Chapter 14
Autoencoder

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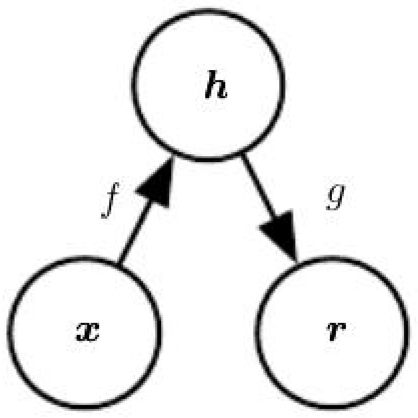
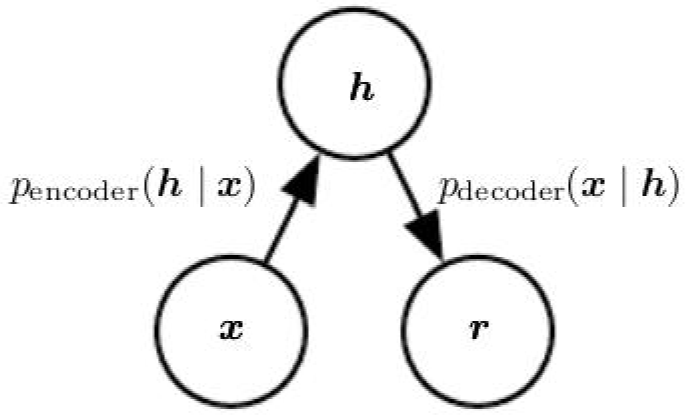
Autoencoder

- A neural network that attempt to copy input to output
- An encoder function **h** = f(**x**), or sotchastic mapping p(**h**|**x**)

  A decoder function **r** = g(**h**), or p(**x**|**h**)

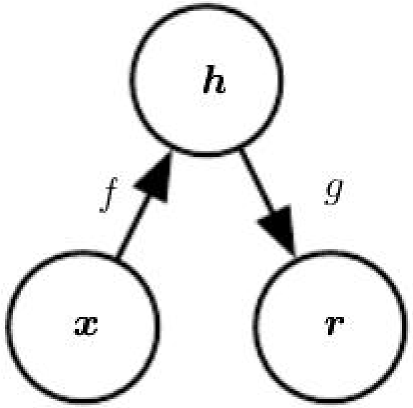




Design Idea

- Disigned to be unable to learn to copy perfectly,
  Just learning a Identity function is useless

- Copy only input that resembles the training data,
  the encoder should capture useful features of
  the training data.



Undercomplete Autoencoder

- Let h to have less dimension than x (bottle-neck)

- When the decoder is linear and loss is the mean squared error, an undercomplete autoencoder span learns the same subspace as PCA.

- When encoder and decoder are too powerful, autoencoder can still learning an just copy network.

Regularized Autoencoder

Use a loss function to have other properties besides copying.

- Sparse Autoencoder: sparsity of the representation $\mathbf{h} = f(\mathbf{x})$
- Denoising Autoencoder: robustness to noise or to missing input
- Contractive Autoencoder: smallness of the derivative of the representation $\mathbf{h} = f(\mathbf{x})$

Sparse Autoencoder (SAE)

- Reconstructive Loss and Sparsity Regularizer

$$L(\boldsymbol{x}, g(f(\boldsymbol{x}))) + \Omega(\boldsymbol{h})$$

$$\Omega(\boldsymbol{h}) = \lambda \sum_i |h_i|$$

- Interpretation of the regularizer:

Weight decay regularizer $\Omega(\theta)$, a Gaussian prior over parameter θ

Sparsity regularizer $\Omega(h)$, a Laplace prior over the activation h

Connection to logZ in maximum likelihood

- For maximum likelihood: $p(\boldsymbol{x}) = \frac{1}{Z}\tilde{p}(\boldsymbol{x})$

  minimizing logZ prevent p(x) large everywhere
- Sparsity prevent h large everywhere

Sparse Autoencoder and Sparse coding

We can think of SAE as a generative model with latent variables, but the latent variable **h** is the output of an parametric encoder rather than the result of an optimization.

So SAE could be optimization use Gradient Descent instead of EM.

Why the features h learned by the autoencoder are useful? They describe the latent variables that explain the input.

Predictive Sparse Decomposition (PSD)

Training SAE just like Sparse coding:

- Alternates minimization with respect to h and minimization with respect to the model.

$$||\boldsymbol{x} - g(\boldsymbol{h})||^2 + \lambda|\boldsymbol{h}|_1 + \gamma||\boldsymbol{h} - f(\boldsymbol{x})||^2$$

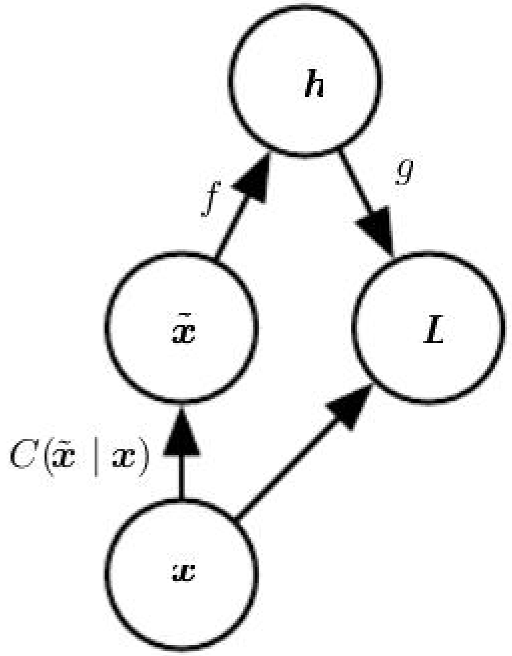
- PSD is kind of learned approximate inference.

Denoising Autoencoder(DAE)

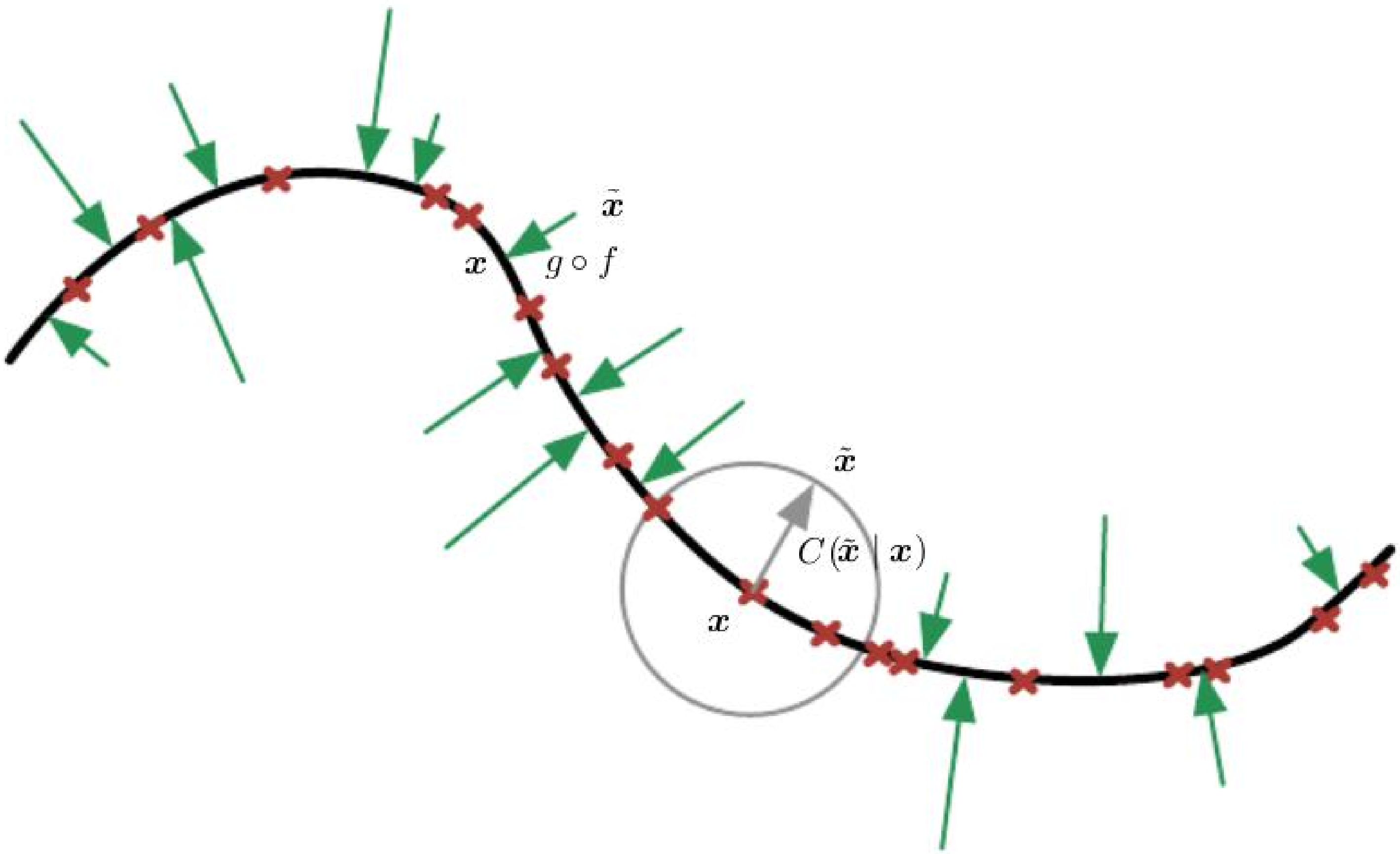
- DAE receives a corrupted data point as input and is trained to predict the original, uncorrupted data point as its output.

- C(x'|x) is a corruption process: add noise or delete part of input.

$$-\mathbb{E}_{\mathbf{x}\sim\hat{p}_{\text{data}}(\mathbf{x})}\mathbb{E}_{\tilde{\mathbf{x}}\sim C(\tilde{\mathbf{x}}|\boldsymbol{x})}\log p_{\text{decoder}}(\boldsymbol{x}\mid\boldsymbol{h}=f(\tilde{\boldsymbol{x}}))$$



Example



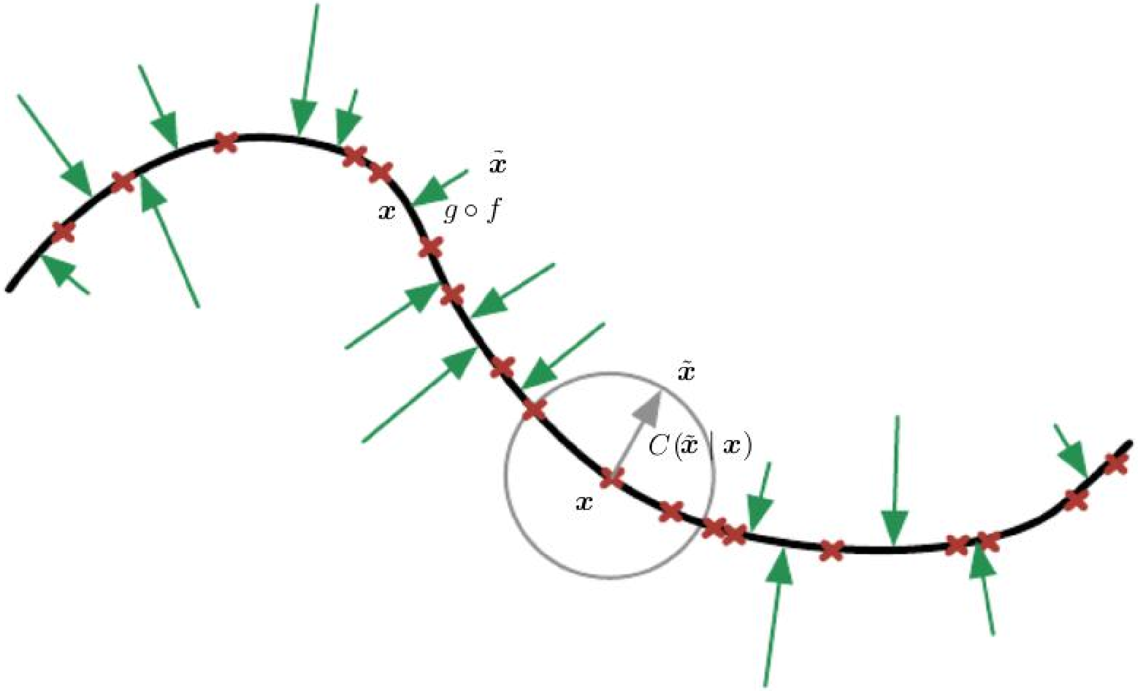
Score Matching

- an alternative to maximum likelihood
- encourage the model have same score as the data distribution at every training point **x**.
- In this context, the score is a particular gradient field:

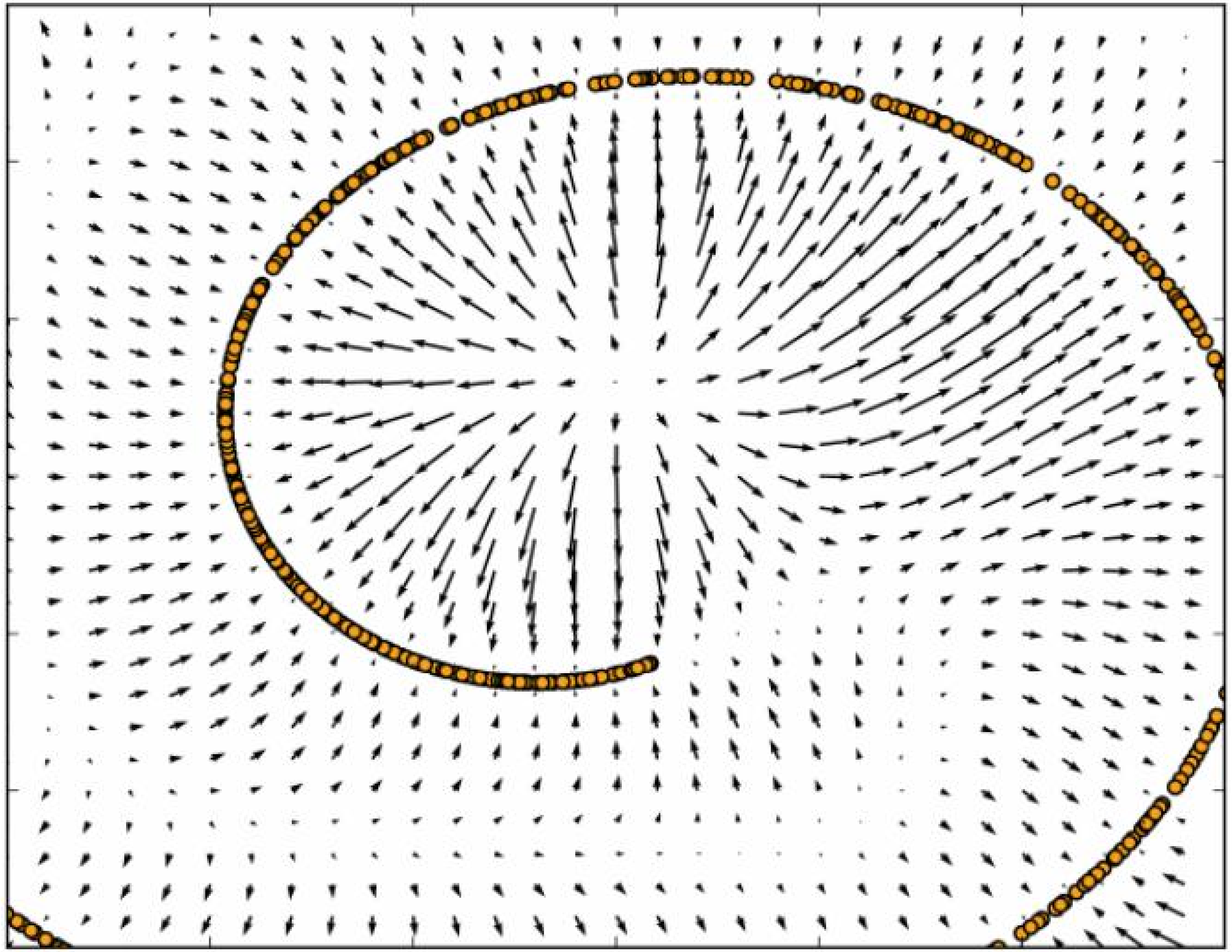
$$\nabla_{\boldsymbol{x}} \log p(\boldsymbol{x})$$

DAE and Score Matching

- DAE learning vector field (g(f(**x**))-**x**)
- The vector field estimates the score of the data distribution



Vector field learned by DAE



Contractive Autoencoder (CAE)

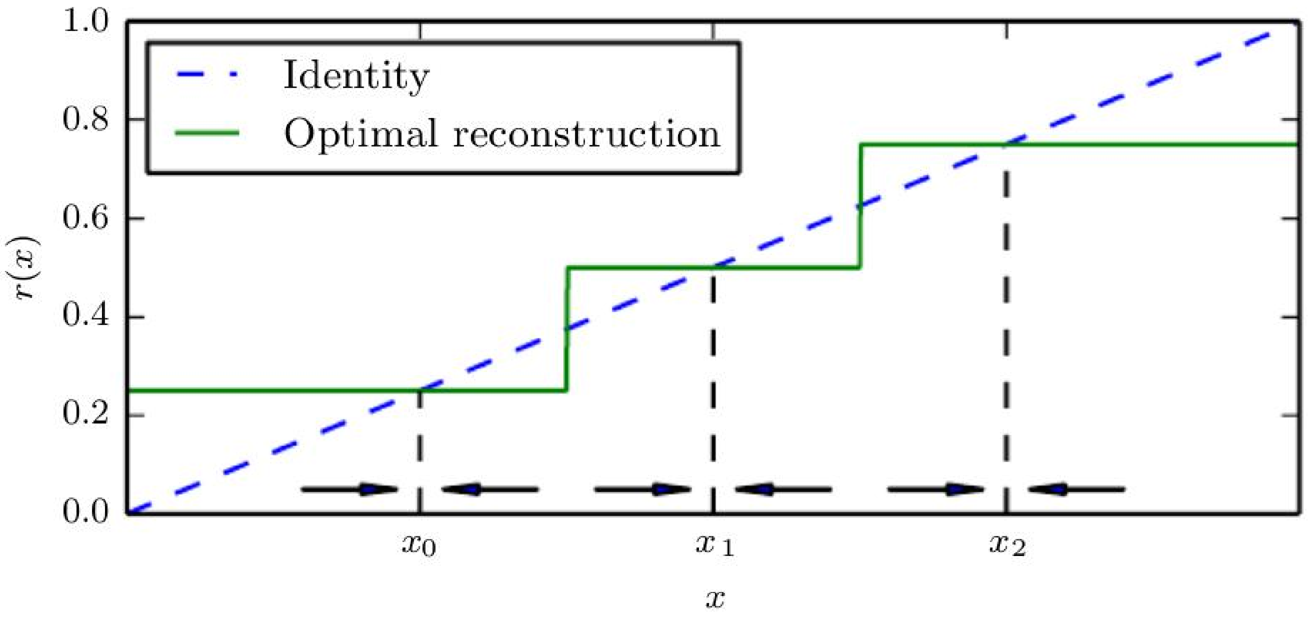
- encouraging the derivatives of encoder f(**x**) to be as small as possible by a regularizer:

$$\Omega(\boldsymbol{h}) = \lambda \left\| \frac{\partial f(\boldsymbol{x})}{\partial \boldsymbol{x}} \right\|_F^2$$

- name contractive because CAE encourage to map two neighbor point more closer.

Contractive Autoencoder

- CAE contractive only locally, all pertubations of a training point **x** are mapped near to f(**x**).
- Globally, two different point x and x' may be map farther than the original point.
- For sigmoidal units, CAE may make h saturate to 0 or 1.



DAE and CAE

- DAE make the reconstruction function ($g(f(\mathbf{x}))$) resist finite-sized pertubations of the input.

- CAE make the feature extraction function ($f(\mathbf{x})$) resist infinitesimal pertubations of the input.

Practical issue with the CAE

Issue 1: It's expensive to compute the regularizer when autoencoder is deep, because gradient is layer-wise related.

Solution: separately train a series of single-layer autoencoder

Practical issue with the CAE

Issue 2: The contraction penalty can obtain useless result if we do not constraint the scale of decoder. (Such as $f'(x)=\varepsilon f(x)$,

$g'(h)=g(h)/\varepsilon$)

Solution: set the weight matrix of g(h) to be the transpose of the

weight matrix of f(x).

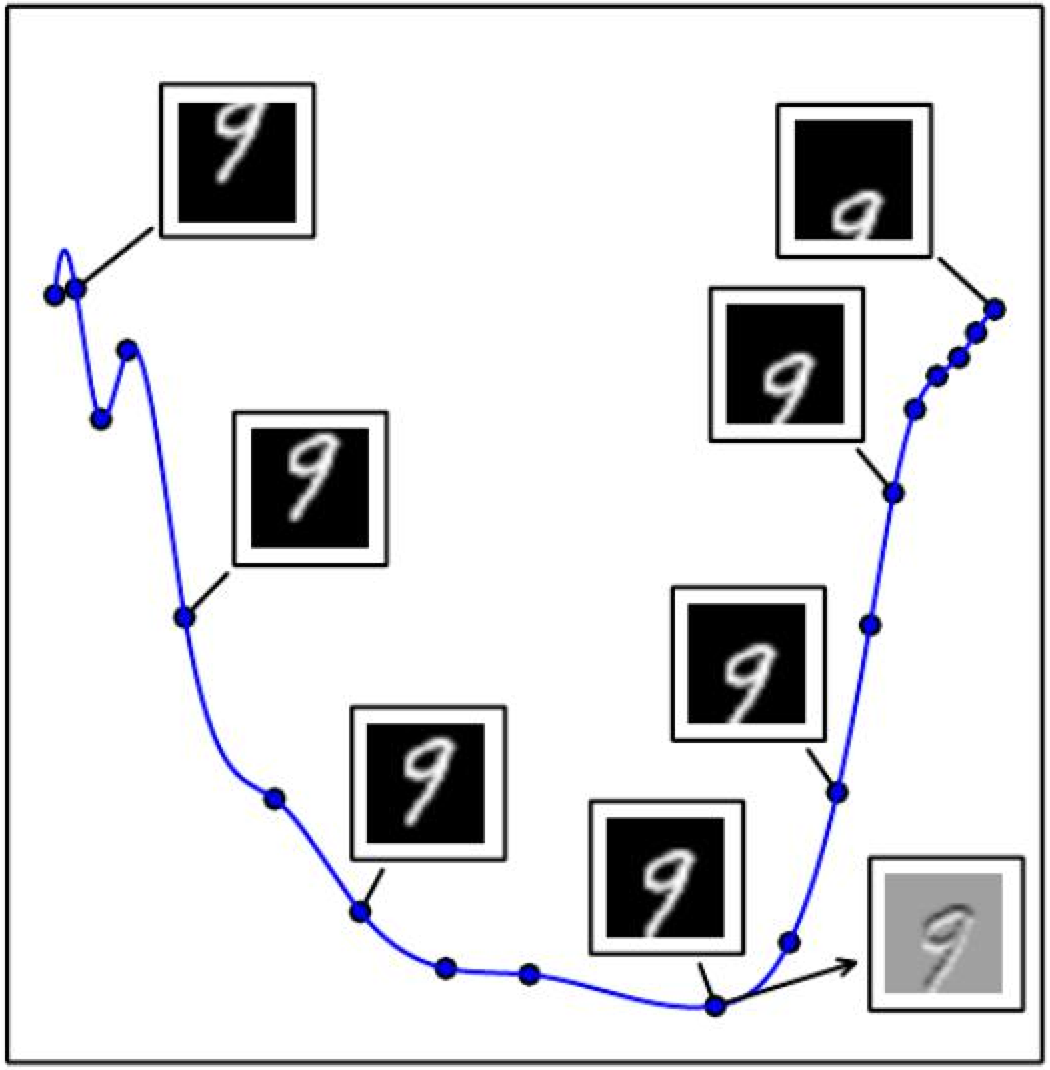
A Characterizaion of Manifolds

- The set of manifolds' tangent planes

- At a point **x** on a d-dimensional manifold, the tangent plane is given by d basis vectors

An illustration of a tangent hyperplane.

- A one-dimension manifold given by transform a image vertically.
- Projected into two-dimension by PCA.
- Change **x** infinitesimally while staying on the manifold.



Learning Manifolds with Autoencoder

Training an autoencoder involve a compromise between two forces:

- Learning a representation **h** of a training example x such that x can be approximately recovered from **h** throught a decoder.
- Satisfying the constraint or regularization penalty.

Learning Manifolds with Autoencoder

Copying the input to the output is not useful, nor is ignoring the input.

- The autoencoder can afford to represent only the variations that are needed to reconstruct training examples.

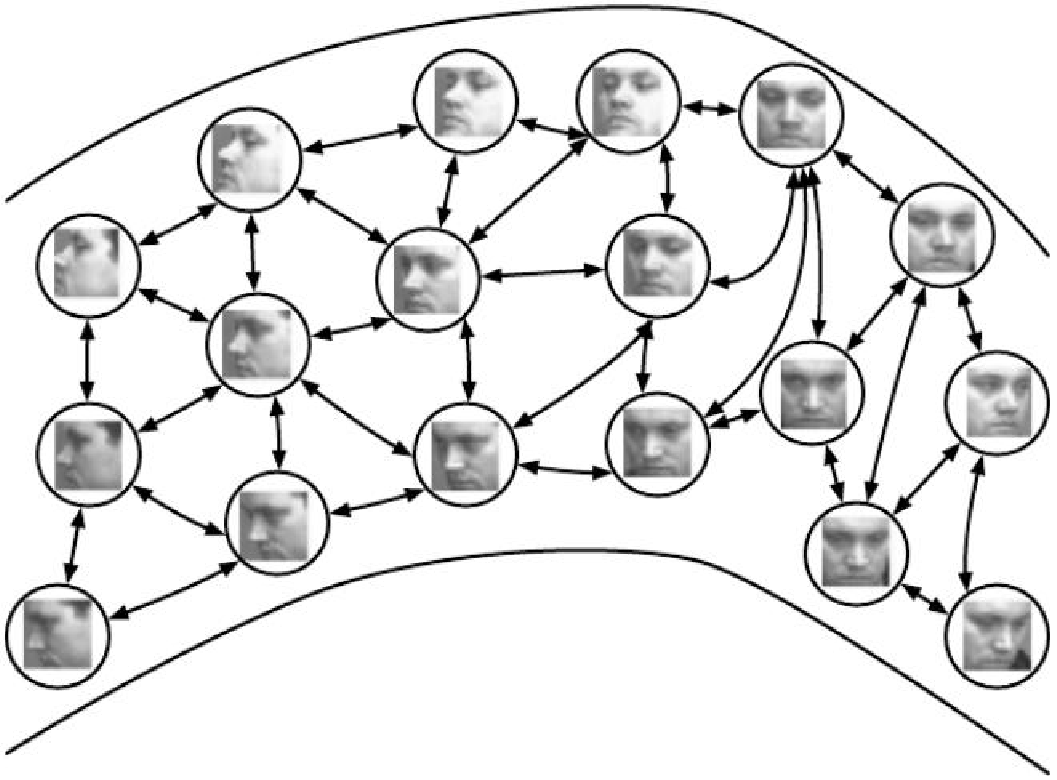
- The variations correspond to changes in $\mathbf{h} = f(\mathbf{x})$.

Learning Manifolds with non-parametric methods

based on the nearest-neighbor graph.

For each point **x** and its neighb
we could get a local coordinate
system.

By an optimization or solving a
linear system, we could get a
global coordinate system.



Learning Manifolds with non-parametric methods

- To get representation to a new example, a form of interpolation may be used.
- But if the manifolds are not very smooth, one may need a very large number of example to cover each one of all variations.
- And interpolation in high-dimension space may be meaningless.

Applications of Autoencoders

- Dimensinality reduction
- As sematic hashing on information retrieval
- Semi-supervised learning
- Pretrain for deep model (no longer needed now)

Thanks