Step 1

Step 1:
Use the following algorithm of Midpoint Method to compute the approximate solution of the differential equation:

1. Specify t_n and y_n
2. Specify a step-size, *h*
3. For *n* from 1 to *k* compute the following

a. $t_{n+1/2} = t_n + h/2$

b. $t_{n+1} = t_n + 0.2$

c. $y_{n+1/2} = y_n + \frac{h}{2} y'(t_n, y_n)$

d. $y_{n+1} = y_n + hy'(t_{n+1/2}, y_{n+1/2})$

4. Repeat No. 1 for $t_n + h$ and y_{n+1} until $y_t \approx \text{Exact Solution}$

Step 2

Step 2: Use the algorithm in Step 1 with $h = 0.1$.
The Excel setup for this problem is given in the following the computed values:

Step 3

	A	B	C	D	E
1	h	0.1			
2	tn	tn+1/2	tn+1	yn	yn+1/2
3	1	=A3+$B$1/2	=A4+$B$1	0	=D3+$B$1/2*(D

Step 2: Use the algorithm in Step 1 with $h = 0.1$.
The Excel setup for this problem is given in the following the computed values:

Step 3

	A	B	C	D	E
1	h	0.1			
2	tn	tn+1/2	tn+1	yn	yn+1/2
3	1	=A3+$B$1/2	=A4+$B$1	0	=D3+$B$1/2*(D
4	=A3+$B$1	=A4+$B$1/2	=A5+$B$1	=F3	=D4+$B$1/2*(D
5	=A4+$B$1	=A5+$B$1/2	=A6+$B$1	=F4	=D5+$B$1/2*(D
6	=A5+$B$1	=A6+$B$1/2	=A7+$B$1	=F5	=D6+$B$1/2*(D
7	=A6+$B$1	=A7+$B$1/2	=A8+$B$1	=F6	=D7+$B$1/2*(D
8	=A7+$B$1	=A8+$B$1/2	=A9+$B$1	=F7	=D8+$B$1/2*(D
9	=A8+$B$1	=A9+$B$1/2	=A10+$B$1	=F8	=D9+$B$1/2*(D

The output is as shown below:

tn	tn+1/2	tn+1	yn
1.0000	1.0500	1.2000	0.0000
1.1000	1.1500	1.3000	0.1105
1.2000	1.2500	1.4000	0.2457
1.3000	1.3500	1.5000	0.4122
1.4000	1.4500	1.6000	0.6200
1.5000	1.5500	1.7000	0.8846
1.6000	1.6500	1.8000	1.2322
1.7000	1.7500	1.9000	1.7107
1.8000	1.8500	2.0000	2.4176
1.9000	1.9500	2.1000	3.5846
2.0000	2.0500	2.2000	5.9076

Step 4

Step 3: As a result, using the Midpoint Method and a step size $h =$ 0.1,

Step 2: Use the algorithm in Step 1 with $h = 0.1$.
The Excel setup for this problem is given in the following the computed values:

Step 3

C	D	E	F
tn+1	yn	yn+1/2	yn+1
A4+$B$1	0	=D3+$B$1/2*(D3^2+A3^2)	=D3+$B$1*(E3^2+B3^2)
A5+$B$1	=F3	=D4+$B$1/2*(D4^2+A4^2)	=D4+$B$1*(E4^2+B4^2)
A6+$B$1	=F4	=D5+$B$1/2*(D5^2+A5^2)	=D5+$B$1*(E5^2+B5^2)
A7+$B$1	=F5	=D6+$B$1/2*(D6^2+A6^2)	=D6+$B$1*(E6^2+B6^2)
A8+$B$1	=F6	=D7+$B$1/2*(D7^2+A7^2)	=D7+$B$1*(E7^2+B7^2)
A9+$B$1	=F7	=D8+$B$1/2*(D8^2+A8^2)	=D8+$B$1*(E8^2+B8^2)
A10+$B$1	=F8	=D9+$B$1/2*(D9^2+A9^2)	=D9+$B$1*(E9^2+B9^2)

tn+1	yn	yn+1/2	yn+1
1.2000	0.0000	0.0500	0.1105
1.3000	0.1105	0.1716	0.2457
1.4000	0.2457	0.3207	0.4122
1.5000	0.4122	0.5052	0.6200
1.6000	0.6200	0.7372	0.8846
1.7000	0.8846	1.0362	1.2322
1.8000	1.2322	1.4362	1.7107
1.9000	1.7107	2.0016	2.4176
2.0000	2.4176	2.8719	3.5846
2.1000	3.5846	4.4076	5.9076
2.2000	5.9076	7.8525	12.4941

Step 4

Step 3: As a result, using the Midpoint Method and a step size $h = 0.1$,

Step 4

Step 3: As a result, using the Midpoint Method and a step size $h = 0.1$,

$$\boxed{y(2) = 5.9076}.$$

Step 5

Step 4: With $h = 0.05$, the computed values are given in the following table:

tn	tn+1/2	tn+1	yn
1.0000	1.0250	1.1000	0.0000
1.0500	1.0750	1.1500	0.0526
1.1000	1.1250	1.2000	0.1107
1.1500	1.1750	1.2500	0.1749
1.2000	1.2250	1.3000	0.2462
1.2500	1.2750	1.3500	0.3252
1.3000	1.3250	1.4000	0.4132
1.3500	1.3750	1.4500	0.5116
1.4000	1.4250	1.5000	0.6220
1.4500	1.4750	1.5500	0.7467
1.5000	1.5250	1.6000	0.8885
1.5500	1.5750	1.6500	1.0513
1.6000	1.6250	1.7000	1.2402
1.6500	1.6750	1.7500	1.4624
1.7000	1.7250	1.8000	1.7281
1.7500	1.7750	1.8500	2.0527
1.8000	1.8250	1.9000	2.4599
1.8500	1.8750	1.9500	2.9888
1.9000	1.9250	2.0000	3.7083
1.9500	1.9750	2.0500	4.7516
[illegible]	[illegible]	[illegible]	[illegible]

Step 5

Step 4: With $h = 0.05$, the computed values are given in the following table:

tn	tn+1/2	tn+1	yn
1.0000	1.0250	1.1000	0.0000
1.0500	1.0750	1.1500	0.0526
1.1000	1.1250	1.2000	0.1107
1.1500	1.1750	1.2500	0.1749
1.2000	1.2250	1.3000	0.2462
1.2500	1.2750	1.3500	0.3252
1.3000	1.3250	1.4000	0.4132
1.3500	1.3750	1.4500	0.5116
1.4000	1.4250	1.5000	0.6220
1.4500	1.4750	1.5500	0.7467
1.5000	1.5250	1.6000	0.8885
1.5500	1.5750	1.6500	1.0513
1.6000	1.6250	1.7000	1.2402
1.6500	1.6750	1.7500	1.4624
1.7000	1.7250	1.8000	1.7281
1.7500	1.7750	1.8500	2.0527
1.8000	1.8250	1.9000	2.4599
1.8500	1.8750	1.9500	2.9888
1.9000	1.9250	2.0000	3.7083
1.9500	1.9750	2.0500	4.7516

Step 6

Step 5: As a result, using the Midpoint Method and a step size $h = 0.05$,

$$\boxed{y(2) = 6.4106}$$

.

Step 5

puted values are given in the

tn+1	yn	yn+1/2	yn+1
1.1000	0.0000	0.0250	0.0526
1.1500	0.0526	0.0802	0.1107
1.2000	0.1107	0.1412	0.1749
1.2500	0.1749	0.2088	0.2462
1.3000	0.2462	0.2837	0.3252
1.3500	0.3252	0.3669	0.4132
1.4000	0.4132	0.4597	0.5116
1.4500	0.5116	0.5637	0.6220
1.5000	0.6220	0.6807	0.7467
1.5500	0.7467	0.8132	0.8885
1.6000	0.8885	0.9645	1.0513
1.6500	1.0513	1.1390	1.2402
1.7000	1.2402	1.3427	1.4624
1.7500	1.4624	1.5839	1.7281
1.8000	1.7281	1.8750	2.0527
1.8500	2.0527	2.2346	2.4599
1.9000	2.4599	2.6922	2.9888
1.9500	2.9888	3.2977	3.7083
2.0000	3.7083	4.1424	4.7516
2.0500	4.7516	5.4111	6.4106

Step 6

Step 5: As a result, using the Midpoint Method and a step size h = 0.05,

$$\boxed{y(2) = 6.4106}$$

.

1.1000	0.0000	0.0250	0.0526
1.1500	0.0526	0.0802	0.1107
1.2000	0.1107	0.1412	0.1749
1.2500	0.1749	0.2088	0.2462
1.3000	0.2462	0.2837	0.3252
1.3500	0.3252	0.3669	0.4132
1.4000	0.4132	0.4597	0.5116
1.4500	0.5116	0.5637	0.6220
1.5000	0.6220	0.6807	0.7467
1.5500	0.7467	0.8132	0.8885
1.6000	0.8885	0.9645	1.0513
1.6500	1.0513	1.1390	1.2402
1.7000	1.2402	1.3427	1.4624
1.7500	1.4624	1.5839	1.7281
1.8000	1.7281	1.8750	2.0527
1.8500	2.0527	2.2346	2.4599
1.9000	2.4599	2.6922	2.9888
1.9500	2.9888	3.2977	3.7083
2.0000	3.7083	4.1424	4.7516
2.0500	4.7516	5.4111	6.4106

Step 6

Step 5: As a result, using the Midpoint Method and a step size h = 0.05,

$$\boxed{y(2) = 6.4106}.$$

The results are not the same.

The exact answer to the differential equation at t = 2 is 6.703787.

Therefore, the Modified Euler Method with step-size h = 0.05 is the most accurate.