Frequency and Duration Method for Reliability Evaluation of Large Scale Power Generation System by Fast Fourier Transform Technique

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Abstract

This paper describes FFT technique for frequency and duration (F&D) method in large-scale power generation system reliability evaluation. In this paper a detailed study of the Fast Fourier Transform (FFT) scheme for calculating cumulative Probability (P), cumulative Frequency (F), cycle Time (T) and mean Duration (D) is done. It is shown that FFT scheme is most efficient, accurate and also quite fast. The proposed FFT scheme is applied to IEEE RTS generating system.

Nomenclature

p	individual probability
f	incremental frequency
P	cumulative probability
F	cumulative frequency
S	multi-state component
G	generation
F&D	frequency and duration
GCR	generating capacity reliability
RTS	reliability test system
FFT	fast Fourier transform
COPFT	capacity outage probability and frequency table

Introduction

The frequency and duration (F&D) approach is undoubtedly a more complete technique to evaluate the static capacity adequacy for a given generation system. Although the loss of load expectation (LOLE) method [1] is very simple to handle, it does not give any indication of the frequency of occurrence of an insufficient capacity condition, nor the duration for which it is likely to exist. This is only achieved by the F & D method.

The use of F & D methods in capacity evaluation was formalized in a sequence of papers published in 1968-1969, beginning in Reference [2]. These papers presented recursive algorithms for capacity model building and load model combination. The capacity model was constructed by adding one generator at a time to the model. The common two-state representation of a generator was initially assumed, and after extended to account for generating units with partial capacity states [3]. In Reference [4], basic formulas have been derived to build up the cumulative probability and frequency of a system by adding elements, whose cumulative probability and frequency are known. Also important concept of incremental frequency was introduced. Reference [5] presents the F & D method described in its general form, i.e., the generation units are represented by multi-state models. Reference [6] describes how to estimate the expected number of starts of generating units in a power system using the F&D approach In the present paper, the application of efficient fast Fourier transform technique is described for evaluating the cumulative probability (P) and cumulative frequency (F) for large scale generating system. The performance of the proposed FFT Technique for F&D method is analyzed with IEEE RTS [7].

Model Parameters and Combination of Multi-state Components

A power system is usually composed of a set of statistically independent components. In GCR evaluation, these components are generating units, which are described by multi-state discrete capacity models. During the GCR evaluation these multi-state components are combined to provide the system model. Those states responsible for the system failure are identified and the long-run probability, frequency or any other statistical measure can be determined. In GCR evaluation, each component state has an associate capacity (generating power), so that the combination process starts from the smallest capacity state to the largest capacity state. Therefore, if state k is being combined, let S_k^+ be the set of states not yet combined, or in other words, the set of states with higher capacities than state k. Also, let S_k^- be the set of states already added, or in other words, the set of states with smaller capacities than state k. The state combination or addition process can be built from the following equations [1].

$$P = P_{k+1} + p_k \tag{1}$$

$$F = F_{k+1} + f_k \tag{2}$$

where, P_k, F_k = probability of residing and frequency of encountering either state k or states belonging to S_k

p_k = limiting state probability of state k

$$f_k = \sum f_{ky} - \sum f_{zk} \text{ with } y \in s_k^+ \text{ and } z \in S_k^- \tag{3}$$

Observe that P_k, and F_k are the cumulative probability and frequency associated with state k, so that p_k and f_k can be viewed as incremental values. It is also interesting to observe that, when the component is frequency balanced, i.e., $f_{zk} = f_{kz}$ then eqn. 3 can be simplified to

$$f_k = p_k \left(\lambda_k^+ - \lambda_k^- \right) \tag{4}$$

where, λ_k^+ and λ_k^- denote the transition rates from state k to states belonging to s_k^+ and S_k^-, respectively. Eqn. 4 has already been used and parameter f_k called incremental frequency [4]. However, Eqn.3 is more general because it covers both frequency-balanced and unbalanced multi-state components.

Usually, in GCR evaluation, the cumulative probabilities and frequencies are the only parameters required to assess the F & D indices. Consequently, a multi-state component S, representing a generating unit is only characterized by its state capacities c, state probabilities p and incremental frequencies f, i.e,

$$S\{c;p;f\} \tag{5}$$

The basic idea is to combine each multi-state component expressed as eqn. 5 to model the whole system, also expressed as Eqn. 5. Therefore consider two system components S(1) and S(2) described by the following parameters:

$$S(1)=\{c(1);p(1);f(1)\} \tag{6}$$

$$S(2)=\{c(2);p(2);f(2)\} \tag{7}$$

It is shown in reference [5] that the model parameters of a combination, S=S(1) + S(2) can be evaluated from the following convolution (*) equations:

$$p = p(1) * p(2) \tag{8}$$

$$f = [p(1) * f(2)] + [p(2) * f(1)] \tag{9}$$

Mathematical Basis of FFT's

Fourier transforms and inverse Fourier transforms are well known mathematical concepts used in Reliability Evaluation [8] and [9]. For computer applications, discrete version of these transforms, which are called the Discrete Fourier Transforms (DFT) and the Inverse Discrete Fourier Transforms (IDFT) respectively, are to be used. These are given by

$$A_l = \sum_{t=0}^{N} B_t \exp\left(-j2\pi tl / N\right) \quad l=0,1,2,3\ldots,N-1 \qquad (10$$

$$B_t = \sum_{l=0}^{N-1} A_l \exp\left(j2\pi tl / N\right) \quad t=0,1,2,3\ldots,N-1 \tag{11}$$

where, $\{B_t\}=\{B_0, B_1\ldots B_{N-1}\}$ and $\{A_l\}=\{A_0, A_1\ldots A_{N-1}\}$ are N-point complex number sequences. These are called time and frequency domains, respectively. The computational complexity of DFT or IDFT is N. So their use in practical situations is time consuming. Algorithms, which evaluate DFT and IDFT very efficiently, are known as FFT algorithms. The pioneering work in this area was done by Runga and Konig & Danielson and Lanczos in 1924 and 1942 respectively. However, they were brought prominently to notice by Cooley and Tukey in their work published only in 1965. In general, FFT algorithms are efficient only when N is a non-prime, say, of the form $N=r_1, r_2\ldots r_m$, m>1. In this case complexity of the algorithm reduces to $(r_1+r_2+\ldots+r_m)$ N, if $N= r^m$, the FFT algorithms are called radix–r algorithms. In particular, radix-2 FFT algorithms reduce the complexity to $(N/2) \log_2(N/2)$.

Simple numerical example

To illustrate the simplicity in applying the FFT Technique for the Equations 8 & 9, consider state model of two components S(1) (two numbers of 25 Mw capacity having FOR= 0.02, λ =0.49 μ=0.01),and S(2) (one number of 50 Mw having FOR=0.02,λ =0.49, μ=0.01). Once the limiting state probabilities have been obtained, Eqn. 4 is used to obtain the incremental frequencies of S(1) and S(2).This is shown in Table 1.

Table 1 Model parameters for S(1) and S(2)

Capacity outage	Individual probability	Incremental frequency
c(1)	**p(1)**	**f(1)**
0	0.9604	-0.019208
25	0.0392	0.018816
50	$4 x 10^{-2}$	0.000392
c(2)	**p(2)**	**f(2)**
0	0.98	-0.098
50	0.02	0.098

Using FFT Technique, the model parameters of S = S(1) +S(2) are evaluated. Table 2 summarizes individual (ind.) probabilities and incremental (inc.) frequencies associated with the states of system S. Applying Eqns. 1 and 2, the cumulative probabilities and cumulative frequencies of the system S are shown in Table 2.

Table 2 Cumulative probability and cumulative frequency

Capacity Outage	Ind. Probability (p)	Inc. Frequency (f)	Cumulative Probability(P)	Cumulative Frequency(F)
0	0.941192	-0.02823576	1	0
25	0.038416	0.01805552	0.058808	0.02823576
50	0.0196	9.408×10^{-3}	0.020392	0.01018024
75	7.84×10^{-4}	7.6048×10^{-4}	0.000792	7.7224×10^{-4}
100	8×10^{-6}	1.176×10^{-5}	8×10^{-6}	1.176×10^{-5}

Application of the Proposed Technique

To evaluate the accuracy and efficiency of the proposed FFT technique for F&D method, IEEE RTS [6] is tested. To obtain the capacity model, all generating units must be combined to produce an equivalent unit G. The parameters of the equivalent unit G can be described by a generating capacity outage probability and frequency table (COPFT) of the same type of Table 2.There are nine sets of identical generating units among the 32 generating units in the IEEE RTS. The amount of calculation can be reduced and the limit on the choice of steps can be relaxed if the identical units are first modeled like S (1) and S (2) .The limiting state probabilities together with the incremental frequencies are determined. The models are added one by one, until the last generating unit is included. Thus the parameters of the equivalent unit G can be expressed as follows:

$$G = \{c_G; p_G; f_G\}$$

Algorithm to Obtain State Model for Identical Generating Units by Radix 2 FFT Technique

- Read the input data availability p, unavailability q, individual capacity c_i , number of units, failure rate λ, repair rate μ
- Choose N point according to total capacity (the N point should be in powers of 2)
- Assign the availability as the first input of a function X (i.e. X(1) = p) and unavailability as $X(1+ c_i) = q$)
- Take FFT and power it with the number of units

- Taking inverse FFT gives the individual probability (p).
- By taking FFT for failure rate(λ), λ_{-n} is calculated
- By taking FFT for repair rate(μ), λ_{+n} is calculated
- Subtracting $\boldsymbol{\lambda}_{-n}$ from λ_{+n} and multiplying with individual probability gives the incremental frequency (f).
- By repeating the steps from 1 to 8 individual probability and incremental frequency for each set of identical capacity unit's state models S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, and S_9 are calculated.

For IEEE RTS 9 state models S1 (c_1, p_1, f_1), …. to S_9(c_9, p_9, f_9) are obtained. The models are then combined one by one. To start this process, first combine S_1 and S_2 as follows:

ifft (fft (p_1, p_2)) gives individual probability
ifft (fft ($p_1 f_2$)) + ifft(fft($p_2 f_1$)) gives incremental frequency

With this obtained individual probability and incremental frequency combine the model S_3. Repeat the process until all 9 models are combined. The last combination is represented as G ={c, p, f }

First, the cumulative parameters P and F of G have to be obtained through equations 1 & 2. The other two reliability indices can be computed from the previously defined ones, as follows:

$$\text{Cycle Time (T)} = 1/F \tag{12}$$
$$\text{Mean Duration (D)} = P/F \tag{13}$$

Test Result

A computer program written in Matlab 6 Version implementing the proposed algorithm was developed to test the IEEE RTS 32 generating system in Pentium III processor. The cumulative probability (P), cumulative frequency (F), cycle time (T) and mean duration (D) are obtained and the results are shown in Table 3.The computation time for the frequency and duration indices evaluation performed with FFT Technique has been compared with conventional method[1] and discrete convolution method[5]. The percentage reduction in computation time has been presented in Table 4. The results shown in Table 3 are almost free of error. Table 3 compares Indices P and F obtained in reference 4 with those obtained by the proposed method. Indices P and F obtained by the proposed method agree with reference 4. In table 3 Indices P less than 10^{-16} are neglected. Hence results are shown up to 2600 MW.

Table 3 Cumulative probability, cumulative frequency, cycle time, mean duration

C.Out. (MW)	Cum.Prob.(P)	Inc. fre.(f)	F by Ref. [4]	F by FFT	Cycle time	D
0	1.000000000	-0.006659959	0	0	-----	------
12	0.763605000	-0.000269350	0.00665996	0.006659959	150.1510751	114.65611
24	0.634418000	0.000005750	0.007554525	0.007554525	132.3710046	83.978548
50	0.604744000	0.000319951	0.007132571	0.007132571	140.2018909	84.786252
100	0.547601000	-0.000188011	0.006086601	0.006086601	164.2953122	89.968277
150	0.492886085	0.000078303	0.005667834	0.005667834	176.43423	86.961977
200	0.381328100	0.000021397	0.005072666	0.005072666	197.1349843	75.173109
250	0.342798259	0.000004926	0.004043841	0.004043841	247.2896724	84.770469
300	0.320653834	0.000000837	0.00333816	0.00333816	299.5662725	96.057074
350	0.311056455	-0.000343128	0.002916199	0.002916199	342.9121167	106.66503
400	0.261873431	-0.001285251	0.002681036	0.002681036	372.9901126	97.676200
450	0.145170190	0.000130955	0.003440087	0.003440087	290.6903325	42.199571
500	0.122516218	1.94329E-05	0.002818902	0.002818902	354.7480017	43.462384
550	0.096401234	2.82731E-05	0.002344567	0.002344567	426.5180771	41.116869
600	0.062112861	9.0203E-06	0.001787916	0.001787916	559.3105413	34.740378
650	0.049418664	1.59553E-06	0.001324533	0.001324533	754.9829641	37.310249
700	0.042461346	1.53653E-06	0.001032823	0.001032823	968.2199082	41.111920
750	0.038490865	-5.0594E-05	0.000836424	0.000836424	1195.565719	46.018359
800	0.024719396	-4.2712E-05	0.00064575	0.000645750	1548.58714	38.280139
850	0.014730846	1.93E-05	0.000533432	0.000533432	1874.651684	27.615205
900	0.014730846	7.69E-06	0.000414319	0.000414319	2413.596707	35.554321
950	0.007491953	3.07E-06	0.000296669	0.000296669	3370.763233	25.253599
1000	0.004340874	9.19E-07	0.000199396	0.000199396	5015.141377	21.770098
1050	0.003020831	1.43E-07	0.000135982	0.000135982	7353.910307	22.214920
1100	0.002353021	4.01E-07	9.95798E-05	9.95798E-05	10042.19512	23.629497
1150	0.001804193	-4.90E-07	6.93026E-05	6.93026E-05	14429.47043	26.033544
1200	0.000791252	1.23E-06	4.34465E-05	4.34465E-05	23016.80674	18.212100
1250	0.000568197	1.01E-06	2.96748E-05	2.96748E-05	33698.57464	19.147437
1300	0.000400522	2.14E-07	2.07417E-05	2.07417E-05	48212.07976	19.310017
1350	0.000179843	9.24E-08	1.18208E-05	1.18208E-05	84596.87995	15.214170
1400	0.000101723	1.20E-08	7.06145E-06	7.06145E-06	141613.9432	14.405409
1450	5.90564E-05	2.33E-09	4.30119E-06	4.30119E-06	232493.8085	13.730239
1500	4.04351E-05	2.84E-08	2.88478E-06	2.88478E-06	346646.7261	14.016687
1550	1.49042E-05	5.17E-09	1.31327E-06	1.31327E-06	761455.3147	11.348853
1600	8.03963E-06	6.40E-09	7.22855E-07	7.22855E-07	1383402.66	11.122050
1650	4.07586E-06	9.74E-10	3.8731E-07	3.8731E-07	2581911.531	10.523505
1700	1.58327E-06	5.35E-10	1.68398E-07	1.68398E-07	5938319.154	9.401984
1750	7.21642E-07	9.07E-11	8.10154E-08	8.10154E-08	12343336.83	8.907469
1800	2.91206E-07	2.24E-11	3.48554E-08	3.48554E-08	28689950.86	8.354674
1850	1.52927E-07	5.54E-12	1.83065E-08	1.83065E-08	54625346.6	8.353664
1900	4.69167E-08	2.10E-11	6.36243E-09	6.36243E-09	157172652.6	7.374016
1950	2.15125E-08	1.70E-10	2.96025E-09	2.96025E-09	337809306.6	7.267129
2000	7.2462E-09	1.68E-11	1.06534E-09	1.06534E-09	938667467.7	6.801772
2100	8.4307E-10	2.11E-12	1.3841E-10	1.3841E-10	7224911495	6.091106
2200	9.27E-11	9.71E-14	1.628E-11	1.628E-11	61425061425	5.694103
2300	7.97E-12	3.15E-15	1.52E-12	1.52E-12	6.57895E+11	5.243421
2400	4.70E-13	2.09E-16	1.00E-13	1.00E-13	9.98864E+12	4.691843
2500	2.01E-14	9.47E-18	4.76E-15	4.76E-15	2.10077E+14	4.219013
2600	6.81E-16	2.10E-19	1.65E-16	1.58E-16	6.34519E+15	4.321175

Table 4 Comparison of computation time

Method	Computation time in seconds	% Reduction in computation time with respect to conventional method
Conventional	77.15	-
Discrete Convolution Technique	41.2	46.6
FFT Technique	1.14	98.5

Conclusion

A general, simple and efficient algorithm for building the generation model is developed by fast Fourier transform technique. This paper presents FFT technique to evaluate the cumulative probabilities and frequencies. This FFT Technique saves computational time and effort. The numerical results have shown that the described FFT Technique was accurate and fast. This method can handle large-scale power generation system without much computational time.

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