1.36.

Section 1.5

36. Most functions can be rearranged in several ways to give $x = g(x)$ with which to begin the fixed-point method. For $f(x) = e^x - 2x^2$, one $g(x)$ is

$$x = \pm\sqrt{(e^x/2)}.$$

a. Show that this converges to the root near 1.5 if the positive value is used and to the root near -0.5 if the negative is used.

b. There is a third root near 2.6. Show that we do not converge to this root even though values near to the root are used to begin the iterations. Where does it converge if $x_0 = 2.5$? If $x_0 = 2.7$?

c. Find another rearrangement that does converge correctly to the third root.

1.47.

47. MATLAB finds six solutions to this system and two are complex valued. Two of the real solutions are near (1, 1, 1) and (1.3, 0.9, -1.2).

$$x - 3y - z^2 = -3,$$
$$2x^3 + y - 5z^2 = -2,$$
$$4x^2 + y + z = 7.$$

a. What are the partial derivatives that would be used in Newton's method?

b. The matrix of partial derivatives is called the Jacobian matrix. For the starting vector [1, 1, 1], what are its elements?

c. Complete getting the two solutions with Newton's method. Find starting values that converge. Is convergence quadratic?