Homework 11[1]

Due before class on November 20, Wednesday

(Professor Wu and Professor Zhang)

Due before class on November 21, Thursday

(Professor Coll, Professor Recio-Mitter and Professor Weintraub)

1. Solve the nonhomogeneous linear system of differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} = \begin{pmatrix} 8 & 3 \\ 3 & 8 \end{pmatrix}\mathbf{u} + \begin{pmatrix} -10 \\ 10 \end{pmatrix} + \begin{pmatrix} 25 \\ -25 \end{pmatrix} t.$$

2. Solve the nonhomogeneous linear system of differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} = \begin{pmatrix} 10 & 7 \\ 8 & 9 \end{pmatrix}\mathbf{u} - 9e^{8t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

3. Solve the homogeneous linear system of differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} = \begin{pmatrix} 7 & -4 \\ 2 & 3 \end{pmatrix}\mathbf{u} + e^{3t}\begin{pmatrix} 4 \\ -6 \end{pmatrix} + e^{4t}\begin{pmatrix} 9 \\ -4 \end{pmatrix}.$$

4. Solve the nonhomogeneous linear system of differential equations

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} = \begin{pmatrix} 9 & -4 \\ 5 & 5 \end{pmatrix}\mathbf{u} \\ + \;& 4t^3e^{7t}\begin{pmatrix} 2\cos(4t) - 4\sin(4t) \\ 5\cos(4t) \end{pmatrix} + 8t^7e^{7t}\begin{pmatrix} 4\cos(4t) + 2\sin(4t) \\ 5\sin(4t) \end{pmatrix}. \end{aligned}$$

5. Solve the initial value problem for the nonhomogeneous linear system of differential equations

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} = \begin{pmatrix} 9 & -4 \\ 5 & 5 \end{pmatrix}\mathbf{u} \\ + \;& 4t^3e^{7t}\begin{pmatrix} 2\cos(4t) - 4\sin(4t) \\ 5\cos(4t) \end{pmatrix} + 8t^7e^{7t}\begin{pmatrix} 4\cos(4t) + 2\sin(4t) \\ 5\sin(4t) \end{pmatrix}, \\ &\mathbf{u}(0) = \begin{pmatrix} 22 \\ 15 \end{pmatrix}. \end{aligned}$$

6. Solve the nonhomogeneous system of differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} = \begin{pmatrix} 14 & 4 & -2 \\ 2 & 12 & 2 \\ 2 & 4 & 10 \end{pmatrix}\mathbf{u} + \begin{pmatrix} 3 \\ 12 \\ -19 \end{pmatrix} + \begin{pmatrix} -16 \\ 32 \\ -40 \end{pmatrix} t.$$

[1]Chapter 9: Systems of Differential Equations, Section 9.4 (Systems of Differential Equations with Diagonalizable Constant Matrices) and Section 9.6 (the Method of Variation of Parameters)

Hint:

$$\det(A - \lambda I) = -(\lambda - 8)(\lambda - 12)(\lambda - 16).$$

7. Solve the nonhomogeneous linear system of differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} = \begin{pmatrix} 7 & -7 & 4 \\ -3 & 11 & -4 \\ -3 & -7 & 14 \end{pmatrix} \mathbf{u} + e^{8t} \begin{pmatrix} 33 \\ 27 \\ 37 \end{pmatrix}.$$

Hint:

$$\det(A - \lambda I) = -(\lambda - 4)(\lambda - 10)(\lambda - 18).$$

8. Solve the nonhomogeneous linear system of differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} = \begin{pmatrix} \alpha & -\alpha & \alpha \\ -\alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha \end{pmatrix} \mathbf{u} + e^{3\alpha t} \begin{pmatrix} 11\alpha \\ 12\alpha \\ 3\alpha \end{pmatrix} + e^{4\alpha t} \begin{pmatrix} 23\alpha \\ 25\alpha \\ 13\alpha \end{pmatrix},$$

where $\alpha \neq 0$ is a real nonzero constant. Hint:

$$\det(A - \lambda I) = -(\lambda - 2\alpha)^2(\lambda + \alpha).$$

9. Solve the nonhomogeneous linear system of differential equations.

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} &= \begin{pmatrix} 2 & 7 & 7 \\ -2 & 11 & 7 \\ 2 & -7 & -3 \end{pmatrix} \mathbf{u} \\ &+ 2te^{4t} \begin{pmatrix} 7 \\ 2 \\ 0 \end{pmatrix} + 3t^2 e^{4t} \begin{pmatrix} 7 \\ 0 \\ 2 \end{pmatrix} + 4t^3 e^{2t} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}. \end{aligned}$$

Hint:

$$\det(A - \lambda I) = -(\lambda - 2)(\lambda - 4)^2.$$

10. Solve the initial value problem for the nonhomogeneous linear system of differential equations

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} &= \begin{pmatrix} 2 & 7 & 7 \\ -2 & 11 & 7 \\ 2 & -7 & -3 \end{pmatrix} \mathbf{u} \\ &+ 2te^{4t} \begin{pmatrix} 7 \\ 2 \\ 0 \end{pmatrix} + 3t^2 e^{4t} \begin{pmatrix} 7 \\ 0 \\ 2 \end{pmatrix} + 4t^3 e^{2t} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \\ \mathbf{u}(0) &= \begin{pmatrix} 39 \\ 8 \\ 2 \end{pmatrix}. \end{aligned}$$