MATH 205, FALL - 2019, PRACTICE EXAM # 1

(1) Solve the linear system

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 2 \\ 2 & 3 & 7 & 6 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 5 \\ 3 \\ 19 \end{bmatrix}.$$

(As a check on your arithmetic, this system is consistent and the matrix of this system has rank 2.)

(2) Find the inverse of the matrix

$$\begin{bmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 1 \\ 2 & 0 & 1 & 3 \\ 0 & 1 & 2 & -4 \end{bmatrix}.$$

(3) Find the determinant of the matrix

$$\begin{bmatrix} 0 & 2 & 3 & 5 \\ 2 & 1 & 1 & 1 \\ 0 & -1 & 2 & 1 \\ 4 & 2 & 2 & 5 \end{bmatrix}.$$

(4) Let W be the subset of P_3 (the vector space of polynomials of degree at most 3) consisting of all polynomials $p(x)$ with $p(0) = 0$. Show that W is a subspace of P_3.

(5) Let $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ be a set of vectors in a vector space V. Carefully define:
(a) S is linearly independent.
(b) S spans V.
(c) S is a basis of V.

(6) Let

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ 11 \\ 5 \end{bmatrix} \right\}.$$

Determine whether S spans R^4. If not, find a vector in R^4 that is not in Span(S).

(7) Let

$$S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \\ 1 \end{bmatrix} \right\}.$$

Determine whether S is linearly independent. If not, find a linear dependence relation between the elements of S.