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Adaptive Semi-Periodically Intermittent and Lag Synchronization Control of Neural Networks With Mixed Delays

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ABSTRACT In this paper, the lag exponential synchronization for a class of neural networks with distributed delays and discrete delays (mixed delays) is studied via adaptive semiperiodically intermittent control. Using the adaptive control theory, the Lyapunov stability theory combined with the method of intermittent control, the simple but robust adaptive semiperiodically intermittent controller and impulse controller are designed. Via the proposed control methods, the response system can lag synchronize with the drive system, and the less conservative results are obtained. Using the adaptive control approach and giving a rigorous proof for the synchronization scheme, the proposed controllers are obviously little costly and more useful in practice than before. In the original references, the control width should be larger than the time delay and the time delay should be smaller than the noncontrol width. While, in this paper, these strict assumptions can be removed. Moreover, the control time and the control rate may not be constants. This leads to a larger application scope for our method. Last, numerical simulations are exploited to show the effectiveness of the results.

INDEX TERMS Neural networks, mixed delays, lag synchronization, adaptive control, semi-periodically intermittent control.

I. INTRODUCTION

In 1983, the competitive neural networks (CNNs) have been proposed by Cohen and Grossberg [1]. Afterward, the neural networks (NNs) have got great development. Meyer-Bäse *et al.* [2]–[4] presented the CNNs with different time scales, which can be regarded as the extensions of primitive neuronal competition [1], [5]. In the last few years, the neural networks have been extensively investigated across many fields of science and engineering since they have been widely applied in associative memories, signal and image processing and combinatorial optimization [6]–[9].

The modelling of neural networks is based on biological neural systems. The neural networks consist of a lot of artificial interconnected neurons and processes information using a connectional approach. The neural networks have represented an alternative method to solve the problems in control engineering via control technique. The most useful

property of control neural networks is their ability to approximate arbitrary nonlinear or linear mapping through learning. Since the early 1990s, there has been more and more interest in using neural networks for control of complex systems, such as robotics, electrical power system and mechatronics systems and so on. Many control neural networks have been developed for the compensation for the effects of nonlinearities and system uncertainties in control systems. So the system performance of the robustness and stability can be improved [10].

Synchronization is a significant dynamical behavior in neural networks [6]–[28]. Many synchronization phenomena are very useful for us, such as the synchronous transfer of digital and analog signals in communication networks [11]. In fact, synchronization is a well-established concept and it is typical in the basic motions of nature. Many control techniques, such as linear feedback control [12]–[14],

adaptive feedback control [8], [11], [15]–[17], impulsive control [18], pinning control [19]–[25] and intermittent control [23], [28]–[32] have been developed to drive the synchronization of networks. The intermittent control is one useful control method in engineering fields. The approach of the intermittent control is implemented easily in practice. In [21], intermittent control has been first introduced to control synchronization of complex network systems. But two restrictive conditions that about the control width, the time delays and the noncontrol width are imposed. From then on, lots of interesting results have been presented for synchronization of complex dynamic networks by periodically intermittent control in [21] and [22]. In [23], the problem of robust synchronization about the neural networks with uncertain parameters and mixed delays was investigated. A novel controller is designed to pin the coupled networks to reach the synchronization state. In [28], the lag exponential synchronization for a class of neural networks with mixed delays is studied by intermittent control. The results of [29] removed the conservative assumptions on control width and time delays mentioned in [20]. The control time and the control rate were constants in most of the previous results. In [31], the synchronization problem via semi-periodically intermittent control technique and mode-dependent average dwell time method has been researched for switched complex networks with delayed coupling. The controllers which were aperiodically intermittent extended the existing results of periodically intermittent control. Unluckily, the linear intermittent control gain which has been obtained may be much larger than the actual needs. Therefore, the study of the semi-periodically intermittent control needs further exploration and improvement.

It is well known that there inevitably exists time delay when the signal travels through the complex neural networks due to the finite speeds of transmission and spreading. Time delay is likely to cause undesirable dynamical behavior, for example, oscillation, chaos and instability. So we often require to synchronize between one neural networks and the other neural networks at a constant time delay. Compared with complete synchronization [8], [28], lag synchronization may be a more appropriate technique to clearly indicate the fragile nature of neural networks. Hence, it is important to effectively lag synchronize two chaotic neural networks for potentially theoretical research and practical application.

For all the above reasons, this paper aims to handle the problem of lag synchronization for the neural networks with mixed delays via adaptive semi-periodically intermittent control. Using the theory of Lyapunov stability combined with adaptive control and intermittent control techniques, improved adaptive semi-periodically intermittent lag synchronization controllers which are little costly and more useful in practice are proposed. In [8] and [21], the control width is supposed to larger than the time delay and the time delay is supposed to smaller than the noncontrol width. While, in this paper, these two restrictive assumptions are not required. In [22], pinning synchronization of complex network with

delayed dynamical nodes was further investigated via periodically intermittent control. Unfortunately, the control gain that have obtained may be much larger than the needed value. Moreover, the control time and the control rate were constants in [22]. While, the control gain which have obtained is smaller than the gain of [22]. In [22] and [30], the control time and the control rate are constants. These greatly limit the scope of application of the controller. While, in this paper, the control time and the control rate may not be constants. This leads to a larger application scope for our method. By utilizing mathematical induction method and the analysis technique, the novel adaptive semi-periodically intermittent controllers and impulse controllers are designed, which can be testified to be less conservative comparing with linear intermittent control. Using the adaptive control approach and giving a rigorous proof for the synchronization scheme, the proposed controllers are obviously little costly and more useful in practice than before. Numerical examples are presented to show the effectiveness of the obtained method.

The rest of this paper is organized as follows. The neural networks model with mixed delays, some necessary assumptions and lemmas are given in Sect. 2. In Sect. 3, lag synchronization controllers of neural networks with mixed delays via adaptive semi-periodically intermittent control and impulse control are designed, respectively. The numerical simulations are given to illustrate the results in Sect. 4. The conclusions are obtained in Sect. 5 at last.

II. NEURAL NETWORKS MODEL AND PRELIMINARIES

We consider the mixed delays neural networks as follows:

$$\begin{aligned} \dot{x}_i(t) = & -c_i x_i(t) + \sum_{j=1}^N a_{ij} f_j(x_j(t)) + \sum_{j=1}^N b_{ij} \\ & \cdot f_j(x_j(t-\theta)) + \sum_{j=1}^N d_{ij} \int_{t-\eta}^t f_j(x_j(s)) ds, \end{aligned} \quad (1)$$

where $i = 1, \dots, N$, $x_i(t)$ is the neuron current activity level, $f_j(x_j(t))$ is the output of neurons, $c_i > 0$ is the time constant of the neuron, a_{ij} represents the connection weight between the i th neuron and the j th neuron, b_{ij} and d_{ij} represent the synaptic weight of delayed feedback. $\theta > 0$ is the discrete, and $\eta > 0$ is the discrete and the distributed time delay.

Let $x(t) = (x_1^T(t), x_2^T(t), \dots, x_N^T(t))^T$, $f(x(t)) = (f(x_1^T(t)), f(x_2^T(t)), \dots, f(x_N^T(t)))^T$, $C = \text{diag}(c_1, c_2, \dots, c_N)^T$, $A = (a_{ij})_{N \times N}$, $B = (b_{ij})_{N \times N}$, $D = (d_{ij})_{N \times N}$. Then, (1) turns into the following form:

$$\dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t-\theta)) + D \int_{t-\eta}^t f(x(s)) ds, \quad (2)$$

The initial value of (2) is denoted by $x(t) = \phi^x(t) \in C([-\mu, 0], R^N)$, where $\mu = \max\{\theta, \eta\}$.

On account of the drive-response synchronization, let take (2) as the drive system. Then we design the response

system as follows:

$$\dot{y}(t) = -Cy(t) + Af(y(t)) + Bf(y(t-\theta)) + D \int_{t-\eta}^t f(y(s))ds + u(t), \quad (3)$$

where $u(t) = (u_1^T(t), u_2^T(t), \dots, u_N^T(t))^T$ is the controller to be designed, $u_i(t) \in R^n$ is the input vector of node i . The initial values of (3) is given as $y(t) = \varphi^y(t) \in C([-μ, 0], R^N)$, where $μ = \max\{\theta, \eta\}$. Assume that solutions of neural networks (2) and (3) are bounded. And the output signals of the NNs (2) can be received by (3) with transmission delay $τ ≥ 0$.

Definition 1: Drive-response neural networks (2) and (3) are said to achieve exponential lag synchronization if there exist $α ≥ 1$ and $ε > 0$ such that

$$\|y(t) - x(t - τ)\| ≤ αe^{-ε(t-τ)} \sup_{-τ ≤ θ ≤ 0} \|\varphi^y(t) - φ^x(t - τ)\|, \quad (4)$$

for any $t > τ$. Here, $ε$ is defined as the degree of exponential lag synchronization.

In order to research the lag synchronization (2) and (3) with the lag time $τ ≥ 0$, the error state is defined $e(t) = y(t) - x(t - τ)$. Subtracting (2) from (3) yields the following error system:

$$\dot{e}(t) = -Ce(t) + Ag(e(t)) + Bg(e(t - θ)) + D \int_{t-\eta}^t g(e(s))ds + u(t), \quad (5)$$

where $g(e(t)) = f(y(t)) - f(x(t - τ))$, the initial condition of (5) is $e(t) = \varphi^y(t) - φ^x(t - τ)$.

In the following, some necessary definitions and assumptions and lemmas are given.

Assumption 1 (A1): There exist positive constants l_i , ($i = 0, 1, \dots, N$) satisfying

$$\|f_i(x) - f_i(y)\| ≤ l_i \|x - y\|, \quad \forall x, y \in R, x ≠ y \quad (6)$$

Lemma 1 [1]: For any vectors $x, y \in R^m$ and positive definite matrix $Q \in R^{m×m}$, the following matrix inequality holds:

$$2x^T y \leq x^T Q x + y^T Q^{-1} y$$

If not specified otherwise, inequality $Q > 0$ ($Q < 0$, $Q ≥ 0$, $Q ≤ 0$) means Q is a positive (or negative, or semi-positive, or semi-negative) definite matrix, where Q is a square matrix.

Lemma 2 [33]: For any constant symmetric matrix $S \in R^{n×n}$, $S > 0$, scalar $h > 0$ and vector function $\dot{x}(\cdot) \in C([-h, 0], R^n)$ the following inequality always holds.

$$h \int_0^h \dot{x}^T(s) S \dot{x}(s) ds \geq \left(\int_0^h \dot{x}(s) ds \right)^T S \left(\int_0^h \dot{x}(s) ds \right).$$

III. MAIN RESULTS

In this section, the adaptive semi-periodically intermittent control which is added to the neural networks with mixed delays (3) such that states of (3) can be exponentially lag-synchronized with (2) is designed. At the same time, the trivial solution of error system (5) is exponential stable. In order to realize lag synchronization of the neural networks with mixed delays by adaptive semi-periodically intermittent control, the controllers are added to nodes of the neural networks. In error system (5), the adaptive semi-periodically intermittent controllers are defined as follows:

$$u_i(t) = \begin{cases} -k_i(t)e_i(t), & t \in [\kappa T + \sum_{m=1}^{\zeta-1} T_m, \\ & \kappa T + \sum_{m=1}^{\zeta-1} T_m + δ_ζ), \\ 0, & t \in [\kappa T + \sum_{m=1}^{\zeta-1} T_m + δ_ζ, \\ & \kappa T + \sum_{m=1}^{\zeta} T_m). \end{cases} \quad (7)$$

and the corresponding updating laws

$$k_i(t) = \begin{cases} α_i e_i(t)^T e_i(t), & t \in [\kappa T + \sum_{m=1}^{\zeta-1} T_m, \\ & \kappa T + \sum_{m=1}^{\zeta-1} T_m + δ_ζ), \\ 0, & t \in [\kappa T + \sum_{m=1}^{\zeta-1} T_m + δ_ζ, \\ & \kappa T + \sum_{m=1}^{\zeta} T_m). \end{cases} \quad (8)$$

where $k_i(0) > 0$ ($i = 1, 2, \dots, N$) are initial value, $α_i$ ($i = 1, 2, \dots, N$) are positive constants. $κ = 0, 1, 2, \dots$. T is the control period which consists of $ζ$ switched small periods T_1, T_2, \dots, T_l , and $ζ = 1, 2, \dots, l$.

We assume that there exists constant $l > 0$, such that one has $\sum_{ζ=κl+1}^{(κ+1)l} σ T_ζ = δ$ for any $κ = 0, 1, 2, \dots$, where $T = \sum_{ζ=κl+1}^{(κ+1)l} T_ζ$, $σ$ ($0 < σ < 1$) is called the control rate, and $δ$ is the control width in period T . $H_{ζ1}^{(κ)} = [\kappa T + \sum_{ζ=1}^{l-1} T_ζ, κ T + \sum_{ζ=1}^{l-1} T_ζ + δ_l)$ ($ζ = 1, 2, 3, \dots, l$) denote the control width in small period $T_ζ$ of $(κ + 1)$ th control period T , $H_{ζ2}^{(κ)} = [\kappa T + \sum_{ζ=1}^{l-1} T_ζ + δ_l, (κ + 1)T)$ ($ζ = 1, 2, 3, \dots, l$) denote the rest width in small period $T_ζ$ of $(κ + 1)$ th control period T . Defined that $σ_ζ = \frac{\sum_{m=1}^ζ δ_m}{\sum_{m=1}^ζ T_m}$ and $σ̄ = \min\{σ, σ_{ζ-1}, σ_ζ\}$.

Let $K(t) = diag(k_1(t), k_2(t), \dots, k_N(t))$. Then we can rewrite the system (5) as,

$$\begin{cases} \dot{e}(t) = -Ce(t) + Ag(e(t)) + Bg(e(t - θ)) + D \int_{t-\eta}^t g(e(s))ds - K(t)e(t), \\ t \in [\kappa T + \sum_{m=1}^{\zeta-1} T_m, κ T + \sum_{m=1}^{\zeta-1} T_m + δ_ζ), \\ \dot{e}(t) = -Ce(t) + Ag(e(t)) + Bg(e(t - θ)) + D \int_{t-\eta}^t g(e(s))ds, \\ t \in [\kappa T + \sum_{m=1}^{\zeta-1} T_m + δ_ζ, κ T + \sum_{m=1}^{\zeta} T_m]. \end{cases} \quad (9)$$

Now we give lag synchronization criteria under the adaptive semi-periodically intermittent controller in neural networks with mixed-delays.

Theorem 1: Suppose Assumptions 1 holds. The neural networks with mixed delays (2) and (3) globally exponentially lag-synchronize under adaptive semi-periodical intermittent controllers (7)-(8) if there exist positive constants $a_1 > L$, a_2, ε and $\alpha_i (i = 1, 2, \dots, N)$, such that

$$\begin{aligned} a_2 &\geq L + \frac{L\eta^2}{a_1} + \lambda_{\max}(\Pi), \\ \varepsilon &= \lambda - a_2(1 - \bar{\sigma}) > 0, \end{aligned} \quad (10)$$

where $\Pi = AA^T + BB^T + DD^T - 2C$, $\lambda > 0$ is the positive solution of the equation $a_1 - \lambda - L \exp\{\lambda\theta\} = 0$.

Proof: First, we introduce a new Lyapunov-Krasovskii functional

$$\begin{aligned} V(t) &= \frac{1}{2} \exp\{-a_1 t\} e^T(t) e(t) + \frac{1}{2} \sum_{i=1}^N \exp\{-a_1 t\} \frac{(k_i(t) - k)^2}{\alpha_i} \\ &\quad + \frac{1}{2} \exp\{-a_1 t\} \int_{t-\eta}^t \int_z^t e^T(s) S e(s) ds dz, \end{aligned} \quad (11)$$

where k is an undetermined sufficiently large positive constant. Calculating the derivative of (11) along the trajectories of the error system (9) under Assumption 1, we get

When $t \in H_{\zeta_1}^{(m)}$, for $m = 0, 1, 2, \dots, \zeta_1 = 1, 2, \dots, l$

$$\begin{aligned} \dot{V}(t) &= \exp\{-a_1 t\} e^T(t) \dot{e}(t) - \frac{a_1}{2} \exp\{-a_1 t\} e^T(t) e(t) \\ &\quad - \frac{a_1}{2} \sum_{i=1}^N \exp\{-a_1 t\} \frac{(k_i(t) - k)^2}{\alpha_i} \\ &\quad + \exp\{-a_1 t\} \sum_{i=1}^N (k_i(t) - k) e_i^T(t) e_i(t) \\ &\quad + \frac{a_1}{2} \exp\{-a_1 t\} e^T(t) S e(t) \\ &\quad - \frac{a_1}{2} \exp\{-a_1 t\} \int_{t-\eta}^t e^T(s) S e(s) ds \\ &\quad - \frac{a_1}{2} \exp\{-a_1 t\} \int_{t-\eta}^t \int_z^t e^T(s) S e(s) ds dz \\ &= \exp\{-a_1 t\} e^T(t) \left[\left(\frac{1}{2} \eta S - C - K \right) e(t) \right. \\ &\quad \left. + A g(e(t)) + B g(e(t-\theta)) + D \int_{t-\eta}^t g(e(s)) ds \right] \\ &\quad - a_1 V(t) - \frac{a_1}{2} \exp\{-a_1 t\} \int_{t-\eta}^t e^T(s) S e(s) ds, \end{aligned} \quad (12)$$

From Lemma 1 and Assumption 1, we obtain

$$\begin{aligned} e^T(t) A g(e(t)) &\leq \frac{1}{2} e^T(t) A A^T e(t) + \frac{1}{2} g^T(e(t)) g(e(t)) \\ &\leq e^T(t) \left[\frac{1}{2} A A^T + \frac{1}{2} L \right] (e(t)), \end{aligned} \quad (13)$$

and

$$\begin{aligned} e^T(t) B g(e(t-\theta)) &\leq \frac{1}{2} e^T(t-\theta) B B^T e(t-\theta) + \frac{1}{2} g^T(e(t-\theta)) g(e(t-\theta)) \\ &\leq \frac{1}{2} e^T(t) B B^T (e(t)) + \frac{1}{2} e^T(t-\theta) L (e(t-\theta)), \end{aligned} \quad (14)$$

where $L = \text{diag}(l_1^2, l_2^2, \dots, l_N^2)$. From Lemma 2, we can get

$$\begin{aligned} e^T(t) D \int_{t-\eta}^t g(e(s)) ds &\leq \frac{1}{2} e^T(t) D D^T (e(t)) \\ &\quad + \frac{1}{2} \left(\int_{t-\eta}^t g^T(e(s)) ds \right)^T \left(\int_{t-\eta}^t g^T(e(s)) ds \right) \\ &\leq \frac{1}{2} e^T(t) D D^T (e(t)) + \int_{t-\eta}^t e^T(s) \left(\frac{1}{2} \eta L \right) e(s) ds. \end{aligned} \quad (15)$$

Substituting (13)-(15) into (12) gives

$$\begin{aligned} \dot{V}(t) &\leq \exp\{-a_1 t\} e^T(t) \left[-C + \frac{1}{2} A A^T + \frac{1}{2} L I_N \right. \\ &\quad \left. + \frac{1}{2} B B^T + \frac{1}{2} D D^T + \frac{1}{2} \eta S - K \right] e(t) \\ &\quad + \frac{1}{2} \exp\{-a_1 t\} e^T(t-\theta) L (e(t-\theta)) \\ &\quad + \exp\{-a_1 t\} \int_{t-\eta}^t e^T(s) \left(\frac{1}{2} \eta L \right) e(s) ds \\ &\quad - a_1 V(t) - \frac{a_1}{2} \exp\{-a_1 t\} \int_{t-\eta}^t e^T(s) S e(s) ds, \end{aligned} \quad (16)$$

where $K = k I_N$. Taking $S = \frac{\eta L}{a_1}$. We select k as

$$k > \frac{L}{2} \left(1 + \frac{\eta^2}{a_1} \right) + \frac{1}{2} \lambda_{\max}(\Pi), \quad (17)$$

So we have

$$\begin{aligned} V(t) &\leq -a_1 V(t) + \frac{1}{2} \exp\{-a_1 t\} e^T(t-\theta) L (e(t-\theta)) \\ &\quad + \frac{L}{2} \exp\{-a_1 t\} \int_{t-\theta-\eta}^{t-\theta} \int_{z-\theta}^{t-\theta} e^T(z) S e(z) dz dz \\ &\quad + \frac{L}{2} \sum_{i=1}^N \exp\{-a_1 t\} \frac{(k_i(t) - k)^2}{\alpha_i} \\ &= -a_1 V(t) + L V(t-\theta) \end{aligned} \quad (18)$$

Similarly, when $t \in H_{\zeta_1}^{(m)}$, for $\zeta_1 = 1, 2, \dots, l$, $m = 0, 1, 2, \dots$, according to condition the first inequality of the (10), we have

$$\begin{aligned} \dot{V}(t) &\leq \exp\{-a_1 t\} e^T(t) \left[-C + \frac{1}{2} A A^T + \frac{1}{2} (L - a_2) I_N \right. \\ &\quad \left. + \frac{1}{2} B B^T + \frac{1}{2} D D^T + \frac{1}{2} \eta S \right] e(t) + \frac{1}{2} a_2 e^T(t) e(t) \\ &\quad + \frac{1}{2} \exp\{-a_1 t\} e^T(t-\theta) L (e(t-\theta)) \end{aligned}$$

$$\begin{aligned}
& + \exp\{-a_1 t\} \int_{t-\eta}^t e^T(s) \left(\frac{1}{2} \eta L\right) e(s) ds \\
& - a_1 V(t) - \frac{a_1}{2} \exp\{-a_1 t\} \int_{t-\eta}^t e^T(s) S e(s) ds \\
\leq & (a_2 - a_1) V(t) + L V(t - \theta), \tag{19}
\end{aligned}$$

Take $P(t) = \exp\{\lambda t\}V(t)$ and $\bar{Q} = \sup_{-\theta \leq t \leq 0} V(t)$. Let $\Omega_{11}^{(0)}(t) = P(t) - \beta \bar{Q}$, where the constant $\beta > 1$, then

$$\Omega_{11}^{(0)}(t) < 0, \quad \text{for all } t \in [-\theta, 0] \tag{20}$$

Then, we want to prove that

$$\Omega_{11}^{(0)}(t) < 0, \quad \text{for all } t \in H_{11}^{(0)}. \tag{21}$$

If (21) dose hold, suppose to exist a $t_{11} \in H_{11}^{(0)}$, such that

$$\Omega_{11}^{(0)}(t_{11}) = 0, \quad \dot{\Omega}_{11}^{(0)}(t_{11}) \geq 0. \tag{22}$$

$$\Omega_{11}^{(0)}(t) < 0, \quad -\tau \leq t \leq t_{11}. \tag{23}$$

Using (20), (22) and (23), we obtain

$$\begin{aligned}
\dot{\Omega}_{11}^{(0)}(t_{11}) &= \lambda P(t_{11}) + \exp\{\lambda t_{11}\} \cdot \dot{V}(t_{11}) \\
&\leq \lambda P(t_{11}) - a_1 \exp\{\lambda t_{11}\} \cdot V(t_{11}) \\
&\quad + L \exp\{\lambda t_{11}\} V(t_{11} - \theta) \\
&\leq (\lambda - a_1) P(t_{11}) + L \exp\{\lambda \theta\} P(t_{11} - \theta) \\
&< (\lambda - a_1) \beta \bar{Q} + L \exp\{\lambda \theta\} \beta \bar{Q} \\
&= (\lambda - a_1 + L \exp\{\lambda \theta\}) \beta \bar{Q} = 0. \tag{24}
\end{aligned}$$

The second inequality in (22) contradicts (24), then (21) holds.

We want to prove that $\Omega_{12}^{(0)}(t) = P(t) - \beta \bar{Q} \exp\{a_2(t - \delta_1)\} < 0$ for $t_{12} \in H_{12}^{(0)}$. Otherwise, there exists a $t_{12} \in H_{12}^{(0)}$, such that

$$\Omega_{12}^{(0)}(t_{12}) = 0, \quad \dot{\Omega}_{12}^{(0)}(t_{12}) \geq 0. \tag{25}$$

$$\Omega_{12}^{(0)}(t) < 0, \quad \delta_1 \leq t \leq t_{12}. \tag{26}$$

For $\theta > 0$, if $\delta_1 \leq t_{12} - \theta \leq t_{12}$, it follows from (26) that

$$P(t_{12} - \theta) < \beta \bar{Q} \exp\{a_2(t_{12} - \delta_1)\},$$

and if $-\theta \leq t_{12} - \theta < \delta_1$, from (20) and (21), we have

$$P(t_{12} - \theta) < \beta \bar{Q} \leq \beta \bar{Q} \exp\{a_2(t_{12} - \delta_1)\},$$

Hence, for $\theta > 0$, we always have

$$P(t_{12} - \theta) < \beta \bar{Q} \exp\{a_2(t_{12} - \delta_1)\}. \tag{27}$$

Then

$$\begin{aligned}
\dot{\Omega}_{12}^{(0)}(t_{12}) &= \lambda P(t_{12}) + \exp\{\lambda t_{12}\} \cdot \dot{V}(t_{12}) \\
&\quad - a_2 \beta \bar{Q} \exp\{a_2(t_{12} - \delta_1)\} \\
&\leq \lambda P(t_{12}) + (a_2 - a_1) \exp\{\lambda t_{12}\} \cdot V(t_{12}) \\
&\quad + L \exp\{\lambda t_{12}\} V(t_{12} - \theta) \\
&\quad - a_2 \beta \bar{Q} \exp\{a_2(t_{12} - \delta_1)\} \\
&\leq (\lambda + a_2 - a_1) P(t_{12}) + L \exp\{\lambda \theta\} P(t_{12} - \theta) \\
&\quad - a_2 \beta \bar{Q} \exp\{a_2(t_{12} - \delta_1)\} \\
&< (\lambda - a_1 + L \exp\{\lambda \theta\}) \beta \bar{Q} \exp\{a_2(t_{12} - \delta_1)\} \\
&= 0. \tag{28}
\end{aligned}$$

The second inequality in (25) contradicts (28), so $\Omega_{12}^{(0)}(t) < 0$ holds. Then we attain for $t \in H_{12}^{(0)}$

$$P(t) < \beta \bar{Q} \exp\{a_2(t_{12} - \delta_1)\} \leq \beta \bar{Q} \exp\{a_2(T_1 - \delta_1)\},$$

Then, for $t \in [-\theta, \delta_1]$, we have

$$P(t) < \beta \bar{Q} < \beta \bar{Q} \exp\{a_2(T_1 - \delta_1)\},$$

So

$$P(t) < \beta \bar{Q} \exp\{a_2(T_1 - \delta_1)\}, \quad \text{for all } t \in [-\theta, T_1]$$

Let $\Omega_{\zeta 1}^{(\kappa)}(t) = P(t) - \beta \bar{Q} \exp\{a_2[\kappa(1 - \sigma) + \sum_{m=1}^{\zeta-1} (T_m - \delta_m)]\}$, where $t \in H_{\zeta 1}^{(\kappa)}$.

By following a similar proof above, we can obtain that for any $\kappa = 1, 2, \dots, \zeta = 1, 2, \dots, l$. When $t \in H_{\zeta 1}^{(\kappa)}$, we have

$$\begin{aligned}
P(t) &< B \bar{Q} \exp\{a_2[\kappa(1 - \sigma)T + (1 - \sigma_{\zeta-1} \sum_{m=1}^{\zeta-1} T_m)]\} \\
&\leq B \bar{Q} \exp\{a_2(1 - \bar{\sigma})(\kappa T + \sum_{m=1}^{\zeta-1} T_m)\} \\
&\leq B \bar{Q} \exp\{a_2(1 - \bar{\sigma})t\}. \tag{29}
\end{aligned}$$

Let $\Omega_{\zeta 2}^{(\kappa)}(t) = P(t) - \beta \bar{Q} \exp\{a_2[\kappa(1 - \sigma) + t - \sum_{m=1}^{\zeta} \delta_m]\}$, where $t \in H_{\zeta 2}^{(\kappa)}$. When $t \in H_{\zeta 2}^{(\kappa)}$, we have

$$\begin{aligned}
P(t) &< B \bar{Q} \exp\{a_2[t - (\sigma \kappa T + \sigma_{\zeta} \sum_{m=1}^{\zeta} T_m)]\} \\
&\leq B \bar{Q} \exp\{a_2[t - \bar{\sigma}(\kappa T + \sum_{m=1}^{\zeta} T_m)]\} \\
&\leq B \bar{Q} \exp\{a_2(1 - \bar{\sigma})t\}. \tag{30}
\end{aligned}$$

Let $\beta \rightarrow 1$, we can obtain

$$\begin{aligned}
V(t) &\leq \bar{Q} \exp\{-[\lambda - a_2(1 - \bar{\sigma})t]\} \\
&= \sup_{-\theta \leq s \leq 0} V(s) \cdot e^{-\varepsilon t}, \quad t \geq 0 \tag{31}
\end{aligned}$$

From condition in the (10), the states of the error system (9) are globally exponentially stable, i.e., the neural networks with mixed delays(3) globally exponentially synchronize to the system (2) under adaptive semi-periodically intermittent controller (7)-(8). This ends the proof.

Remark 1: Compare with [21] and [28], Theorem 1 does not need some unnecessary restrictions about control rate and control period. This leads to a larger application scope for our method. Moreover, our results have been more adaptive and robust than [31].

Remark 2: In this paper, the control time and the control rate may not be constants. In order to achieve this, we introduce the method of proof by contradiction and take the method of dividing the subintervals of different lengths within each period.

Remark 3: The transcendental equation $a_1 - \lambda - L \exp\{\lambda \theta\} = 0$ has no analytical solutions. However, we can

obtain its numerical solution by the Mathematica software. Let $F = a_1 - \lambda - L \exp\{\lambda\theta\}$. Using the function of *FindMinimum*[F, λ, λ_0], the Mathematica software can search for a local minimum in F and corresponding lambda, starting from the point $\lambda = \lambda_0$, that is, we can obtain the numerical solution lambda of the transcendental equation indirectly.

Remark 4: Especially, when $\delta_\zeta \rightarrow 0$, $\zeta = 0, 1, 2, \dots, l$, the controller (7) becomes the impulsive controller (32).

$$u_i(t) = \begin{cases} -k_{ir}e_i(t_r^-), & t = t_r, r = 1, 2, \dots; \\ 0, & t \neq t_r. \end{cases} \quad (32)$$

The general mixed delays neural networks (3) becomes the impulsive delays neural networks (33).

$$\begin{aligned} \dot{y}_i(t) = & -c_i y_i(t) + \sum_{j=1}^N b_{ij} f_j(y_j(t-\theta)) \\ & + \sum_{j=1}^N a_{ij} f_j(y_j(t)) + \sum_{j=1}^N d_{ij} \int_{t-\eta}^t f_j(y_j(s)) ds, \quad t \neq t_r. \end{aligned} \quad (33)$$

$$\text{and } \Delta y_i(t_r) = y_i(t_r^+) - y_i(t_r^-) = k_{ir} e_i(t_r^-), \quad t = t_r$$

Applying the method similar to the Theorem 1, the impulsive synchronization control criterion of impulsive delays neural networks (33) can obtained as follows.

Corollary 1: Under impulsive controller (32), if Assumption 1 holds, and there exist positive constants $a_1 > L$ and a_2 , such that

$$L + \frac{L\eta^2}{a_1} + \lambda_{\max}(\Pi) \leq a_2 < \lambda, \quad (34)$$

where $\lambda > 0$ is the positive solution of the equation $a_1 - \lambda - L \exp\{\lambda\theta\} = 0$. Then the global exponential lag synchronization is achieved between the neural networks with mixed delays (2) and the response neural networks (33).

IV. NUMERICAL SIMULATION

In this section, numerical examples are given to illustrate that our analytical results are valid for exponential lag synchronization.

Example 1: The chaotic neural networks with mixed delays drive system is given by

$$\begin{aligned} \dot{x}(t) = & -Cx(t) + Af(x(t)) + Bf(x(t-1)) \\ & + D \int_{t-1}^t f(x(s)) ds, \end{aligned} \quad (35)$$

where $x(t) = (x_1(t), x_2(t))^T$, $f(x(t)) = (\tanh(x_1(t)), \tanh(x_2(t)))^T$, $A = \begin{bmatrix} 1.3 & -0.1 \\ -1.5 & 0.2 \end{bmatrix}$, $B = \begin{bmatrix} -1.5 & -0.4 \\ 0.1 & -2 \end{bmatrix}$, $C = \text{diag}(1, 1)$, $D = \begin{bmatrix} -0.4 & 0.1 \\ -0.1 & -0.6 \end{bmatrix}$.

Figure 1 is the numerical simulation of system (35), which shows that system (35) has a chaotic attractor. Next, the

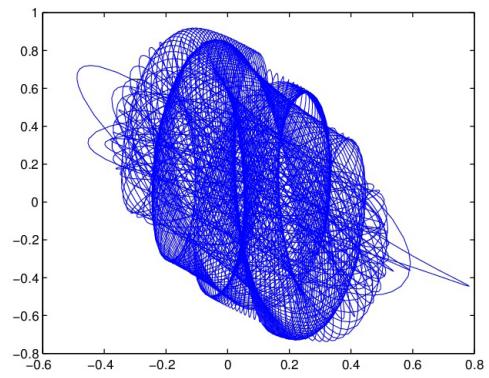


FIGURE 1. Chaotic trajectory of model (35).

response system is described by

$$\begin{aligned} \dot{y}(t) = & -Cy(t) + Af(y(t)) + Bf(y(t-1)) \\ & + D \int_{t-1}^t f(y(s)) ds + u(t), \end{aligned} \quad (36)$$

the parameters A, B, C , and D are the same in system (35).

In this simulation, the values of the parameters are taken as $\tau = 6$, $T = 1$, $T_1 = 0.4$, $T_2 = 0.6$, $\delta_1 = 0.15$, $\delta_2 = 0.2$, $\alpha_i = 5$ ($i = 1, 2$). If we choose $a_1 = 70$ and $a_2 = 6.5$, it is easy to testify that (10) in Theorem 1 of Section 3 are satisfied, hence drive system (35) and response system (36) are exponentially lag synchronized under the adaptive semi-periodically intermittent controller (7) and updating laws (8). Denoting $e_i(t) = y_i(t) - x_i(t-6)$, Figure 2 and Figure 3 shows the lag synchronization of neural networks with mixed delays (35) and (36) under the controllers (7)–(8) when lag $\tau = 6$ is selected.

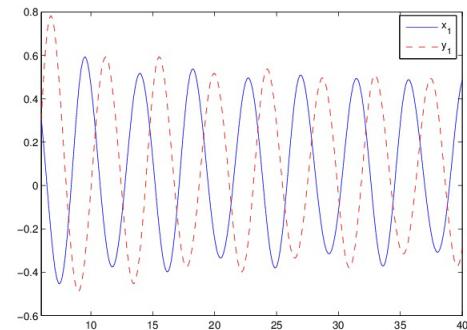


FIGURE 2. Lag synchronization of x_1 and y_1 of neural network under the adaptive semi-periodically intermittent controllers (7)–(8).

Example 2: Consider a delayed coupled neural network with $N = 5$ nodes described by

$$\dot{y}_i(t) = F(y_i(t), y_i(t-1)) + \sum_{j=1}^5 a_{ij} \Gamma_j y_j(t), \quad (37)$$

where the single delayed dynamical equation $F(\cdot)$ is described by the form of (35). In practice, the weights and coupling structures in large-scale complex dynamical networks are always to be uncertain or unknown. In comparison

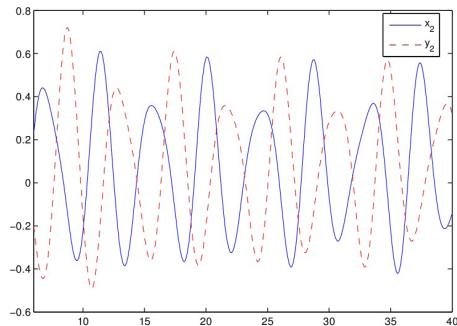


FIGURE 3. Lag synchronization of x_2 and y_2 of neural network under the adaptive semi-periodically intermittent controllers (7)–(8).

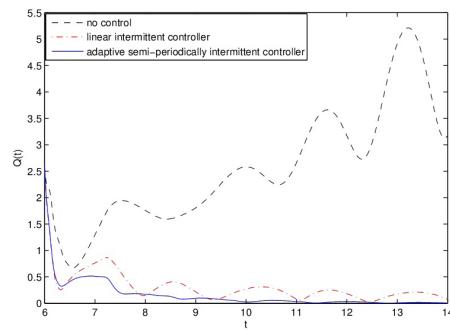


FIGURE 4. Comparison of linear intermittent control and adaptive semi-periodically intermittent control qualities $Q(t)$ with the disturbance signal.

with the robustness, we introduce the disturbance signal to the complex network with adaptive semi-periodically intermittent controller and the linear intermittent controller, respectively. For validating the result, we assume that coupling matrix $\mathcal{A} = (a_{ij})$ is included mismatched disturbance. Let

$$\mathcal{A} = \begin{pmatrix} 1 - 2\sin^2 t & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} - 2\cos^2 t & \frac{1}{4} \\ \frac{1}{2} & -1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & -1 & 0 & 0 \\ \frac{1}{3} - 4\sin^2 t & 0 & \frac{1}{3} & 3 - 4\cos^2 t & \frac{1}{3} \\ \arctan t & \frac{1}{2} & \frac{1}{2} & 0 & -1 - \arctan t \end{pmatrix},$$

The individual coupling matrix is $\Gamma = \text{diag}\{1, 1, 1\}$. The other parameters are same the system (35).

The quantity $Q(t) = \sqrt{\sum_{i=1}^5 [y_i(t) - x(t-6)]^2 / 5}$ is used to measure the quality of the synchronization process. We plot the synchronization quantity in the figure 3. Obviously, the states of the network (37) can not achieve lag-synchronization without controller. The states of the network (37) do not lag-converge to the desired manifold (35) under the common linear intermittent controller in [21] and [28], while the adaptive semi-periodically intermittent controller (7)–(8) can still guarantee the lag synchronization of the controlled network (37) onto $x(t)$. It is shown that the adaptive semi-periodically intermittent

controllers have strong robustness against uncertainties in comparison with linear intermittent controllers.

V. CONCLUSIONS

Lag synchronization of a class of neural networks with discrete delays and distributed delays with adaptive semi-periodically intermittent control is investigated in this paper. Based on the stability theory, combined with semi-periodically intermittent control method, the technique of adaptive control and some general criteria for ensuring neural networks with mixed delays lag synchronization have been derived. And the corresponding adaptive semi-periodically intermittent feedback synchronization controllers and impulse synchronization controllers are designed. The results obtained can be used in the mechatronics systems. Moreover, our results are rather general and less conservative. In this paper, we introduce the notation of adaptive semi-periodically intermittent control, which remove the strict assumptions on control width and time delays. And the conclusions in this paper enhance and generalize the previous results. Finally, an illustrative example is presented to show the feasibility and effectiveness of the presented method.

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