Homework 9 (for Sections 8.3, 8.5, 8.6 and 1.5. Autumn 2019)

Due before class on November 4, Monday (Professor Wu, Zhang)

Due before class on November 5, Tuesday

(Professor Coll, Professor Recio-Mitter and Professor Weintraub)

There are 3 problems in the first part of this homework assignment.

Consider the following second order nonhomogeneous linear differential equations

$$
\begin{aligned}
&(1) \quad y'' + 100y = 36\cos(8x) + 72\sin(8x),\\
&(2) \quad y'' - 4y' + 3y = (2 - 12x)e^{x} + (60 + 40x)e^{3x},\\
&(3) \quad y'' - 2\alpha y' + \alpha^2 y = 60xe^{\alpha x} + 60x^2 e^{\alpha x},
\end{aligned}
$$

where $\alpha \neq 0$ is a real nonozero constant. Use the method of undetermined coefficients to find a particular solution for each equation. Then solve each equation for real general solution.

(4) The number of bacteria in a culture grows at a rate that is proportional to the number present. Initially, there were 100 bacteria in the culture. If the doubling time of the culture is 10 hours, find the number of bacteria that were present after 65 hours.

(5) The population of a certain city at time t is increasing at a rate that is proportional to the number of residents in the city at that time. On January 1, 2000, the population of the city is 1000 and on January 1, 2010, the population of the city is 10000.

(5-1) What will the population of the city be on January 1, 2020?

(5-2) In what year the population will reach one million?

(6) Consider the spring-mass system whose motion is governed by the initial value problem

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + 16y = 0, \qquad y(0) = 24, \qquad y'(0) = -28.$$

Determine the circular frequency of the system and the amplitude.

Problems 7 and 8 are based on the following initial value problems.

Consider the spring-mass system whose motion is governed by the initial value problems

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + \frac{c}{m}\frac{\mathrm{d}y}{\mathrm{d}t} + \frac{k}{m}y = \frac{1}{m}f(t), \qquad y(0) = y_0, \qquad y'(0) = y_1,$$

where $m > 0$, $c \geq 0$ and $k > 0$ are real constants.

(7) Consider the case where there is no damping: $c = 0$. Let

$$\omega_0 = \sqrt{\frac{k}{m}}, \qquad f(t) = 10m\omega_0[\cos(\omega_0 t) - \sin(\omega_0 t)].$$

Is the motion of the system oscillatory? What happens to the amplitude as $t \to \infty$?

(8) Consider the case there is no damping and the external force is $f(t) = 60\cos(2t) + 60\sin(2t)$. Let $m = 1$ and $k = 16$. Determine the period of the motion for the spring-mass system.

(9) In the RLC circuit

$$\frac{\mathrm{d}^2 q}{\mathrm{d}t^2} + \frac{R}{L}\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{1}{LC}q = \frac{1}{L}E(t),$$

Let

$$R = 4, \qquad L = 1, \qquad C = \frac{1}{4}, \qquad E(t) = 8\cos(2t) - 8\sin(2t).$$

Determine the steady-state current.

(10) In the RLC circuit

$$\frac{\mathrm{d}^2 q}{\mathrm{d}t^2} + \frac{R}{L}\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{1}{LC}q = \frac{1}{L}E(t),$$

let $R^2 C < 4L$ and $E(t) = 1$. Determine the electric charge.