



Chapter 6 Problem 11E



Step 1

Step 1:

Use the following algorithm of Midpoint Method to compute the approximate solution of the differential equation:

- 1. Specify t_a and y_a
- 2. Specify a step-size, h
- 3. For n from 1 to k compute the following

$$\mathbf{a}. \ \mathbf{t_{n+1/2}} = \mathbf{t_n} + \mathbf{h}/2$$

$$b. t_{n+1} = t_n + 0.2$$

$$y_{n+1/2} = y_n + \frac{h}{2}y'(t_n, y_n)$$

$$y_{n+1} = y_n + hy' (t_{n+1/2}, y_{n+1/2})$$

4. Repeat No. 1 for $t_n + h$ and y_{n+1} until $y_k \approx Exact Solution$

Step 2

Step 2: Use the algorithm in Step 1 with h = 0.1.

The Excel setup for this problem is given in the following the computed values:

Step 3

	D	C	В	A	
			0.1	h	1
VI	yn	tn+1	tn+1/2	tn	2







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Step 2: Use the algorithm in Step 1 with h = 0.1.

The Excel setup for this problem is given in the following the computed values:

Step 3

-	A	В	C	D	E
1	h	0.1			
2	tn	tn+1/2	tn+1	yn	yn+1/
3	1	=A3+\$B\$1/2	=A4+\$B\$1	0	=D3+\$B\$1/2*(D)
4	=A3+\$B\$1	=A4+\$B\$1/2	=A5+\$B\$1	=F3	=D4+\$B\$1/2*(D-
5	=A4+\$B\$1	=A5+\$B\$1/2	=A6+\$B\$1	=F4	=D5+\$B\$1/2*(D)
6	=A5+\$B\$1	=A6+\$B\$1/2	=A7+\$B\$1	=F5	=D6+\$B\$1/2*(D)
7	=A6+\$B\$1	=A7+\$B\$1/2	=A8+\$B\$1	=F6	=D7+\$B\$1/2*(D)
8	=A7+\$B\$1	=A8+\$B\$1/2	=A9+\$B\$1	=F7	=D8+\$B\$1/2*(D
9	=A8+\$B\$1	=A9+\$B\$1/2	=A10+\$B\$1	=F8	=D9+\$B\$1/2*(D)

The output is as shown below:

tn	tn+1/2	tn+1	yn
1.0000	1.0500	1.2000	0.0000
1.1000	1.1500	1.3000	0.1105
1.2000	1.2500	1.4000	0.2457
1.3000	1.3500	1.5000	0.4122
1.4000	1.4500	1.6000	0.6200
1.5000	1.5500	1.7000	0.8846
1.6000	1.6500	1.8000	1.2322
1.7000	1.7500	1.9000	1.7107
1.8000	1.8500	2.0000	2.4176
1.9000	1.9500	2.1000	3.5846
2.0000	2.0500	2.2000	5.9076

Step 4

Step 3: As a result, using the Midpoint Method and a step size h =0.1,





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Step 2: Use the algorithm in Step 1 with h = 0.1.

The Excel setup for this problem is given in the following the computed values:

Step 3

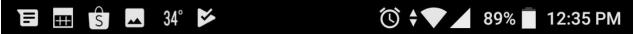
С	D	E	F
tn+1	yn	yn+1/2	yn+1
A4+\$B\$1	0	=D3+\$B\$1/2*(D3^2+A3^2)	=D3+\$B\$1*(E3^2+B3^2)
A5+\$B\$1	=F3	=D4+\$B\$1/2*(D4^2+A4^2)	=D4+\$B\$1*(E4^2+B4^2)
A6+\$B\$1	=F4	=D5+\$B\$1/2*(D5^2+A5^2)	=D5+\$B\$1*(E5^2+B5^2)
A7+\$B\$1	=F5	=D6+\$B\$1/2*(D6^2+A6^2)	=D6+\$B\$1*(E6^2+B6^2)
A8+\$B\$1	=F6	=D7+\$B\$1/2*(D7^2+A7^2)	=D7+\$B\$1*(E7^2+B7^2)
A9+\$B\$1	=F7	=D8+\$B\$1/2*(D8^2+A8^2)	=D8+\$B\$1*(E8^2+B8^2)
A10+\$B\$1	=F8	=D9+\$B\$1/2*(D9^2+A9^2)	=D9+\$B\$1*(E9^2+B9^2)

tn+1	yn	yn+1/2	yn+1
1.2000	0.0000	0.0500	0.1105
1.3000	0.1105	0.1716	0.2457
1.4000	0.2457	0.3207	0.4122
1.5000	0.4122	0.5052	0.6200
1.6000	0.6200	0.7372	0.8846
1.7000	0.8846	1.0362	1.2322
1.8000	1.2322	1.4362	1.7107
1.9000	1.7107	2.0016	2.4176
2.0000	2.4176	2.8719	3.5846
2.1000	3.5846	4.4076	5.9076
2.2000	5.9076	7.8525	12.4941

Step 4

Step 3: As a result, using the Midpoint Method and a step size h = 0.1,









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Step 4

Step 3: As a result, using the Midpoint Method and a step size h = 0.1,

$$y(2) = 5.9076$$

Step 5

Step 4: With h = 0.05, the computed values are given in the following table:

following table.			
tn	tn+1/2	tn+1	yn
1.0000	1.0250	1.1000	0.0000
1.0500	1.0750	1.1500	0.0526
1.1000	1.1250	1.2000	0.1107
1.1500	1.1750	1.2500	0.1749
1.2000	1.2250	1.3000	0.2462
1.2500	1.2750	1.3500	0.3252
1.3000	1.3250	1.4000	0.4132
1.3500	1.3750	1.4500	0.5116
1.4000	1.4250	1.5000	0.6220
1.4500	1.4750	1.5500	0.7467
1.5000	1.5250	1.6000	0.8885
1.5500	1.5750	1.6500	1.0513
1.6000	1.6250	1.7000	1.2402
1.6500	1.6750	1.7500	1.4624
1.7000	1.7250	1.8000	1.7281
1.7500	1.7750	1.8500	2.0527
1.8000	1.8250	1.9000	2.4599
1.8500	1.8750	1.9500	2.9888
1.9000	1.9250	2.0000	3.7083
1.9500	1.9750	2.0500	4.7516
0.0000			0.4400 101





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Step 5

Step 4: With h = 0.05, the computed values are given in the following table:

tn	tn+1/2	tn+1	yn
1.0000	1.0250	1.1000	0.0000
1.0500	1.0750	1.1500	0.0526
1.1000	1.1250	1.2000	0.1107
1.1500	1.1750	1.2500	0.1749
1.2000	1.2250	1.3000	0.2462
1.2500	1.2750	1.3500	0.3252
1.3000	1.3250	1.4000	0.4132
1.3500	1.3750	1.4500	0.5116
1.4000	1.4250	1.5000	0.6220
1.4500	1.4750	1.5500	0.7467
1.5000	1.5250	1.6000	0.8885
1.5500	1.5750	1.6500	1.0513
1.6000	1.6250	1.7000	1.2402
1.6500	1.6750	1.7500	1.4624
1.7000	1.7250	1.8000	1.7281
1.7500	1.7750	1.8500	2.0527
1.8000	1.8250	1.9000	2.4599
1.8500	1.8750	1.9500	2.9888
1.9000	1.9250	2.0000	3.7083
1.9500	1.9750	2.0500	4.7516

Step 6

Step 5: As a result, using the Midpoint Method and a step size h = 0.05,







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Step 5

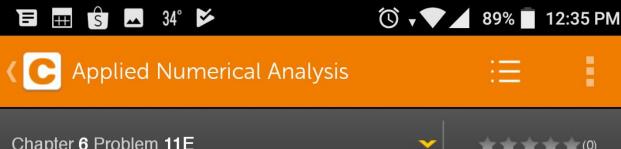
outed values are given in the

tn+1	yn	yn+1/2	yn+1
1.1000	0.0000	0.0250	0.0526
1.1500	0.0526	0.0802	0.1107
1.2000	0.1107	0.1412	0.1749
1.2500	0.1749	0.2088	0.2462
1.3000	0.2462	0.2837	0.3252
1.3500	0.3252	0.3669	0.4132
1.4000	0.4132	0.4597	0.5116
1.4500	0.5116	0.5637	0.6220
1.5000	0.6220	0.6807	0.7467
1.5500	0.7467	0.8132	0.8885
1.6000	0.8885	0.9645	1.0513
1.6500	1.0513	1.1390	1.2402
1.7000	1.2402	1.3427	1.4624
1.7500	1.4624	1.5839	1.7281
1.8000	1.7281	1.8750	2.0527
1.8500	2.0527	2.2346	2.4599
1.9000	2.4599	2.6922	2.9888
1.9500	2.9888	3.2977	3.7083
2.0000	3.7083	4.1424	4.7516
2.0500	4.7516	5.4111	6.4106
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Step 6

Step 5: As a result, using the Midpoint Method and a step size h = 0.05,

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1.1000	0.0000	0.0250	0.0526
1.1500	0.0526	0.0802	0.1107
1.2000	0.1107	0.1412	0.1749
1.2500	0.1749	0.2088	0.2462
1.3000	0.2462	0.2837	0.3252
1.3500	0.3252	0.3669	0.4132
1.4000	0.4132	0.4597	0.5116
1.4500	0.5116	0.5637	0.6220
1.5000	0.6220	0.6807	0.7467
1.5500	0.7467	0.8132	0.8885
1.6000	0.8885	0.9645	1.0513
1.6500	1.0513	1.1390	1.2402
1.7000	1.2402	1.3427	1.4624
1.7500	1.4624	1.5839	1.7281
1.8000	1.7281	1.8750	2.0527
1.8500	2.0527	2.2346	2.4599
1.9000	2.4599	2.6922	2.9888
1.9500	2.9888	3.2977	3.7083
2.0000	3.7083	4.1424	4.7516
2.0500	4.7516	5.4111	6.4106
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Step 6

Step 5: As a result, using the Midpoint Method and a step size h = 0.05,

The results are not the same.

The exact answer to the differential equation at t = 2 is 6.703787.

Therefore, the Modified Euler Method with step-size h = 0.05 is the most accurate.