12. Construct the divided-difference table from these data:

x	-0.2	0.3	0.7	-0.3	0.1
$f(x)$	1.23	2.34	-1.05	6.51	-0.06

14. Use the divided-difference table from Exercise 12 to interpolate for $f(0.4)$

a. Using the first three points.
b. Using the last three points.
c. Using the best set of three points. Which points should be used?
d. Using the best set of four points.
e. Using all of the points.
f. Explain why the results are not all the same.

43. Is $f(x)$ a linear spline?

$$f(x) = \begin{cases} 1 - x, & -1 \le x \le 1 \\ 2(x - 1), & 1 \le x \le 2 \\ (x + 2)/2, & 2 \le x \le 4 \end{cases}$$

▶**44.** For the function of Exercise 43, fit the four points $f(-1)$, $f(1)$, $f(2)$, and $f(4)$ with a cubic spline. What is the maximum deviation of this spline from $f(x)$ in the interval $[-1, 4]$? At what x-value does this occur?

53. If these four points are connected in order by straight lines, a zigzag line is created:

$$(0, 0), (1, 0.3), (2, 1.7), (3, 1.5).$$

a. Using the two interior points as controls, find the cubic Bezier curve. Plot this together with the zigzag line.
b. Use this cubic equation to find interpolates at $x = 0.5$, $x = 0.75$, and $x = 2.5$. How close are these to the zigzag line?
c. If the second and third points (the control points) are moved, the Bezier curve will change. If these are moved vertically, where should they be located so that the Bezier curve passes through all of the original four points?

54. Repeat Exercise 53, but for B-spline curves. Add fictitious points at the end so the end portions are completed.

▶**62.** From this table, estimate $z(x, y)$ for $x = 2.8$ and $y = 0.54$ using an array of nine points nearest to the point of interpolation to construct interpolating polynomials. (There may be several ways to choose these points; try them all.) The function whose values are tabulated is $z = x + e^y$.

$x \backslash y$	0.2	0.4	0.5	0.7	0.9
1.3	2.521	2.792	2.949	3.314	3.760
2.5	3.721	3.992	4.149	4.514	4.960
3.1	4.321	4.592	4.749	5.114	5.560
4.7	5.921	6.192	6.349	6.714	7.160
5.5	6.721	6.992	7.149	7.514	7.960

63. Using the data from Exercise 62, construct the B-spline surface from the rectangular array of 16 points nearest to (2.8, 0.54) and find $z(2.8, 0.54)$. Compare to the result of Exercise 62.

64. Repeat Exercise 63, but now for a Bezier surface.

73. $y = ax^2 + bx + c$ is a quadratic equation, of course. Compute $z = y +$ a random number within the range $[0, .2]$ for six x-values chosen randomly within the range $[2, 7]$.

a. Fit the least-squares line to these points.
b. Fit the least-squares quadratic to them.
c. Fit the least-squares cubic to them.
d. Compare the sum of squares of the deviations for each part.

APP2. The cost of government welfare programs adds significantly to our taxes. The table below gives data for several years:

Year	Expenditures in billions of dollars
1985	731
1986	782
1987	833
1988	886
1989	956
1990	1049
1991	1159
1992	1267
1993	1367
1994	1436
1995	1505

Use the data between 1991 to 1994 to estimate what the value would be in 1995 and compare to the value in the table. Do this

a. From a cubic interpolating polynomial.
b. From the least-squares line.
c. From the least-squares quadratic.
d. From a cubic spline.

From each of these, project to find what one would anticipate the expenditures for the year 2000 might be; then find what the actual expenditures were for comparison.

APP3. Use the data of APP2 with several approaches to extrapolate backward to estimate the expected expenditure for 1980. How do these values compare to 492 billion, the amount actually spent?