Fall 2019 - Math 205
Homework 7

Due at the beginning of class on Monday Oct. 21 (Profs. Zhang and Wu), Tuesday Oct. 22 (Profs. Coll, Weintraub, Recio-Mitter). Write your name and section number on your homework. You must show your work in order to receive full credit.

> We share a philosophy about linear algebra:
> we think basis-free, we write basis-free,
> but when the chips are down we close the office door
> and compute with matrices like fury.
>
> *Irving Kaplansky about Paul Halmos*

1. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} -4 & 2 \\ -15 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

   Find a basis $\mathcal{B}$ of eigenvectors and find the matrix $[T]_{\mathcal{B}}^{\mathcal{B}}$.

2. Is the matrix

$$A = \begin{bmatrix} -9 & 4 \\ -30 & 13 \end{bmatrix}$$

   similar to the matrix

$$B = \begin{bmatrix} -4 & 2 \\ -15 & 7 \end{bmatrix}$$

   from exercise 1? In other words, does there exist a matrix S such that $SAS^{-1} = B$?

3. Find the eigenvalues and eigenspaces of the following matrix and determine whether the matrix is diagonalizable.

$$A = \begin{bmatrix} 2 & 5 \\ 0 & 2 \end{bmatrix}$$

4. Determine the eigenvalues and eigenspaces of the following matrix .

$$A = \begin{bmatrix} 6 & 2 & 8 \\ -2 & 1 & -4 \\ -2 & -1 & -2 \end{bmatrix}$$

   Hint: You may use that the characteristic polynomial of A is $(\lambda - 1)(\lambda - 2)^2$.

5. Determine the algebraic and geometric multiplicities of the eigenvalues of the matrix A in problem 4. Is A diagonalizable? If yes give a basis of eigenvectors and the corresponding diagonalization. For the diagonalization no further computations are needed.

6. Determine the eigenvalues and eigenspaces of the following matrix .

$$A = \begin{bmatrix} 9 & -3 & 5 \\ 4 & 1 & 4 \\ -6 & 3 & -2 \end{bmatrix}$$

   Hint: You may use that the characteristic polynomial of A is $(\lambda - 1)(\lambda - 3)(\lambda - 4)$.

7. Determine the algebraic and geometric multiplicities of the eigenvalues of the matrix A in problem 6. Is A diagonalizable? If yes give a basis of eigenvectors and the corresponding diagonalization. For the diagonalization no further computations are needed.

8. Find the eigenvalues and eigenspaces of the following matrix.

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

   Is the matrix diagonalizable? If the answer is yes, find a basis of eigenvectors and the corresponding diagonalization.

9. Find the eigenvalues and a basis of eigenvectors of the following matrix.

$$\begin{bmatrix} 7 & -5 \\ 13 & 8 \end{bmatrix} \begin{bmatrix} 6 & 0 \\ 0 & 17 \end{bmatrix} \begin{bmatrix} 7 & -5 \\ 13 & 8 \end{bmatrix}^{-1}$$

10. Use diagonalization to compute

$$\begin{bmatrix} 13 & -42 \\ 4 & -13 \end{bmatrix}^{999}.$$