# **Numerical Model of Flat Solar Collector**

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#### Abstract

The use of solar energy to obtain heat is object of detailed investigations. The thermal analysis of flat solar collectors is an important element of the modern technology of solar radiation heat transfer for domestic and industrial needs. The new methods proposed are used to complete the education in EUC (European University Centre) for Renewable Energies of Technical University of Varna "Energy-Nature-Balkan" in the field of solar thermal energy exploitation in order to investigate different shapes of absorbers and materials.

#### Introduction

The classical thermal calculations are based on thermal balance between incoming radiation heat, losses and useful heat transferred to the fluid inside absorbers tube. The factors which is necessary to be have in mind to realize a correct calculation are a large in number. All the factors cause difficulties when analyzing the collector behavior efficiently working daily in different conditions. The method proposed makes calculations easy without loosing precision. On the contrary – using a computer realization of a numerical method with combination with experimental temperature measurement makes possible to perform a real time efficiency estimation of a flat solar collector with arbitrary geometry of the absorber. The numerical method used is based on BIEM technique – Boundary Integral Equation Method, with the main advantage to calculate precisely the temperature gradient (also heat flux) on the boundary where temperatures are given as boundary conditions. A steady state heat conduction problem with linear and nonlinear thermal characteristics of the absorber can be solved in order to perform calculation of the useful heat flux on the inner diameter of the absorbers tube.

## Efficiency Calculation of Flat Solar Collectors with the Energy Balance Equations

The well known and officially used (and standardized) methods are based on the balance of the collectors energy [1].

$$I_{c}A_{c}\overline{\tau_{s}}\alpha_{s} = q_{u} + q_{l} + \frac{dI_{c}}{dt}$$
(1)

where,

 $I_c$  - density of the solar irradiance, falling on the collectors surface,

A<sub>c</sub> - collectors area,

 $au_{s}$  - the optical characteristic of the collectors cover,

 $\alpha_{\rm s}$  - absorbing capability of the frames plate of the absorber,

 $\mathbf{q}_{\mathbf{u}}$  - the useful heat flux from the plate to the thermal fluid,

 $\mathbf{q_l}$  - heat losses of the plate of the absorber outside the collector,

 $\frac{dl_c}{dt}$  heat flux accumulated from collectors internal energy.

Finally this method calculates the useful heat flux  $\mathbf{q}_{\mathbf{u}}$ 

$$\mathbf{q}_{u} = \mathbf{A}_{c} \mathbf{F}_{R} \left[ \alpha_{s} \mathbf{I}_{s} - \mathbf{U}_{c} \left( \mathbf{T}_{f,in} - \mathbf{T}_{a} \right) \right], \tag{2}$$

where.

 $\mathbf{F}_{\mathbf{R}}$  means coefficient of heat extracting from the collector:

$$F_R = f(G_c, c_p, U_c, F'), I_s = \overline{\tau_s}.I_c$$

 $G_c$  the rate of thermal fluid delivery,  $c_p$  - thermal fluid heat capacity,

U - total coefficient of heat losses,

F' - coefficient of efficiency of the collector,

T<sub>f in</sub> - the incoming temperature of thermal fluid,

T<sub>f in</sub> - ambient temperature.

F' depends on collectors geometrical characteristics, Uc, the convection coefficient in collectors tubes, frame parameters and oth.

For some of the parameters mentioned above like the total coefficient of heat losses Uc and the convection coefficient in collectors tubes there is no possibility always to obtain a precise value for the analysis, so there may be a decay in precision. The collectors heat efficiency can be calculated with less number of parameters, compensated with an experimental measurement used for a proper numerical model like boundary conditions.

# Heat Efficiency of a Flat Solar Collector Calculation with a Numerical Model

The processes in the solar collectors of the flat type may be treated like steady state or quasi steady state, so they are steady state for a long period of time and after the parameters have changed. Also the longitude cross-section of the frame is the same. Different may be the thermal loading. So the problem in computational aspect can be simplified as a steady state heat transfer problem in Cartesian coordinate system. Something more, when the irradiance is also symmetrical, only the half of the frame may be of interest.

The heat load for the numerical model needed may be defined Fig.1 and Fig. 2. Incoming heat flow by solar radiation, decreased with the influence of optical characteristics of the cover and the absorber surface values - contour  $G_1$ . Heat flow corresponding to the heat losses of the lower part of the absorber, contacting with the insulation - contour  $G_2$ . Temperature of the internal side of the absorbers tube - contour  $G_3$  and zones of a zero value of the heat flow in symmetry line - contour  $G_4$ .

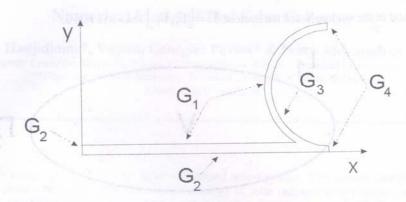


Fig. 1 Cross-section of the absorber of a flat solar collector

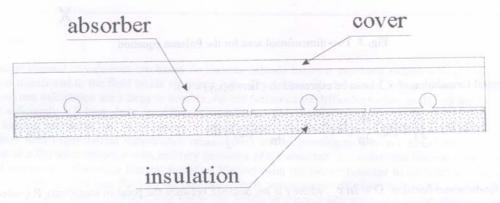


Fig. 2 Cross-section of a flat solar collector

Practically it is difficult to obtain the values for the contour  $G_3$ . To avoid this difficulty may be perform temperature measurement of the outer side of the absorbers tube. The collector materials are more often Al or Cu with low thickness of the tube which corresponds to a temperature difference between outer and internal side of the tube about a 0.1 - 0.3 °C. On the other hand a correct temperature measurement of the outer side of the tube may be performed. This measurement is very important. It renders the integral impact of solar irradiance and temperature of the thermal fluid (not used in the numerical model). With so formulated thermal loading a mathematical method has to be found to obtain the "heat flux" on the internal side of the absorbers tube (contour  $G_3$ ). This opportunity gives the formulation of the Boundary Integral Equation Method (BIEM) [2], [3]. The equation of the steady state heat transfer is used.

$$\frac{\partial^2 \mathbf{T}}{\partial \mathbf{x}^2} + \frac{\partial^2 \mathbf{T}}{\partial \mathbf{y}^2} = \mathbf{b}(\mathbf{x}, \mathbf{y})$$

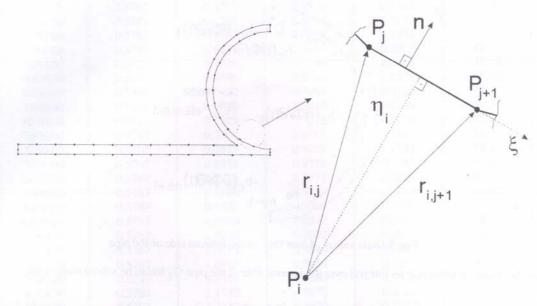


Fig. 4 Discretization of the boundary

The boundary integral (4) for the segment  $j \rightarrow j+1$  is expressed as follows:

$$I_{e} = \int_{\xi_{i}}^{\xi_{j+1}} \left( \frac{\Phi}{\mathbf{r}_{i}} \frac{\partial \mathbf{r}_{i}}{\partial \mathbf{n}} - \frac{\partial \Phi}{\partial \mathbf{n}} \ln \mathbf{r}_{i} \right) \cdot d\xi = \left| \mathbf{K}_{1}^{e} \right| \left\{ \frac{\Phi_{j}}{\Phi_{j+1}} \right\} - \left| \mathbf{K}_{2}^{e} \right| \left\{ \frac{(\partial \Phi / \partial \mathbf{n})_{j}}{(\partial \Phi / \partial \mathbf{n})_{j+1}} \right\}. \tag{5}$$

 $|\mathbf{K}_1^e|$  and  $|\mathbf{K}_2^e|$  are the matrices of the nodes "j" and "j+1" respectively for the linear element (segment). Finally a system of algebraic equations is formed with unknowns  $\Phi_j$  and  $(\partial \Phi / \partial n)_j$ , where  $j = 1 \dots n$ , n is the number of the nodes.

$$[\mathbf{A}]\{\mathbf{x}\} = \{\mathbf{b}\}\tag{6}$$

where,

 $\{x\}$  is the vector of the unknowns  $\Phi_j$  on  $\Gamma_{\!_2}$  and  $\big(\partial\Phi\,/\,\partial n\big)_j$  on  $\Gamma_{\!_1}$  ,

 $\{b\}$  is the vector of the known values  $\Phi_j$  on  $\Gamma_l$  and  $\left(\partial\Phi/\partial n\right)_j$  on  $\Gamma_2$ .

## Useful Heat Flow for the Collector Efficiency Calculation

After getting the decision of (6) we have the node values of the heat flux on the internal side of the pipe  $G_3$  shown in Fig. 5.

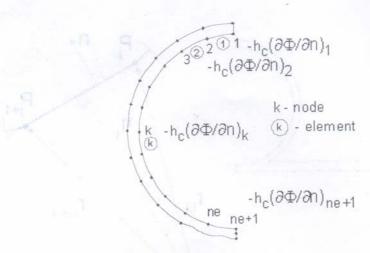


Fig. 5 Node values of heat flux on the internal side of the pipe

For the whole cross-section an integral over the internal side of the pipe G<sub>3</sub> has to be solved numerically.

$$\mathbf{q_u} \approx -2\mathbf{h_c} \sum_{i=1}^{\text{ne-l}} \left( \frac{\left( \partial \Phi / \partial \mathbf{n} \right)_i + \left( \partial \Phi / \partial \mathbf{n} \right)_{i+1}}{2} . \mathbf{l_i} \right)$$
 (7)

where,

ne are the number of the elements on  $G_3$  and  $l_i$  is the length of the element i. The number 2 is for the symmetry of the cross-section ( the model consists only of one half ).

### **Numerical Example**

A simple absorber of flat solar collector is analyzed with the following characteristics:

	material of the absorber:	Alluminium, $h_c = 230$ , W/(m.K);
_	frame length:	50, mm;
41	frame thickness :	2, mm;
-	tube internal diameter:	14, mm;
-	tube thickness:	1, mm;
_	solar radiation : Tana (n(\ an) box of no	215, W/m <sup>2</sup> ;
_	emissivity of the plate:	0,92, -
	optical characteristic of the cover :	0,86, -
-	temperature on the inner surface	30°C.

For comparison two numerical models are realized. One is with above mentioned BIEM formulation, the other is a Finite Element model, based on COSMOS/M software [3]. The discretization of both models is shown on Fig. 6 and Fig. 7.

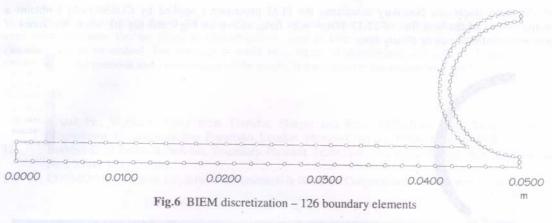




Fig.7 FEM discretization - 967 triangle elements, 630 nodes

The BIEM formulation (linear steady state heat conduction problem, b(x,y) = 0, (3.1) ) gives the following distribution of heat flux on the inner surface of the tube, Fig. 8.

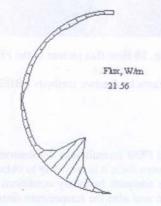


Fig.8 Heat flux distribution and useful heat flux, BIEM formulation. Maximum value of the flux  $-11,72~\text{W/m}^2$ 

With the same shape and boundary conditions the FEM procedure (applied by COSMOS/M) obtains a maximum value of the heat flux of 12,17  $W/m^2$  with field, shown on Fig.9 and Fig 10, where the zones of maximal heat influence ca be clearly seen.



Fig.9 Heat flux picture of the FEM model, whole picture

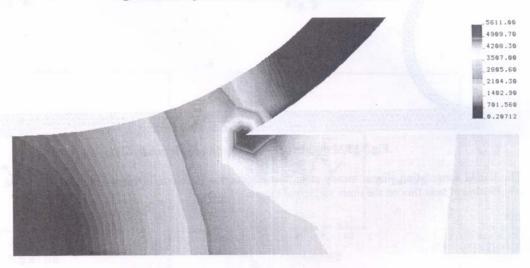


Fig. 10 Heat flux picture of the FEM model, zoomed

The temperature on the end of the frame by the two methods ( BIEM and FEM ) is almost equal : 30,39 °C for BIEM and 30,4 °C for FEM.

### Conclusions

The comparison between BIEM and FEM formulations demonstrates the capabilities of the modern numerical methods. With small difference between them it is very easy to obtain a correct decision for the useful heat flux. The only obstacle is to find out the adequate boundary conditions in order to satisfy the real behavior of the absorber, exposed on solar radiation and also the temperature distribution on the internal surface of the tube. With a precise temperature and solar radiation measurement it will be very easy to build a 3-Dimensional model

of a collector with arbitrary shape of the absorber, which cause no problems implementing a numerical method even with nonlinear thermo physical characteristics. This is important in the cases where the composite absorbers are to be treated. For instance it could be a frame of Aluminium and tube of Copper. Also the realization of the methods and visualization of the results is very suitable for educational needs.

### References

- [1] Kreith Fr., W.Black, Basic Heat Transfer, Harper and Row, Publishers, New York, Cambridge, Hagerstown, Philadelphia, San Francisko, London, Mexico City, Sao Paulo, Sidney, 1984.
- [2] Brebbia C., J.Telles, L.Wrobel, Boundary Element Techniques, Springer-Verlag, Berlin, Heidelberg, New York, Tokyo, 1984.
- [3] COSMOS/M, Version 1.7, Structural Research & Analysis Corporation, Los Angeles, California.