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Prospects of Using Time Series to Find the Maximum Power Point of a Photovoltaic Array from its Load Currents and Voltages

S. Shahnawaz Ahmed*, Mahbubul Hasan*, Md. Enamul Haque Khondker*, A. K. M. Moniruzzaman* and Azhar B. Khairuddin**

**Department of Electrical & Electronic Engineering, Bangladesh University of Engineering & Technology, Dhaka 1000, Bangladesh*
***Faculty of Electrical Engineering, Universiti Teknologi Malaysia, Malaysia*
Email: ssahmed@eee.buet.ac.bd
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Abstract

In this paper an algorithm based on autoregressive (AR) third order time series has been proposed to forecast a photovoltaic (PV) array's short circuit current and open circuit voltage at a given instant from directly and easily measurable load currents and voltages in shortly spaced three preceding intervals of time. The maximum power point (MPP) of the PV array's current-voltage characteristic curve corresponding to that instant can then be obtained from the forecasted short circuit current and open circuit voltage. This paper has emphasized on laying the theoretical foundation, achieving preliminary results, their comparison with measured values and identifying further investigations needed for the proposed real-time application of time series which has so far been used mainly for forecasting long term events. The methodology presented in this paper does not lack in generality and can easily be extended to include refinements. It is applicable for a PV array installed at any geographical location either in stand-alone or in grid connected mode.

Introduction

Depleting fossil fuel reserves (gas, oil and coal) coupled with their contributions towards formation of the polluting green house gases are gradually paving the way for photovoltaic (PV) electricity. This is one of the major renewable energy sources, and requires only the first (capital) cost but zero fuel cost and very little maintenance. With continued research on the line of solar cell materials, power conditioning devices (DC to AC conversion) and battery technology, the initial expenses of installing a PV array is expected to be affordable by the common people in near future. Its application horizon [1] is now not only confined within small-scale and stand-alone use by the isolated and off-grid rural or coastal communities rather it is also making inroad to the grid connected urban people. Under the emerging competitive market environment or deregulation [2] the non-utility generation (NUG) companies have started operating PV plants which are installed in the grid systems as distributed or embedded generation sources.

A PV systems is by nature a non linear power sources which delivers the maximum power output with highest efficiency only for a particular load current and voltage which is termed the maximum power point (MPP). This point depends on the PV array's current-voltage (I-V) characteristics which in its turn varies instant to instant mainly due to random changes in insolation (incoming solar radiation). Moreover, the load connected to the PV array also changes as per consumers' needs and does not correspond to the MPP. A prior knowledge of the MPP will be useful for operating a PV array at this point, irrespective of the connected load magnitude, through on-line control [3] of the firing angle or any other controllable parameter of the power electronic interfaces (e.g. inverter or chopper) between the load and the PV array. MPP information will also be useful for a grid connected [4] PV array for the purpose of load dispatching i.e. buying deficit power from the grid or selling surplus power to the grid. Hence there is a need for predetermination of the maximum power point in real time i.e. within a short interval of time.

It appears from a review of the literature that mainly different variants of artificial neural network (ANN) have been applied [5-6] to find the MPP of a PV array. However, the methods suffer from one or more of the drawbacks such as requiring sophistication in the choice of various parameters which influence the success of ANN, use of difficult-to-measure input variables, comparison of the output with those obtained through conventional modeling based calculations but not with the measured values, and prediction of MPP on hourly basis whereas solar insolation changes every few minutes.

The present paper proposes determining the MPP at an instant from the immediate past basic data using a time series e.g. a third order Autoregressive (AR) model [7]. The basic data for the time series are the directly measurable load currents and voltages in closely spaced three preceding intervals of time. This work has considered this interval as 10 minutes. However, this interval can even be made less. The load currents and voltages will be transformed into corresponding interval's open circuit voltage (V_{oc}) and short circuit current (I_{sc}). Then the subject interval's I_{sc} and V_{oc} will be forecasted from these transformed ones. The forecasted I_{sc} and V_{oc} will then be used to compute the PV array's maximum power output in that interval.

It is noteworthy that AR model has widely been applied for other long term events such as forecasting the hourly, weekly, monthly or annual demand [8] of a power utility. But the potentials of AR or any time series have not yet been investigated into for forecasting the real-time events like determining MPP. Therefore, this paper has presented the theoretical basis, reported the results of a preliminary analysis and compared those with the physical data collected from an actual PV system in stand alone mode, and discussed further refinements needed for the proposed forecasting algorithm which is perhaps the first of its kind.

I-V Characteristics and Maximum Power of PV Array

An elementary solar cell is basically a p-n junction diode usually made of semiconductors like silicon or gallium arsenide. To have an appreciable current output and a practical open circuit voltage, large number of elementary cells are assembled in series-parallel blocks and each such assembly is called a photovoltaic panel. Several panels are electrically connected in parallel to make a PV array.

The refined, simplified and final [3] equivalent circuit model for a single PV cell is shown in Fig.1.

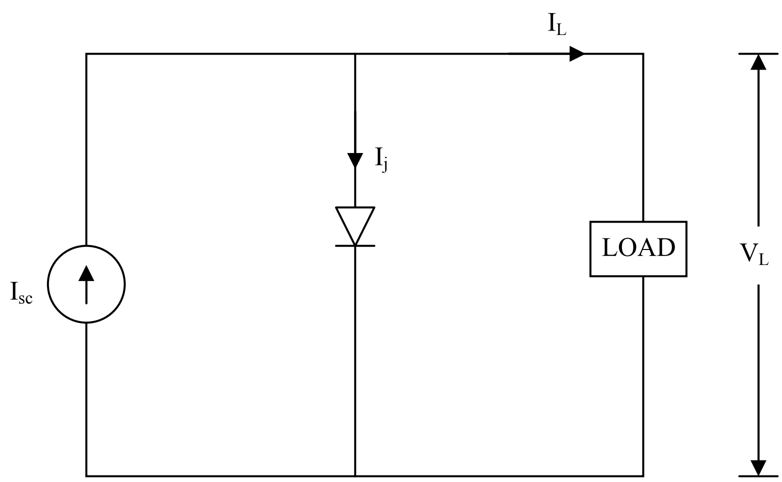

Fig.1 Equivalent circuit of a PV cell

The equation governing the relationship of I_L (the load current) with I_{sc} (the light generated diode current without bias i.e. short circuit) and I_j (the diode current with bias in the dark i.e. absence of light) is given as follows.

$$I_L = I_{sc} - I_j = I_{sc} - I_o\left(e^{\frac{qV_L}{\gamma kT}} - 1\right) \quad (1)$$

where,

I_j is an exponential function of the diode reverse saturation current I_o depending upon the load voltage V_L

q = charge of an electron = 1.5992 x 10^{-19} coulombs

γ = a factor having a value usually between 1.0 and 3.0.

k = Boltzman's constant = 1.38066 x 10^{-23} Joules/°K

T = (ambient temperature, t_a°C +273) °K

Eqn. 1 leads to an I-V characteristics of an elementary PV cell as shown in Fig.2. The characteristic curve of a complete PV array is similar in shape to that in Fig.2 with the only exception that the complete array current and voltages are just the current and voltage of a single cell multiplied by appropriate factors considering respectively the number of parallel and series paths in the array of panels.

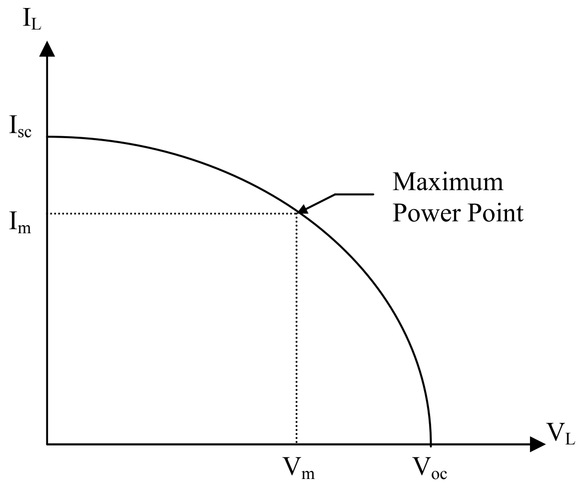

Fig. 2 I-V characteristics of a PV cell

The load voltage at the maximum power point denoted by V_m (Fig.2) can be obtained by differentiating with respect to V_L the expression of power output i.e. the product of I_L (eqn.1) and V_L and then setting the derivative to zero. This results in a final expression for V_m as follows.

$$e^{K_1V_m}(1 + K_1V_m) = e^{K_1V_{oc}} \quad (2)$$

where,

$$K_1 = q / (\gamma\, k\, T) \tag{3}$$

Once V_m is obtained from eqn. 2 it can be substituted into eqn.1 to get the corresponding I_L i.e. I_m as follows.

$$I_m = I_{sc} - \left(\frac{I_{sc}}{e^{K_1 V_{oc}} - 1}\right)(e^{K_1 V_m} - 1) \tag{4}$$

It should be noted that the quantity V_{oc} termed open circuit voltage and used in eqns. 2 and 4 is expressed as follows.

$$V_{oc} = \frac{1}{K_1} \ln\left(\frac{I_{sc}}{I_o} + 1\right) \tag{5}$$

The above expression of V_{oc} can be found by substituting $I_L = 0$ into eqn.1 and denoting the corresponding V_L as V_{oc} .

Eqns. 2 and 4 show that at a given instant the maximum power which is the product of V_m and I_m can only be known if the corresponding instant's short circuit current I_{sc} and the open circuit voltage V_{oc} are available.

Proposed Methodology

The method proposed in this paper for determining the MPP of a PV array consists of the following steps.

a) Transformation of load currents and load voltages measured in three intervals into corresponding instant's short circuit currents and open circuit voltages.
b) Forecasting the I_{sc} and V_{oc} for the required instant from those computed for the three immediate past intervals using a third order AR time series.
c) Identifying the MPP by finding V_m and I_m from forecasted I_{sc} and V_{oc}.

Transformation of load currents and load voltages

The load to be served by a stand-alone or a grid connected PV array is always variable and changes from instant to instant by a magnitude whatever small it may be. This aspect has been utilized in the proposed method to avoid measurement of I_{sc} and V_{oc} requiring disconnecting the PV array from load and instead, derive those in every interval in terms of two pairs of load currents and voltages which are very easy-to-measure. The values of I_{sc} and V_{oc} derived so in the last three intervals will be kept stored for use by a 3rd order AR model to forecast, well in advance, the I_{sc} and V_{oc} of the immediately succeeding interval.

If at a given instant two pairs of load currents and voltages respectively I_1, V_1 and I_2, V_2 are recorded in quick succession so that the same I_{sc} can be considered for both pairs, then it follows from eqns.1 and 3 that

$$I_{sc} = I_1 + I_0(e^{K_1 V_1} - 1) = I_2 + I_0(e^{K_1 V_2} - 1) \tag{6}$$

It follows from eqn.6 that

$$I_0 = \frac{(I_1 - I_2)}{(e^{K_1 V_2} - e^{K_1 V_1})} \tag{7}$$

Using eqn.7 and substituting $I_L = I_1$ or I_2 and $V_L = V_1$ or V_2 back into eqn.1 provides I_{sc} of an interval in terms of the corresponding two pairs of measured values of load currents and voltages, as expressed below.

$$I_{sc} = I_1 + \left(\frac{I_1 - I_2}{e^{K_1 V_2} - e^{K_1 V_1}}\right)(e^{K_1 V_1} - 1) \tag{8}$$

Substituting I_{sc} from eqn.8 and I_0 from eqn.7 into eqn. 5 will provide the corresponding's instant V_{oc} in terms of the known pairs of load currents and voltages.

Forecasting the I_{sc} and V_{oc} using AR model

To forecast the short circuit current for any instant of time t from the short circuit currents (computed using eqn. 8) for the three immediately preceding and equally spaced instants (t-1), (t-2) and (t-3), a third order (p=3) AR model [7] can be applied as follows.

$$\hat{I}_{sc}(t) = \bar{I}_{sc} + \sum_{i=1}^{p=3}[\alpha_{p,i}\, y(t-i)] + w(t) \tag{9}$$

where,

$\bar{I}_{sc}$ is the average of the short circuit currents obtained for the three preceding intervals.

$\alpha_{p,i}$ are the AR model parameters determined for each forecast instant (t) using successively the eqns. 14 to 18 shown in the Appendix.

$$y(t\text{-}i) = \bar{I}_{sc} - I_{sc}(t\text{-}i) \tag{10}$$

w(t) is white noise represented by normally distributed random variables with zero mean and a variance σ^2_w .

Similarly the open circuit voltage for instant t is forecasted using the following equation from those computed for three preceding intervals.

$$\hat{V}_{oc}(t) = \overline{V}_{oc} + \sum_{i=1}^{p=3}[\alpha_{v\,p,i}\, y_v(t-i)] + w_v(t) \tag{11}$$

where α_v, y_v, w_v have significance respectively similar to those of α, y and w in eqn.9 and are determined similarly.

Identifying MPP

The open circuit voltage forecasted for the instant 't' using eqn.11 is substituted into eqn.2 and the natural logarithm is taken for both sides so that following equation results.

$$K_1 V_m + \ln(1 + K_1 V_m) - K_1 \hat{V}_{oc} = 0 \tag{12}$$

Eqn.12 is solved for V_m , the voltage at the MPP for the instant 't', using any numerical method e.g. Newton-Raphson method in the present work.

The open circuit voltage forecasted for the instant 't' using eqn.11, the short circuit current forecasted for instant 't' using eqn.9 and V_m obtained from eqn.12 are substituted into eqn.4 to find I_m, the current at the MPP for the instant 't'.

The maximum power at instant 't' is then

$$P_m = V_m I_m \tag{13}$$

Results and Discussion

The various steps of the proposed method have been coded in FORTRAN 77 and run on a PC. The method has been tested using some readily available data sets on load currents and voltages which were recorded in different days from a live stand-alone PV array. The PV array consisted of three panels in parallel, each of which was designed for a maximum output of 43 watts subject to the availability of the correspondingly required insolation. Each panel again consisted of 30 blocks in series and each block comprises 53 cells in parallel so that the current through a single cell is the total PV array current divided by (3x53=) 159 and the voltage across a single cell is the array voltage divided by 30. The PV array was connected to a variable load.

Forecasted vs. actual I_{sc} and V_{oc}

Table1 shows, for some typical cases, 10 minutes ahead forecasts on the short circuit currents and open circuit voltages for an interval of time from the recorded load currents and voltages of three 10 minutes spaced preceding intervals.

Table 1 Forecasted vs. actual short circuit current and open circuit voltages of the PV array

Days	Time of inter-val	I_1 amps	I_2 amps	V_1 volts	V_2 volts	Ambi-ent temp. t_a°C	Forec-asted $\hat{I}_{sc}$ amps	Actual I_{sc} amps	Foreca-sted $\hat{V}_{oc}$ Volts	Actual Voc volts
Day 1	08.50	2.10	2.00	11.8	12.0	19.0	for 09.20: 3.16	at 09.20: 3.54	For 09.20: 14.16	at 09.20: 16.1
	09.00	1.88	1.85	12.6	12.7	20.5				
	09.10	1.78	1.70	12.8	13.0	21				
Day 2	09.40	1.44	1.40	13.9	14.0	22	for 10.10: 4.6	at 10.10: 4.3	For 10.10: 14.83	at 10.10: 16.0
	09.50	1.35	1.30	14.0	14.05	22.5				
	10.00	1.27	1.25	14.08	14.1	23				
Day 3	11.30	0.29	0.28	15.0	15.1	28.3	for 12.00: 1.75	at 12.00: 2.01	For 12.00: 13.31	at 12.00: 15.1
	11.40	5.25	4.91	5.0	9.0	28.2				
	11.50	1.14	1.04	13.7	14.0	28				

The forecasted values have been compared with the actual values of short circuit current and open circuit voltage at the corresponding instant. These actual values were measured (for the purpose of comparison) respectively by short circuiting and open circuiting the PV array just after every recording of the load currents and voltages.

It should be noted in Table 1 that the two pairs of load currents and voltages I_1, V_1 and I_2, V_2 shown in the same interval were recorded just at a gap of 1 minute.

It is evident that the forecasted and actual short circuit current and open circuit voltages for the PV array are close to each other. Even the proposed method was able to accommodate interval to interval erratic variation in the input i.e. load currents and voltages as may be seen in the third one (Day 3) of the typical cases reported in Table 1.

Maximum power using forecasted and actual values of I_{sc} and V_{oc}

The load voltage and current at the MPP point, respectively V_m and I_m , were computed in the way mentioned in Section 3.3 using the forecasted as well as actual I_{sc} and V_{oc} for the same interval of time. Then the maximum power output of the PV array, P_m was computed using those V_m and I_m. Table 2 shows a comparison of the maximum power outputs corresponding to the typical cases reported in Table 1.

Table 2 Maximum power using forecasted vs. actual short circuit current and open circuit voltages of the PV array

Day and interval of time	Ambient temp. t_a°C	P_m using forecasted I_{sc} and V_{oc}					P_m using actual I_{sc} and V_{oc}				
		Forecasted $\hat{I}_{sc}$ amps	Forecasted $\hat{V}_{oc}$ volts	V_m volts	I_m amps	Max. power P_m watts	Actual I_{sc} amps	Actual Voc volts	V_m volts	I_m amps	Max. power P_m watts
Day 1 09.20	21.0	3.16	14.16	10.67	2.70	28.83	3.54	16.1	12.37	3.09	38.22
Day 2 10.10	23.0	4.6	14.83	11.24	3.96	44.51	4.3	16.0	12.27	3.74	45.96
Day 3 12.00	28.0	1.75	13.31	9.89	1.48	14.59	2.01	15.1	11.49	1.73	19.81

It should be noted that in the Newton-Raphson solution of eqn. 12 for each V_m , the starting value has been chosen as the corresponding open circuit voltage (forecasted or actual as the case may be) and for a tolerance margin of 0.001 volts the convergence was achieved in only 3 iterations for each case.

The ambient temperature (t_a°C) shown in Table 2 at an interval for which P_m is being calculated, was considered as equal to the last one among the temperatures recorded for the preceding three intervals. This is because the temperature did not vary significantly over the closely spaced (10 minutes) consecutive three intervals. However, during actual implementation of the algorithm, if needed, the temperature for the forecast interval can always be approximated using some simple extrapolation technique.

It is evident from Table 2 that the two values of P_m computed using forecasted and actual short circuit current and open circuit voltages for the PV array are close to each other.

Computer time requirement

In each of the cases investigated by the present work, the proposed algorithm required on average only about 0.5 seconds for all the three steps on a 225 MHz PC. The requirement of such a less time stems mainly from simple transformation equations and use of a short order (i.e. 3rd) AR model.

Conclusions

The widely used time series is still in its infancy so far as determining the MPP of a photovoltaic array is concerned. In exploring the prospects of the time series, a third order autoregressive model based algorithm has been proposed. This algorithm finds the MPP at an instant from the directly measurable load currents and voltages of the PV array in shortly spaced three preceding intervals of time.

The proposed algorithm has been presented in details. To have a preliminary assessment, it has been tested on several available sets of real data for a live PV system. The obtained results have been compared with the actually measured values and found very much promising.

The difference of the forecasts from the actual ones can further be reduced by adopting the following two means which could not have been done in the time frame set for the present preliminary investigation.

i) The results shown in Table 1 were achieved without using the random noise w(t) or $w_v(t)$ of the AR model (eqns. 9 and 11). As regards a PV system, this random noise is important because it takes into account a lot of uncertainties which affect insolation and may vary even minute to minute e.g. cloud coverage of the sky, moisture and dust content in the surrounding atmosphere. However, these two noise terms' standard deviations can be determined off-line from a long database on short circuit currents and open circuit voltages computed from recorded load current and voltages. Once the standard deviations are known a random number generator will add the noise terms on-line with the respective AR model (eqns.9 and 11) at every instant of forecast.

ii) The factor $K_1 = q / (\gamma k T)$ used in various equations requires proper choice of the value for γ parameter of a PV cell. In the absence of manufacturer- supplied information, γ had to be determined by the present work. It was computed in the way shown in the Appendix for a few sets of measurements on short circuit current, load current and load voltage of the used PV array. From these the minimum one found to be 2.3807 was chosen for γ. Indeed, for accuracy γ is to be determined by the best curve fitting method from a large number of data sets on load currents and voltages. However, the method followed in the present work and also the method of curve fitting for γ, both need just off-line implementation.

It should be noted that in actual implementation of the proposed algorithm, load currents and voltages will be recorded through on-line data acquisition system. Then the three steps viz. transformation, forecasting and MPP identification will be done on-line. However, the on-line time needed then would be much trivial as confirmed by the requirement of only 0.5 seconds in each case on a 225 MHz PC in the present investigation.

However, further time saving could have been made by avoiding use and transformation of load current and voltages after first few intervals and instead, forecasting the I_{sc} and V_{oc} for an interval from those forecasted for the three immediately preceding intervals. But this may cost the accuracy to some extent. This is why the present work has used measured load currents and voltages to make the forecasting as realistic as possible.

The proposed method does not lack in generality and can easily be applied for a stand-alone or grid-interfaced PV array in any location. The suggested improvements can form the basis of an elaborate research and further investigations.

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Appendices

Equations successively used to determine AR model parameters α (in eqn.9) and α_v (in eqn.11) at the instant 't'

$$C_k = \frac{1}{p}\sum_{i=1}^{p-k}[y(t-i)-\bar{y}][y(t-i-k)-\bar{y}];\ p=3, k=0,1,..p \qquad (14)$$

where y (t-i) or y(t-i-k) are as in eqn. 10 and $\bar{y}$ is the average of y(t-i), i=1,2....p.

$$r(j) = \frac{C_j}{C_0};\ j=1,....p \qquad (15)$$

$$\alpha_{1,1} = r(1) \qquad (16)$$

$$\alpha_{l+1,l+1} = \frac{r(l+1)-\sum_{j=1}^{l}\alpha_{l,j}r(l+1-j)}{1-\sum_{j=1}^{l}\alpha_{l,j}r(j)};\ l=1,...p-1 \qquad (17)$$

$$\alpha_{l+1,j} = \alpha_{l,j} - \alpha_{l+1,l+1}\times\alpha_{l,l+1-j};\ l=1,...p-1,\ j=1,...l \qquad (18)$$

The AR model parameters α_v for open circuit voltage forecast are determined using a set of equations similar to eqns. 14 through 18 and using y_v instead of y.

Determination of γ in the present work

From eqn.6 it can be written that

$$I_0 = \frac{(I_{sc} - I_1)}{\left(e^{\frac{qV_1}{\gamma kT}} - 1\right)} = \frac{(I_{sc} - I_2)}{\left(e^{\frac{qV_2}{\gamma kT}} - 1\right)} \tag{19}$$

It can be rearranged as

$$(I_{sc} - I_1)e^{\frac{qV_2}{\gamma kT}} - (I_{sc} - I_2)e^{\frac{qV_1}{\gamma kT}} = I_2 - I_1 \tag{20}$$

Replacing (I_{sc}-I_1) by A, (I_{sc}-I_2) by B, (I_2-I_1) by C and (q/kT) by K_0 it can be written that

$$Ae^{\frac{K_0 V_2}{\gamma}} - Be^{\frac{K_0 V_1}{\gamma}} - C = 0 \tag{21}$$

Using a measured value of I_{sc} and two pairs of load currents and voltages (I_1,V_1) and (I_2,V_2) recorded in quick succession, and downscaling them for a single cell, eqn.21 can be solved for γ through Newton-Raphson technique.