Homework Assignment 1

Due at the start of class, Weds. Sept. 4 (Profs. Zhang and Wu), Thurs. Sept. 5 (Profs. Coll, Weintraub, Recio-Mitter).

1. Consider the matrices

$$A = \begin{bmatrix} -1 & 2 & -4 \\ 0 & 2 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 3 & -3 \\ 4 & 1 & 0 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 2 & 3 & -3 & 5 \\ 1 & 0 & -1 & -2 \\ 4 & 1 & 0 & -1 \end{bmatrix}.$$

   (a) Find $2A + 3B$.
   (b) Find AC.

2. Consider the matrices

$$D = \begin{bmatrix} 1 & 3 & -5 & 6 \\ 2 & -1 & 4 & 3 \\ 1 & 2 & 0 & 2 \end{bmatrix}, \qquad E = \begin{bmatrix} -2 & 4 & -5 & 2 \\ 0 & 6 & -2 & 1 \\ 5 & 3 & 2 & -3 \end{bmatrix} \quad \text{and} \quad F = \begin{bmatrix} 3 & 2 & -4 \\ 2 & 0 & -3 \\ 0 & 5 & 2 \\ 4 & 1 & 2 \end{bmatrix}.$$

   (a) Find $3D - E$.
   (b) Find FD.

3. Consider the matrices

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \\ 2 & 1 & 5 \end{bmatrix}, \qquad N = \begin{bmatrix} 2 & 0 & -1 \\ 3 & -2 & 0 \\ 4 & 1 & 3 \end{bmatrix} \quad \text{and} \quad P = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 5 & 6 \\ -1 & 2 & 8 \end{bmatrix}.$$

   Show by direct computation that $(MN)P = M(NP)$.

4. Write the following linear system as a matrix equation:

$$\begin{aligned} 3x_1 + 4x_2 - 5x_3 + 6x_4 &= 9 \\ 5x_1 + 2x_2 + 7x_3 - 2x_4 &= 7 \\ 2x_1 - 3x_2 + 6x_3 + 7x_4 &= 3 \end{aligned}$$

5. Write the following linear system as a matrix equation:

$$\begin{aligned} 5x_1 + 6x_2 - 8x_3 &= 2 \\ 4x_1 + 5x_2 - 7x_3 &= 3 \\ 3x_1 - 5x_2 - 9x_3 &= 5 \\ 2x_1 + 3x_2 - 4x_3 &= 6 \end{aligned}$$

6. Write the following matrix equation as a linear system:

$$\begin{bmatrix} 2 & 7 & -3 \\ 3 & -2 & 5 \\ 4 & 6 & 9 \\ 5 & 2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \\ 5 \\ 4 \end{bmatrix}.$$

7. Write the following matrix equation as a linear system:

$$\begin{bmatrix} 4 & -6 & 9 & 7 \\ 2 & -4 & 3 & 5 \\ 7 & 8 & -5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 7 \end{bmatrix}.$$

8. Consider a homogeneous system $Ax = 0$.

(a) If y and z are any solutions of this system, show that $y + z$ is also a solution of this system.

(b) If y is any solution of this system and c is any scalar, show that cy is also a solution of this system.

9. Consider a nonhomogeneous system $Ax = b$.

(a) If y and z are any solutions of this system, show that $y + z$ is *not* a solution of this system.

(b) If y is any solution of this system and c is any scalar with $c \neq 1$, show that cy is *not* a solution of this system.

10. Fix a matrix A and a vector b. Suppose that y is any solution of the homogeneous system $Ax = 0$ and that z is any solution of the system $Ax = b$. Show that $y + z$ is also a solution of the system $Ax = b$.