Numerical Methods, 2018 Spring

Assignment 5
Due in the class of 6/4

Note: you should explain how you obtain your solution in your submission. If you use MATLAB or any other software to compute your results, you should provide your code or describe the solving process. This is a good practice for you to explain things in a logical, organized, and concise way!

8. (25%) Find the solution to $\frac{dy}{dt} = y^2 + t^2$, $y(1) = 0$, at $t = 2$ by the Euler method, using $h = 0.1$. Repeat with $h = 0.05$. From the two results, estimate the accuracy of the second computation. Hints: use the Romberg Integration (Slide 12 of Chap5_2) to estimate the error.

11. (25%) Repeat the previous problem but use the midpoint method. Are the results the same? If not, which is more accurate?

47. (25%) For the third-order equation

$$y''' + ty' - 2y = t \ , y(0) = y''(0) = 0 \ , y'(0) = 1,$$

(a) Solve for $y(0.2)$, $y(0.4)$, $y(0.6)$ by RKF (you can use ode45() in Matlab to run RKF, but you need to list the system of ODE equations and explain how you solve this equation. Please upload you code to E3).

(b) Advance the solution to $t = 1.0$ with the Adams-Moulton method.

(c) Estimate the accuracy of $y(1.0)$ in part (b).

60. (25%) Given the boundary-value problem:

$$\frac{d^2y}{d\theta^2} + \frac{y}{4} = 0 \ , y(0) = 0 \ , y(\pi) = 2,$$

which has the solution $y = 2\sin\left(\frac{\theta}{2}\right)$

(a) Solve using finite difference approximations to the derivative with $h = \frac{\pi}{4}$ and tabulate the errors.

(b) Solve again by finite differences but with a value of h small enough to reduce the maximum error to 0.5%.

(c) Solve again by the shooting method. Find how large h can be to have maximum error of 0.5%. (Please use the secant method to find $y'(0)$)

66. (25%) (Bonus) The most general form of boundary condition normally encountered in second-order boundary-value problems is a linear combination of the function and its derivatives at both ends of the region. Solve through finite difference approximations with four subintervals:

$$\begin{aligned}
&x'' - tx' + t^2 x = t^3, \\
&x(0) + x'(0) - x(1) + x'(1) = 3, \\
&x(0) - x'(0) + x(1) - x'(1) = 2.
\end{aligned}$$