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Assignment 2.1

Que-1]

solution -> considering markov's chain given in the Question

$\{1, 2, 3, 4\}$

(a)

$$Q = \begin{pmatrix} 0.5 & 0.5 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 \\ 0 & 0 & 0.25 & 0.75 \\ 0 & 0 & 0.75 & 0.25 \end{pmatrix}$$

(b) our chain splits into two closed communicating classes $\{1,2\}$ and $\{3,4\}$. all four states are recurrent and there are no transient states.

$$\underline{\underline{P_{12} = 0.5}}$$

(c) on class $\{1,2\}$

$$\pi_1 = \pi_1 \times 0.5 + \pi_2 \times 0.25 \quad -(1)$$

$$\pi_2 + \pi_1 = 1 \quad -(2)$$

on solving equation (1) and (2)

$$\pi_1 = \frac{1}{3}, \quad \pi_2 = \frac{2}{3}$$

$$\pi^{(A)} = \left(\frac{1}{3}, \frac{2}{3}, 0, 0\right)$$

on class $\{3,4\}$

$$\pi_3 = \pi_3 \times 0.25 + \pi_4 \times 0.75 \quad \text{--- (3)}$$

$$\pi_4 + \pi_3 = 1 \quad \text{--- (4)}$$

on solving (3) and (4)

$\pi_3 = \frac{1}{2}$, $\pi_4 = \frac{1}{2}$ $\quad \pi^{(B)} = \left(0, 0, \frac{1}{2}, \frac{1}{2}\right)$

$$\pi = \lambda \pi^{(A)} + (1-\lambda)\pi^{(B)}, \quad 0 \le \lambda \le 1,$$

so, the two distinict stationary distribution giving Infinitely many solutions are

$$\pi^{(A)} = \left(\frac{1}{3}, \frac{2}{3}, 0, 0\right), \quad \pi^{(B)} = \left(0, 0, \frac{1}{2}, \frac{1}{2}\right).$$

Que-$\boxed{4}$

Solution→ $\deg(i)$ = number of legal moves from i

we have

1) corners = 4 squares , $\deg(\text{corners}) = 3$
2) Edge non-corners = 24 squares $\deg(\text{Edge.nc}) = 5$
3) Interior squares = 36 squares $\deg(\text{Interior}) = 8$

$$\pi_i \propto \deg(i)$$

and

$$\pi_i = \frac{\deg(i)}{\sum_{p\in S} \deg(p)}$$

$$\sum_{p\in S} \deg(p) = 4\times 3 + 24\cdot 5 + 36\times 8 = 420$$

π_i (for corner) $= \frac{1}{140}$

π_i (for edge non-corner) $= \frac{1}{84}$

π_i (for interior) $= \frac{2}{105}$

so the Stationary probablity of each type is

1) corner (total 4 squares)

π_i (corner) $= \frac{1}{140}$ total π per type $= 4\times\frac{1}{140} = \frac{\cancel{4}}{\cancel{140}_{35}} = \frac{1}{35}$

(2) Edge non-corner (total 24 squares)

$\pi_i = \frac{1}{84}$ total π per type $= \cancel{24}^{2} \times \frac{1}{\cancel{84}_{7}} = \frac{2}{7}$

(3) Interior (total 36 squares)

$\pi_i = \frac{2}{105}$ total π per type $= 36 \times \frac{2}{105} = \frac{72}{105} = \frac{24}{35}$

Q2 Win $\to$ W
Loss $\to$ L

Transition matrix = $\begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$

Let stationary distribution be

$\pi = [\pi_W, \pi_L]$

Solving $\pi P = \pi$ gives:

$\pi_W = 0.6$ & $\pi_L = 0.4$

(a) Long run proportion of wins = 0.6
(b) Proportion of dinners:

$0.6 \times 0.7 + 0.4 \times 0.2 = 0.5$

(c) $E[\text{no. of games for dinner}] = \frac{1}{0.5} = 2$

Q3

(a) Cat's transition matrix $= \begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix}$

$\pi_{cat} = [0.5 \quad 0.5]$

Mouse's Transition matrix $= \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix}$

$\therefore \pi_{mouse} = \left[\frac{2}{3}, \frac{1}{3}\right]$

(b) Since cat and mouse move independently

$Z_n = (cat, mouse)$

is a markov chain in 4^{th} stage

Md. Waqar Mohd,

Q.1) The Wandering King

Ans. • Corner (4) = 3 moves → $n = 3c$

• Edge (24) = 5 moves → $n = 5c$

• edge - adjacent (20) = 8 moves → $n = 8c$

• Center (16) : 8 moves → $n = 8c$

weights = $(4 \times 3 + 24 \times 5 + 20 \times 8 + 16 \times 8)c = 1$

$c = \frac{1}{420}$

center has sq. probability is $8 \times \frac{1}{420} = \frac{2}{105}$

Q.5) Size = 0.01

time = 5 hrs = 18,000 sec

steps = $\frac{18000}{5}$ = 3600

initial price = ₹120.

$P(X_{n+1} = X_n + 1) = 0.1$

$P(X_{n+1} = X_n - 1) = 0.05$

$P(X_{n+1} = X_n) = 0.85$

This is a biased walk,

as drift $(\mu) = (+1).0.1 + 0.(0.85) + (-1).0.05 = 0.1 - 0.05 = 0.05$

positive drift.

In a biased random walk with random drift, the chain is transient.

No. The stock price is not recurrent

b) No stationary distribution as walk is biased.