Homework Assignment 1–Solutions

1. (a)

$$2A + 3B = \begin{bmatrix} 4 & 13 & -17 \\ 12 & 7 & -4 \end{bmatrix}.$$

(b)

$$AC = \begin{bmatrix} -16 & -7 & 1 & -5 \\ -6 & -2 & -2 & -2 \end{bmatrix}.$$

2. (a)

$$3D - E = \begin{bmatrix} 5 & 5 & -10 & 16 \\ 6 & -9 & 14 & 8 \\ -2 & 3 & -2 & 9 \end{bmatrix}.$$

(b)

$$FD = \begin{bmatrix} 3 & -1 & -7 & 16 \\ -1 & 0 & -10 & 6 \\ 12 & -1 & 20 & 19 \\ 8 & 15 & -16 & 31 \end{bmatrix}.$$

3.

$$MN = \begin{bmatrix} 20 & -1 & 8 \\ 12 & 1 & -1 \\ 27 & 3 & 13 \end{bmatrix}, \quad \text{and then} \quad (MN)P = \begin{bmatrix} 9 & 51 & 38 \\ 16 & 27 & -14 \\ 23 & 95 & 95 \end{bmatrix}.$$

$$NP = \begin{bmatrix} 3 & 2 & -10 \\ -3 & -4 & -15 \\ 4 & 19 & 26 \end{bmatrix}, \quad \text{and then} \quad M(NP) = \begin{bmatrix} 9 & 51 & 38 \\ 16 & 27 & -14 \\ 23 & 95 & 95 \end{bmatrix}.$$

We then see that $(MN)P = M(NP)$.

4.

$$\begin{bmatrix} 3 & 4 & -5 & 6 \\ 5 & 2 & 7 & -2 \\ 1 & -3 & 6 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \\ 3 \end{bmatrix}.$$

5.

$$\begin{bmatrix} 5 & 6 & -8 \\ 4 & 5 & -7 \\ 3 & -5 & -9 \\ 1 & 3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \\ 6 \end{bmatrix}.$$

6.

$$\begin{aligned} 2x_1 + 7x_2 - 3x_3 &= 7 \\ 3x_1 - 2x_2 + 5x_3 &= 9 \\ 4x_1 + 6x_2 + 9x_3 &= 5 \\ 5x_1 + 2x_2 - 4x_3 &= 4 \end{aligned}$$

7.

$$\begin{aligned}
4x_1 - 6x_2 + 9x_3 + 7x_4 &= 6 \\
2x_1 - 4x_2 + 3x_3 + 5x_4 &= 1 \\
7x_1 + 8x_2 - 5x_3 - 3x_4 &= 7
\end{aligned}$$

8.(a) Since y and z are both solutions of $Ax = 0$, we have $Ay = 0$ and $Az = 0$. Then

$$A(y + z) = Ay + Az = 0 + 0 = 0$$

so $y + z$ is also a solution of $Ax = 0$.

(b) Since y is a solution of $Ax = 0$, we have $Ay = 0$. Then

$$A(cy) = c(Ay) = c0 = 0$$

so cy is also a solution of $Ax = 0$.

9. Since this system is nonhomogeneous, $b \neq 0$.

(a) Since y and z are both solutions of $Ax = b$, we have $Ay = b$ and $Az = b$. Then

$$A(y + z) = Ay + Az = b + b = 2b \neq b$$

since $b \neq 0$, so $y + z$ is *not* a solution of $Ax = b$.

(b) Since y is a solution of $Ax = b$, we have $Ay = b$. Then

$$A(cy) = c(Ay) = cb \neq b$$

since $b \neq 0$ and $c \neq 1$, so cy is *not* a solution of $Ax = b$.

10. Since y is a solution of $Ax = 0$, we have $Ay = 0$. Since z is a solution of $Ax = b$, we have $Az = b$. Then

$$A(y + z) = Ay + Az = 0 + b = b$$

so $y + z$ is also a solution of $Ax = b$.