

## Application of Genetic Algorithm to Optimal Reactive Power Dispatch including Voltage Stability Constraint

S. Durairaj\*, P. S. Kannan\* and D. Devaraj\*\*

*Power System Automation Laboratory*

*\*Arulmigu Kalasalingam College of Engineering, Srivilliputtur-626 190, India*

*\*\*Thiagarajar College of Engineering, Srivilliputtur-626 190, India*

*Email: [rajsdr@rediffmail.com](mailto:rajsdr@rediffmail.com)*

*(Received on 7 Feb 2005, revised on 17 Apr 2005)*

---

### Abstract

This paper presents a Genetic Algorithm (GA) - based approach for solving optimal Reactive Power Dispatch (RPD) including voltage stability limit in power systems. The monitoring methodology for voltage stability is based on the L-index of load buses. Bus voltage magnitudes, transformer tap settings and reactive power generation of capacitor banks are the control variables. A binary-coded GA with tournament selection, two point crossover and bit-wise mutation is used to solve this complex optimization problem. The proposed algorithm has been applied to the IEEE 30-bus system to find the optimal reactive power control variables while keeping the system under safe voltage stability limit, and found to be more effective for this task.

---

### Nomenclature

$P_{loss}$	Network real power loss
$P_i, Q_i$	Real and reactive powers injected into network at bus i
$G_{ij}, B_{ij}$	Mutual conductance and susceptance between bus i and bus j
$G_{ii}, B_{ii}$	Self- conductance and susceptance of bus i
$Q_{gi}$	Reactive power generation at bus i
$Q_{Ci}$	Reactive power generated by $i^{th}$ capacitor bank
$t_k$	Tap setting of transformer at branch k
$V_i$	Voltage magnitude at bus i
$V_j$	Voltage magnitude at bus j
$\theta_{ij}$	Voltage angle difference between bus i and bus j
$S_l$	Apparent power flow through the $l^{th}$ branch
$g_k$	Conductance of branch k
$N_B$	Total number of buses
$N_{B-1}$	Total number of buses excluding slack bus
$N_{PQ}$	Number of PQ buses
$N_g$	Number of generator buses
$N_c$	Number of capacitor banks
$N_T$	Number of tap-setting transformer branches
$N_l$	Number of branches in the system
$\delta_i$	Voltage phase angle of $i^{th}$ generator bus

## Introduction

The purpose of the reactive power dispatch (RPD) in power system is to identify the control variables that minimize a given objective function while satisfying the unit and system constraints. Scheduling of reactive power in an optimum manner reduces circulating VAR (volt ampere reactive), thereby promoting a uniform voltage profile which leads to appreciable power saving on account of reduced system losses. RPD is a complex non-linear optimization problem.

Linear programming (LP), non-linear programming and gradient based techniques have been proposed in the literature [1-4] for solving RPD problems. However, due to the approximations introduced by linearized models, the LP results may not represent the optimal solution for inherently non-linear objective functions such as the one used in the reactive power dispatch problem. It is very difficult to calculate the gradient variables and a large volume of computations is involved in this approach. Also, these conventional techniques are known to converge to a local optimal solution rather than the global one. Lately, expert system approach [5] has been proposed for the reactive power control computations. This approach is based on “If-then” based production rules. The construction of such rules requires extensive help from skilled knowledge engineers.

Evolutionary computational techniques like Genetic algorithm (GA) [6], Evolutionary programming (EP) [7] and Evolutionary strategy [8] have also been proposed to solve the optimal reactive power dispatch problems. Generator voltages, transformer tap positions and number of switchable shunt capacitor banks were used as the control variables in the work [6] and they are represented as integer vector in the genetic population. In addition to the crossover and mutation operations, AI-based rules were applied to improve the solutions. Evolutionary programming was applied in [7] to solve the reactive power dispatch problem with minimization of active power loss as the objective function. IEEE 30-bus system was employed to carryout the simulations and the results obtained using the EP-based approaches were found to be better than the results obtained using the conventional method. Although, these works have solved the RPD problem successfully, none of them has considered the line flow and voltage stability constraints, which are important for any practical implementation of RPD. If a contingency occurs in an already stressed system both angular and voltage stability may be lost. Many voltage instability i.e. voltage collapse events have been experienced by the utilities in the recent years. This was mainly due to reactive power shortage during the peak load. These events warrant inclusion of the voltage stability constraint in the RPD for maintaining the security of modern power systems.

This paper is concerned with application of GA for optimal reactive power dispatch with line flow and voltage stability constraints. The L-index defined in the work [9] is used in this paper to compute the voltage stability level of the system. This index uses information from a normal power flow and is in the range of zero to one. In the present work some restrictions are applied on the maximum value of L-index in the normal operating condition so that even if a contingency occurs on the system the L-index value does not reach an alarming level. Thus voltage stability constrained reactive power dispatch problem is solved in this paper using the genetic algorithm. IEEE 30-bus test system has been used to carry out the simulation study.

## Problem Formulation

The RPD problem aims at minimizing the real power loss in a power system while satisfying the unit and system constraints. This goal is achieved by proper adjustment [10-12] of reactive power variables like generator voltage magnitudes ( $V_{gi}$ ), reactive power generation of capacitor banks ( $Q_{ci}$ ) and transformer tap settings ( $t_k$ ).

This is mathematically stated as;

$$\text{Minimize } P_{loss} = \sum_{\substack{k \in N_l \\ k=(i,j)}} g_k (V_i^2 + V_j^2 - 2V_i V_j \cos \theta_{ij}) \quad (1)$$

The real power loss given by (1) is a non-linear function of bus voltages and phase angles which are a function of control variables. The minimization problem is subjected to the following equality and inequality constraints:

(i) Load flow constraints:

$$P_i - V_i \sum_{j=1}^{N_B} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0, \quad i = 1, 2, \dots, N_B - 1 \quad (2)$$

$$Q_i - V_i \sum_{j=1}^{N_B} V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0, \quad i = 1, 2, \dots, N_{PQ} \quad (3)$$

(ii) Voltage constraints:

$$V_i^{\min} \leq V_i \leq V_i^{\max}; \quad i \in N_B \quad (4)$$

(iii) Generator reactive power capability limit:

$$Q_{gi}^{\min} \leq Q_{gi} \leq Q_{gi}^{\max}; \quad i \in N_g \quad (5)$$

(iv) Reactive power generation limit of capacitor banks

$$Q_{Ci}^{\min} < Q_{Ci} \leq Q_{Ci}^{\max}; \quad i \in N_c \quad (6)$$

(v) Transformer tap setting limit:

$$t_k^{\min} \leq t_k \leq t_k^{\max}; \quad k \in N_T \quad (7)$$

(vi) Transmission line flow limit

$$S_l \leq S_l^{\max}; \quad l \in N_l \quad (8)$$

(vii) Voltage stability constraint

$$L^{\max} \leq L^{\text{ime}} \quad (9)$$

The voltage stability index given in Equation (9) is evaluated as follows:

First, the L-indices [9] of all the load buses in the system are computed using the expression:

$$L_j = \left| 1 - \sum_{i=1}^{N_g} F_{ji} \frac{V_i}{V_j} \angle (\theta_{ji} + \delta_i - \delta_j) \right| \quad (10)$$

The values of  $F_{ji}$  are obtained from the matrix  $F_{LG}$ ,

$$\text{where, } F_{LG} = -[Y_{LL}]^{-1} [Y_{LG}] \quad (11)$$

The maximum of the L indices ( $L^{\max}$ ) gives the proximity of the system to voltage collapse. The bus with the highest L index value will be the most vulnerable bus in the system which need critical reactive power support.

The equality constraints given by Equations (2) and (3) are satisfied by running the Newton Raphson Power flow algorithm. Generator bus terminal voltages ( $V_{gi}$ ), transformer tap settings ( $t_k$ ) and the reactive power generation of capacitor bank ( $Q_{ci}$ ) are the optimization variables and are self-restricted between the minimum and maximum value by the optimization algorithm. The limits on active power generation at the slack bus ( $P_{gs}$ ), load bus voltages ( $V_{load}$ ) and reactive power generation ( $Q_{gi}$ ), line flow ( $S_l$ ) and voltage stability level ( $L^{\max}$ ) are state variables which are satisfied by adding a penalty function to the objective function and minimizing the combined function.

### Genetic Algorithm Solution Technique

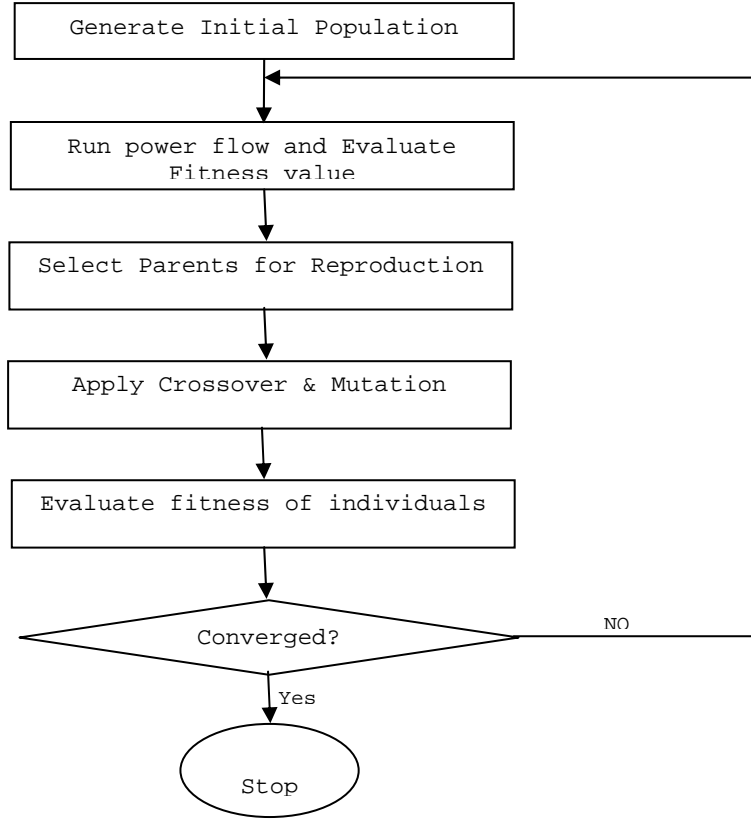
Genetic Algorithm (GA) is a generalized search and optimization technique inspired by the theory of biological evolution [13-14]. GA maintains a population of individuals that represent candidate solutions. Each individual is evaluated to give some measure of its fitness to the problem from the objective function. In each generation, a new population is formed by selecting the more fit individuals based on a particular selection strategy. Some members of the new population undergo genetic operations to form new solutions. The two commonly used genetic operations are crossover and mutation. Crossover is a mixing operator that combines genetic material from selected parents. Mutation acts as a background operator and is used to search the unexplored search space by randomly changing the values at one or more positions of the selected chromosome. Fig. 1 shows the various components of the proposed algorithm used to solve the RPD problem, the details of which are presented in the following subsections:

#### Representation

Each individual in the genetic population represents a candidate solution. In the binary-coded GA, the solution variables are represented by a string of binary alphabets. The size of the string depends on the precision of the solution required. For problems with more than one decision variables, each variable is usually represented by a sub-string. All the sub-strings are concatenated together to form a bigger string. In the RPD problem, the elements of the solution consist of all the control variables namely generator bus voltages ( $V_{gi}$ ), the transformer tap setting ( $t_k$ ), and the reactive power generation ( $Q_{ci}$ ). These variables are represented as binary strings in the GA population.

With binary representation, an individual in the GA population for the reactive power optimization problem will look like the following.

$$\begin{array}{ccccccc}
 100001 & 101111 \dots 111111 & & 001 & 110 \dots & 100 & & 101 & 100 \dots & 111 \\
 \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} \\
 V_{g1} & V_{g2} \dots & V_{gn} & t_1 & t_2 \dots & t_n & & Q_{c1} & Q_{c2} \dots & Q_{cn}
 \end{array}$$



**Fig. 1** Flow chart of GA based RPD algorithm

### *Fitness function*

In the RPD problem under consideration the objective is to minimize the total power loss satisfying the constraints (2) to (9). For each individual, the equality constraints (2) and (3) are satisfied by running Newton-Raphson algorithm and the constraints on the state variables are taken into consideration by adding a quadratic penalty function to the objective function. With the inclusion of penalty function, the new objective function then becomes,

$$\text{Min. } F = P_{\text{loss}} + K_v \sum_{i=1}^{N_{PQ}} (V_i - V_i^{\text{lim}})^2 + K_q \sum_{g=1}^N (Q_{gi} - Q_{gi}^{\text{lim}})^2 + K_f \sum_{l=1}^N (S_l - S_l^{\text{lim}})^2 + K_l \sum_{j=1}^{N_{PQ}} (L^{\text{max}} - L^{\text{lim}})^2 \quad (12)$$

where,  $K_v$ ,  $K_q$ ,  $K_f$  and  $K_l$  are the penalty factors for the bus voltage limit violation, generator reactive power limit violation, line flow violation and voltage stability limit violation respectively. In the above objective function  $V_i^{\text{lim}}$  and  $Q_{gi}^{\text{lim}}$  are defined as;

$$V_i^{\text{lim}} = \begin{cases} V_i^{\text{min}} & \text{if } V_i < V_i^{\text{min}} \\ V_i^{\text{max}} & \text{if } V_i > V_i^{\text{max}} \end{cases}$$

$$Q_{gi}^{\text{lim}} = \begin{cases} Q_{gi}^{\text{min}} & \text{if } Q_{gi} < Q_{gi}^{\text{min}} \\ Q_{gi}^{\text{max}} & \text{if } Q_{gi} > Q_{gi}^{\text{max}} \end{cases}$$

The minimization objective function given by (12) is transformed to a fitness function (f) to be maximized as,

$$f = k / F \quad (13)$$

where, 'k' is a large constant. This is used to amplify (1/F), the value of which is usually small, so that the fitness value of the chromosome will be in a wider range.

### ***Selection strategy***

The selection of parents to produce successive generations plays an important role in the GA. The goal is to allow the "fittest" individuals to be selected more often to reproduce. There are a number of selection methods proposed in the literature [14]; fitness proportionate selection, ranking and tournament selection. Tournament selection is used in this work. In tournament selection, 'n' individuals are selected randomly from the population, and the best of the 'n' is inserted into the new population for further genetic processing. Tournaments are often held between pairs of individuals, although larger tournaments can be used. This procedure is repeated until the mating pool is filled.

### ***Crossover operation***

The crossover operator is mainly responsible for the global search property of the GA. The operator basically combines substructures of two parent chromosomes to produce new structures, with the chosen probability ( $P_c$ ). For binary-coded GA, there exist a number of crossover operators. Crossover can occur at a single position (single crossover), or at number of different positions (multiple crossover). In this work two point crossovers is employed in which two crossover sites are chosen and offspring are created by swapping the bits between the chosen crossover sites.

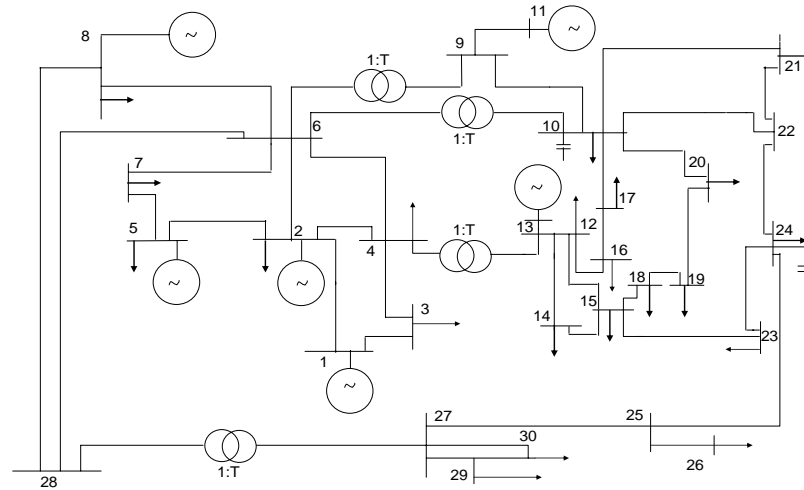
### ***Mutation***

The final operator in the genetic algorithm is mutation. The mutation operator is used to inject new genetic material into the population. Mutation changes randomly the new offspring. For binary encoding bit-wise mutation is preferred which switches a few randomly chosen bits from 1 to 0 or from 0 to 1 with a small mutation probability ( $P_m$ ). After mutation, the new generation is complete and the procedure begins again with the fitness evaluation of the population.

### **Simulation Results**

This section presents the details of the simulation study carried out on IEEE 30-bus system (Fig. 2) using the proposed GA-based method. IEEE 30-bus system consists of 6 generator buses, 24 load buses and 41

transmission lines of which 4 branches (6-9), (6-10), (4-12) and (28-27) are with the tap setting transformer. Generator parameters are given in the Appendix. The transmission line parameters of this system and the base loads are given in [10]. For the RPD problem, the candidate buses for reactive power compensation are 10, 12, 15, 17, 20, 21, 23, 24 and 29. The GA- based RPD algorithm was implemented using MATLAB code and was executed on a PC. Three different cases were considered to show the effectiveness of the proposed method. In case1, RPD problem is solved by the proposed method with base load condition. In case 2, reactive power dispatch is done with 150% of the base load without considering the voltage stability level of the system. In case 3, voltage stability limit is incorporated to improve the voltage stability in the contingency state. The results of these simulations are presented below.



**Fig. 2** IEEE 30 bus system

### Case 1

In this case, the optimal reactive power dispatch problem is solved under base load condition using the genetic algorithm. The real power settings of the generator are taken from [2]. To obtain the optimal values of the control variables the GA-based algorithm was run with different control parameter settings. The optimal values of the GA control parameters are given below:

Max. No. of Generations	: 50
Population size	: 30
Crossover probability	: 0.6
Mutation probability	: 0.01
Tournament size	: 2

The optimal values of the control variables and power loss obtained using the above settings are presented in Table 1. To illustrate the convergence of the algorithm, the relationship between the best fitness value of the population and the average fitness are plotted against the number of generations in Fig. 3. From the figure it can be seen that the proposed algorithm converges rapidly towards the optimal solution. The minimum transmission loss obtained is **4.6501 MW**. For comparison, the RPD problem was solved using

the BFGS (Broyden, Fletcher, Goldfarb and Shannon) method [7] which is a quasi-Newton optimization technique. It does not require the second order derivatives of the objective function directly and is able to approach the inverse Hessian matrix through iterations. In this method the control variables are updated in the optimization process and the range of optimum step length is chosen to be very small, otherwise oscillations will occur and the algorithm will diverge. Table 2 gives the comparison between the results obtained using BFGS method and the proposed GA approach. From this Table it is found that the minimum loss obtained using the proposed GA-based approach is less than the value obtained using the conventional BFGS method. Notably the loss obtained here is also less than the value reported in the literature [7-8] using the evolutionary computation techniques. This shows the effectiveness of the proposed approach to solve the RPD problem.

### *Case 2*

In this case the load on each bus is uniformly increased to 150% of the base load condition to analyze the voltage stability level of the system under severe conditions. Again generator bus voltage magnitudes, reactive power generation of capacitor bank and transformer taps position are taken as control variables. The optimal values of the control variables obtained by the GA-based algorithm in this case are given in column three of Table 1. For this case the minimum loss obtained is **20.80MW**, and the maximum value of L-index value is **0.2027**.

To test the ability of the system to withstand the contingencies, the system response for the worst case contingency namely, line outage (4-12) was checked. In this case it was found that the L-index value reached a maximum value of **0.3319**. To keep the L-index value under acceptable value even under contingency condition it was decided to put some restriction on the L-index value in the normal condition. The simulation results of this case are presented next.

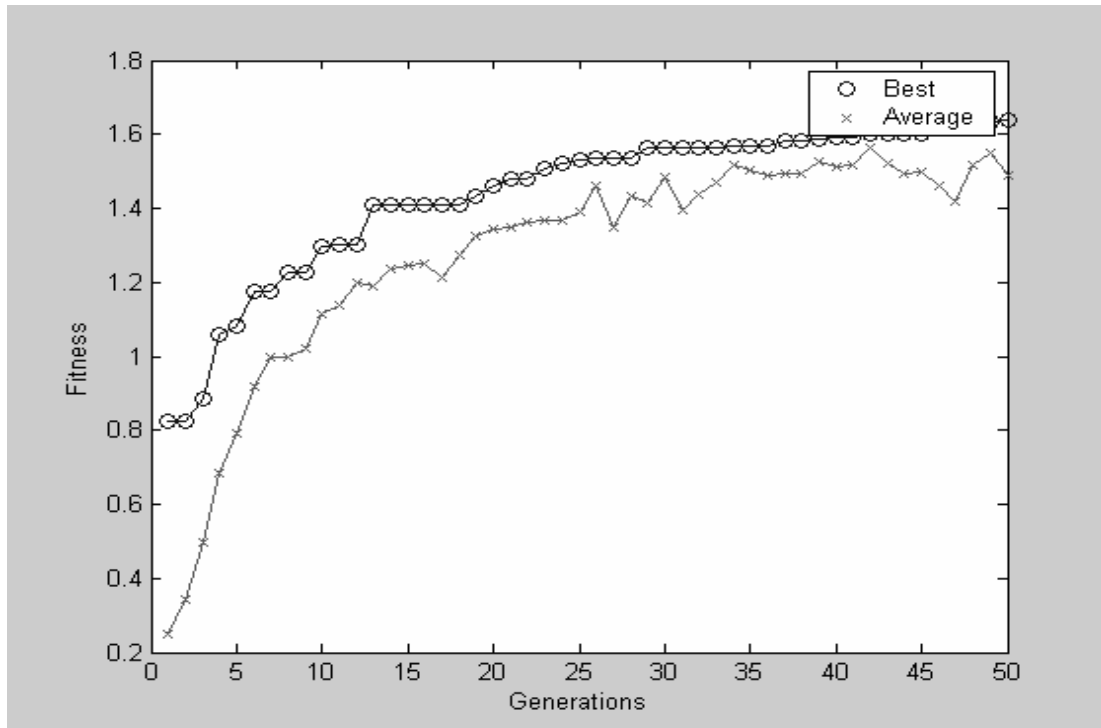
### *Case 3*

Again in this case, the same values of load condition and generator setting as in case 2 are followed. But an additional constraint in the form of limit on the maximum value of L-index under normal condition is incorporated. This is done to restrict the maximum value of L-index under contingency condition from reaching a dangerously high value. For the same contingency, namely line outage (4-12), with the inclusion of the voltage stability constraint the GA-based algorithm was applied to obtain the optimal values of the control variables under normal condition, the result of which is given in the fourth column of Table 1. For these optimal values of control variables when line (4-12) was removed it was found that the maximum value of L-index reached by the system is **0.2815** only. This improvement in voltage stability was achieved because of the restriction put on the maximum L-index value in the base case condition. This shows the effectiveness of the proposed algorithm for voltage security enhancement.



**Table 1 Optimal control variable**

Control variables	Optimal control variable settings		
	Case1 (Base case)	Case 2 (150% load)	Case 3 (with $L^{\max}$ constraint)
$V_1$	1.0373	1.0341	1.0357
$V_2$	1.0310	1.0040	1.0103
$V_5$	1.0119	0.9722	0.9738
$V_8$	1.0143	0.9802	0.9754
$V_{11}$	1.0071	1.0405	1.0468
$V_{13}$	1.0262	1.0500	1.0484
$t_{11}$	1.0500	1.0429	0.9000
$t_{12}$	1.0750	1.0429	0.9000
$t_{15}$	1.1000	1.0143	0.9000
$t_{36}$	0.9250	0.9571	0.9857
$Qc_{10}$	0	20	20
$Qc_{12}$	0	17.1429	20
$Qc_{15}$	2.8571	8.5714	11.4286
$Qc_{17}$	2.8571	17.1429	20
$Qc_{20}$	2.8571	8.5714	11.4286
$Qc_{21}$	8.5714	20	2.8571
$Qc_{23}$	2.8571	2.8571	14.2857
$Qc_{24}$	0	17.1429	11.4286
$Qc_{29}$	5.7143	5.7143	17.1429
$P_L(\text{MW})$	4.6501	20.8074	21.4004
$L_{\max}$ (Base case)	0.1828	0.2027	0.1702
$L_{\max}$ (Contingency case)	0.2237	0.3319	0.2815



**Fig. 3** Convergence characteristics

**Table 2** Comparison of results

Parameter	Case 1		Case 2	Case 3
	Conventional method(Quasi Newton method)	GA	GA	GA
$P_{\text{loss}}$ (MW)	5.54	4.6501	20.8074	24.104
$Q_{\text{gen}}$ (MVAR)	116.5	95.1	93.0	105.7
L-index(max)	0.1112	0.1828	0.3319	0.2819
Total $Q_c$ (MVAR)	0	24.71	95.1389	151.8
Load bus voltage violations	No	No	No	No
Line flow violations	Branch (28-27) only	No	No	No

## Conclusion

The continuous demand in electric power system network has caused the system to be heavily loaded leading to voltage instability. Voltage instability condition in a stressed power system could be improved by implementing an effective Reactive Power Dispatch (RPD) procedure. In this paper, Genetic Algorithm (GA), a stochastic optimization technique was employed to obtain the optimum values of the reactive

power variables in the IEEE 30-bus test system. With the inclusion of voltage stability constraint, the algorithm has helped to improve the voltage security of the system. Also the transmission loss obtained with the proposed algorithm is less than that reported in the literature. In future, the GA based approach can be combined with the conventional method to speed up the convergence of the algorithm.

## References

- [1] H. W. Dommel and W. F. Tinny, "Optimal power flow solutions," IEEE trans. on power app & systems, 87, 1968, pp 1866-1876.
- [2] K. Y. Lee, Y. M. Park, and J. L. Ortiz, "A united approach to optimal real and reactive power dispatch," IEEE trans. on PAS, 104, 1985, pp. 1147-1153.
- [3] G. R. M. Da Costa, "Optimal reactive dispatch through primal-dual method," IEEE trans. on power systems, Vol. 12, No. 2, May 1997, pp 669-674.
- [4] L. D. B. Terra and M. J. Short "Security constrained reactive power dispatch," IEEE Trans. on power systems, Vol. 6, No. 1, February 1991.
- [5] K. H. A. Rahman and S. M. Shahidehpour, "Application of fuzzy set theory to optimal reactive power planning with security constraints," IEEE Trans. on power systems, Vol. 9, No.2, May 1994, pp. 589-597.
- [6] K. Iba, "Reactive power optimization by genetic algorithm," IEEE trans. on power systems, Vol. 9 No. 2, 1994, pp 685-692.
- [7] Q. H. Wu and J. T. Ma , "Power system optimal reactive power dispatch using evolutionary programming," IEEE trans. on power systems, Vol. 10, No. 3, August 1995, pp 1243-1248.
- [8] B. Das and C. Patvardhan, "A new hybrid evolutionary strategy for reactive power dispatch," Electric power system research, Vol. 65, 2003, pp 83-90.
- [9] P. Kessel and H. Glavitsch, "Estimating the voltage stability of power systems," IEEE Trans. on Power systems, Vol. 1, No.3, 1986, pp 346-354.
- [10] O. Alsac and B. Scott, "Optimal load flow with steady-state security," IEEE Trans. on power systems, Vol. 93, 1974, pp 745-751.
- [11] K. Y. Lee, Y. M. Park and J. L. Ortiz, "Optimal real and reactive power dispatch" Electric power system research, Vol. 7, 1984, pp 201-212.
- [12] K. Y. Lee, Y. M. Park and J. L. Ortiz, "Fuel-cost minimization for both real –and reactive power Dispatches," in IEE Proc., Vol. 131, Pt. C, No.3, May 1984.
- [13] K. Deb, "Optimization for engineering design algorithms and examples," PHI, 2002.
- [14] D. E. Gold berg "Genetic Algorithms in search optimization and machine learning," Addison Wesley, 1989.

## Appendix

**Generator data**

Generator Bus Number	Cost Coefficients		
	A	B	C
1	0.0	2.00	0.00375
2	0.0	1.75	0.01750
5	0.0	1.00	0.06250
8	0.0	3.25	0.00834
11	0.0	3.00	0.02500
13	0.0	3.00	0.02500