Homework 8

Due on October 28(Prof. Zhang and Wu)/October 29(Prof. Weintraub, Coll and Recio-Mitter), before class

Problem 1

Determine whether the following matrices are diagonalizable or not.

1. $A = \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}$.
2. $B = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$.
3. $C = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$.
4. $D = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$.

Problem 2

Determine whether the following matrices are diagonalizable or not.

1. $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 4 & 0 \end{bmatrix}$.
2. $B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

Problem 3

Determine whether $A = \begin{bmatrix} -1 & 2 & 2 \\ -4 & 5 & 2 \\ -4 & 2 & 5 \end{bmatrix}$ is diagonalizable or not. The characteristic polynomial is $p(\lambda) = (3 - \lambda)^3$.

Problem 4

Diagonalize the following matrices:

1. $A = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$.
2. $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.
3. $C = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$.
4. $D = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix}$.

Problem 5

Assume A is an invertible matrix.

1. Prove that 0 is not an eigenvalue of A.
2. Assume λ is an eigenvalue of A. Show that λ^{-1} is an eigenvalue of A^{-1}.

Problem 6

Prove that $\mathrm{e}^x \sin(x)$ and $\mathrm{e}^x \cos(x)$ are linearly independent.

Problem 7

Find the general solution $y(x)$ to the equation $y'' - 6y' + 9y = 0$.

Problem 8

Find the general solution $y(x)$ to the equation $y'' - 2y' + 2y = 0$.

Problem 9

Find the general solution $y(x)$ to the equation $2y'' + 3y' - 2y = 0$.

Problem 10

1. Find the general solution $y(x)$ to the equation $y'' + 6y' + 9y = 0$.
2. Find a particular solution $y_p(x)$ that has the form $y_p(x) = Dx^2 \mathrm{e}^{-3x}$ for some constant D to the equation $y'' + 6y' + 9y = 2\mathrm{e}^{-3x}$.
3. Find the general solution $y(x)$ to the equation $y'' + 6y' + 9y = 2\mathrm{e}^{-3x}$.