4-bit Arithmetic Logic Unit

 ${\tt Group\text{-}3}$

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1 Designing an ALU

Truth Table : Arithmetic Operations

cs_2	cs_1	cs_0	Arithmetic Operation	x_i	y_i
0	0	0	Transfer A	A_i	0
0	0	1	Increment A	A_i	0
0	1	0	Subtraction with Borrow	A_i	$\overline{B_i}$
0	1	1	Subtraction	A_i	$\overline{B_i}$

Subtract with borrow explanation:

$$= A - B - 1$$

$$= (A + \overline{B} + 1) - 1$$

$$=A+\overline{B}$$

So,
$$Y = CS_1\overline{B_i}$$

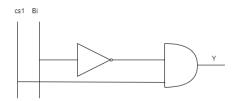


Figure 1: $Y = CS_1\overline{B_i}$

2 Design of Arithmetic Logic Unit

Truth Table : Logical Operations

cs_2	cs_1	cs_0	x_i	y_i	$f_i = X_i \oplus Y_i$	Operation
1	0	0	A_i	0	A_i	OR
1	0	1	A_i	0	$A_i + 1$	OR
1	1	0	A_i	B_i	$A_i - B_i - 1$	AND
1	1	1	A_i	B_i	$A_i - B_i$	AND

Explanation:

we can't modify Y_i because that would change the arithmetic operations and neither can omit A_i in any input, So we change X_i ,

$$X_i = A_i + K_i$$

$$F_i = X_i \oplus Y_i$$

$$F_i = X_i \oplus 0$$

$$F_i = X_i$$

$$F_i = A_i + K_i$$

But the desired output is $A_i + B_i$. So putting $K_i = B_i$

$$F_i = X_i \oplus Y_i$$

$$F_i = (A_i \oplus K_i) \oplus \overline{B_i}$$

$$F_i = (A_i \oplus K_i) \oplus \overline{B_i}$$

$$F_i = (A_i \oplus K_i)B_i + \overline{(A_i \oplus K_i)}.\overline{B_i}$$

$$F_i = A_i B_i + K_i B_i + \overline{A_i} \cdot \overline{K_i} \cdot \overline{B_i}$$

Here our desired operation is A_iB_i

So,
$$A_iB_i + K_iB_i + \overline{A_i}.\overline{K_i}.\overline{B_i} = A_iB_i$$

if
$$K_i = \overline{B_i}$$
 Then $F_i = A_i B_i$

So we need $K_i = B_i$ when we will do OR operation and $K_i = \overline{B_i}$ for AND operation.

cs_2	cs_1	cs_0	В
1	0	0	B_i
1	0	1	B_i
1	1	0	$\overline{B_i}$
1	1	1	$\overline{B_i}$

So from the truth table we can derive,

$$X_i = A_i + CS_2(\overline{CS_1}.B(CS_0 + \overline{CS_0}) + CS_1.\overline{B_i}(CS_0 + \overline{CS_0})))$$

$$X_i = A_i + CS_2(CS_1 \oplus B_i)$$

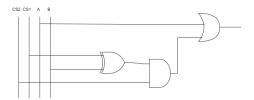


Figure 2: $X_i = A_i + CS_2(CS_1 \oplus B_i)$

3 Final Diagram

$$X_{i} = A_{i} + CS_{2}(\overline{CS_{1}}.B(CS_{0} + \overline{CS_{0}}) + CS_{1}.\overline{B_{i}}(CS_{0} + \overline{CS_{0}})))$$

$$X_{i} = A_{i} + CS_{2}(CS_{1} \oplus B_{i})$$

$$Y = CS_{1}\overline{B_{i}}$$

$$C_{out} = \overline{CS_{2}}.C_{i}$$