Homework 10 (Chapter 9: Systems of Differential Equations, Sections 9.1, 9.2, 9.3, 9.4.)

Due before class on November 11, Monday

(Professor Wu and Professor Zhang)

Due before class on November 12, Tuesday

(Professor Coll, Professor Recio-Mitter and Professor Weintraub)

1. Solve the homogeneous linear system of differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} = \begin{pmatrix} 5 & 4 \\ 6 & 7 \end{pmatrix} \mathbf{u}.$$

2. Consider the nonhomogeneous linear system of differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} = \begin{pmatrix} 12 & 2 \\ -4 & 18 \end{pmatrix} \mathbf{u} - \begin{pmatrix} 14 \\ 14 \end{pmatrix}.$$

Given the particular solution

$$\mathbf{u}_p(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Find the general solution.

3. Find the real solution of the homogeneous linear system

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} = \begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix} \mathbf{u}.$$

Then find a real fundamental matrix and the inverse of the fundamental matrix. Hint: $\det(A - \lambda I) = \lambda^2 - 4\lambda + 5$.

4. Consider the nonhomogeneous linear system of differential equations

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} &= \begin{pmatrix} 4 & 3 \\ -3 & 4 \end{pmatrix} \mathbf{u} \\ &+ \quad 2te^{4t} \begin{pmatrix} \cos(3t) \\ -\sin(3t) \end{pmatrix} + 4t^3 e^{4t} \begin{pmatrix} \sin(3t) \\ \cos(3t) \end{pmatrix}. \end{aligned}$$

Given a particular solution

$$\mathbf{u}_p(t) = t^2 e^{4t} \begin{pmatrix} \cos(3t) \\ -\sin(3t) \end{pmatrix} + t^4 e^{4t} \begin{pmatrix} \sin(3t) \\ \cos(3t) \end{pmatrix}.$$

Find the general solution of the nonhomogeneous system. Hint: $\det(A - \lambda I) = \lambda^2 - 8\lambda + 25$.

5. Solve the homogeneous system of differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} = \begin{pmatrix} 7 & 3 & 4 \\ -1 & 11 & 4 \\ 2 & -3 & 5 \end{pmatrix} \mathbf{u}.$$

Hint: $\det(A - \lambda I) = -(\lambda - 6)(\lambda - 8)(\lambda - 9)$.

6. Consider the nonhomogeneous system of differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} = \begin{pmatrix} 15 & 8 & -4 \\ 4 & 11 & 4 \\ 4 & 8 & 7 \end{pmatrix} \mathbf{u} + \begin{pmatrix} 5 \\ 14 \\ 5 \end{pmatrix}.$$

Given a particular solution

$$\mathbf{u}_p(t) = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}.$$

Find the general solution of the nonhomogeneous system of differential equations. Hint: $\det(A - \lambda I) = -(\lambda - 3)(\lambda - 11)(\lambda - 19)$.

7. Solve the following first order homogeneous linear system of differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} = \begin{pmatrix} \alpha & -\beta & -\beta \\ -\beta & \alpha & -\beta \\ -\beta & -\beta & \alpha \end{pmatrix} \mathbf{u},$$

where α and β are real nonzero constants. Find a fundamental matrix and the inverse matrix of the fundamental matrix. Hint: $\det(A - \lambda I) = -(\lambda - \alpha - \beta)^2(\lambda - \alpha + 2\beta)$.

8. Consider the nonhomogeneous linear system of differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \mathbf{u} - e^t \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

First of all, find a fundamental matrix and the inverse matrix of the fundamental matrix of the corresponding homogeneous linear system. Then given a particular solution

$$\mathbf{u}_p(t) = e^t \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix},$$

find the general solution of the nonhomogeneous linear system of differential equations. Hint:

$$\det(A - \lambda I) = -(\lambda - 2)^2(\lambda + 1)$$

9. Solve the initial value problem for the homogeneous linear system of differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} = \begin{pmatrix} 4 & 3 & 3 \\ -2 & 9 & 3 \\ 2 & -3 & 3 \end{pmatrix} \mathbf{u}, \qquad \mathbf{u}(0) = \begin{pmatrix} 1 \\ 3 \\ -3 \end{pmatrix}.$$

The characteristic polynomial is $\det(A - \lambda I) = (\lambda - 6)^2(4 - \lambda)$.

10. Solve the initial value problem for the homogeneous linear system of differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} = \begin{pmatrix} 4 & 2 & 3 \\ -4 & 10 & 3 \\ 4 & -2 & 5 \end{pmatrix} \mathbf{u}, \qquad \mathbf{u}_0 = \begin{pmatrix} 2 \\ 0 \\ 6 \end{pmatrix}.$$

The characteristic polynomial is $\det(A - \lambda I) = (\lambda - 8)^2(3 - \lambda)$.