Step 1

Use the following algorithm to compute the approximate of the differential equation using the Modified Euler's Method:

1. Specify t_n and y_n
2. Specify a step-size, *h*
3. For *n* from 1 to *k* compute the following

a. $y'(t_n, y_n)$

b. $hy'(t_n, y_n)$

c. $y_{n+1,p} = y_n + hy'(t_n, y_n)$

d. $hy'_{n+1,p} = hy'(t_{n+1}, y_{n+1})$

e. $hy'_{av} = (hy'_n + y'_{n+1,p})/2$

f. $y'_{n+1,c} = y_n + hy'_{av}$

4. Repeat No. 1 for $t_n + h$ and $y_{n+1,c}$ until $y_k \approx$ Exact Solution

Step 2

Use the algorithm in Step 1 with $h = 0.1$. The Excel setup for this problem is given in the following figure followed by the computed values:

	A	B	C	D	E	F	G	H
1	h	0.1						
2	tn	yn	y'n	h*y'	yn+1,p	hy'n+1	hy'av	yn+1,c
3	1	0	=(A3)^2+B3^2	=$B$1*C3	=B3+D3	=((A4)^2+E3^2)*$B$1	=(D3+F3)/2	=B3+G3
4	=A3+$B$1	=H3	=(A4)^2+B4^2	=$B$1*C4	=B4+D4	=((A5)^2+E4^2)*$B$1	=(F4+D4)/2	=B4+G4
5	=A4+$B$1	=H4	=(A5)^2+B5^2	=$B$1*C5	=B5+D5	=((A6)^2+E5^2)*$B$1	=(F5+D5)/2	=B5+G5
6	=A5+$B$1	=H5	=(A6)^2+B6^2	=$B$1*C6	=B6+D6	=((A7)^2+E6^2)*$B$1	=(F6+D6)/2	=B6+G6
7	=A6+$B$1	=H6	=(A7)^2+B7^2	=$B$1*C7	=B7+D7	=((A8)^2+E7^2)*$B$1	=(F7+D7)/2	=B7+G7
8	=A7+$B$1	=H7	=(A8)^2+B8^2	=$B$1*C8	=B8+D8	=((A9)^2+E8^2)*$B$1	=(F8+D8)/2	=B8+G8
9	=A8+$B$1	=H8	=(A9)^2+B9^2	=$B$1*C9	=B9+D9	=((A10)^2+E9^2)*$B$1	=(F9+D9)/2	=B9+G9

tn	yn	y'n	h*y'	yn+1,p	hy'n+1	hy'av	yn+1,c	error
1	0	1	0.1000	0.1000	0.1220	0.1110	0.1110	1.59375
1.10000	0.1110	1.222321	0.1222	0.2332	0.1494	0.1358	0.2468	1.48275
1.20000	0.2468	1.500928	0.1501	0.3969	0.1848	0.1674	0.4143	1.34691
1.30000	0.4143	1.861611	0.1862	0.6004	0.2321	0.2091	0.6234	1.17949
1.40000	0.6234	2.348585	0.2349	0.8582	0.2987	0.2668	0.8901	0.97038
1.50000	0.8901	3.042318	0.3042	1.1944	0.3986	0.3514	1.2416	0.70363
1.60000	1.2416	4.101477	0.4101	1.6517	0.5618	0.4860	1.7275	0.35219
1.70000	1.7275	5.874407	0.5874	2.3150	0.8599	0.7237	2.4512	-0.13379
1.80000	2.4512	9.248487	0.9248	3.3761	1.5008	1.2128	3.6640	-0.85747
1.90000	3.6640	17.03518	1.7035	5.3676	3.2811	2.4923	6.1563	-2.07029
2.00000	6.1563	41.90041	4.1900	10.3464	11.1457	7.6679	13.8242	-4.56258

As a result, using the Modified Euler's Method and a step size $h = 0.1$, $\boxed{y(2) = 6.1563}$.

Step 3

Use the algorithm in Step 1 with $h = 0.05$, the computed values are given in the following figure:

tn	yn	y'n	h*y'	yn+1,p	hy'n+1	hy'av	yn+1,c
1	0	1	0.0500	0.0500	0.0553	0.0526	0.0526
1.05000	0.0526	1.105269	0.0553	0.1079	0.0611	0.0582	0.1108
1.10000	0.1108	1.222276	0.0611	0.1719	0.0676	0.0644	0.1752
1.15000	0.1752	1.35318	0.0677	0.2428	0.0749	0.0713	0.2465
1.20000	0.2465	1.500742	0.0750	0.3215	0.0833	0.0792	0.3256
1.25000	0.3256	1.668531	0.0834	0.4091	0.0929	0.0881	0.4138
1.30000	0.4138	1.861206	0.0931	0.5068	0.1040	0.0985	0.5123
1.35000	0.5123	2.084936	0.1042	0.6165	0.1170	0.1106	0.6229
1.40000	0.6229	2.348019	0.1174	0.7403	0.1325	0.1250	0.7479
1.45000	0.7479	2.661819	0.1331	0.8810	0.1513	0.1422	0.8901
1.50000	0.8901	3.042232	0.1521	1.0422	0.1744	0.1633	1.0533
1.55000	1.0533	3.512039	0.1756	1.2289	0.2035	0.1896	1.2429
1.60000	1.2429	4.104813	0.2052	1.4481	0.2410	0.2231	1.4660
1.65000	1.4660	4.871703	0.2436	1.7096	0.2906	0.2671	1.7331
1.70000	1.7331	5.89373	0.2947	2.0278	0.3587	0.3267	2.0598
1.75000	2.0598	7.305414	0.3653	2.4251	0.4561	0.4107	2.4705
1.80000	2.4705	9.343357	0.4672	2.9377	0.6026	0.5349	3.0054

1.25000	0.3256	1.668531	0.0834	0.4091	0.0929	0.0881	0.4138
1.30000	0.4138	1.861206	0.0931	0.5068	0.1040	0.0985	0.5123
1.35000	0.5123	2.084936	0.1042	0.6165	0.1170	0.1106	0.6229
1.40000	0.6229	2.348019	0.1174	0.7403	0.1325	0.1250	0.7479
1.45000	0.7479	2.661819	0.1331	0.8810	0.1513	0.1422	0.8901
1.50000	0.8901	3.042232	0.1521	1.0422	0.1744	0.1633	1.0533
1.55000	1.0533	3.512039	0.1756	1.2289	0.2035	0.1896	1.2429
1.60000	1.2429	4.104813	0.2052	1.4481	0.2410	0.2231	1.4660
1.65000	1.4660	4.871703	0.2436	1.7096	0.2906	0.2671	1.7331
1.70000	1.7331	5.89373	0.2947	2.0278	0.3587	0.3267	2.0598
1.75000	2.0598	7.305414	0.3653	2.4251	0.4561	0.4107	2.4705
1.80000	2.4705	9.343357	0.4672	2.9377	0.6026	0.5349	3.0054
1.85000	3.0054	12.45487	0.6227	3.6281	0.8387	0.7307	3.7361
1.90000	3.7361	17.56842	0.8784	4.6145	1.2548	1.0666	4.8027
1.95000	4.8027	26.86856	1.3434	6.1461	2.0888	1.7161	6.5188
2.00000	6.5188	46.4948	2.3247	8.8435	4.1205	3.2226	9.7414

As a result, using the Modified Euler's Method and a step size $h = 0.05$, $\boxed{y(2) = 6.5188}$. Halving the step size tends to decrease the numerical error in the Modified Euler's Method by one-quarter. Therefore, the accuracy of the second computation is going to be one-quarter of the difference between the solutions from each step size (6.5188 – 6.1563)/4 = 0.090625.