MATH 205 - Fall, 2019
HOMEWORK #2

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MATERIAL COVERED

**Section 2.4** Row-Echelon Matrices and Elementary Row Operations
**Section 2.5** Gaussian Elimination
**Section 2.6** The Inverse of a Square Matrix

Use the following system for problems 1 and 2.

$$\begin{array}{rcrcrl} x_1 & + & 3x_2 & + & 3x_3 & = 4 \\ x_1 & + & 4x_2 & + & 3x_3 & = 2 \\ x_1 & + & 3x_2 & + & 4x_3 & = 6 \end{array}$$

1. Reduce the augmented matrix for this system to reduced row-echelon form. Indicate the elementary row operations used for this reduction.
2. Is the system consistent? Justify your assertion.

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3. Solve the matrix equation using Gaussian elimination.

$$A\mathbf{x} = \mathbf{0},$$

where

$$A = \begin{bmatrix} 1 & -3 & -4 & 3 \\ -1 & 3 & 6 & 1 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 4 & 8 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

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Use the following matrix for problems 4 and 5.

$$A = \begin{bmatrix} 1 & -3 \\ -2 & 2 \end{bmatrix}$$

4. Determine $A^{-1}$ (if possible).
5. Show that $A$ satisfies the following equation:

$$A^2 - 3A - 4I_2 = 0$$

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Use the following matrices for problems 6-8. For each of these problems, find (if possible) the matrix indicated. If a certain algebraic operation is not permitted, explain why.

$$A = \begin{bmatrix} 2 & -4 & 6 \\ -1 & 2 & 2 \\ 2 & -5 & 10 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

6. Find $A^{-1}$.
7. Find the inverse of $B^5$.
8. Find $B^{-1}$ and $C^{-1}$. Use these matrices to find $(BC)^{-1}$.

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9. Consider the following system of linear equations

$$\begin{array}{rcrcrl} x_1 & + & 2x_2 & + & 3x_3 & = 4 \\ 2x_1 & - & x_2 & - & 2x_3 & = a^2 \\ -x_1 & - & 7x_2 & - & x_3 & = a \end{array}$$

where $a$ is a real number. Determine all the values of $a$ so that this system is consistent.

10. Consider the matrix equation:

$$A\mathbf{x} = \mathbf{c},$$

where

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

Find $A^{-1}$ and use it to solve the given matrix equation.