Applied Numerical Analysis (7th Edition)

Problem

For the function of Exercise, fit the four points $f(-1)$, $f(1)$, $f(2)$, and $f(4)$ with a cubic spline. What is the maximum deviation of this spline from $f(x)$ in the interval $[-1, 4]$? At what x-value does this occur?

Exercise

Is $f(x)$ a linear spline?

$$f(x) = \begin{cases} 1 - x, & -1 \le x \le 1 \\ 2(x-1), & 1 \le x \le 2 \\ (x+2)/2, & 2 \le x \le 4 \end{cases}$$

Step-by-step solution

Step 1 of 5

To fit a natural cubic spline (end condition 1) of the form

$$g_i(x) = a_i(x-x_i)^3 + b_i(x-x_i)^2 + c_i(x-x_i) + d_i$$
$$g(x) = g_i(x) \text{ on } [x_i, x_i+1] \text{ for } i = 1, 2, \ldots, n-1$$

to the data points, create the Matlab m-file “cubspline1.m” to compute the coefficients on the spline polynomials based upon the algorithm in the text

```
function rtn = cubspline1(x,f)
%this function takes in a set of x-data values and f-data values
%and produces a natural cubic spline (end condition 1)
%S_0=S_n=0. The output is a coeffcient matrix of the cubic polynomials
%where the first row represents the cubic spline
%a_1(x-x_1)^3+b_1(x-x_1)^2+c_1(x-x_1)+d_1 on [x_1,x_2]
%and the ith rows similary represents the ith cubic spline
%on [x_i,x_(i+1)]
%generate h-vector, differences of x's
h=zeros(1,length(x)-1);
for i=1:length(h)
h(i)=x(i+1)-x(i);
end
%generate coefficient matrix
A=sparse(length(h)-1,length(h)-1);
%A=zeros(length(h)-1,length(h)-1);
%diagonal
for i=1:length(h)-1
A(i,i)=2*(h(i)+h(i+1));
end
%superdiagonal
for i=1:length(h)-2
A(i,i+1)=h(i+1);
end
%subdiagonal
for i=1:length(h)-2
A(i+1,i)=h(i+1);
end
%generate right-hand side vector
y=zeros(length(h)-1,1);
for i=1:length(h)-1
y(i)=6*((f(i+2)-f(i+1))/h(i+1)-(f(i+1)-f(i))/h(i));
end
S=A\y;
S=[0; S; 0];
a=zeros(length(h),1);
b=zeros(length(h),1);
c=zeros(length(h),1);
d=zeros(length(h),1);
for i=1:length(h)
a(i)=(S(i+1)-S(i))/(6*h(i));
b(i)=S(i)/2;
c(i)=(f(i+1)-f(i))/h(i)-(2*h(i)*S(i)+h(i)*S(i+1))/6;
d(i)=f(i);
end
polysplinecoeff=[a b c d]
end
```

Comment

Step 2 of 5

Using Matlab, give the following commands to calculate the cubic spline polynomials

Comment

Step 3 of 5

INPUT:

```
>> x=[-1,1,2,4];
>> f=[2,0,2,3];
>> cubspline1(x,f)
```

OUTPUT:

```
polysplinecoeff =
0.2786 0 -2.1143 2.0000
-0.9000 1.6714 1.2286 0
0.1714 -1.0286 1.8714 2.0000
```

which yields the spline function

$$\boxed{fs1(x) = \begin{cases} 0.2786(x+1)^3 - 2.1143(x+1) + 2 & -1 \le x \le 1 \\ -.9(x-1)^3 + 1.6714(x-1)^2 + 1.2286(x-1) & 1 \le x \le 2 \\ 0.1714(x-2)^3 - 1.0286(x-2)^2 + 1.8714(x-2) + 2 & 2 \le x \le 4 \end{cases}}$$

Comment

Step 4 of 5

To find the maximum deviation of the spline approximation from the given function

$$f(x)=\begin{cases} 1-x & -1\le x\le 1 \\ 2(x-1) & 1\le x\le 2 \\ (x+2)/2 & 2\le x\le 4 \end{cases}$$

which is $|f(x)-fs1(x)|$, make use of the following Mathematica commands

INPUT:

```
f[x_]:=Piecewise[{{1-x,-1<x<1},{2*(x-1),1<x<2},{(x+2)/2,2<x<4}}]
```

To input the spline function

$$fs1(x)=\begin{cases} 0.2786(x+1)^3-2.1143(x+1)+2 & -1\le x\le 1 \\ -.9(x-1)^3+1.6714(x-1)^2+1.2286(x-1) & 1\le x\le 2 \\ 0.1714(x-2)^3-1.0286(x-2)^2+1.8714(x-2)+2 & 2\le x\le 4 \end{cases}$$

into Mathematica, make use of the following command

INPUT:

```
fs1[x_]:=Piecewise[{{0.2786*(x+1)^3-2.1143*(x+1)+2,-1<x<1},
{-0.9*(x-1)^3+1.6714*(x-1)^2+1.2286*(x-1),1<x<2},
{0.1714*(x-2)^3-1.0286*(x-2)^2+1.8714(x-2)+2,2<x<4}}]
```

Comment

Step 5 of 5

To maximize $|f(x)-fs1(x)|$, make use of the following Mathematica command

INPUT:

```
NMaximize[Abs[f[x]-fs1[x]],x]
```

OUTPUT:

```
{0.85775,{x → 0.154649}}
```

From the Mathematica output, it appears that the maximum deviation is 0.85775, which occurs at $x=0.154649$.

Comment

Was this solution helpful? 0 0