Homework Assignment 5

Due at the start of class, Weds. Oct. 2 (Profs. Zhang and Wu), Thurs. Oct. 3 (Profs. Coll, Weintraub, Recio-Mitter).

1. Let A be the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 & 4 \\ 1 & 2 & 1 & 0 & 10 \\ 2 & 4 & 1 & 1 & 9 \\ 5 & 10 & 1 & 1 & 21 \end{bmatrix}.$$

   (a) Find a basis for the nullspace of A.
   (b) a basis for the column space of A.

2. Let B be the matrix

$$B = \begin{bmatrix} 1 & 0 & 1 & 1 & 3 & 0 \\ 3 & 1 & 5 & 3 & 8 & 0 \\ 5 & 1 & 7 & 4 & 14 & -1 \\ 6 & 1 & 8 & 6 & 17 & -1 \end{bmatrix}.$$

   (a) Find a basis for the nullspace of B.
   (b) a basis for the column space of B.

3. Let C be a 7 by 9 matrix.

   (a) If $\text{rank}(C) = 3$, what is $\text{nullity}(C)$?
   (b) If $\text{nullity}(C) = 5$, what is $\text{rank}(C)$?

4. (a) Is there an 8 by 5 matrix with rank 6? Justify your answer.
   (b) Is there a 3 by 9 matrix with nullity 5? Justify your answer.

5. Let V be a vector space and let $\mathcal{B} = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ be a basis of V.

   (a) Find $[\mathbf{0}]_{\mathcal{B}}$, the coordinate vector of the zero vector of V in the basis $\mathcal{B}$.
   (b) Find $[\mathbf{v}_i]_{\mathcal{B}}$, the coordinate vector of the vector $\mathbf{v}_i$ in the basis $\mathcal{B}$.

6. Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ 8 \end{bmatrix} \right\}.$$

$\mathcal{B}$ is a basis of $\mathbb{R}^2$.
   (a) If $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$, find $\mathbf{v}$.
   (b) If $\mathbf{w} = \begin{bmatrix} 5 \\ 24 \end{bmatrix}$, find $[\mathbf{w}]_{\mathcal{B}}$.

7. Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \quad \begin{bmatrix} 7 \\ 9 \end{bmatrix} \right\}.$$

$\mathcal{B}$ is a basis of $\mathbb{R}^2$.

(a) If $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} -6 \\ 17 \end{bmatrix}$, find $\mathbf{v}$.

(b) If $\mathbf{w} = \begin{bmatrix} -8 \\ 29 \end{bmatrix}$, find $[\mathbf{w}]_{\mathcal{B}}$.

8. Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ 4 \\ 8 \end{bmatrix} \right\}.$$

$\mathcal{B}$ is a basis of $\mathbb{R}^3$.

(a) If $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} 5 \\ 7 \\ 0 \end{bmatrix}$, find $\mathbf{v}$.

(b) If $\mathbf{w} = \begin{bmatrix} -9 \\ 14 \\ 3 \end{bmatrix}$, find $[\mathbf{w}]_{\mathcal{B}}$.

9. Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ 7 \\ 13 \end{bmatrix}, \quad \begin{bmatrix} 4 \\ 6 \\ 13 \end{bmatrix} \right\}.$$

$\mathcal{B}$ is a basis of $\mathbb{R}^3$.

(a) If $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} 9 \\ 5 \\ 2 \end{bmatrix}$, find $\mathbf{v}$.

(b) If $\mathbf{w} = \begin{bmatrix} 3 \\ -1 \\ -5 \end{bmatrix}$, find $[\mathbf{w}]_{\mathcal{B}}$.

10. Let

$$\mathcal{B} = \{1 + 2x + 3x^2, x + x^2, 1 + 2x + 4x^2\}.$$

$\mathcal{B}$ is a basis of P_2 (the vector space of polynomials of degree at most 2).

(a) If $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$, find $\mathbf{v}$.

(b) If $\mathbf{w} = 2 + x^2$, find $[\mathbf{w}]_{\mathcal{B}}$.