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Power Allocation and Performance Analysis of Cooperative Spatial Modulation in Wireless Relay Networks

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ABSTRACT In this paper, the error performance analysis of an amplify-and-forward (AF) relay-aided cooperative multi-antenna system using spatial modulation (SM) over Rayleigh channels is presented. The SM is a simple and spectral efficient modulation technique that can increase the data rate by using input data bits to select a transmit antenna to send a constellation point. The relay transmission deploying SM not only can enhance the coverage and but also can improve the spectral efficiency. In the performance analysis, the overall average bit error rate (BER), which is related to the error probability of an antenna index detection (P_a) and the error probability of symbol detection (P_d) given transmit antenna index, is analyzed. With the derivation of the probability density function and the moment generating function of output signal-to-noise ratio (SNR), closed-form expressions of P_a and P_d are obtained to compute the overall BER. With the obtained BER expression, asymptotical BER expressions are derived to reveal the BER performance of the system at high SNR. By minimizing the asymptotical BER, a suboptimal power allocation (PA) scheme is developed. The diversity gain is analyzed at high SNR. The results indicate that the cooperative AF-SM can achieve full diversity order. Simulation results verify the effectiveness of the theoretical BER analysis, and the cooperative AF-SM with PA scheme outperforms upon the equal PA scheme.

INDEX TERMS Spatial modulation, cooperative communication, amplify and forward relaying, power allocation, average BER.

I. INTRODUCTION

With the increasing demand of wireless data communication services, it becomes imperative for next generation wireless communication to provide high data rate and link reliability. Cooperative communication technology is proposed to increase the network coverage of wireless communication systems, and has earned much attention [1]. Amplify-and-forward (AF) protocol [2], as a simple and effective cooperative strategy, has been widely used for cooperative communication. In this protocol, a relay amplifies and transfers the signals received from the source, and the destination combines the signals from the source and the relay to estimate the transmitted symbol.

Spatial modulation (SM), of which the transmitted data bits are used to select a transmit antenna and a constellation point for transmission, can avoid inter-channel interference and synchronization requirement of transmit antennas. The SM scheme was firstly proposed in [3]. In this scheme, only one active transmit antenna is used for transmitting information at each time slot, and the transmitted bits are encoded to represent the constellation symbols and active antenna index. At the receiver, the active transmit antenna index and the transmitted symbol are estimated by a maximum likelihood (ML) detector. However, the conventional SM scheme does not consider the superiority of cooperative communication, and thus its performance will be limited, especially for long-distance communication.

For this reason, considering the simplicity of Space Shift Keying (SSK) scheme (a special SM, where only active antenna index conveys information), some relay-aided

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SSK schemes were presented in [4]–[6]. In [4], the bit error rate (BER) performance of cooperative SSK in multi-input-multi-output (MIMO) system with decode-and-forward relaying was analyzed. In [5] and [6] cooperative SSK schemes with AF relaying were proposed, and the corresponding upper bound of BER was derived. Since the SSK only uses the transmit antenna index for carrying information and does not consider the higher order modulation, the throughput of the system using these cooperative SSK schemes is limited. Moreover, the schemes above basically consider a single receive antenna at the destination for simplicity. This further limits the system reliability performance.

Equal power allocation is commonly applied to the source node and relay node in cooperative communication for simplicity, which is not effective since the path gains of the respective channels from the source and the relay to the destination are mostly different. In [7], by minimizing an asymptotically tight approximation of symbol error rate (SER), optimal power allocation schemes were obtained for AF and decode-and-forward cooperative systems. In [8], considering channel path loss, the energy distribution ratio between the relay and direct link was optimized such that the quality of received signal is maintained with minimum total transmission energy consumption. However, these works do not consider the superiority of the SM scheme, which can further enhance the spectral efficiency. To the best of our knowledge, there are very few works addressing the power allocation in cooperative spatial modulation system.

Based on the discussion above, the relay-aided cooperative AF system using SM (AF-SM) with optimized PA over Rayleigh channels is investigated in this paper. For this cooperative AF-SM system, we derive a closed-form BER formula and develop a suboptimal power allocation (PA) scheme based on the asymptotic analysis at high SNR. With this PA scheme, the BER performance is greatly improved in comparison with the equal power scheme. The main contributions of this paper are summarized as follows:

- The error probability of the relay-aided AF-SM system in a Rayleigh fading channel is derived. The probability density function (PDF) and moment generating function (MGF) of output SNR are developed. With these functions, the error probability of antenna index detection and the error probability of symbol detection, which constitute the overall average BER, are obtained. Based on these results, a closed-form average BER is derived. Simulation results show that the obtained theoretical expressions are valid and agree well with computer simulations.
- By asymptotic performance analysis of the system at high SNR, the asymptotic average BER is derived. As a result, an approximate closed-form asymptotic BER expression is obtained. By minimizing this approximate BER expression, a suboptimal power allocation to the source and relay is derived, and its effect on the BER performance of the AF-SM system can be well evaluated. Simulation results indicate that the cooperative AF-SM system with the developed power allocation scheme outperforms upon the conventional equal power allocation scheme.
- The diversity gain of the cooperative AF-SM system is analyzed. Based on the obtained asymptotic BER formula at high SNR, we derive the diversity order of the AF-SM system. The result shows that the diversity order of N_r+1 is obtained for the AF-SM system with N_r receive antennas.

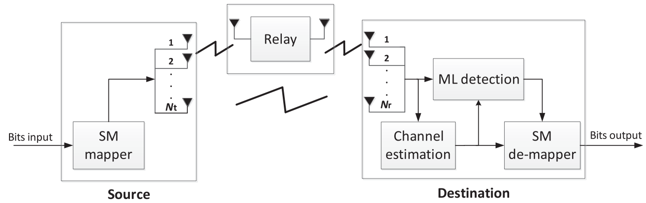


FIGURE 1. Cooperative AF-SM system model.

The notations in this paper are summarized as follows. The superscripts $(.)^T$ and $(.)^H$ represent matrix transposition and conjugate transposition, respectively. Bold lower case and upper case letters represent column vectors and matrices, respectively. $\|.\|_F^2$ is the Frobenius norm.

II. SYSTEM AND CHANNEL MODELS

By combining spatial modulation and cooperative communication with AF protocol, a two-hop half-duplex cooperative AF-SM system with direct link is presented. The system model is illustrated in Fig.1, and it includes one source node with N_t transmit antennas, one destination node with N_r receive antennas, and one relay node with single antenna. In the first hop, the source transmits signals to the destination and the relay, while in the second hop, only the relay transmits signals to the destination.

As shown in Fig. 1, at the source, the SM mapper takes $m = \log_2(N_t) + \log_2(M)$ input bits to select one transmit antenna and one M-ary constellation point for transmission. The output of SM mapper that selects the i-th transmit antenna for transmission can be expressed as [9]

$$\mathbf{x}_{iq} = [0\;0\;\cdots\;x_q\;\cdots\;0]^T \tag{1}$$

where $\mathbf{x}_{iq}$ is an N_t-dimensional symbol vector, x_q is the i-th element of $\mathbf{x}_{iq}$ representing the q-th element of the M-ary constellation with $q \in [1:M]$.

The two-hop transmission can be described as follows. In the first hop, the i-th transmit antenna of the source transmits x_q to the relay and the destination over the i-th path of $N_t \times 1$ $\mathbf{h}_{sr}$ and the i-th column of $N_r \times N_t$ $\mathbf{H}_{sd}$ Rayleigh fading channels, respectively. Let P_s denote the transmitted power of the source, so the received signals at the relay and the destination can be, respectively, written as

$$y_{sr} = \sqrt{P_s}\mathbf{h}_{sr}^T\mathbf{x}_{iq} + n_{sr} \tag{2}$$

$$\mathbf{y}_{sd} = \sqrt{P_s}\mathbf{H}_{sd}\mathbf{x}_{iq} + \mathbf{n}_{sd}. \tag{3}$$

In the second hop, the relay amplifies and transfers the signal received in the first hop to the destination over an $N_r \times 1$ fading channel $\mathbf{h}_{rd}$. The received signal at the destination can be expressed as

$$\mathbf{y}_{rd} = \mathbf{h}_{rd}(Ay_{sr}) + \mathbf{n}_{rd}. \quad (4)$$

The entries of $\mathbf{h}_{sr}$, $\mathbf{H}_{sd}$, and $\mathbf{h}_{rd}$ respectively in (2), (3), and (4) are independent and identically distributed ($i.i.d.$) zero-mean complex Gaussian random variables (r.v.s) with respective variances δ_{sd}^2, δ_{sr}^2, and δ_{rd}^2. The variance of each channel coefficient is modeled as $\delta_{xy}^2 = d_{xy}^{-\alpha}$, where d_{xy} is the distance between nodes x and y, and α is the path-loss exponent. The noise n_{sr} in (2) and the entries of noise vectors $\mathbf{n}_{sd}$ and $\mathbf{n}_{rd}$ respectively in (3) and (4) are zero-mean complex Gaussian r.v.s with variance N_0. We define A as amplification factor and it can be expressed as

$$A = \sqrt{P_r/(P_s\delta_{sr}^2 + N_0)}, \quad (5)$$

where P_r is the transmission power of the relay. The total transmission power $P_t = P_s + P_r$, and SNR is defined as $\overline{\gamma} = P_t/N_0$.

Substituting (2) and (5) into (4) yields

$$\mathbf{y}_{rd} = \sqrt{P_s}A\mathbf{h}_{rd}(\mathbf{h}_{sr}^T\mathbf{x}_{iq}) + \mathbf{n}, \quad (6)$$

where $\mathbf{n} = (A\mathbf{h}_{rd}n_{sr} + \mathbf{n}_{rd})$, and its covariance matrix is given by $\mathbf{D} = N_0(A^2\mathbf{h}_{rd}\mathbf{h}_{rd}^H + \mathbf{I}_{N_r})$.

The correlated noise vector $\mathbf{n}$ in (6) is whitened by $N_0^{1/2}\mathbf{D}^{-1/2}$ to render the entries to be $i.i.d.$ Gaussian r.v.s of variance N_0 to facilitate ML detection. The optimal SM detector is thus given by $[\hat{i}, x_{\hat{q}}] = \arg\min_{i,q} \big[\|\mathbf{y}_{sd} - \sqrt{P_s}\mathbf{h}_{sd}^i x_q\|_F^2 - N_0\|\mathbf{D}^{-1/2}(\mathbf{y}_{rd} - A\sqrt{P_s}\mathbf{h}_{rd}h_{sr}^i x_q)\|_F^2\big]$. Explicitly,

$$[\hat{i}, x_{\hat{q}}] = \arg\min_{i,q} \big[P_s\|\mathbf{h}_{sd}^i x_q\|_F^2 - 2\sqrt{P_s}\Re\{\mathbf{y}_{sd}^H\mathbf{h}_{sd}^i x_q\} + G\|\mathbf{h}_{rd}\|_F^2|h_{sr}^i x_q|^2 - 2\sqrt{G}\Re\{(\tilde{\mathbf{y}}_{rd})^H\mathbf{h}_{rd}h_{sr}^i x_q\}\big] \quad (7)$$

where $x_{\hat{q}}$ and $\hat{i}$ denote the estimated symbol and transmit antenna index, respectively, $\mathbf{h}_{sd}^i$ and h_{sr}^i denote the i-th column of $\mathbf{H}_{sd}$ and the i-th entry of $\mathbf{h}_{sr}$, respectively, $\Re\{x\}$ is the real part of complex number x, $G = P_sA^2/(A^2\|\mathbf{h}_{rd}\|_F^2 + 1)$, and $\tilde{\mathbf{y}}_{rd} = \mathbf{y}_{rd}/\sqrt{A^2\|\mathbf{h}_{rd}\|_F^2 + 1}$. In (7), $\mathbf{D}^{-1} = N_0^{-1}(\mathbf{I}_{N_r} - \frac{A^2\mathbf{h}_{rd}\mathbf{h}_{rd}^H}{1+A^2\mathbf{h}_{rd}^H\mathbf{h}_{rd}})$ is applied. With this optimal detector, the transmit antenna index and the transmitted symbol can be optimally detected.

III. PERFORMANCE ANALYSIS OF COOPERATIVE AF-SM

In this section, we analyze the performance of the cooperative AF-SM system over a Rayleigh fading channel and derive a tight approximate BER expression. At the destination, the transmitted symbol and the active transmit antenna index are jointly estimated by the ML detector. Thus, the error performance depends on the error probability of the joint detection. Let P_c denote the probability of correct detection, of which both the symbol x_q and the transmit antenna index i are correctly detected. The probability of error is thus given as

$$P_e = 1 - P_c = 1 - Pr(x_q|\text{index} = i)Pr(\text{index} = i). \quad (8)$$

Let P_a be defined as the error probability of detecting the transmit antenna index, while P_d is defined as the error probability of symbol detection given the transmit antenna index. With (8), the overall average BER of the AF-SM system, P_e, can be expressed as

$$P_e = 1 - (1 - P_d)(1 - P_a) = P_a + P_d - P_aP_d. \quad (9)$$

A. ERROR PROBABILITY OF TRANSMITTED SYMBOL DETECTION (P_d)

In this subsection, the error probability of transmitted symbol detection given the transmit antenna index, P_d, is analyzed and derived. Given the transmit antenna index, P_d of the system for MQAM modulation can be expressed as

$$P_d = \int_0^\infty BER(\gamma)f_{\gamma_{out}}(\gamma)d\gamma, \quad (10)$$

where $BER(\gamma)$ is the BER expression of MQAM in the AWGN channel with the system output SNR, γ, given by

$$BER(\gamma) = \sum_{l}^{\pi(M)} \alpha_l \text{erfc}(\sqrt{\beta_l\gamma}), \quad (11)$$

where erfc(.) denotes the complementary error function, $\{\alpha_l, \beta_l, \pi(M)\}$ are the coefficients associated with specific MQAM of constellation size M [10], [11].

Based on (3) and (6), using the whitening filter, the system output SNR at the destination, γ_{out}, can be written as

$$\begin{aligned}\gamma_{out} &= \gamma_{sd} + \gamma_{srd} = P_s\|\mathbf{h}_{sd}^i\|_F^2/N_0 + A^2P_s|h_{sr}^i|^2\mathbf{h}_{rd}^H\mathbf{D}^{-1}\mathbf{h}_{rd} \\ &= \frac{P_s\|\mathbf{h}_{sd}^i\|_F^2}{N_0} + \frac{P_sP_r\|\mathbf{h}_{rd}\|_F^2|h_{sr}^i|^2}{N_0(P_r\|\mathbf{h}_{rd}\|_F^2 + P_s\delta_{sr}^2 + N_0)} \\ &= \gamma_{sd} + \gamma_{sr}\gamma_{rd}/(\gamma_{rd} + C) = \gamma_{sd} + \gamma_{srd},\end{aligned} \quad (12)$$

where $\gamma_{srd} = \gamma_{sr}\gamma_{rd}/(\gamma_{rd} + C)$, $\gamma_{sd} = P_s\|\mathbf{h}_{sd}^i\|_F^2/N_0$, $\gamma_{sr} = P_s|h_{sr}^i|^2/N_0$, $\gamma_{rd} = P_r\|\mathbf{h}_{rd}\|_F^2/N_0$, and $C = (P_s\delta_{sr}^2 + N_0)/N_0 = P_r/(A^2N_0)$.

Considering the Rayleigh fading channel, the PDFs of γ_{sd} and γ_{rd}, and γ_{sr} can be, respectively, given by

$$f_{\gamma_{sd}}(\gamma) = \frac{1}{\Gamma(N_r)\overline{\gamma}_{sd}}(\gamma\ /\overline{\gamma}_{sd})^{N_r-1}\exp(-\gamma/\overline{\gamma}_{sd}), \quad (13)$$

$$f_{\gamma_{rd}}(\gamma) = \frac{1}{\Gamma(N_r)\overline{\gamma}_{rd}}(\gamma\ /\overline{\gamma}_{rd})^{N_r-1}\exp(-\gamma/\overline{\gamma}_{rd}), \quad (14)$$

$$f_{\gamma_{sr}}(\gamma) = \exp(-\gamma/\overline{\gamma}_{sr})/\overline{\gamma}_{sr}, \quad (15)$$

where $\overline{\gamma}_{sd} = P_s\delta_{sd}^2/N_0$, $\overline{\gamma}_{sr} = P_s\delta_{sr}^2/N_0$, $\overline{\gamma}_{rd} = P_r\delta_{rd}^2/N_0$. By the definition of C in (12), we have $C = \overline{\gamma}_{sr} + 1$.

Correspondingly, the CDF of γ_{sr} is expressed as

$$F_{\gamma_{sr}}(\gamma) = 1 - \exp(-\gamma/\overline{\gamma}_{sr}). \quad (16)$$

With (13), the MGF of γ_{sd} is given by

$$M_{\gamma_{sd}}(s) = \mathscr{L}\{f_{\gamma_{sd}}(\gamma)\} = (1 + \overline{\gamma}_{sd}s)^{-N_r}, \quad (17)$$

where $\mathscr{L}\{.\}$ denotes the Laplace transform.

Using (14) and (16), the cumulative distribution function (CDF) of γ_{srd} is obtained as follows:

$$\begin{aligned} F_{\gamma_{srd}}(\gamma) &= Pr\left(\frac{\gamma_{sr}\gamma_{rd}}{\gamma_{rd}+C} < \gamma\right) \\ &= \int_0^\infty Pr\left(\frac{\gamma_{sr}\gamma_{rd}}{\gamma_{rd}+C} < \gamma|\gamma_{rd}\right) f_{\gamma_{rd}}(\gamma_{rd})d\gamma_{rd} \\ &= \int_0^\infty F_{\gamma_{sr}}\left(\gamma(\gamma_{rd}+C)/\gamma_{rd}\right) f_{\gamma_{rd}}(\gamma_{rd})d\gamma_{rd} \\ &= 1 - \frac{2}{\Gamma(N_r)} e^{-\frac{\gamma}{\overline{\gamma}_{sr}}} \left(\frac{\gamma C}{\overline{\gamma}_{sr}\overline{\gamma}_{rd}}\right)^{\frac{N_r}{2}} K_{N_r}\left(\sqrt{\frac{4\gamma C}{\overline{\gamma}_{sr}\overline{\gamma}_{rd}}}\right). \end{aligned} \tag{18}$$

where $K_v(.)$ is the v-th order modified Bessel function of the second kind [12].

With (18), the MGF of γ_{srd} is given by

$$\begin{aligned} M_{\gamma_{srd}}(s) &= s\mathscr{L}\{F_{\gamma_{srd}}(\gamma)\} \\ &= 1 - \frac{s\overline{\gamma}_{sr}\overline{\gamma}_{rd}N_r}{C} z^{(N_r+1)/2} e^{z/2} W_{-(N_r+1)/2, N_r/2}(z), \end{aligned} \tag{19}$$

where $z = C/[\overline{\gamma}_{rd}(s\overline{\gamma}_{sr}+1)]$, $W_{\lambda,\mu}(z)$ is the Whitakker function [12].

Substituting (11) and (12) into (10) yields

$$\begin{aligned} P_d &= \sum_{l}^{\pi(M)} \alpha_l \int_0^\infty \int_0^\infty erfc\left(\sqrt{\beta_l(x+y)}\right) f_{\gamma_{sd}}(x) f_{\gamma_{srd}}(y) dxdy \\ &= \sum_{l}^{\pi(M)} \frac{2\alpha_l}{\pi} \int_0^\infty \int_0^\infty \int_0^{\pi/2} \exp\left(-\frac{\beta_l}{\sin^2\theta}(x+y)\right) \\ &\quad \times f_{\gamma_{sd}}(x) f_{\gamma_{srd}}(y) d\theta dxdy \\ &= \sum_{l}^{\pi(M)} \frac{2\alpha_l}{\pi} \int_0^{\pi/2} M_{\gamma_{sd}}\left(\frac{\beta_l}{\sin^2\theta}\right) M_{\gamma_{srd}}\left(\frac{\beta_l}{\sin^2\theta}\right) d\theta \end{aligned} \tag{20}$$

Let $t = \sin\theta$, (20) can be rewritten as

$$\begin{aligned} P_d &= \sum_{l}^{\pi(M)} \frac{2\alpha_l}{\pi} \int_0^1 M_{\gamma_{sd}}\left(\frac{\beta_l}{t^2}\right) M_{\gamma_{srd}}\left(\frac{\beta_l}{t^2}\right) \frac{1}{\sqrt{1-t^2}} dt \\ &\cong \sum_{l}^{\pi(M)} \alpha_l N_p^{-1} \sum_{u=1}^{N_p} M_{\gamma_{sd}}(\beta_l/\phi_u^2) M_{\gamma_{srd}}(\beta_l/\phi_u^2), \end{aligned} \tag{21}$$

where N_p is the order of the Chebyshev polynomial and the abscissas ϕ_u is expressed as [13]

$$\phi_u = \cos((2u-1)\pi/(2N_p)). \tag{22}$$

Substituting (17) and (19) into (21) yields

$$\begin{aligned} P_d &\cong \sum_{l}^{\pi(M)} \alpha_l \sum_{u=1}^{N_p} N_p^{-1} (1+\overline{\gamma}_{sd}\frac{\beta_l}{\phi_u^2})^{-N_r} \\ &\quad \times \left[1 - \frac{\beta_l\overline{\gamma}_{sr}\overline{\gamma}_{rd}N_r}{\phi_u^2 C} \check{z}^{(N_r+1)/2} e^{\check{z}/2} W_{-(N_r+1)/2, N_r/2}(\check{z})\right], \end{aligned} \tag{23}$$

where $\check{z} = \phi_u^2 C/[\overline{\gamma}_{rd}(\beta_l\overline{\gamma}_{sr}+\phi_u^2)]$, and (21) is a closed-form expression of the error probability of symbol detection given transmit antenna index, P_d, and it will be shown to agree with simulation results well.

B. ERROR PROBABILITY OF TRANSMIT ANTENNA INDEX DETECTION (P_a)

In this subsection, we give the analysis of the error probability of transmit antenna index detection, P_a. The P_a is tightly bounded as [3] and [11]

$$P_a \le P_a^u = \sum_{i=1}^{N_t} \sum_{q=1}^{M} \sum_{\hat{i}=1,\neq i}^{N_t} \frac{N(i,\hat{i})}{MN_t \log_2(N_t)} PEP(i \to \hat{i}|x_q), \tag{24}$$

where $N(i,\hat{i})$ is the number of error bits between the estimated transmit antenna index $\hat{i}$ and transmit antenna index i, and $PEP(i \to \hat{i}|x_q)$ denotes its pairwise error probability (PEP) given that the transmitted symbol x_q is transmitted at the source. Equation (24) is also an accurate expression of P_a for $N_t = 2$, as shown in Appendix. According to the results in Appendix, the PEP can be expressed as

$$\begin{aligned} &PEP(x_i \to x_{\hat{i}}) \\ &= \frac{1}{\pi}\int_0^{\pi/2} M_{\tilde{\gamma}_{srd}}\left(|x_q|^2/(2\sin^2\theta)\right) \\ &\quad \times M_{\tilde{\gamma}_{sd}}\left(|x_q|^2/(2\sin^2\theta)\right) d\theta \\ &\cong \frac{1}{2N_p}\sum_{u=1}^{N_p} M_{\tilde{\gamma}_{srd}}\left(|x_q|^2/(2\phi_u^2)\right) M_{\tilde{\gamma}_{sd}}\left(|x_q|^2/(2\phi_u^2)\right) \end{aligned} \tag{25}$$

where ϕ_u is given in (22).

According to the definitions of $\tilde{\gamma}_{sd}$, $\tilde{\gamma}_{sr}$ and $\tilde{\gamma}_{rd}$ in the line below (65) for Rayleigh fading channel, the PDFs of $\tilde{\gamma}_{sd}$ and $\tilde{\gamma}_{rd}$ as well as the CDF of $\tilde{\gamma}_{sr}$ can, respectively, be expressed as

$$f_{\tilde{\gamma}_{sd}}(\gamma) = \frac{1}{\Gamma(N_r)\overline{\gamma}_{sd}} (\gamma/\overline{\gamma}_{sd})^{N_r-1} \exp(-\gamma/\overline{\gamma}_{sd}), \tag{26}$$

$$f_{\tilde{\gamma}_{rd}}(\gamma) = \frac{1}{\Gamma(N_r)\overline{\gamma}_{rd}} (\gamma\ /\overline{\gamma}_{rd})^{N_r-1} \exp(-\gamma/\overline{\gamma}_{rd}), \tag{27}$$

$$F_{\tilde{\gamma}_{sr}}(\gamma) = 1 - \exp(-\gamma/\overline{\gamma}_{sr}). \tag{28}$$

Using (26) and the Laplace transform, the MGF of $\tilde{\gamma}_{sd}$ is expressed as

$$M_{\tilde{\gamma}_{sd}}(s) = (1+\overline{\gamma}_{sd}s)^{-N_r}. \tag{29}$$

Like the derivation in (18), the CDF of $\tilde{\gamma}_{srd}$ in (65) can be derived as

$$F_{\tilde{\gamma}_{srd}}(\gamma) = 1 - \frac{2}{\Gamma(N_r)} e^{-\frac{\gamma}{\overline{\gamma}_{sr}}} \left(\frac{\gamma C}{\overline{\gamma}_{sr}\overline{\gamma}_{rd}}\right)^{\frac{N_r}{2}} K_{N_r}\left(\sqrt{\frac{4\gamma C}{\overline{\gamma}_{sr}\overline{\gamma}_{rd}}}\right). \tag{30}$$

Hence, by using the Laplace transform, we can obtain the MGF of $\tilde{\gamma}_{srd}$ as

$$\begin{aligned}M_{\tilde{\gamma}_{srd}}(s) &= s\mathscr{L}\{F_{\tilde{\gamma}_{srd}}(\gamma)\}\\ &= 1-\frac{s\overline{\gamma}_{sr}\overline{\gamma}_{rd}N_r}{C}z^{(N_r+1)/2}e^{z/2}W_{-(N_r+1)/2,N_r/2}(z),\end{aligned} \tag{31}$$

where $C=\overline{\gamma}_{sr}+1$, $z=C/[\overline{\gamma}_{rd}(s\overline{\gamma}_{sr}+1)]$.

Substituting (29) and (31) into (25) yields

$$\begin{aligned}PEP(i\to\hat{i}|x_q) &\approx \sum_{u=1}^{N_p}\frac{1}{2N_p}\Big(1+\overline{\gamma}_{sd}\frac{|x_q|^2}{2\phi_u^2}\Big)^{-N_r}\\ &\times\Big[1-\frac{|x_q|^2}{2\phi_u^2}\frac{\overline{\gamma}_{sr}\overline{\gamma}_{rd}N_r}{C}\tilde{z}^{\frac{N_r+1}{2}}e^{\frac{\tilde{z}}{2}}W_{-(N_r+1)/2,N_r/2}(\tilde{z})\Big],\end{aligned} \tag{32}$$

where $\tilde{z}=2\phi_u^2\,C/[\overline{\gamma}_{rd}(|x_q|^2\overline{\gamma}_{sr}+2\phi_u^2)]$.

Substituting (32) into (24) gives

$$\begin{aligned}P_a \le \sum_{i=1}^{N_t}\sum_{q=1}^{M}\sum_{\hat{i}=1,\ne i}^{N_t}\frac{N(i,\hat{i})}{2MN_tN_p\log_2(N_t)}\sum_{u=1}^{N_p}\Big(1+\frac{|x_q|^2\overline{\gamma}_{sd}}{2\phi_u^2}\Big)^{-N_r}\\ \times\Big[1-\frac{\overline{\gamma}_{sr}\overline{\gamma}_{rd}N_r|x_q|^2}{2\phi_u^2C}\tilde{z}^{\frac{N_r+1}{2}}e^{\frac{\tilde{z}}{2}}W_{-(N_r+1)/2,N_r/2}(\tilde{z})\Big].\end{aligned} \tag{33}$$

Thus, a tight upper bound of P_a is achieved.

Substituting (23) and (33) into (9), the overall average BER of the system is obtained, and its values are shown to be close to simulation.

It is worth mentioning that though the derived BER is the error performance only for MQAM, the analysis method can be applied to MPSK to obtain the corresponding approximate average BER.

IV. POWER ALLOCATION SCHEME AND DIVERSITY GAIN

In this section, we develop a suboptimal power allocation scheme for the cooperative AF-SM system by asymptotic performance analysis. Based on the obtained P_a and P_d in Section III, an asymptotic average BER at high SNR is derived. With the asymptotic BER formula, a suboptimal power allocation scheme is developed for achieving superior performance. Also, the diversity gain of the cooperative AF-SM system can be obtained from the asymptotic BER analysis.

A. ASYMPTOTIC BER ANALYSIS

As the total BER consists of P_a and P_d, the asymptotic analysis firstly needs to derive approximate formulae of P_a and P_d at high SNR. For large SNR, the argument of the v-th order modified Bessel function of the second kind in (18) becomes small. Using (8.446) in [12], an approximate expression of the function $K_v(2x)$ for small x can be given by

$$\begin{aligned}K_v(2x) \approx \frac{1}{2}\sum_{j=0}^{v-1}(-1)^j\frac{(v-j-1)!}{j!}x^{2j-v}\\ +(-1)^{v+1}\frac{x^v}{v!}\Big[\ln(x)-\frac{1}{2}\psi(1)-\frac{1}{2}\psi(v+1)\Big],\end{aligned} \tag{34}$$

where $\psi(.)$ is the psi function [12].

Substituting (34) into (18), the CDF of γ_{srd} for large SNR can be approximated as

$$\begin{aligned}&F_{\gamma_{srd}}(\gamma)\\ &\approx 1-\frac{1}{\Gamma(N_r)}e^{-\frac{\gamma}{\overline{\gamma}_{sr}}}\sum_{j=0}^{N_r-1}\frac{(N_r-j-1)!}{j!}\Big(-\frac{\gamma}{\overline{\gamma}_{rd}}\Big)^j\\ &\quad-\Big(-\frac{\gamma}{\overline{\gamma}_{rd}}\Big)^{N_r}\frac{\ln(\gamma/\overline{\gamma}_{rd})-\psi(1)-\psi(N_r+1)}{N_r!}\\ &\approx\frac{\gamma}{\overline{\gamma}_{sr}}-\frac{1}{\Gamma(N_r)}\sum_{j=1}^{N_r-1}\frac{(N_r-j-1)!}{j!}\Big(-\frac{\gamma}{\overline{\gamma}_{rd}}\Big)^j\\ &\quad-\Big(-\frac{\gamma}{\overline{\gamma}_{rd}}\Big)^{N_r}\frac{\ln(\gamma)-\ln(\overline{\gamma}_{rd})-\psi(1)-\psi(N_r+1)}{N_r!}.\end{aligned} \tag{35}$$

Based on the approximation of the CDF $F_{\gamma_{srd}}(\gamma)$, the approximate MGF of γ_{srd} is given by

$$\begin{aligned}M_{\gamma_{srd}}(s) &\approx s\mathscr{L}\{F_{\gamma_{srd}}(s)\}\\ &=\frac{1}{\overline{\gamma}_{sr}s}-\frac{1}{\Gamma(N_r)}\Big(\sum_{j=1}^{N_r-1}(-1)^j\frac{\Gamma(N_r-j)}{(\overline{\gamma}_{rd}s)^j}\\ &\quad+(-1)^{N_r}\frac{\ln(\overline{\gamma}_{rd}s)+\psi(1)}{(\overline{\gamma}_{rd}s)^{N_r}}\Big).\end{aligned} \tag{36}$$

Substituting (36) and (17) into (20) yields

$$\begin{aligned}P_d \approx \sum_{l}^{\pi(M)}\frac{\alpha_l}{\pi\overline{\gamma}_{sd}^{Nr}}\int_0^{\pi/2}\Big[\frac{1}{\overline{\gamma}_{sr}}\Big(\frac{\beta_l}{\sin^2\theta}\Big)^{-N_r-1}\\ -\sum_{j=1}^{N_r-1}\frac{\Gamma(N_r-j)}{\Gamma(N_r)}\Big(-\frac{1}{\overline{\gamma}_{rd}}\Big)^j\Big(\frac{\beta_l}{\sin^2\theta}\Big)^{-N_r-j}\\ -\frac{1}{\Gamma(N_r)}\Big(-\frac{1}{\overline{\gamma}_{rd}}\Big)^{N_r}\Big(\frac{\beta_l}{\sin^2\theta}\Big)^{-2N_r}\\ \times\Big(\psi(1)+\ln\Big(\frac{\beta_l\overline{\gamma}_{rd}}{\sin^2\theta}\Big)\Big)\Big]d\theta.\end{aligned} \tag{37}$$

Using the integral formulae [i.e., [12], eqs. (3.621.3) and (4.387.4)], (37) can be further simplified as

$$\begin{aligned}P_d \approx \sum_{l}^{\pi(M)}\frac{\alpha_l}{\overline{\gamma}_{sd}^{Nr}}\Big[\frac{1}{\overline{\gamma}_{sr}\beta_l^{N_r+1}}\frac{(2N_r+1)!!}{(2N_r+2)!!}\\ -\sum_{j=1}^{N_r-1}\frac{\Gamma(N_r-j)}{\Gamma(N_r)\beta_l^{N_r+j}}\Big(-\frac{1}{\overline{\gamma}_{rd}}\Big)^j\frac{(2N_r+2j-1)!!}{(2N_r+2j)!!}\\ -\frac{1}{\Gamma(N_r)\beta_l^{2N_r}}\Big(-\frac{1}{\overline{\gamma}_{rd}}\Big)^{N_r}\frac{(4N_r-1)!!}{(4N_r)!!}\\ \times\Big(\psi(1)+\ln(4\beta_l\overline{\gamma}_{rd})-2\sum_{m=1}^{4N_r}\frac{(-1)^{m+1}}{m}\Big)\Big],\end{aligned} \tag{38}$$

where $n!! = \prod_{k=0}^{\lceil n/2\rceil -1}(n-2k)$ is double factorial of n.

Equation (38) is an asymptotic approximation of P_d at high SNR. Similarly, we can derive an asymptotic approximation of P_a at high SNR as

$$P_a \approx \sum_{i=1}^{N_t}\sum_{q=1}^{M}\sum_{\hat{i}=1,\neq i}^{N_t} \frac{N(i,\hat{i})\overline{\gamma}_{sd}^{-Nr}}{2MN_t\log_2(N_t)}\Big[\frac{1}{\overline{\gamma}_{sr}\mu_q^{N_r+1}}\frac{(2N_r+1)!!}{(2N_r+2)!!} - \sum_{j=1}^{N_r-1}\frac{(-1)^j\Gamma(N_r-j)}{\overline{\gamma}_{rd}^{j}\mu_q^{N_r+j}\Gamma(N_r)}\frac{(2N_r+2j-1)!!}{(2N_r+2j)!!} - \frac{(-1)^{N_r}}{\Gamma(N_r)\overline{\gamma}_{rd}^{N_r}\mu_q^{2N_r}}\frac{(4N_r-1)!!}{(4N_r)!!} \times\Big(\psi(1)+\ln(4\overline{\gamma}_{rd}\mu_q)-2\sum_{m=1}^{4N_r}\frac{(-1)^{m+1}}{m}\Big)\Big] \quad (39)$$

where $\mu_q = |x_q|^2/2$.

For $N_r = 1$, with (38) and (39), the P_a and P_d can be respectively reduced to

$$P_d \approx \sum_{l}^{\pi(M)}\frac{3\alpha_l}{8\overline{\gamma}_{sd}}\beta_l^{-2} \times\Big(\frac{1}{\overline{\gamma}_{sr}}+\frac{1}{\overline{\gamma}_{rd}}\big[\ln(\overline{\gamma}_{rd})+\ln(4\beta_l)+\psi(1)-\frac{7}{6}\big]\Big), \quad (40)$$

$$P_a \approx \sum_{i=1}^{N_t}\sum_{q=1}^{M}\sum_{\hat{i}=1,\neq i}^{N_t}\frac{N(i,\hat{i})}{MN_t\log_2(N_t)}\frac{3\mu_q^{-2}}{16\overline{\gamma}_{sd}} \times\Big(\frac{1}{\overline{\gamma}_{sr}}+\frac{1}{\overline{\gamma}_{rd}}\big[\ln(\overline{\gamma}_{rd})+\ln(4\mu_q)+\psi(1)-\frac{7}{6}\big]\Big) \quad (41)$$

For large SNR $\overline{\gamma}$, the constants in comparing with $\ln(\overline{\gamma}_{rd})$ in the square brackets of (40) and (41) can be neglected, and thus they can be further simplified as

$$P_d \approx \sum_{l}^{\pi(M)}\frac{3\alpha_l}{8\overline{\gamma}_{sd}}\beta_l^{-2}(\overline{\gamma}_{sr}^{-1}+\overline{\gamma}_{rd}^{-1}\ln\overline{\gamma}_{rd}) \quad (42)$$

and

$$P_a \approx \sum_{i=1}^{N_t}\sum_{q=1}^{M}\sum_{\hat{i}=1,\neq i}^{N_t}\frac{N(i,\hat{i})}{MN_t\log_2(N_t)}\frac{3\mu_q^{-2}}{16\overline{\gamma}_{sd}}(\overline{\gamma}_{sr}^{-1}+\overline{\gamma}_{rd}^{-1}\ln\overline{\gamma}_{rd}). \quad (43)$$

At high SNR, the product of P_a and P_d in (9) is very small and can be neglected when compared to the values of P_a and P_d. Hence, P_e in (9) can be approximated as $P_e \approx P_a + P_d$. Thus, with (42) and (43), P_e can be asymptotically approximated as

$$P_e \approx P_a + P_d = \frac{3}{8\overline{\gamma}_{sd}}\Big(\frac{1}{\overline{\gamma}_{sr}}+\frac{1}{\overline{\gamma}_{rd}}\ln\overline{\gamma}\Big) \times\Big(\sum_{i=1}^{N_t}\sum_{q=1}^{M}\sum_{\hat{i}=1,\neq i}^{N_t}\frac{N(i,\hat{i})\mu_q^{-2}}{2MN_t\log_2(N_t)}+\sum_{l}^{\pi(M)}\alpha_l\beta_l^{-2}\Big). \quad (44)$$

For $N_r \geq 2$, considering large SNR, (38) and (39) can be further approximated as

$$P_d \approx \sum_{l}^{\pi(M)}\frac{\alpha_l}{\overline{\gamma}_{sd}^{N_r}}\beta_l^{-N_r-1}\frac{(2N_r+1)!!}{(2N_r+2)!!}\Big(\frac{1}{\overline{\gamma}_{sr}}+\frac{1}{N_r-1}\frac{1}{\overline{\gamma}_{rd}}\Big), \quad (45)$$

$$P_a \approx \sum_{i=1}^{N_t}\sum_{q=1}^{M}\sum_{\hat{i}=1,\neq i}^{N_t}\frac{N(i,\hat{i})}{MN_t\log_2(N_t)}\frac{1}{2\overline{\gamma}_{sd}^{N_r}}(\mu_q)^{-N_r-1} \times\frac{(2N_r+1)!!}{(2N_r+2)!!}\Big(\frac{1}{\overline{\gamma}_{sr}}+\frac{1}{N_r-1}\frac{1}{\overline{\gamma}_{rd}}\Big). \quad (46)$$

Thus, the P_e can be asymptotically approximated as

$$P_e \approx \frac{1}{\overline{\gamma}_{sd}^{N_r}}\Big(\frac{1}{\overline{\gamma}_{sr}}+\frac{1}{N_r-1}\frac{1}{\overline{\gamma}_{rd}}\Big)\frac{(2N_r+1)!!}{(2N_r+2)!!} \times\Big(\sum_{i=1}^{N_t}\sum_{q=1}^{M}\sum_{\hat{i}=1,\neq i}^{N_t}\frac{N(i,\hat{i})(\mu_q)^{-N_r-1}}{2MN_t\log_2(N_t)}+\sum_{l}^{\pi(M)}\frac{\alpha_l}{\beta_l^{N_r+1}}\Big). \quad (47)$$

B. POWER ALLOCATION SCHEME

Based on the asymptotic expressions of P_e at high SNR in (44) and (47), suboptimal power allocation schemes for $N_r = 1$ and $N_r \geq 2$ are developed by minimizing the error probability.

Let $P_s = r_1P_t$ and $P_r = r_2P_t$, then the power control coefficients r_1 and r_2 are constrained as $r_1 + r_2 = 1$, r_1, $r_2 \in [0, 1]$. In the following, the optimal r_1 and r_2 are derived for cases $N_r = 1$ and $N_r \geq 2$.

For $N_r = 1$, using $P_s = r_1P_t$ and $P_r = r_2P_t = (1-r_1)P_t$, P_e in (44) can be rewritten as

$$P_e \approx \Big(\frac{1}{r_1^2\delta_{sr}^2}+\frac{1}{r_1(1-r_1)\delta_{rd}^2}\ln((1-r_1)\delta_{rd}^2\overline{\gamma})\Big)\Delta_1 \quad (48)$$

where $\Delta_1 = (\overline{\gamma}\delta_{sd})^{-2}\frac{3}{8}\Big(\sum_{i=1}^{N_t}\sum_{q=1}^{M}\sum_{\hat{i}=1,\neq i}^{N_t}\frac{N(i,\hat{i})}{2MN_t\log_2(N_t)} \times\Big(\frac{|x_q|^2}{2}\Big)^{-2}+\sum_{l}^{\pi(M_n)}\alpha_l\beta_l^{-2}\Big)$.

Taking the first derivative with respect to r_1 yields

$$\Upsilon(r_1) = \frac{\partial P_e}{\partial r_1} = \Big[-\frac{2}{r_1^3\delta_{sr}^2}+\frac{2r_1-1}{r_1^2(1-r_1)^2\delta_{rd}^2} \times\ln((1-r_1)\delta_{rd}^2\overline{\gamma})-\frac{1}{r_1(1-r_1)^2\delta_{rd}^2}\Big]\Delta_1. \quad (49)$$

By (49), we have:

$$\Upsilon(r_1) < 0 \text{for } r_1 \in (0, 0.5] \text{ or } r_1 \in [1-1/(\delta_{rd}^2\overline{\gamma}), 1)$$
$$\geq 0 \text{for } r_1 \in (0.5, 1-1/(\delta_{rd}^2\overline{\gamma})).$$

Thus, the P_e has a unique minimum value with the optimized $r_1 \in (0.5, 1-1/(\delta_{rd}^2\overline{\gamma}))$ for large $\overline{\gamma}$. Based on this result, the optimal r_1 for minimizing P_e can be obtained by using the following gradient descent method.

$$r_1^{(k+1)} = r_1^{(k)} - \tau \Upsilon(r_1^{(k)}), \tag{50}$$

where the initial value of r_1 is set equal to $0.5+\varepsilon$ with small ε, and τ is a step size. With the obtained r_1, the value of r_2 can be computed as $r_2 = 1 - r_1$. The resulting suboptimal power allocations for the system with single receive antenna ($N_r = 1$) are calculated as $P_s = r_1 P_t$ and $P_r = (1 - r_1)P_t$.

For $N_r \geq 2$, the P_e in (47) can be rewritten as

$$P_e \approx \Big(\frac{1}{r_1^{N_r+1}\delta_{sr}^2} + \frac{1}{N_r - 1}\frac{1}{r_1^{N_r}(1-r_1)\delta_{rd}^2}\Big)\Delta_2, \tag{51}$$

where $\Delta_2 = \frac{1}{\delta_{sd}^{2N_r}\overline{\gamma}^{N_r+1}}\frac{(2N_r+1)!!}{(2N_r+2)!!}\Big(\sum_{i=1}^{N_t}\sum_{q=1}^{M}\sum_{\hat{i}=1,\neq i}^{N_t}\frac{N(i,\hat{i})}{2MN_t\log_2(N_t)}(\frac{|x_q|^2}{2})^{-N_r-1} + \sum_l^{\pi(M)}\frac{\alpha_l}{\beta_l^{N_r+1}}\Big)$.

Using (51), the first and second order derivatives with respect to r_1 are computed as

$$\Upsilon(r_1) = \frac{\partial P_e}{\partial r_1} = \Big[-\frac{N_r+1}{r_1^{N_r+2}\delta_{sr}^2} + \frac{1}{N_r-1}\frac{1}{\delta_{rd}^2} \times \Big(-\frac{N_r}{r_1^{Nr+1}(1-r_1)} + \frac{1}{r_1^{Nr}(1-r_1)^2}\Big)\Big]\Delta_2 \tag{52}$$

and

$$\Upsilon'(r_1) = \frac{\partial^2 P_e}{\partial r_1^2} = \Big[\frac{(N_r+2)(N_r+1)}{r_1^{N_r+3}\delta_{sr}^2} + \frac{1}{(N_r-1)r_1^{N_r+2}(1-r_1)^3\delta_{rd}^2} \times \Big(N_r(N_r+1)(1-r_1-\frac{r_1}{N_r+1})^2 + \frac{r_1^2(N_r+2)}{N_r+1}\Big)\Big]\Delta_2. \tag{53}$$

As $\lim_{r_1\to 0}\Upsilon(r_1) < 0$, $\lim_{r_1\to 1}\Upsilon(r_1) > 0$, and $\Upsilon'(r_1) > 0$, P_e in (51) has a unique minimum for $r_1 \in [0, 1]$. By setting $\Upsilon(r_1) = 0$, we can obtain a quadratic equation as

$$r_1^2\big[N_r\delta_{sr}^2 + \delta_{sr}^2 - (N_r^2-1)\delta_{rd}^2\big] + r_1\big[2(N_r^2-1)\delta_{rd}^2 - N_r\delta_{sr}^2\big] - (N_r^2-1)\delta_{rd}^2 = 0. \tag{54}$$

Solving (54) yields

$$r_1 = \frac{2(N_r^2-1)\delta_{rd}^2}{2(N_r^2-1)\delta_{rd}^2 - N_r\delta_{sr}^2 + \sqrt{N_r^2\delta_{sr}^4 + 4\delta_{sr}^2\delta_{rd}^2(N_r^2-1)}}. \tag{55}$$

Obviously, the optimal r_1 obtained by (55) is between zero and one.

With the obtained r_1, the suboptimal power allocations $P_s = r_1P_t$ and $P_r = (1-r_1)P_t$ for the system with multiple receive antennas can be computed.

The suboptimal power allocation scheme will be shown to enhance the error performance of the cooperative system. In fact, the P_e in (9) with P_d in (23) and P_a in (33), the optimal power allocation coefficients can be minimized by means of the function fminbnd in Matlab. Compared to this optimal PA, the proposed suboptimal scheme has nearly optimal BER performance and can provide the closed-form calculation of PA coefficients. Thus, it is an efficient scheme.

Remark: From (49) and (55), it is observed that r_1 is larger than 0.5 (which corresponding to equal power allocation), and it can improve as N_r increases and tend to be one for very large N_r. Thus, the performance gain of the proposed PA scheme over the equal power scheme is also increased with the N_r increasing. In other words, when N_r is large, more power will be allocated to the source node, and correspondingly, less power will be allocated to the relay node. As a result, the relay node will be no longer needed for very large N_r.

C. DIVERSITY GAIN

In this subsection, we give the diversity gain analysis of the cooperative AF-SM system, and derive the diversity order. Let $P_s = r_1P_t$ and $P_r = r_2P_t$, $r_1, r_2 \in [0, 1]$, the asymptotic P_e in (47) at high SNR can be written as

$$P_e \approx \Big(\frac{1}{r_1^{N_r+1}\delta_{sr}^2} + \frac{1}{N_r-1} \times \frac{1}{r_1^{N_r}r_2\delta_{rd}^2}\Big)\frac{1}{\overline{\gamma}^{N_r+1}\delta_{sd}^{2N_r}}\frac{(2N_r+1)!!}{(2N_r+2)!!} \times \Big[\sum_{i=1}^{N_t}\sum_{q=1}^{M}\sum_{\hat{i}=1,\neq i}^{N_t}\frac{0.5N(i,\hat{i})}{MN_t\log_2(N_t)}(\frac{|x_q|^2}{2})^{-N_r-1} + \sum_l^{\pi(M)}\frac{\alpha_l}{\beta_l^{N_r+1}}\Big]. \tag{56}$$

With (56), the system diversity gain can be analyzed. The diversity gain is an important indicative of error performance, and is defined as the slope of the log-log curve of the average BER versus the SNR at high SNR [14], [15]. Thus, the diversity gain is

$$G_d = \lim_{\overline{\gamma}\to\infty} -\frac{\log(P_e)}{\log(\overline{\gamma})} = N_r + 1. \tag{57}$$

According to (57), the system can obtain the full diversity order of $N_r + 1$. Thus, G_d is increased with the number of receive antennas (N_r) at the destination.

V. SIMULATION RESULTS

In this section, we employ Monte Carlo simulation to verify the performance analysis and evaluate the error performance of the cooperative AF-SM system over Rayleigh fading channels, where the developed power allocation scheme is applied. In the simulation, the number of transmit antennas is $N_t = 2$, and the number of receive antennas N_r is set equal to 2 or 4. The normalized distances of the source-to-destination link, source-to-relay link, and relay-to-destination link are denoted by d_{sd}, d_{sr}, and d_{rd}, respectively, which are set to different ratios in the simulation. The path-loss exponent is $\alpha = 3$.

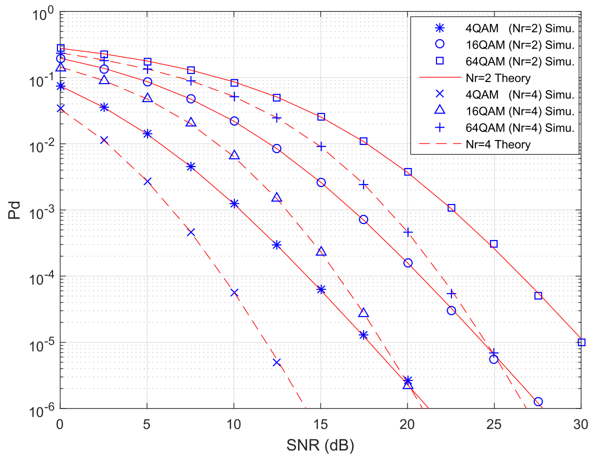


FIGURE 2. **The error probability of symbol detection (P_d) for $d_{SD} : d_{SR} : d_{RD} = 1 : 0.5 : 0.5$.**

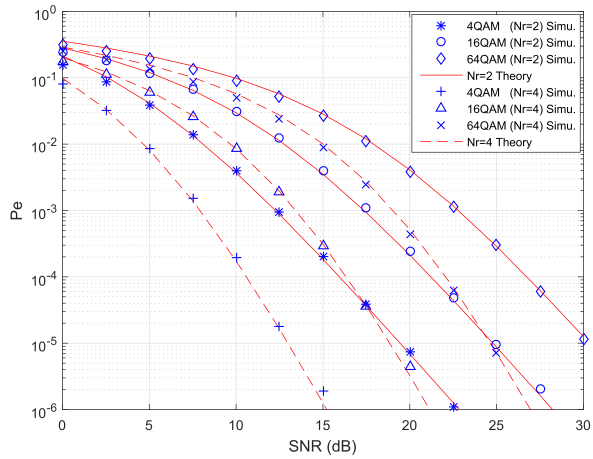


FIGURE 4. **Average BER of cooperative AF-SM system for $d_{SD} : d_{SR} : d_{RD} = 1 : 0.5 : 0.5$.**

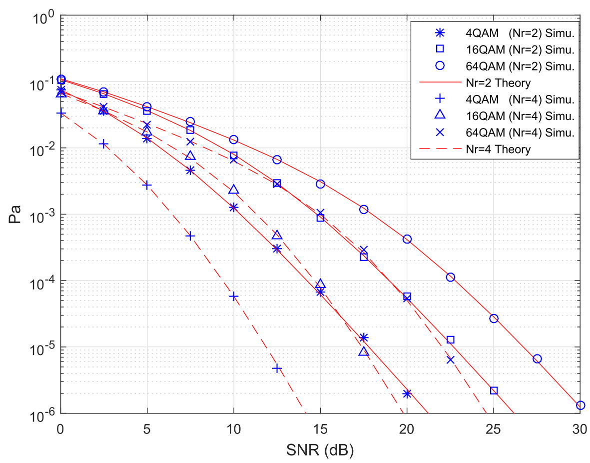


FIGURE 3. **The error probability of transmit antenna index detection (P_a) for $d_{SD} : d_{SR} : d_{RD} = 1 : 0.5 : 0.5$.**

The order N_p of Chebyshev polynomial in (21) and (25) is set equal to 5. With the equal PA scheme (i.e., $r_1 = r_2 = 0.5$), Fig.2, Fig.3, and Fig.4 show the simulation and theoretical values of P_a, P_d, and P_e, respectively. Figures 5 and 6 compare the performances of the optimal PA scheme, suboptimal PA scheme, and equal PA scheme for different receive antennas.

Figure 2 illustrates the error probability of symbol detection, P_d, of the cooperative AF-SM system for different receive antennas and modulation modes. The theoretical P_d is calculated by (23). As shown in Fig.2, the theoretical values agree well with the simulations. Moreover, the P_d of 64QAM is higher than that of 16QAM, and the P_d of 16QAM is higher than that of 4QAM. This is because higher-order modulation has lower output SNR due to the smaller Euclidean distance between constellation points. Besides, the P_d of the system with $N_r = 4$ is lower than that with $N_r = 2$ due to more diversity gain. The above results indicate the derived P_d is valid.

Figure 3 shows the error probability of the transmit antenna index detection, P_a, of the cooperative AF-SM system for different receive antennas and modulation modes. The theoretical P_a is calculated by (33). From Fig. 3, it is found that the analytical P_a matches the corresponding simulated one well. This is because P_a in (24) is an accurate expression for $N_t = 2$. Moreover, as the modulation size decreases, the P_a is decreased accordingly. Namely, the P_a of 4QAM is lower than that of 16QAM and the P_a of 16QAM is lower than that of 64QAM due to the larger Euclidean distance between the constellation points. Besides, the P_a of the system with $N_r = 4$ is lower than that with $N_r = 2$ because more receive antennas are employed with larger diversity gain. Based on the results above, the theoretical expression of P_a is also valid.

Figure 4 plots the overall average BER of the cooperative AF-SM system for different receive antennas, where 4QAM, 16QAM, and 64QAM are used for modulation. The theoretical P_e is calculated by (9) with (23) and (33). As Fig.4 shows, the derived average BER has the values close to the simulation due to the better approximation of theoretical P_a and P_d. Besides, as the modulation order increases, the BER performance becomes worse because of the increased P_a and P_d. Namely, the system with 4QAM has the lowest BER, and the system with 64QAM has the highest BER. As the number of receive antennas increases, the overall BER performance of the cooperative AF-SM system is increased. Thus, the system with $N_r = 4$ is superior to that with $N_r = 2$ due to larger diversity order.

In Fig. 5, Fig.6(a), and Fig.6(b), we plot the asymptotically approximate average BERs of the cooperative SM system with 64QAM for using different PA schemes for $N_r = 1$, $N_r = 2$, and $N_r = 4$, respectively. The equal PA scheme, optimal PA scheme and proposed suboptimal PA scheme are considered for comparison. In the evaluation, the PA coefficients of the optimal scheme are optimized to minimize the P_e in (9) based on (23) and (33) by using the function fminbnd in Matlab, while the suboptimal PA is obtained by the gradient descent method in (50) with step size set equal to 0.01 for $N_r = 1$. For Fig.5, the asymptotical P_e is calculated by (44).

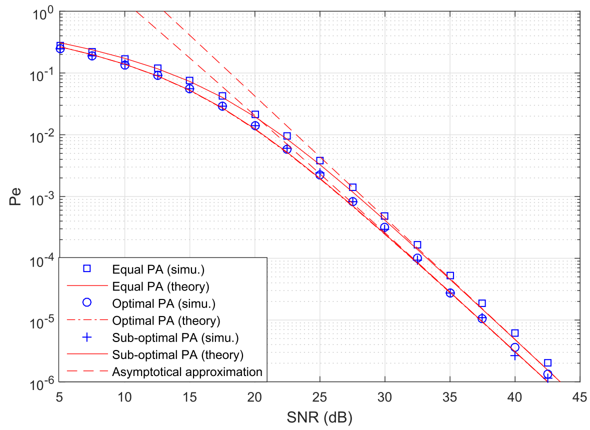
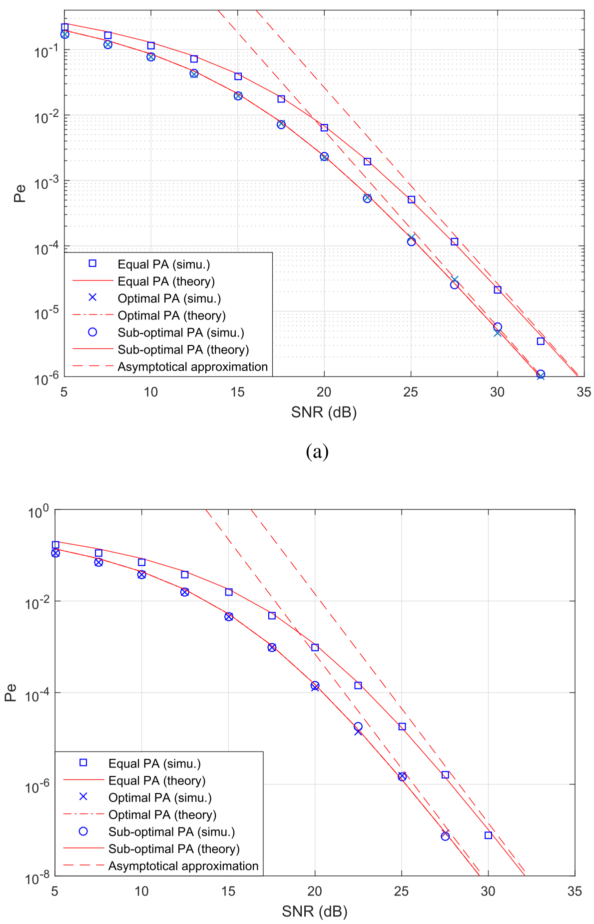


FIGURE 5. Average BER of different power allocation schemes with $N_r = 1$ for $d_{SD} : d_{SR} : d_{RD} = 1 : 0.75 : 0.25$.



(b)

FIGURE 6. Average BER of different power allocation schemes for $d_{SD} : d_{SR} : d_{RD} = 1 : 0.75 : 0.25$. (a) $N_r = 2$. (b) $N_r = 4$

From Fig.5, it is found that the average BER of the system with the suboptimal PA scheme is obviously lower than that of the equal PA scheme, and is quite close to that of the optimal scheme, but its run time is much faster. Specifically, the optimal scheme takes about the average run time of 92.5s, while the average run time of the suboptimal scheme is about 0.8ms, where the computer we used in simulation is equipped with an AMD 2.30-GHz dual core and 2GB RAM. The results illustrate that the proposed suboptimal PA scheme is valid for improving the performance while maintaining the lower complexity.

In Fig.6, the asymptotical P_e is calculated by (47). The PA coefficient r_1 of of the suboptimal PA scheme is computed by (55) and $r_2 = 1 - r_1$. In Fig.6(a) and 6(b), $N_r = 2$ and $N_r = 4$ are respectively considered. Similar results as in Fig.5 can be observed, that is, the suboptimal scheme outperforms the conventional equal power scheme, and has the BER values very close to that of the optimal scheme. Besides, the derived asymptotic BERs are also valid, and agree well with the simulation at high SNR. Moreover, from the asymptotic BER curves, it is found that the diversity order of the system with $N_r = 1$ is 2 in Fig.5, and the diversity orders of the system for $N_r = 2$ and $N_r = 4$ are 3 and 5 as shown in Figs.6(a) and 6(b), respectively, which accords with the diversity gain analysis in section IV-C. Namely, the system can achieve the diversity order of $N_r + 1$. This explains why the BER curves in Fig.6 are lower than that in Fig.5. Furthermore, by comparing Fig.5 and Fig.6, it is found that the performance advantage of the proposed PA scheme over the equal power scheme is improved as the N_r increases. Compared to the equal PA scheme, the proposed scheme gives about 1.1dB gains for $N_r = 1$, 2.1dB gains for $N_r = 2$, and 2.6dB gains for $N_r = 4$ at the BER level of 10^{-6}.

VI. CONCLUSION

The error performance of the AF relay-aided cooperative communication with spatial modulation over Rayleigh fading channels is analyzed and evaluated. In the performance analysis, the PDF and MGF of the system output SNRs are derived. Using these results, the error probability of antenna index detection and the error probability of symbol detection given transmit antenna index are developed and their closed-form expressions are obtained. With these expressions, the overall average BER of AF-SM can be efficiently computed. Computer simulation shows that the theoretical analysis is valid and in agreement with the simulation results. Furthermore, an asymptotically tight approximation of average BER at high SNR is also derived. With the asymptotic BER expression, the diversity gain is analyzed and the suboptimal power allocation scheme is developed. It is shown that the AF-SM system can obtain the diversity order of $N_r + 1$. The system with the suboptimal PA scheme has better BER performance than that with the equal power scheme, and the corresponding performance advantage is enhanced as N_r increases. Simulation results verify the validity of the theoretical error analysis and diversity gain, and the effectiveness of the suboptimal power allocation scheme.

Appendix

In this appendix, we present the derivation of PEP in (24), $PEP(i \to \hat{i}|x_q)$. Given the transmitted symbol, the detection of transmit antenna index is given as

$$\hat{i} = \underset{k=1,2,\cdots,N_t}{\arg\min}\ (D_{i,k}), \tag{58}$$

where $D_{i,k}$ denotes a decision metric for the case that the transmitter uses the i-th transmit antenna for transmission and the receiver decides it was the k-th transmit antenna, expressed as

$$\begin{aligned} D_{i,k} &= P_s\|\mathbf{h}_{sd}^k x_q\|_F^2 - 2\sqrt{P_s}\Re\{\mathbf{y}_{sd}^H\mathbf{h}_{sd}^k x_q\} \\ &\quad + G\|\mathbf{h}_{rd}\|_F^2|h_{sr}^i x_q|^2 - 2\sqrt{G}\Re\{\tilde{\mathbf{y}}_{rd}^H\mathbf{h}_{rd}h_{sr}^k x_q\}. \end{aligned} \tag{59}$$

For $k = i$, substituting (3) and (4) into (59), we have

$$\begin{aligned} D_{i,i} &= -P_s\|\mathbf{h}_{sd}^i x_q\|_F^2 - 2\sqrt{P_s}\Re\{\mathbf{n}_{sd}^H\mathbf{h}_{sd}^i x_q\} \\ &\quad - G\|\mathbf{h}_{rd}\|_F^2|h_{sr}^i x_q|^2 - 2G\Re\{\mathbf{n}^H\mathbf{h}_{rd}h_{sr}^i x_q\}/(A\sqrt{P_s}), \end{aligned} \tag{60}$$

while for $k \neq i$, the decision metric is expressed as

$$\begin{aligned} D_{i,k} &= P_s\|\mathbf{h}_{sd}^k x_q\|_F^2 - 2\sqrt{P_s}\Re\{\sqrt{P_s}(\mathbf{h}_{sd}^i x_q)^H\mathbf{h}_{sd}^k x_q + \mathbf{n}_{sd}^H\mathbf{h}_{sd}^k x_q\} \\ &\quad + G\|\mathbf{h}_{rd}\|_F^2|h_{sr}^k x_q|^2 - 2G\Re\{(h_{sr}^i x_q)^H\|\mathbf{h}_{rd}\|_F^2 h_{sr}^k x_q\} \\ &\quad - 2G\Re\{\mathbf{n}^H\mathbf{h}_{rd}h_{sr}^k x_q\}/(A\sqrt{P_s}). \end{aligned} \tag{61}$$

Given the channel sate information and transmitted symbol, $\boldsymbol{\phi} = \{x_q, \mathbf{h}_{sd}^i, h_{sr}^i, \mathbf{h}_{sd}^k, h_{sr}^k, \mathbf{h}_{rd}\}$, the conditional PEP, $PEP(i \to k|\boldsymbol{\phi})$, can be evaluated as

$$PEP(i \to k|\boldsymbol{\phi}) = Pr(D_{i,k} - D_{i,i} < 0|\boldsymbol{\phi}) = Pr(d_{i,k} < 0|\boldsymbol{\phi}), \tag{62}$$

where $d_{i,k} = D_{i,k} - D_{i,i}$ is a Gaussian r.v. with mean $\bar{d}_{i,k} = P_s\|\mathbf{h}_{sd}^i - \mathbf{h}_{sd}^k\|_F^2|x_q|^2 + G\|\mathbf{h}_{rd}\|_F^2|h_{sr}^i - h_{sr}^k|^2|x_q|^2$ and variance given as

$$\sigma_{d_{i,k}}^2 = 2N_0(P_s\|\mathbf{h}_{sd}^i - \mathbf{h}_{sd}^k\|_F^2|x_q|^2 + G\|\mathbf{h}_{rd}\|_F^2|h_{sr}^i - h_{sr}^k|^2|x_q|^2). \tag{63}$$

Using the Q-function for the Gaussian r.v. $d_{i,k}$, (62) can be expressed as

$$\begin{aligned} PEP(i \to k|\boldsymbol{\phi}) &= \frac{1}{\sqrt{2\pi\sigma_{d_{i,k}}^2}}\int_{\bar{d}_{i,k}}^{\infty}\exp(-v^2/(2\sigma_{d_{i,k}}^2))dv \\ &= Q\big(\sqrt{\bar{d}_{i,k}/(2N_0)}\big). \end{aligned} \tag{64}$$

With $G = P_sA^2/(A^2\|\mathbf{h}_{rd}\|_F^2 + 1)$, (64) can be further expressed as

$$\begin{aligned} &PEP(i \to k|\boldsymbol{\phi}) \\ &= Q\Bigg(\sqrt{\frac{P_s\|\mathbf{h}_{sd}^i - \mathbf{h}_{sd}^k\|_F^2|x_q|^2}{2N_0} + \frac{\frac{P_r\|\mathbf{h}_{rd}\|_F^2}{N_0}\frac{P_s|h_{sr}^i - h_{sr}^k|^2|x_q|^2}{2N_0}}{\frac{P_r\|\mathbf{h}_{rd}\|_F^2}{N_0} + \frac{P_r}{A^2N_0}}}\Bigg) \\ &= Q\Big(\sqrt{\tilde{\gamma}_{sd}|x_q|^2 + \frac{\tilde{\gamma}_{sr}\tilde{\gamma}_{rd}}{\tilde{\gamma}_{rd} + C}|x_q|^2}\Big) \\ &= Q\Big(\sqrt{|x_q|^2(\tilde{\gamma}_{sd} + \tilde{\gamma}_{srd})}\Big), \end{aligned} \tag{65}$$

where $\tilde{\gamma}_{sd} = \frac{P_s\|\mathbf{h}_{sd}^i - \mathbf{h}_{sd}^k\|_F^2}{2N_0}$, $\tilde{\gamma}_{sr} = \frac{P_s|h_{sr}^i - h_{sr}^k|^2}{2N_0}$, $\tilde{\gamma}_{rd} = \frac{P_r\|\mathbf{h}_{rd}\|_F^2}{N_0}$, $C = \frac{P_r}{A^2N_0}$, and $\tilde{\gamma}_{srd} = \frac{\tilde{\gamma}_{sr}\tilde{\gamma}_{rd}}{(\tilde{\gamma}_{rd}+C)}$.

As the conditional PEP in (65) is a function of r.v.s $\tilde{\gamma}_{sd}$ and $\tilde{\gamma}_{srd}$, the $PEP(i \to \hat{i}|x_q)$ in (24) can be computed as

$$\begin{aligned} &PEP(i \to \hat{i}|x_q) \\ &= \int_0^\infty\int_0^\infty Q\Big(\sqrt{|x_q|^2(\tilde{\gamma}_{sd} + \tilde{\gamma}_{srd})}\Big) \\ &\quad \times f_{\tilde{\gamma}_{sd}}(\tilde{\gamma}_{sd})f_{\tilde{\gamma}_{srd}}(\tilde{\gamma}_{srd})d\tilde{\gamma}_{srd}d\tilde{\gamma}_{sd} \\ &= \int_0^\infty\int_0^\infty\frac{1}{\pi}\int_0^{\pi/2}\exp\Big(\frac{-|x_q|^2(\tilde{\gamma}_{sd} + \tilde{\gamma}_{srd})}{2\sin^2\theta}\Big) \\ &\quad \times f_{\tilde{\gamma}_{sd}}(\tilde{\gamma}_{sd})f_{\tilde{\gamma}_{srd}}(\tilde{\gamma}_{srd})d\theta d\tilde{\gamma}_{srd}d\tilde{\gamma}_{sd} \\ &= \frac{1}{\pi}\int_0^{\pi/2}M_{\tilde{\gamma}_{srd}}\big(|x_q|^2/(2\sin^2\theta)\big)M_{\tilde{\gamma}_{sd}}\big(|x_q|^2/(2\sin^2\theta)\big)d\theta, \end{aligned} \tag{66}$$

where $M_{\tilde{\gamma}_{srd}}(\cdot)$ and $M_{\tilde{\gamma}_{sd}}(\cdot)$ are the MGFs of $\tilde{\gamma}_{srd}$ and $\tilde{\gamma}_{sd}$, respectively.

For two transmit antennas at the source (i.e., $N_t = 2$), it can be shown that $PEP(1 \to 2|x_q) = PEP(2 \to 1|x_q)$. Thus, the PEP in (24) can be computed as

$$P_a = \sum_{q=1}^{M}\frac{PEP(1 \to 2|x_q)}{M}. \tag{67}$$

This result shows that (24) is an accurate formula for $N_t = 2$.

If the direct link of the source to the destination is not considered and the modulation is binary, (65) is reduced to [5, eq. (12)], i.e., $PEP(1 \to 2|x_q = \pm 1) = Q\big(\sqrt{\tilde{\gamma}_{srd}}\big) = Q\big(\sqrt{\tilde{\gamma}_{sr}\tilde{\gamma}_{rd}/(\tilde{\gamma}_{rd} + C)}\big)$. Thus, (65) includes [(12), 5] as a special case.

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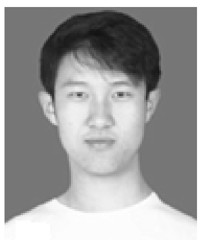

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