18. Solve the system

$$\begin{aligned} 2.51x + 1.48y + 4.53z &= 0.05, \\ 1.48x + 0.93y - 1.30z &= 1.03, \\ 2.68x + 3.04y - 1.48z &= -0.53. \end{aligned}$$

a. Use Gaussian elimination, but use only three significant digits and do no interchanges. Observe the small divisor in reducing the third column. The correct solution is $x = 1.45310$, $y = -1.58919$, $z = -0.27489$.

b. Repeat part (a) but now do partial pivoting.

c. Repeat part (b) but now chop the numbers rather than rounding.

d. Substitute the solutions found in (a), (b), and (c) into the equations. How well do these match the original right-hand sides?

▶**34.** Given this tridiagonal system:

$$\begin{bmatrix} 4 & -1 & 0 & 0 & 0 & 0 & 100 \\ -1 & 4 & -1 & 0 & 0 & 0 & 200 \\ 0 & -1 & 4 & -1 & 0 & 0 & 200 \\ 0 & 0 & -1 & 4 & -1 & 0 & 200 \\ 0 & 0 & 0 & -1 & 4 & -1 & 200 \\ 0 & 0 & 0 & 0 & -1 & 4 & 100 \end{bmatrix}.$$

a. Solve the system using the algorithm for a compacted system matrix that has n rows but only four columns.

b. How many arithmetic operations are needed to solve a tridiagonal system of n equations in this compacted arrangement? How does this compare to solving such a system with Gaussian elimination without compacting?

35. The system of Exercise 34 is an example of a symmetric matrix. Because the elements at opposite positions across the diagonal are exactly the same, it can be stored as a matrix with n rows but only three columns.

a. Write an algorithm for solving a symmetric tridiagonal system that takes advantage of such compacting.

b. Use the algorithm from part (a) to solve the system in Exercise 34.

c. How many arithmetic operations are needed with this algorithm for a system of n equations?

81. Solve this system of equations, starting with the initial vector of [0, 0, 0]:

$$\begin{aligned} 4.63x_1 - 1.21x_2 + 3.22x_3 &= 2.22, \\ -3.07x_1 + 5.48x_2 + 2.11x_3 &= -3.17, \\ 1.26x_1 + 3.11x_2 + 4.57x_3 &= 5.11. \end{aligned}$$

a. Solve using the Jacobi method.

b. Solve using the Gauss–Seidel method.