Least-squares Algorithm

$$(x_i - x_c)^2 + (y_i - y_c)^2 = r^2$$

Equation of circle:

- x_c is the x-coordinate of the center of the circle
- y_c is the y-coordinate of the center of the circle
- r is the radius of the circle

$$2x_c x_i + 2y_c y_i + r^2 - x_c^2 - y_c^2 = x_i^2 + y_i^2$$

Rearrange the circle equation

Let's suppose:

- $a = 2x_c$
- $b = 2y_c$
- $c = r^2 - x_c^2 - y_c^2$

Substitute a, b, c in circle equation.

$$ax_i + by_i + c = x_i^2 + y_i^2$$

The whole system (for all the points) can be rewritten as:

$$
\begin{aligned}
ax_1 + by_1 + c &= x_1^2 + y_1^2 \\
ax_2 + by_2 + c &= x_2^2 + y_2^2 \\
&\cdots \\
ax_n + by_n + c &= x_n^2 + y_n^2
\end{aligned}
$$

The matrix form of the system is given by:

$$
\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ \cdots & \cdots & \cdots \\ x_n & y_n & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} x_1^2 + y_1^2 \\ x_2^2 + y_2^2 \\ \cdots \\ x_n^2 + y_n^2 \end{bmatrix}
$$

$$A.X = B$$

Solve the system for X:

$$X = A^{-1}B$$

Find the center and radius from X:

$$x_c = \frac{a}{2}$$

$$y_c = \frac{b}{2}$$

$$r = \frac{\sqrt{4c + a^2 + b^2}}{2}$$

MATLAB CODE:

```
%% Measurement Results (unit: mm)
Points=[75.048  12.019
66.332  44.536
42.542  68.323
10.004  77.000
-22.465 68.329
-46.265 44.514
-54.999  12.022
-46.275 -20.490
-22.475 -44.254
10.035  -52.955
42.524  -44.278
66.323  -20.458
75.030  12.034];


%% Find Centriod and Radius of circle

% Guess Center of the circle
C = [5 , 20];

% Guess Raduis of the circle
R = 60;

% uncertainty of x and y coordinates is 20 µm convert to mm
k = 0.002;

%lenght of the Measurement Results
l=length(Points);

%Matric A = [x1, y2, 1] there will same number of row as x have
A = [Points(:,1) Points(:,2) ones(l,1)];
```

```
%Matric A = [x1^2, y2^2] there will same number of row as x have
B = [Points(:,1).*Points(:,1) + Points(:,2).*Points(:,2)];
% solve equation for Unknown
X=pinv(A)*B;

% Find the value of center and radius
xc = X(1)/2;
yc = X(2)/2;
r = sqrt(4*X(3) + X(1)*X(1) + X(2)*X(2) )/2;

fprintf("\nThe Center of circle is: ( %f , %f ) ",xc,yc)
fprintf("\nThe Radius of circle is:  %f \n",r)
%% Uncertainty calculation
% Add Uncertainity with point to get Uncertianty in centriod
x1=Points+k;
A = [x1(:,1) x1(:,2) ones(l,1)];
B = [x1(:,1).*x1(:,1) + x1(:,2).*x1(:,2)];
X=pinv(A)*B;
% Gives centriod when the Uncertianity is added
xc1 = X(1)/2;
yc1 = X(2)/2;
% Substract center from Uncertianity center
un_x= abs(xc1-xc);
un_y= abs(yc1-yc);
% Formula for Uncertianity of Radius
un_r= (k^2+k^2)^(1/2);
fprintf("\n\nUncertainty  in the Center of circle is: ( %f , %f ) ",un_x,un_y)
fprintf("\nThe Uncertainty  in the Radius of circle is:  %f \n",un_r)
```

Results:

```
>> Bhavana_Task2

The Center of circle is: ( 10.028101 , 12.027256 )
The Radius of circle is:  65.003507


Uncertainty  in the Center of circle is: ( 0.002000 , 0.002000 )
The Uncertainty  in the Radius of circle is:  0.002828
>>
```