MATH 205, FALL - 2019, EXAM # 1

Name:___________________________________ .

Instructor/Section: Circle one

Prof. Zhang - Section 10, MW 0920-1035
Prof. Zhang - Section 11, MW 1325-1440
Prof. Wu - Section 12, MW 1500-1615
Prof. Coll - Section 13, TR 1210-1325
Prof. Weintraub - Section 14, TR 1045-1200
Prof. Recio-Mitter - Section 15, TR 1625-1740

Instructions: Do all work on the test paper. ***Show all work***. You may receive no credit, even for a correct answer, if no work is shown. You may use the back or the extra page at the end if you need extra space. But please reference this in your answer if you do.

You may ***not*** use any electronic device, calculator, computer, the assistance of any other students, any notes, crib sheets, or texts during this exam. All cellphones must be silenced and out of sight.

This exam has 10 pages. ***Make sure*** your exam is complete.

You have 60 minutes to complete this exam.

Do not turn to the next page until you are instructed to do so.

Grading:

1._______________/16	5._______________/16
2._______________/16	6._______________/16
3._______________/8	7._______________/16
4._______________/12	

Total._______________/100

(1) (16 points)
Solve the linear system

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 5 & 6 & 1 \\ 3 & 5 & -1 & 9 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}.$$

(As a check on your arithmetic, this system is consistent and the matrix of this system has rank 2.)

(2) (a) (12 points) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 & 1 \\ 3 & 3 & 11 & 3 \\ 4 & 5 & 13 & 4 \\ 2 & 2 & 6 & 3 \end{bmatrix}.$$

(b) (4 points) *Use your answer to part (a)* to solve the system

$$A\mathbf{x} = \begin{bmatrix} 2 \\ 4 \\ 1 \\ 5 \end{bmatrix}.$$

(3) (8 points) Find the determinant of the matrix

$$\begin{bmatrix} 0 & 5 & 6 & 7 \\ 2 & 4 & 6 & 5 \\ 4 & 8 & 12 & 13 \\ 0 & 10 & 14 & 13 \end{bmatrix}.$$

(4) (12 points) Let $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ be a set of vectors in a vector space V. Carefully define:

(a) S is linearly independent.

(b) S spans V.

(c) S is a basis of V.

(5) (16 points) Let

$$S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 9 \\ 7 \end{bmatrix} \right\}.$$

Determine whether S spans $\mathbb{R}^4$. If not, find a vector $\mathbf{v}$ in $\mathbb{R}^4$ that is not in the span of S.

(6) (16 points) Let

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ 5 \end{bmatrix} \right\}.$$

Determine whether S is linearly independent. If not, find a linear dependence relation between the elements of S.

(7) (16 points) Let

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 10 \\ 12 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \\ 12 \\ 15 \end{bmatrix} \right\}.$$

Determine whether S is a basis of $\mathbb{R}^4$.

Extra page for additional work, if needed.