Homework 4

Due on September 25(Prof. Zhang and Wu)/September 26(Prof. Weintraub, Coll and Recio-Mitter), before class

Problem 1

Let $S = \left\{ \mathbf{x} \in \mathbb{R}^2 : \mathbf{x} = \begin{pmatrix} 2k \\ -3k \end{pmatrix}, k \in \mathbb{R} \right\}$. Show that S is a subspace of $\mathbb{R}^2$.

Problem 2

Let $S = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 : x^2 = y^2 \right\}$. Determine whether S is a subspace of $\mathbb{R}^2$ and explain your answer.

Problem 3

Find the null space of $A = \begin{bmatrix} 2 & 2 & 4 \\ 1 & 2 & 3 \end{bmatrix}$.

Problem 4

Show that $\mathbf{v}_1 = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$ and $\mathbf{v}_2 = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$ span $\mathbb{R}^2$, and express the vector $\mathbf{v} = \begin{pmatrix} 3 \\ 18 \end{pmatrix}$ as a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$.

Problem 5

Prove that e^x does not belong to $P_1(\mathbb{R})$.

Problem 6

Determine whether the following set of vectors are linearly independent or linearly dependent in $\mathbb{R}^3$: $\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$.

Problem 7

Show that the vectors $p_1(x) = a+bx$ and $p_2(x) = c+dx$ are linearly independent in $P_1(\mathbb{R})$ if and only if the constants satisfy $ad - bc \neq 0$.

Problem 8

Determine whether $S = \left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \\ -2 \end{pmatrix}, \begin{pmatrix} 3 \\ 11 \\ -5 \end{pmatrix} \right\}$ is a basis of $\mathbb{R}^3$.

Problem 9

Find the dimension of the null space of $A = \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}$.

Problem 10

Find a vector $\mathbf{v} \in \mathbb{R}^3$ such that $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \mathbf{v} \right\}$ constitutes a basis of $\mathbb{R}^3$.