MATH 205, FALL - 2019, EXAM # 2

Name:__________________________________ .

Instructor/Section: Circle one

Prof. Zhang - Section 10, MW 0920-1035
Prof. Zhang - Section 11, MW 1325-1440
Prof. Wu - Section 12, MW 1500-1615
Prof. Coll - Section 13, TR 1210-1325
Prof. Weintraub - Section 14, TR 1045-1200
Prof. Recio-Mitter - Section 15, TR 1625-1740

Instructions: Do all work on the test paper. ***Show all work***. You may receive no credit, even for a correct answer, if no work is shown. You may use the back or the extra page at the end if you need extra space. But please reference this in your answer if you do.

You may ***not*** use any electronic device, calculator, computer, the assistance of any other students, any notes, crib sheets, or texts during this exam. All cellphones must be silenced and out of sight.

This exam has 11 pages. ***Make sure*** your exam is complete.

You have 60 minutes to complete this exam.

Do not turn to the next page until you are instructed to do so.

Grading:

1._______________/12

2._______________/8

3._______________/8

4._______________/16

5._______________/20

6._______________/36

Total._______________/100

(1) Let A be the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 & 4 \\ 3 & 6 & 1 & 10 & 14 \\ 7 & 14 & 2 & 23 & 33 \end{bmatrix}.$$

A has reduced row-echelon form

$$R = \begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

(a) (6 points) Find a basis for the nullspace of A.

(b) (6 points) Find a basis for the column space of A.

(2) Let V be a 2-dimensional vector space with basis $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$. Let $T : V \longrightarrow V$ be a linear transformation with

$$T(\mathbf{v}_1) = 3\mathbf{v}_1 + 2\mathbf{v}_2,$$
$$T(\mathbf{v}_2) = 5\mathbf{v}_1 - 4\mathbf{v}_2.$$

(a) (4 points) Find $T(2\mathbf{v}_1 + \mathbf{v}_2)$.

(b) (4 points) Find $[T]_{\mathcal{B}}$, the matrix of T in the basis $\mathcal{B}$.

(3) Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \\ 14 \end{bmatrix} \right\}.$$

We know that $W = \text{Span}(\mathcal{B})$ is a subspace of $\mathbb{R}^3$, and then $\mathcal{B}$ is a basis of W.

Let

$$\mathbf{u}_1 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \quad \text{and} \quad \mathbf{u}_2 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}.$$

(a) (4 points) Is $\mathbf{u}_1$ in W? If so, find the coordinate vector $[\mathbf{u}_1]_{\mathcal{B}}$.

(b) (4 points) Is $\mathbf{u}_2$ in W? If so, find the coordinate vector $[\mathbf{u}_2]_{\mathcal{B}}$.

(4) Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 10 \end{bmatrix} \right\}.$$

$\mathcal{B}$ is a basis of $\mathbb{R}^2$.

(a) (4 points) If $[\mathbf{w}]_{\mathcal{B}} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$, find $\mathbf{w}$.

(b) (4 points) If $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, find $[\mathbf{v}]_{\mathcal{B}}$.

(c) (8 points) Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be the linear transformation $T(\mathbf{x}) = A\mathbf{x}$, where A is the matrix

$$A = \begin{bmatrix} 11 & 13 \\ 17 & 19 \end{bmatrix}.$$

Find $[T]_{\mathcal{B}}$. (You may leave your answer expressed as a product of matrices and their inverses.)

(5) (20 points) Let A be the matrix

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 1 & 3 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

A has characteristic polynomial $-(\lambda-1)(\lambda-2)(\lambda-4)$.

Find an invertible matrix P and a diagonal matrix D with $D = P^{-1}AP$.

More space for the answer to problem 5.

(6) (36 points) Solve each of the following differential equations/initial value problems:

(a) $y'''' + 2y''' - 5y'' - 6y' = 0$.

(In operator notation, this is the differential equation $(D^4 + 2D^3 - 5D^2 - 6D)y = 0$. Note that $D^4 + 2D^3 - 5D^2 - 6D = D(D+1)(D-2)(D+3)$.)

(b) $y'''' - 13y''' + 60y'' - 122y' + 64y = 0$.

(In operator notation, this is the differential equation $(D^4 - 13D^3 + 60D^2 - 112D + 64)y = 0$. Note that $D^4 - 13D^3 + 60D^2 - 112D + 64 = (D-1)(D-4)^3$.)

(c) $y'' - 4y' + 29y = 0.$
(In operator notation, this is the differential equation $(D^2 - 4D + 29)y = 0.$)

(d) $y'' - 4y' + 3y = 0, \quad y(0) = 3,\ y'(0) = 7.$
(In operator notation, this is the differential equation $(D^2 - 4D + 3)y = 0$. Note that $(D^2 - 4D + 3) = (D - 1)(D - 3)$.)

(e) $y'' + 4y' + 3y = 60e^{2x}$.
(In operator notation, this is the differential equation $(D^2 + 4D + 3)y = 60e^{2x}$. Note that $(D^2 + 4D + 3) = (D + 1)(D + 3)$.)

(f) $y'' - 4y = 12e^{2x}$.
(In operator notation, this is the differential equation $(D^2 - 4)y = 12e^{2x}$. Note that $(D^2 - 4) = (D - 2)(D + 2)$.)

Extra page for additional work, if needed.