Numerical Methods, 2017 Spring

Assignment 1
Due in the class on Mar 20

Note: you should explain how you obtain your solution in your submission. If you use MATLAB or any other software to compute your results, you should provide your code and describe your solving process. This is a good practice for you to explain things in a logical, organized, and concise way! **Please hand in your assignment in the class**.

1. (25%) Where do the curves of $y = \sin(x)$ and $y = x^3 - 1$ intersect? Please compute your answer to attain an accuracy of 10^{-5} using (a) the bisection method (b) the secant method, and (c) Newton's method.
2. (20%) Use Newton's method on the polynomial $P(x) = (x-2)^3(x-4)^2$ with $x_0 = 3$. Does it converge? To which root? Is convergence quadratic?
3. (35%) For the equation
$$f(x) = x^2 - x - 2 = 0,$$
each of the following functions yields an equivalent fixed-point problem:
$$g_1(x) = x^2 - 2,$$
$$g_2(x) = \sqrt{x+2},$$
$$g_3(x) = 1 + 2/x$$
$$g_4(x) = (x^2+2)/(2x-1)$$

   (a) Analyze the convergence properties of each of the corresponding fixed-point iteration schemes for the root $x = 2$ by considering $|g'_i(2)|$.

   (b) Confirm your analysis by implementing each of the schemes and verifying its convergence (or lack thereof) and approximate convergence rate. (**Please submit your code with notes to the E3 web**.)

4. (25%) Express the Newton iteration for solving the system of nonlinear equations
$$x_1^2 - x_2^2 = 0$$
$$2x_1x_2 = 1$$
with the starting value $x_0 = [0 \quad 1]^T$. How many iterations are needed to attain an accuracy of 10^{-5}? List the results at each iteration.