

# **CSE 259 - Logic in Computer Science**

**Recitation-10**

## **Project 3: Wang and Kobsa's Algorithm**

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# Project 3

- The partial implementation **wangs\_algorithm.pl** is runnable
- Sample input/output:
  - Premises as list ( $[p \wedge q]$ )
  - A formula as conclusion
  - We can derive the conclusion from the premise

```
?- run([p^q], [p]).  
[p^q]=>[p]  
=      [p,q]=>[p]      (by and/left)  
=      Done (sharing a/f)
```

We can derive the conclusion from the premises.  
**true.**

# Project 3

- Write your codes here
- I have implemented some. The rest is your task

```
%%%%%%%%%% YOUR CODE STARTS %%%%%%%%%%  
  
% Implement all other non-branching rules below by following Wangs algorithm  
  
> /* ...  
> prove(L => R):- ...  
> prove(L => R):- ...  
> /* ...  
> prove(L => R) :- ...  
> prove(L => R) :- ...  
% Implement all branching rules below by following Wangs algorithm  
> /* ...  
> prove(L => R) :- ...  
%%%%%%%%% YOUR CODE ENDS %%%%%%%%%%
```

# Project 3

- Two types of rules
  - Non-branching
  - Branching

# Project 3: Non-branching Rules

## Rule-1

If one of the formulae separated by commas is the negation of a formula, drop the negation sign and move it to the other side of the arrow.

**Example:**

**Formula:**  $p, \sim(q \wedge r) \Rightarrow p \wedge r$

**Change to:**  $p \Rightarrow p \wedge r, q \wedge r$

# Project 3: Non-branching Rules

## Rule-2

If the last connective of a formula on the left is  $\wedge$  (and), or on the right of the arrow is  $\vee$  (or), replace the connective by a comma.

### Example-1:

Formula:  $p, p \wedge q \Rightarrow r, s$

Change to:  $p, p, q \Rightarrow r, s$

### Example-2:

Formula:  $p, q \Rightarrow r \vee p$

Change to:  $p, q \Rightarrow r, p$

# Project 3: Non-branching Rules

## Rule-3

If the last connective of a formula on the right is  $A \rightarrow B$ , remove  $A \rightarrow B$  from the right and then add  $A$  to the left and  $B$  to the right.

.

### Example-1:

Formula:  $p \vee q \Rightarrow q \rightarrow p$

Change to:  $p \vee q, q \Rightarrow p$

# Project 3: Branching Rules

## Rule-4

If the last connective of a formula on the left is  $\vee$  (or), or on the right of the arrow is  $\wedge$  (and), then produce two new lines, each with one of the two sub formulae replacing the formula. Both of these must be proved in order to prove the original theorem.

### Example-1:

Formula:  $p \vee q, r, s \Rightarrow q \vee p$

Change to:

$p, r, s \Rightarrow q \vee p$

$q, r, s \Rightarrow q \vee p$

### Example-2:

Formula:  $p, r, s \Rightarrow q \wedge p$

Change to:

$p, r, s \Rightarrow q$

$p, r, s \Rightarrow p$



# Project 3: Branching Rules

## Rule-5

If the last connective of a formula on the left is  $A \rightarrow B$ , remove  $A \rightarrow B$  from the left and then create two new lines, one with  $B$  added to the left, and the other with  $A$  added to the right.

### Example-1:

Formula:  $p, p \rightarrow q \Rightarrow p \vee q$

Change to:

$p, q \Rightarrow p \vee q$

$p \Rightarrow p \vee q, p$

# Project 3: Non-branching Rules – Rule 1

```
/*
 * Rule-1
 * example rule: negation
 * non-branching rule
 * If one of the formulae separated by commas is the
 * negation of a formula, drop the negation sign and
 * move it to the other side of the arrow.
 */
prove(L => R):-
    member(~X, L),
    del(~X, L, NewL),
    nl, write('=\\t'), write(NewL => [X | R]),
    write('\\t (by negation/left)'),
    prove(NewL => [X | R]).
prove(L => R):-
    member(~X, R),
    del(~X, R, NewR),
    nl, write('=\\t'), write([X | L] => NewR),
    write('\\t (by negation/right)'),
    prove([X | L] => NewR).
```

Example:

Formula:  $p, \sim(q \wedge r) \Rightarrow p \wedge r$

Change to:  $p \Rightarrow p \wedge r, q \wedge r$

Output:

```
?- prove([p, ~(q ^ r)] => [p ^ r]).
= [p] => [q ^ r, p ^ r] (by negation/left)
```

# Project 3: Non-branching Rules – Rule 2

```
/*
 * Rule-2
 * example rule: left conjunction
 * non-branching rule
 * If the last connective of a formula on the left is  $\wedge$  (and),
 * or on the right of the arrow is  $\vee$  (or), replace the connective by a comma.
 */
prove(L  $\Rightarrow$  R) :-
  member(A  $\wedge$  B, L),
  del(A  $\wedge$  B, L, NewL),
  nl, write('=\\t'), write([A, B | NewL]  $\Rightarrow$  R),
  write('\\t (by and/left)'),
  prove([A, B | NewL]  $\Rightarrow$  R).
prove(L  $\Rightarrow$  R) :-
  member(A  $\vee$  B, R),
  del(A  $\vee$  B, R, NewR),
  nl, write('=\\t'), write(L  $\Rightarrow$  [A, B | NewR]),
  write('\\t (by or/right)'),
  prove(L  $\Rightarrow$  [A, B | NewR]).
```

# Project 3: Non-branching Rules – Rule 2 contd.

## Example-1:

Formula:  $p, p \wedge q \Rightarrow r, s$

Change to:  $p, p, q \Rightarrow r, s$

```
?- prove([p, p ^ q] => [r, s]).  
=      [p,q,p]=>[r,s]    (by and/left)
```

## Example-2:

Formula:  $p, q \Rightarrow r \vee p$

Change to:  $p, q \Rightarrow r, p$

```
?- prove([p, q] => [r v p]).  
=      [p,q]=>[r,p]      (by or/right)  
=      Done (sharing a/f)  
.
```

# Project 3: Branching Rules – Rule 5

```
/*
 * Rule-5
 * example rule: left implication
 * branching rule
 * If the last connective of a formula on the left is  $A \rightarrow B$ ,
 * remove  $A \rightarrow B$  from the left and then create two new lines,
 * one with  $B$  added to the left, and the other with  $A$  added to the right.
 */
```

```
prove(L => R) :-
  member(A -> B, L),
  del(A -> B, L, NewL),
  nl,
  write('\tFirst branch: '),
  nl,
  write('= \t'),
  write([B | NewL] => R),
  write('\t (by arrow/left)'),
  prove([B | NewL] => R),
  nl,
  write('\tSecond branch: '),
  nl,
  write('= \t'),
  write(NewL => [A | R]),
  write('\t (by arrow/left)'),
  prove(NewL => [A | R]).
```

# Project 3: Your Tasks

- Rule-3: Non-branching rule
- Rule-4: Branching rule