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Finite Time Optimization of an Irreversible Airconditioning System Using Method of Lagrangian Multiplier

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Abstract

This communication presents finite time thermodynamic analysis and optimization using method of Lagrangian multiplier for a vapour compression air conditioning system with external and internal irreversibilities. A general expression for the coefficient of performance (COP) of the air conditioning system is derived at minimum input power and given cooling load condition for the system. It is shown that external irreversibility in the air conditioning system due to finite temperature difference or finite heat transfer between the cycle fluid and source/sink reservoirs causes the end-reversible COP lower than the reversible COP while internal irreversibility of air conditioning system is characterized by a single irreversibility parameter ($R_{\Delta S}$) representing non-isentropic compression in terms of the ratio of entropy differences. The presence of this parameter in the equations for minimum input power and coefficient of performance of the system clearly shows that a real air conditioning system needs more input power and has a lower coefficient of performance than end-reversible air conditioning system

Nomenclature:

C	=	Heat capacitance rate of external fluid (kW/K).
C_P	=	Specific heat of external fluid (kJ/kg-K).
COP	=	Coefficient of performance.
m	=	Mass flow rates of external fluid (kg/s).
P	=	Power input to airconditioning system (kW).
P_L	=	Cooling load (kW).
Q	=	Lagrangian operator.
Q_C	=	Heat rejected by the system (kJ).
Q_E	=	Heat absorbed by the system (kJ).
$R_{\Delta S}$	=	Irreversibility parameter, $R_{\Delta S} = (S_1 - S'_4) / (S'_2 - S_3)$.
S	=	Entropy (kJ/K).
t	=	$[t = t_E + t_C]$, Cycle time (s).
T	=	Temperature (K).
W	=	Work input to airconditioning system (kJ).
ε	=	Effectiveness of heat exchanger.
λ	=	Langrangian multiplier.
C	=	Condenser or heat sink.
E	=	Evaporator or heat source.

H	=	High temperature side heat reservoir (heat sink).
L	=	Low temperature side heat reservoir (heat source).
h	=	Warm working fluid.
l	=	Cold working fluid.
1, 2	=	Inlet, outlet.
o	=	Optimal.

Introduction

Carnot proposed in 1824 a reversible heat engine which, produces the maximum possible work for a given heat source and sink but generates zero output power because it has to operate at an infinite slow pace. Finite time thermodynamic analysis for a heat engine was carried out by Curzon & Ahlborn and other workers [1-6] who derived an efficiency formula which agrees much better with measured efficiencies of operating installations.

The concept of finite time thermodynamics is equally applicable for an air conditioning system where cooling load is obtained at the expense of work input. It is also desirable to have a corresponding result for coefficient of performance (COP) at minimum power input to the air conditioning system. Although, COP is highest in case of Carnot air conditioning system but cooling load is zero because reversible processes are always infinite time executable processes. However, actual processes are always irreversible, consequently, the performance of the air conditioning system must then be re-examined by means of finite time thermodynamics [7,8,]. Leff and Teeters [9] have noted that the straight forward C & A analysis will not be applicable as such for an air conditioning system because there is no "natural maximum" and We seek the minimum input power (P) for a given cooling power ($P_L = Q_E/t$) therefore, we have to use an alternative method (viz. Lagrangian multiplier method) for power input minimization to the air conditioning system [10,11].

Blanchard [10] has studied the case for an endoreversible heat pump system which is connected to thermal reservoirs (heat source /sink) of infinite heat capacity and gave a formula for optimal COP at minimum input power. However, in real-practice, heat pump/air conditioning systems are irreversible therefore, the system has to deal with both external as well as internal irreversibilities.

In this paper, we have extended Blanchard's analysis for air conditioning systems to include internal irreversibility, finite heat capacitance of the external fluids and fixed effectiveness of the heat exchangers for source/sink heat reservoirs. The effect of operating temperatures, heat capacitance rates of external fluids, effectiveness of source / sink side heat exchangers and internal irreversibility parameter on internal working fluid temperatures, performance and power input to the system are investigated. It is found that for an air conditioning system, the internal irreversibilities may be characterized by a single parameter defined in terms of entropy differences. Typical numerical calculations are presented.

System Description and Analysis

The endoreversible air conditioning cycle

During heat rejection process, temperature of the working fluid will vary in superheat region. But on the T-S diagram, the amount of heat rejected by the heat pump system can be made as equal to the area under a horizontal line with an entropic average temperature (T_h) of heat rejection. Thus, Ideal vapour compression air conditioning cycles can be considered as modified/equivalent reversed Carnot cycles using entropic average temperature to achieve simplified theoretical analysis [11]. A T-s diagram of an endoreversible reversed Carnot cycle is shown in Fig. 1. In this section, we are considering the endoreversible cycle where heat transfer processes, which occur externally across finite temperature difference, are irreversible.

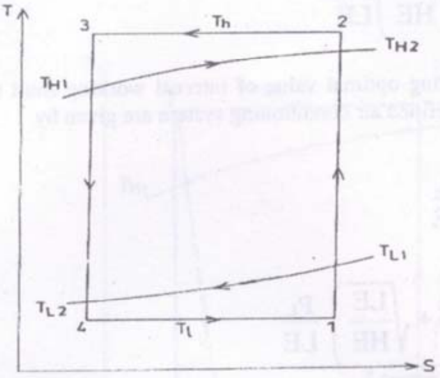

Fig. 1 T-S diagram of Endoreversible Air conditioning Cycle

Following Kaushik [11], we have $W = Q_C - Q_E$ and then the power input to the air conditioning system is given by

$$P = \frac{W}{t} = \frac{HE\ LE\ x\ y\ (T_{H1} + x - T_{L1} + y)}{(T_{H1} + x)\ y\ LE + (T_{L1} - y)\ HE\ x} \tag{1}$$

where,

$x = T_h - T_{H1}$, $y = T_{L1} - T_L$, $HE = C_C\,\varepsilon_C$, $LE = C_E\,\varepsilon_E$, $C_E = m_E\,C_{PE}$ and $C_C = m_C\,C_{PC}$ and cooling load is given by

$$P_L = \frac{Q_E}{t} = \frac{HE\ LE\ x\ y(T_{L1} - y)}{LE\ y\,(T_{H1} + x) + HE\ x\ (T_{L1} - y)} \tag{2}$$

We therefore, seek the unrestricted extremum of

$$Q = P + \lambda\,P_L = \frac{HE\ LE\ x\ y\,[((T_{H1} + x) - (1 - \lambda)(T_{L1} - y)]}{LE\ y\,(T_{H1} + x) + HE\ x\,(T_{L1} - y)} \tag{3}$$

with respect to 'x' and 'y'. The Lagrangian multiplier λ is introduced to maintain a given value of cooling load P_L

$$\text{therefore, } \frac{dQ}{dx} = 0 \text{ gives } x = \frac{T_{H1}\sqrt{\frac{LE}{HE}}\left(1 + \sqrt{\frac{LE}{HE}}\right)\frac{P_L}{LE}}{T_{L1} - \left(1 + \sqrt{\frac{LE}{HE}}\right)^2 \frac{P_L}{LE}} \tag{4}$$

$$\text{and } \frac{dQ}{dy} = 0 \text{ gives } y = \left(1 + \sqrt{\frac{LE}{HE}}\right)\frac{P_L}{LE} \tag{5}$$

By using Eqns.(4) and (5), the resulting optimal value of internal working fluid temperatures, coefficient of performance and power input to the defined air conditioning system are given by

$$(T_l)_o = T_{L1} - \left(1 + \sqrt{\frac{LE}{HE}}\right)\frac{P_L}{LE} \tag{6}$$

$$(T_h)_O = T_{H1} + \frac{T_{H1}\sqrt{\frac{LE}{HE}}\left(1 + \sqrt{\frac{LE}{HE}}\right)\frac{P_L}{LE}}{T_{L1} - \left(1 + \sqrt{\frac{LE}{HE}}\right)^2 \frac{P_L}{LE}} \tag{7}$$

$$(COP)_o = \left[\frac{T_{H1}}{T_{L1} - \left[1 + \sqrt{\frac{LE}{HE}}\right]^2 \frac{P_L}{LE}} - 1\right]^{-1} \tag{8}$$

$$P = \frac{P_L\left[T_{H1} - T_{L1} + \left(1 + \sqrt{\frac{LE}{HE}}\right)^2 \frac{P_L}{LE}\right]}{\left[T_{L1} - \left(1 + \sqrt{\frac{LE}{HE}}\right)^2 \frac{P_L}{LE}\right]} \tag{9}$$

Internal irreversible air conditioning cycle

If internal irreversibility (such as due to friction) is accounted for in the airconditioning system then the two isentropic processes become adiabatic processes with entropy generations. On T-s diagram, the four processes of such an air conditioning system constitute the cycle 1-2'-3-4' as shown in Fig. 2.

Following Kaushik [11], we have the power input to the air conditioning system given by

$$P = \frac{W}{t} = \frac{HE\ LE\ x\,y\,[T_{H1} + x - (T_{L1} - y)\,R_{\Delta s}]}{LE\ y\,(T_{H1} + x) + R_{\Delta s}\,HE\ x\,(T_{L1} - y)} \tag{10}$$

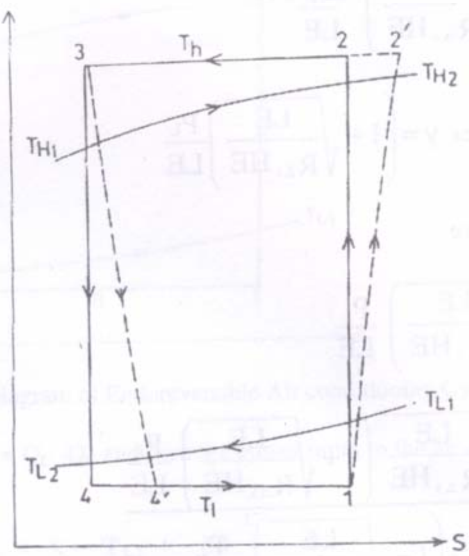

Fig. 2 T-S diagram of Irreversible Air-conditioning Cycle

and cooling load given by

$$P_L = \frac{Q_E}{t} = \frac{HE\ LE\ x\,y(T_{L1} - y)}{LE\ y\,(T_{H1} + x) + R_{\Delta s}\ HE\ x\ (T_{L1} - y)} \tag{11}$$

Where $R_{\Delta S}$ is internal irreversibility parameter which represents nonisentropic compression in terms of the ratio of entropy differences. Now let us define a Lagrangian

$$Q = P + \lambda\ P_L = \frac{HE\ LE\ x\,y\,[(T_{H1} + x) - R_{\Delta s}\ (1 - \lambda)(T_{L1} - y)]}{R_{\Delta s}\ HE\,x\,(T_{L1} - y) + LE\ y\,(T_{H1} + x)} \tag{12}$$

It is thus seen that Q is a function of x and y. For maximizing we differentiate Q w.r.t. 'x' and 'y' and then equate to zero we have, dQ/dx = 0 which gives

$$x = \frac{T_{H1}\sqrt{\frac{LE}{R_{\Delta s}HE}}\left(1+\sqrt{\frac{LE}{R_{\Delta s}HE}}\right)\frac{P_L}{LE}}{T_{L1}-\left(1+\sqrt{\frac{LE}{R_{\Delta s}HE}}\right)^2\frac{P_L}{LE}} \tag{13}$$

and dQ/dy = 0 which gives $y = \left(1+\sqrt{\frac{LE}{R_{\Delta s}HE}}\right)\frac{P_L}{LE}$ (14)

By using Eqns.(13-14), we have

$$(T_l)_o = T_{L1} - \left(1+\sqrt{\frac{LE}{R_{\Delta s}HE}}\right)\frac{P_L}{LE} \tag{15}$$

$$(T_h)_O = T_{H1} + \frac{T_{H1}\sqrt{\frac{LE}{R_{\Delta s}HE}}\left(1+\sqrt{\frac{LE}{R_{\Delta s}HE}}\right)\frac{P_L}{LE}}{T_{L1}-\left(1+\sqrt{\frac{LE}{R_{\Delta s}HE}}\right)^2\frac{P_L}{LE}} \tag{16}$$

$$(COP)_o = \left[\frac{\frac{T_{H1}}{R_{\Delta s}}}{T_{L1} - \left[1+\sqrt{\frac{LE}{R_{\Delta s}HE}}\right]^2\frac{P_L}{LE}} - 1\right]^{-1} \tag{17}$$

and

$$P = \frac{P_L\left[\frac{T_{H1}}{R_{\Delta S}} - T_{L1} + \left(1+\sqrt{\frac{LE}{R_{\Delta S}\ HE}}\right)^2\frac{P_L}{LE}\right]}{\left[T_{L1}-\left(1+\sqrt{\frac{LE}{R_{\Delta S}\ HE}}\right)^2\frac{P_L}{LE}\right]} \tag{18}$$

Discussion of Results:

In order to have a numerical appreciation of the theoretical analysis of vapour compression air conditioning system, we have studied the effect of various input parameters on the performance of the airconditioning system and results are shown in tables (1-4). During the variation of any one parameter, all other parameters are assumed to be constant as given below.

$T_{HI} = 318$ K $T_{LI} = 275$ K $C_C = C_E = 1.00$ kW/K $\varepsilon_C = \varepsilon_E = 0.75$ $R_{\Delta S} = 1.00$

It is seen from the Table-1(a) that by increasing the inlet temperature of the external source side fluid, the power input to the system decreases and consequently the coefficient of performance increases. Table-1(b) shows that by increasing sink inlet temperature, the temperature of internal working fluid on high temperature side and the power input to the system both increase and hence the COP of the system decreases whereas the temperature of the internal working fluid on the low temperature side remains constant.

Table-1 (a & b) Effect of condenser/evaporator heat capacitance rate (C_C / C_E), on working fluid temperatures, power input and performance of the air-conditioning system.

T_{LI} K	$(T_h)_O$ K	$(T_l)_O$ K	$(COP)_O$	P kW	T_{HI} K	$(T_h)_O$ K	$(T_l)_O$ K	$(COP)_O$	P kW
265	328.22	257.00	3.61	0.83	310	319.58	267.00	5.08	0.59
267	328.14	259.00	3.75	0.80	312	321.64	267.00	4.89	0.61
269	328.06	261.00	3.89	0.77	314	323.70	267.00	4.71	0.64
271	327.98	263.00	4.05	0.74	316	325.76	267.00	4.54	0.66
273	327.90	265.00	4.21	0.71	318	327.82	267.00	4.39	0.68
275	327.82	267.00	4.39	0.68	320	329.88	267.00	4.25	0.71
277	327.75	269.00	4.58	0.66	322	331.95	267.00	4.11	0.73
279	327.67	271.00	4.78	0.63	324	334.01	267.00	3.98	0.75
281	327.60	273.00	5.00	0.60	326	336.07	267.00	3.87	0.78
283	327.53	275.00	5.24	0.57	328	338.13	267.00	3.75	0.80

Tables-2(a & b) show similar effects of heat capacitance rates of external source/sink side fluid. By increasing any one of them, the sink side internal working fluid temperature and the power input to the system both decrease consequently performance of the system increases and the source side internal working fluid temperature is also increased

Table-2 (a & b) Effect of condenser/evaporator heat capacitance rate (C_C / C_E), on working fluid temperatures, power input and performance of the air-conditioning system.

C_C KW/K	$(T_h)_O$ K	$(T_l)_O$ K	$(COP)_O$	P kW	C_E kW/K	$(T_h)_O$ K	$(T_l)_O$ K	$(COP)_O$	P kW
0.6	332.81	265.84	3.97	0.76	0.6	329.47	263.17	3.97	0.76
0.7	331.05	266.22	4.11	0.73	0.7	328.92	264.50	4.11	0.73
0.8	329.72	266.53	4.22	0.71	0.8	328.48	265.53	4.22	0.71
0.9	328.67	266.78	4.31	0.70	0.9	328.12	266.34	4.31	0.70
1.0	327.82	267.00	4.39	0.68	1.0	327.82	267.00	4.39	0.68
1.1	327.12	267.19	4.46	0.67	1.1	327.57	267.55	4.46	0.67
1.2	326.53	267.35	4.52	0.66	1.2	327.35	268.02	4.52	0.66
1.3	326.03	267.49	4.57	0.66	1.3	327.15	268.41	4.57	0.66
1.4	325.59	267.62	4.62	0.65	1.4	326.98	268.76	4.62	0.65
1.5	325.21	267.73	4.66	0.64	1.5	326.83	269.07	4.66	0.64

Tables-3(a & b) show the effect of effectiveness ε_C, ε_E of source/sink side heat exchangers of the air conditioning system. By increasing either value with the other being kept constant, the power input to the system decreases and consequently the performance of the system increases because cooling load is fixed and the temperature of the sink side internal working fluid decreases while the source side internal working fluid temperature increases with increasing effectiveness.

Table 3 (a & b) Effect of condenser / evaporator side heat exchanger effectiveness (ε_C / ε_E), on working fluid temperatures, power input and performance of the air-conditioning system.

ε_C	$(T_h)_O$ K	$(T_l)_O$ K	$(COP)_O$	P kW	ε_E	$(T_h)_O$ K	$(T_l)_O$ K	$(COP)_O$	P kW
0.20	348.07	263.25	3.10	0.97	0.20	333.53	252.25	3.10	0.97
0.30	338.90	264.68	3.57	0.84	0.30	331.22	258.68	3.57	0.84
0.40	334.34	265.52	3.86	0.78	0.40	329.93	262.02	3.86	0.78
0.50	331.58	266.10	4.06	0.74	0.50	329.09	264.10	4.06	0.74
0.60	329.72	266.53	4.22	0.71	0.60	328.48	265.53	4.22	0.71
0.70	328.37	266.86	4.34	0.69	0.70	328.02	266.57	4.34	0.69
0.75	327.82	267.00	4.39	0.68	0.75	327.82	267.00	4.39	0.68
0.80	327.34	267.13	4.44	0.68	0.80	327.65	267.38	4.44	0.68
0.90	326.53	267.35	4.52	0.66	0.90	327.35	268.02	4.52	0.66
1.00	325.87	267.54	4.59	0.65	1.00	327.09	268.54	4.59	0.65

Table-4 shows the effect of the internal irreversibility parameter $R_{\Delta S}$. By increasing this parameter, the source side internal working fluid temperature increases whereas the sink side internal working fluid temperature and the power input to the system decrease and hence performance of the system increases. The case $R_{\Delta S}$ = 1.0 corresponds to the endoreversible case

Table 4 Effect of internal irreversibility parameter ($R_{\Delta S}$), on working fluid temperatures, power input and performance of the air-conditioning system.

$R_{\Delta S}$	$(T_h)_O$ K	$(T_l)_O$ K	$(COP)_O$	P kW
0.50	335.26	265.34	0.65	4.58
0.55	333.92	265.61	0.78	3.86
0.60	332.81	265.84	0.92	3.26
0.65	331.87	266.04	1.09	2.76
0.70	331.05	266.22	1.29	2.33
0.75	330.34	266.38	1.53	1.96
0.80	329.72	266.53	1.83	1.64
0.85	329.16	266.66	2.21	1.36
0.90	328.67	266.78	2.71	1.11
0.95	328.22	266.90	3.40	0.88
1.00	327.82	267.00	4.39	0.68

Conclusions

Finite time thermodynamic analysis of an irreversible air conditioning system has been carried out using the method of Lagrangian multiplier by minimizing power input for a given cooling load. It has been shown that internal irreversibility in an air conditioning system can be characterized by a single parameter representing the ratio of two entropy differences. This parameter appears in both the equations for minimum input power and coefficient of performance of the air conditioning system. The equations clearly show that an air conditioning system with internal irreversibility needs more input power and has a lower coefficient of performance than the corresponding endoreversible air conditioning system. In our continuing search for a realistic theoretical upper limit for performance of the system this new equation for performance represents a further improvement to Blanchard's equation [10].

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