

Question 1)

Forward Pass

$$z_1, z_2, z_3 = [0.5726 \quad 0.5833] \begin{bmatrix} -3.6 & 1.5 & 0.99 \\ -1.23 & -3.6 & 1.4 \end{bmatrix}$$

$$= [-2.7788 \quad -1.2409 \quad 1.3834]$$

$$\text{Sigmoid } A_1, A_2, A_3 = \begin{bmatrix} 0.0584 & 0.2242 & 0.7995 \end{bmatrix}$$

$$z_4 = \begin{bmatrix} 0.0584 & 0.2242 & 0.7995 \end{bmatrix} \begin{bmatrix} 0.11 \\ 2.22 \\ 1.4 \end{bmatrix}$$

$$= [1.6234]$$

$$\text{Sigmoid } \hat{y} = 0.8352$$

$$\approx 83.52\%. \quad (\text{Prediction})$$

$$\text{Loss} = \frac{1}{2} (0.75 - 0.8352)^2$$

$$> 0.00362$$

Backpropagation Pass

$$\delta = (\hat{y} - Y)(\hat{y})(1 - \hat{y})$$

$$= (0.8352 - 0.75)(0.8352)(1 - 0.8352)$$

$$= 0.01172$$

$$W_{21}^* = W_{21} - \eta \delta (A_1)$$

$$= 0.11 - 0.5(0.01172)(0.0584)$$

$$= 0.1096$$

$$W_{12}^* = 2.22 - 0.5(0.01172)(0.2242)$$

$$= 2.2186$$

$$W_{13}^* = 1.4 - 0.5(0.01172)(0.7995)$$

$$= 1.3953$$

$$W_{11}^* = W_{11} - \eta \delta (W_{21})(A_1)(1 - A_1)(Z_{11})$$

$$= -3.6 - (0.5)(0.01172)(0.11)(0.0584)(1 - 0.0584)(0.5726)$$

$$= -3.6$$

$$W_{12}^* = 1.5 - (0.5)(0.01172)(0.5726)(0.2242)(1 - 0.2242)(2.22)$$

$$= 1.4987$$

$$W_{13}^* = 0.99 - (0.5)(0.01172)(0.5726)(0.7995)(1 - 0.7995)(1.4)$$

$$= 0.9892$$

$$W_{14}^* = -1.23 - (0.5)(0.01172)(0.5833)(0.11)(0.0584)(1 - 0.0584)$$

$$= -1.23$$

$$W_{15}^* = -3.6 - (0.5)(0.01172)(0.5833)(2.22)(0.2242)(1 - 0.2242)$$

$$= -3.6013$$

$$W_{16}^* = 1.4 - (0.5)(0.01172)(0.5833)(1.4)(0.7995)(1 - 0.7995)$$

$$= 1.3992$$

Forward Pass (2nd Times)

$$z_1, z_2, z_3 = \begin{bmatrix} 0.5726 & 0.5833 \end{bmatrix} \begin{bmatrix} -3.6 & 1.4987 & 0.9892 \\ -1.23 & -2.6013 & 1.3992 \end{bmatrix}$$

$$= \begin{bmatrix} -2.7788 & -1.2424 & 1.3826 \end{bmatrix}$$

$$\text{Sigmoid } A_1, A_2, A_3 = \begin{bmatrix} 0.0584 & 0.224 & 0.7994 \end{bmatrix}$$

$$z_4 = \begin{bmatrix} 0.0584 & 0.224 & 0.7994 \end{bmatrix} \begin{bmatrix} 0.1096 \\ 2.2186 \\ 1.3953 \end{bmatrix}$$

$$= 1.6187$$

$$\text{Sigmoid } \hat{y} = 0.8346$$

$$\approx 83.46\%$$

$$\text{Loss} = \frac{1}{2} (0.75 - 0.8346)^2$$

$$= 0.00357$$

\therefore From 0.00362 to 0.00357, it decrease "0.00005"

Question 1 (using tanh)

We know that

$$z_1, z_2, z_3 = \begin{bmatrix} -2.7288 & -1.24098 & 1.283494 \\ \cancel{-2.7288} & \cancel{-1.24098} & \end{bmatrix}$$

$$\therefore \tanh A_1, A_2, A_3 = \frac{1 - e^{-2(z_i)}}{1 + e^{-2(z_i)}}$$

$$= \begin{bmatrix} -0.9923 & -0.84573 & 0.88173 \end{bmatrix}$$

$$z_4 = \begin{bmatrix} -0.9923 & -0.84573 & 0.88173 \end{bmatrix} \begin{bmatrix} 0.11 \\ 2.22 \\ 1.4 \end{bmatrix}$$

$$= -0.752262$$

$$\hat{y} = \frac{1}{1 + e^{-z_4}}$$

$$= 0.32032$$

$$\approx 32.03\%$$

$$\text{Loss} = \frac{1}{2} (0.75 - 0.3203)^2$$

$$= 0.0923$$

Backpropagation (tanh)

$$\begin{aligned}\delta &= (\hat{y} - Y)(\hat{y})(1 - \hat{y}) \\ &= (0.3203)(1 - 0.3203)(0.3203 - 0.75) \\ &= -0.09354\end{aligned}$$

$$\begin{aligned}W_{21} &= W_{21} - \eta \delta A_1 \\ &= 0.11 - (0.5)(-0.09354)(-0.9923) \\ &= 0.06358\end{aligned}$$

$$\begin{aligned}W_{22} &= 2.22 - (0.5)(-0.09354)(-0.8457) \\ &= 2.1804\end{aligned}$$

$$\begin{aligned}W_{23} &= 1.4 - (0.5)(-0.09354)(0.88173) \\ &= 1.4412\end{aligned}$$

$$\begin{aligned}W_{11} &= W_{11} - \eta \delta W_{21} (1 - A_1^2) 2n_1 \\ &= -3.6 - (0.5)(-0.09354)(0.11)(1 - (-0.9923)^2)(0.5726) \\ &= -3.5999\end{aligned}$$

$$\begin{aligned}W_{12} &= 1.5 - (0.5)(-0.09354)(2.22)(1 - (-0.8457)^2)(0.5726) \\ &= 1.5169\end{aligned}$$

$$\begin{aligned}W_{13} &= 0.99 - (0.5)(-0.09354)(1.4)(1 - (0.88173)^2)(0.5726) \\ &= 0.9983\end{aligned}$$

$$\begin{aligned}W_{14} &= -1.23 - (0.5)(-0.09354)(0.11)(1 - (-0.9923)^2)(0.5833) \\ &= -1.2299\end{aligned}$$

$$\begin{aligned}W_{15} &= -3.6 - (0.5)(-0.09354)(2.22)(1 - (-0.8457)^2)(0.5833) \\ &= -3.5827\end{aligned}$$

$$\begin{aligned}W_{16} &= 1.4 - (0.5)(-0.09354)(1.4)(1 - (0.88173)^2)(0.5833) \\ &= 1.4085\end{aligned}$$

2nd Forward Pass (tanh)

$$z_1, z_2, z_3 = \begin{bmatrix} 0.5726 & 0.5833 \end{bmatrix} \begin{bmatrix} -3.5999 & 1.5169 & 0.9983 \\ -1.2299 & -3.5827 & 1.4085 \end{bmatrix}$$

$$= \begin{bmatrix} -2.7787 & -1.2212 & 1.3932 \end{bmatrix}$$

$$\tanh A_1, A_2, A_3 = \begin{bmatrix} -0.9923 & -0.84 & -0.8838 \end{bmatrix}$$

$$z_4 = \begin{bmatrix} -0.9923 & -0.84 & 0.8838 \end{bmatrix} \begin{bmatrix} 0.0635 \\ 2.1804 \\ \underline{2} \\ 1.4412 \end{bmatrix}$$

$$= -0.6208$$

$$\hat{y} = 0.3495$$

$$\approx 34.95\%$$

$$\text{Loss} = \frac{1}{2} (0.75 - 0.3495)^2$$

$$= 0.08016$$

Forward Pass (Relu)

$$\therefore z_1, z_2, z_3 = [-2.7788 \quad -1.24098 \quad 1.3834]$$

$$A_1 = 0, A_2 = 0, A_3 = 1.3834$$

↳ since negative, so equal 0.

* When value ≤ 0 , change ^{value} number = 0;

when value > 0 , remain the value as the same.

$$z_4 = (0 \times 0.11) + (0 \times 2.22) + (1.3834 \times 1.4)$$

$$= 1.9368$$

$$\text{Sigmoid } \hat{y} = 0.87401$$

$$\approx 87.401\%$$

$$\text{Loss} = \frac{1}{2} (0.75 - 0.87401)^2$$

$$= 0.00768$$

Backpropagation (Relu)

$$\text{loss} = \frac{1}{2} (Y - \hat{y})^2 = \frac{1}{2} (0.75 - 0.874)^2$$

$$= 0.00768$$

$$\delta = (\hat{y} - Y) (\hat{y}) (1 - \hat{y}) = (0.874 - 0.75) (0.874) (1 - 0.874)$$

$$= 0.01365$$

$$\therefore W_{21} = W_{21} - \eta \delta (A_1)$$

$$= 0.11 - (0.5) (0.01365) (0)$$

$$= 0.11$$

$$W_{22} = 2.22 - (0.5) (0.01365) (0)$$

$$= 2.22$$

$$W_{23} = 1.4 - (0.5) (0.01365) (1.3834)$$

$$= 1.39317$$

$$\rightarrow f'(\text{Relu}(A_i)) \begin{cases} \text{if } 0 = 0; & \therefore A_1 = 0 \\ \text{else} = 1 & A_2 = 0 \\ & A_3 = 1 \end{cases}$$

$$W_{11} = W_{11} - \eta \delta W_{21} A_1 z_{n1}$$

$$= -3.6 - (0.5) (0.01365) (0.11) (0) (0.5726)$$

$$= -3.6$$

$$W_{12} = 1.5 - (0.5) (0.01365) (2.22) (0) (0.5726)$$

$$= 1.5$$

$$W_{13} = 0.99 - (0.5) (0.01365) (1.4) (1) (0.5726)$$

$$= 0.9896$$

$$W_{14} = -1.23 - (0.5) (0.01365) (0.11) (0) (0.5833)$$

$$= -1.23$$

$$W_{15} = -3.6 - (0.5) (0.01365) (2.22) (0) (0.5833)$$

$$= -3.6$$

$$W_{16} = 1.4 - (0.5) (0.01365) (1.4) (1) (0.5833)$$

$$= 1.3944$$

2nd forward Pass (Rem)

$$z_1, z_2, z_3 = \begin{bmatrix} 0.5726 & 0.5833 \end{bmatrix} \begin{bmatrix} -3.6 & 1.5 & 0.9896 \\ -1.23 & -3.6 & 1.3944 \end{bmatrix}$$

$$= \begin{bmatrix} -2.7788 & -1.2409 & 1.3771 \end{bmatrix}$$

↓ ↓ ↓

$$\text{Rem } A_1, A_2, A_3 = \begin{bmatrix} 0 & 0 & 1.3771 \end{bmatrix}$$

$$z_4 = (0 \times 0.11) + (0 \times 2.22) + (1.3771 \times 1.4) \\ = 1.9149$$

$$\hat{y} = 0.87157$$

$$\approx 87.16\%$$

$$\text{Loss} = \frac{1}{2} (0.75 - 0.87157)^2$$

$$= 0.00739$$