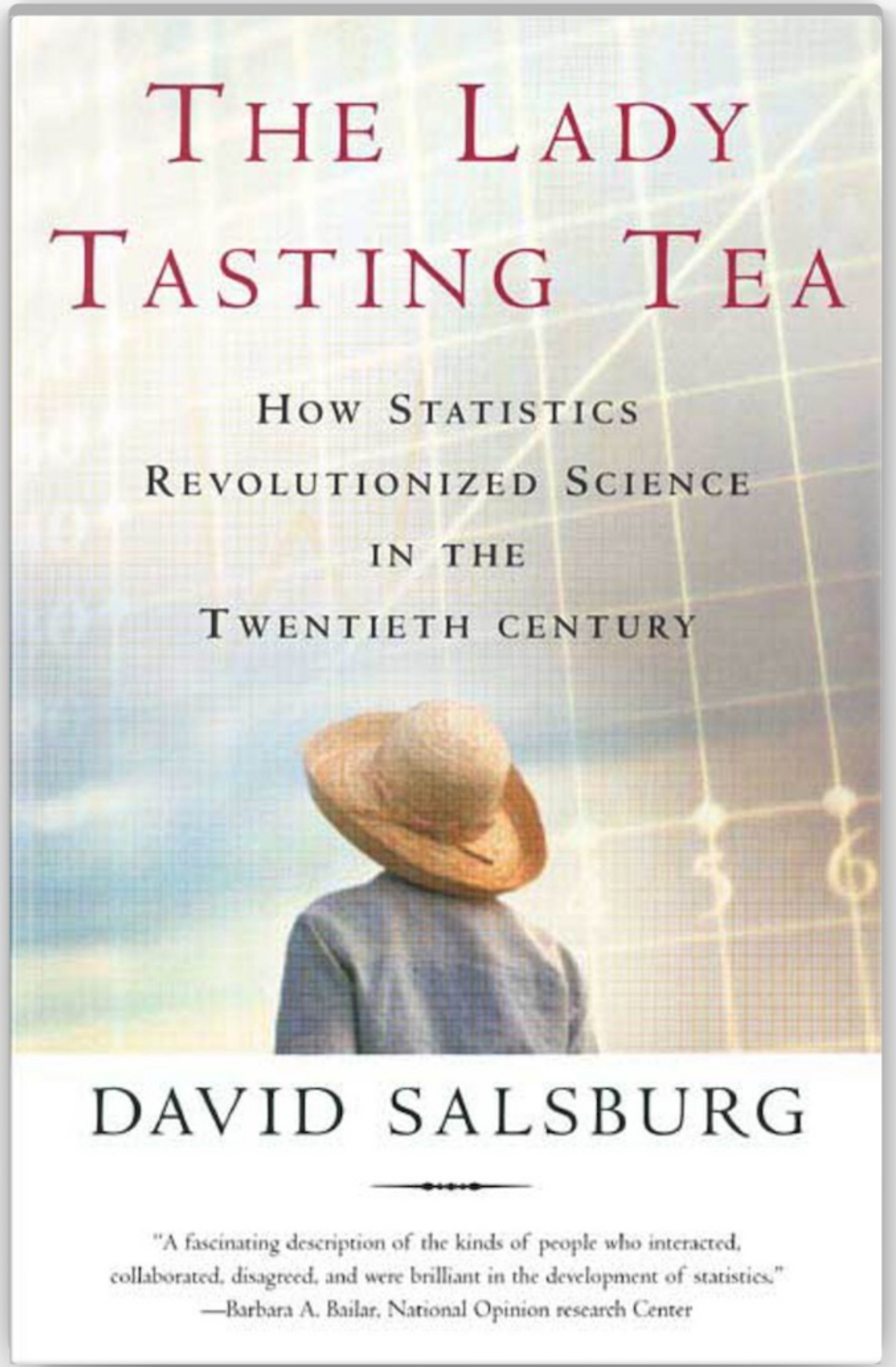
THE LADY TASTING TEA
HOW STATISTICS REVOLUTIONIZED SCIENCE IN THE TWENTIETH CENTURY
DAVID SALSBURG
"A fascinating description of the kinds of people who interacted, collaborated, disagreed, and were brilliant in the development of statistics."
—Barbara A. Bailar, National Opinion research Center

The botanist tasting tea (1920s)

Ronald Fisher

Muriel Bristol

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What's the probability that a *chance guess* would be *perfect*?

$1/70 \approx 0.014$

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This is a p-value for H_0 : Muriel cannot distinguish btwn MT and TM.

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The probability that a *chance guess* would yield *at most one error* is 17/70 ≈ 0.24, which is not so impressive.

If she had made exactly one mistake, could we just let her keep tasting tea?
No, this is blatant p-hacking.

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In fact, it is typically the case that

$$\mathbb{P}_{H_0}\left(P_\tau \leq \alpha\right) \approx 1!$$

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An e-value is a function E of the data (MT or TM) so that

$$E \geq 0 \quad \text{and} \quad \mathbb{E}_{H_0}(E) \leq 1.$$

Suppose Muriel keeps tasting tea:

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Then, $\mathbb{P}_{H_0}\left(P_\tau^\star \leq \alpha\right) \leq \alpha$, regardless of how τ is chosen!

where $P_n^\star := \left(E_1 \cdot E_2 \ \cdots \ E_n\right)^{-1}$.

Thank you!

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