

Applied Numerical Analysis | (7th Edition)

Chapter 3, Problem 3APP

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Problem

Use the data of APP with several approaches to extrapolate backward to estimate the expected expenditure for 1980. How do these values compare to 492 billion, the amount actually spent?

APP

The cost of government welfare programs adds significantly to our taxes. The table below gives data for several years:

Year	Expenditures in billions of dollars
1985	731
1986	782
1987	833
1988	886
1989	956
1990	1049
1991	1159
1992	1267
1993	1367
1994	1436
1995	1505

Use the data between 1991 to 1994 to estimate what the value would be in 1995 and compare to the value in the table. Do this

- From a cubic interpolating polynomial.
- From the least-squares line.
- From the least-squares quadratic.
- From a cubic spline.

From each of these, project to find what one would anticipate the expenditures for the year 2000 might be; then find what the actual expenditures were for comparison.

Step-by-step solution

Step 1 of 11

a) When using numerical approximation techniques, it is useful to have programs written to aid in numerical calculations to avoid calculations by hand. Using Matlab, create an M-file named 'lagrange.m' which will contain Matlab code that implements the algorithm to interpolate Lagrange polynomials and return the coefficients of the polynomial in Lagrange form. Inside this file, write the code to implement the method. One example of a correctly executable routine based on the algorithm description in the text is as follows:

```
function rn = lagrange (X, F, x0)
%given a vector list of x-values and a vector
%list of y-values, the Lagrangian interpolation of
%x0 is calculated.
n = length (X);
coefficient = zeros(1,n);
f=0;
%Lagrangian Interpolation loop
for i = 1:n
    P=1; coeff=1;
    for j = 1:n
        if j ~= i
            P=P*(x0-X(j))/(X(i)-X(j));
            coeff=coeff*(X(i)-X(j));
        end
    end
    f = f + P * F(i);
    coefficient(i)=coeff*P(i);
end
%outputs
f
```

[Comment](#)

Step 2 of 11

Letting 0 correspond to 1985, 1 correspond to 1986, and so forth, use the following Matlab commands to extrapolate the value at -5 corresponding to 1980.

```
INPUT:
>> format shortG
>> x=[0,1,2,3,4,5,6,7,8,9,10];
>> f=[731,782,833,886,956,1049,1159,1267,1367,1436,1505];
>> lagrange(x,f,5)
OUTPUT:
f =
640228
```

[Comment](#)

Step 3 of 11

b) For almost all least-squares problems in this text, the data can be fit to a **linear combination of basis functions** $\{1, x_1, x_2, \dots, x_n\}$ (these are function vectors such as $x, y, z^2, \ln x, e^x$, and so forth) of the form

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n$$

which when fitting the equation to m data points (leads to the system of equations (expressed compactly in matrix notation).

$$\sum_{j=1}^m X_{ij} \beta_j = y_i, (i=1,2,\dots,m),$$

$$y = X\beta$$

$$X = \begin{bmatrix} X_{11} & X_{12} & \dots & X_{1(n+1)} \\ X_{21} & X_{22} & \dots & X_{2(n+1)} \\ \vdots & \vdots & \ddots & \vdots \\ X_{m1} & X_{m2} & \dots & X_{m(n+1)} \end{bmatrix}, \quad \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_n \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix},$$

which is known as the **general linear least-squares model**.

[Comment](#)

Step 4 of 11

The get the least squares solution to the system, the deviation function

$$\begin{aligned} S(\beta) &= \|y - X\beta\|^2 \\ &= (y - X\beta)^T (y - X\beta) \\ &= y^T y - \beta^T X^T y - y^T X \beta + \beta^T X^T X \beta \\ &= y^T y - 2\beta^T X^T y + \beta^T X^T X \beta. \end{aligned}$$

must be minimized, so differentiating with respect to β and setting $S(\beta) = 0$ yields the normal equation

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and solving for β yields
$$\beta = (X^T X)^{-1} X^T y$$

This form leads to a computationally efficient solution to the problems and numerical calculations can be directly entered into the Matlab command window.

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Step 5 of 11

To get the solution using a least-squares line, use the following sequence of Matlab commands

INPUT:

```
>> X=[ones(size(X')) X];  
>> B=(X\X)(X\Y)
```

OUTPUT:

```
B =  
676.77  
82.3
```

So the model is

$$f(x) = 82.3x + 676.77$$

[Comment](#)

Step 6 of 11

To extrapolate at $x = -5$, use the Matlab command

[Comment](#)

Step 7 of 11

INPUT:

```
>> 676.77+82.3*(-5)
```

OUTPUT:

```
ans =  
265.57
```

[Comment](#)

Step 8 of 11

c) To get the solution using a least-squares quadratic, use the following sequence of Matlab commands

[Comment](#)

Step 9 of 11

INPUT:

```
>> X=[ones(size(X')) X X.^2];  
>> B=(X\X)(X\Y)
```

OUTPUT:

```
B =  
715.32  
56.601  
2.5699
```

So the model is

$$f(x) = 2.5699x^2 + 56.601x + 715.32$$

[Comment](#)

Step 10 of 11

To extrapolate at $x = -5$, use the Matlab command

[Comment](#)

Step 11 of 11

INPUT:

```
>> 715.32+56.601*(-5)+2.5699*(-5)^2
```

OUTPUT:

```
ans =  
496.56
```

Looking at the Matlab outputs, the least-square quadratic is the closest approximation to the true value 492.

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Recommended solutions for you in Chapter 3

Chapter 3, Problem 76E

If the data of Exercise are plotted on log-log paper, the points appear to be nearly linear with a slope of 2. That means that a quadratic, $F = aP^2 + bP + c$, should fit the data. a. Get the coefficients of this quadratic by least squares. b. Is...

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Chapter 3, Problem 69E

The equation of a plane is $z = ax + by + c$. We can fit experimental data to such a plane using the least-squares technique. Here are some data for $z = f(x, y)$...

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