

# Applied Numerical Analysis (7th Edition)

Chapter 3, Problem 44E

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## Problem

For the function of Exercise, fit the four points  $f(-1)$ ,  $f(1)$ ,  $f(2)$ , and  $f(4)$  with a cubic spline. What is the maximum deviation of this spline from  $f(x)$  in the interval  $[-1, 4]$ ? At what  $x$ -value does this occur?

### Exercise

Is  $f(x)$  a linear spline?

$$f(x) = \begin{cases} 1-x, & -1 \leq x \leq 1 \\ 3x-1, & 1 \leq x \leq 2 \\ 3x+2, & 2 \leq x \leq 4 \end{cases}$$

## Step-by-step solution

### Step 1 of 5

To fit a natural cubic spline (end condition 1) of the form

$$g_i(x) = a_i(x-x_i)^3 + b_i(x-x_i)^2 + c_i(x-x_i) + d_i$$

$$g(x) = g_i(x) \text{ on } [x_i, x_{i+1}] \text{ for } i = 1, 2, \dots, n-1$$

to the data points, create the Matlab m-file 'cubspline1.m' to compute the coefficients on the spline polynomials based upon the algorithm in the text

function rtn = cubspline1(x,f)

%this function takes in a set of x-data values and f-data values

%and produces a natural cubic spline (end condition 1)

%S\_0=S\_n=0. The output is a coefficient matrix of the cubic polynomials

%where the first row represents the cubic spline

%a\_1(x-x\_1)^3+b\_1(x-x\_1)^2+c\_1(x-x\_1)+d\_1 on [x\_1,x\_2]

%and the ith rows similarly represents the ith cubic spline

%on [x\_i,x\_{i+1}]

%generate h=vector, differences of x's

h=zeros(1,length(x)-1);

for i=1:length(h)

h(i)=(i+1)-x(i);

end

%generate coefficient matrix

A=sparse(length(h)-1,length(h)-1);

%A=zeros(length(h)-1,length(h)-1);

%diagonal

for i=1:length(h)-1

A(i,i)=2\*(h(i)+h(i+1));

end

%supdiagonal

for i=1:length(h)-2

A(i,i+1)=h(i+1);

end

%subdiagonal

for i=1:length(h)-2

A(i+1,i)=h(i+1);

end

%generate right-hand side vector

y=zeros(length(h)-1,1);

for i=1:length(h)-1

y(i)=6\*((f(i+2)-f(i+1))\*h(i+1)-(f(i+1)-f(i))\*h(i));

end

S=A\y;

S=[0; S; 0];

a=zeros(length(h),1);

b=zeros(length(h),1);

c=zeros(length(h),1);

d=zeros(length(h),1);

for i=1:length(h)

a(i)=(S(i+1)-S(i))/(6\*h(i));

b(i)=S(i)/2;

c(i)=(f(i+1)-f(i))/h(i)-(2\*h(i)\*S(i)+h(i)\*S(i+1))/6;

d(i)=f(i);

end

polysplinecoeff=[a b c d]

end

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### Step 2 of 5

Using Matlab, give the following commands to calculate the cubic spline polynomials

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### Step 3 of 5

INPUT:

>> x=[-1,2,4];

>> f=[2,0,2,3];

>> cubspline1(x,f)

OUTPUT:

polysplinecoeff =

0.2786 0 -2.1143 2.0000

-0.9000 1.6714 1.2286 0

0.1714 -1.0286 1.8714 2.0000

which yields the spline function

$$f_1(x) = \begin{cases} 0.2786(x+1)^3 - 2.1143(x+1) + 2 & -1 \leq x \leq 1 \\ -9(x-1)^3 + 1.6714(x-1)^2 + 1.2286(x-1) & 1 \leq x \leq 2 \\ 0.1714(x-2)^3 - 1.0286(x-2)^2 + 1.8714(x-2) + 2 & 2 \leq x \leq 4 \end{cases}$$

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To find the maximum deviation of the spline approximation from the given function

$$f(x) = \begin{cases} 1-x & -1 \leq x \leq 1 \\ 2(x-1) & 1 \leq x \leq 2 \\ (x+2)/2 & 2 \leq x \leq 4 \end{cases}$$

which is  $|f(x) - fs1(x)|$ , make use of the following Mathematica commands

INPUT:

```
f[x_]:=Piecewise[{{1-x,-1<x<1},{2*(x-1),1<x<2},{(x+2)/2,2<x<4}}]
```

To input the spline function

$$fs1(x) = \begin{cases} 0.2786(x+1)^3 - 2.1143(x+1) + 2 & -1 \leq x \leq 1 \\ -9(x-1)^3 + 1.6714(x-1)^2 + 1.2286(x-1) & 1 \leq x \leq 2 \\ 0.1714(x-2)^3 - 1.0286(x-2)^2 + 1.8714(x-2) + 2 & 2 \leq x \leq 4 \end{cases}$$

into Mathematica, make use of the following command

INPUT:

```
fs1[x_]:=Piecewise[{{0.2786*(x+1)^3-2.1143*(x+1)+2,-1<x<1},
{-0.9*(x-1)^3+1.6714*(x-1)^2+1.2286*(x-1),1<x<2},
{0.1714*(x-2)^3-1.0286*(x-2)^2+1.8714*(x-2)+2,2<x<4}}]
```

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#### Step 5 of 5

To maximize  $|f(x) - fs1(x)|$ , make use of the following Mathematica command

INPUT:

```
NMaximize[Abs[f[x]-fs1[x]],x]
```

OUTPUT:

```
{0.85775,{x -> 0.154649}}
```

From the Mathematica output, it appears that the maximum deviation is 0.85775, which occurs at  $x = 0.154649$ .

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### Recommended solutions for you in Chapter 3

#### Chapter 3, Problem 3E

Multiply out the Lagrangian polynomials in Exercises 1 and 2 to get the quadratics in the form  $ax^2 + bx + c$ . How different are the values for  $a$ ,  $b$ , and  $c$ ? Exercise 1 Write out the Lagrangian polynomial from this table....

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#### Chapter 3, Problem 48E

Compute the connected Bezier curve from this set of points: Point # 0123456789x10507590105150180190160130y101560 Draw the graph determined by the ten points,  $b$ . Why is the graph smoothly connected at points 3 and 6?...

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