

# FALL 2019 - MATH 205

## HOMEWORK 3

Due at the beginning of class on Weds. Sept. 18 (Profs. Zhang and Wu), Thurs. Sept. 19 (Profs. Coll, Weintraub, Recio-Mitter). Write your name and section number on your homework. You must show your work in order to receive full credit.

*Neo:* What is the Matrix?

*Trinity:* The answer is out there, Neo, and it's looking for you, and it will find you if you want it to.

Dialogue from "The Matrix"

1. Find the rank of  $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ .
2. Let  $A$  and  $B$  be  $n \times n$  matrices. Write the following determinants in terms of  $\det(A)$  and  $\det(B)$ .
  - (a)  $\det(5A)$
  - (b)  $\det(AB^2)$
  - (c)  $\det((AB)^{-1})$
3. Compute the determinant of the following matrices.
  - (a)  $A = \begin{bmatrix} 1 & -1 \\ 3 & 4 \end{bmatrix}$
  - (b)  $B = \begin{bmatrix} 3 & -5 \\ 9 & -15 \end{bmatrix}$
  - (c)  $A^{-1}B^2$
4. Compute the determinant of the matrix

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 7 & 3 & -1 \\ 2 & 1 & 4 \end{bmatrix}.$$

5. Compute the determinant of the matrix

$$A = \begin{bmatrix} -7 & 3 & 1 \\ 2 & 0 & -2 \\ 1 & 0 & 5 \end{bmatrix}.$$

6. (a) Compute the determinant of the matrix

$$A = \begin{bmatrix} 8 & -1 & 4 \\ 6 & 1 & 3 \\ 1 & 2 & 0 \end{bmatrix}.$$

(b) Determine the rank of the matrix  $A$ .

(c) What is the solution set of the homogeneous system  $A\mathbf{x} = \mathbf{0}$ ? Justify your answer.

7. Compute the determinant of the matrix

$$A = \begin{bmatrix} 3 & -3 & 7 & 5 \\ 2 & 0 & 5 & 6 \\ 1 & -1 & 3 & 1 \\ 2 & -2 & 1 & 5 \end{bmatrix}.$$

*Hint:* Use row operations.

8. Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 3 & 3 & 3 \\ 7 & 9 & -5 & 1 \\ 0 & -4 & 3 & 2 \end{bmatrix}.$$

How many solutions does the homogeneous system  $A\mathbf{x} = \mathbf{0}$  have?

9. (a) Is the set

$$S = \left\{ \begin{bmatrix} 2s + t \\ t - 3s \\ 3t \end{bmatrix} \mid s, t \in \mathbb{R} \right\}$$

a subspace of the vector space  $\mathbb{R}^3$ ?

- (b) Is the set

$$S = \left\{ \begin{bmatrix} 2s \\ 3t \\ 1 \end{bmatrix} \mid s, t \in \mathbb{R} \right\}$$

a subspace of the vector space  $\mathbb{R}^3$ ?

10. The real-valued continuous functions on the real numbers form a vector space  $C(\mathbb{R}) = \{f: \mathbb{R} \rightarrow \mathbb{R} \mid f \text{ is continuous}\}$  with:

- Addition  $(f + g)(x) = f(x) + g(x)$ .
- Scalar multiplication  $(kf)(x) = kf(x)$ .

Is the set

$$S = \{f \in C(\mathbb{R}) \mid f(3) = 0\}$$

a subspace of the vector space  $C(\mathbb{R})$ ?

(b) Is the set

$$S = \{f \in C(\mathbb{R}) \mid f(3) = 2\}$$

a subspace of the vector space  $C(\mathbb{R})$ ?