A Comprehensive Review of Image Restoration Using Partial Differential Equations

Project Portfolio

Acknowledgments

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Table of Contents

Portfolio on Image Restoration Using Partial Differential Equations

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***Abstract*— This paper presents a comprehensive evaluation of image restoration techniques using Partial Differential Equations (PDEs), focusing on methods such as Total Variation (TV) Inpainting, Curvature-Driven Diffusion (CDD), Gaussian Filtering, and Nonlinear Diffusion. The need for high-quality visuals in fields like astrophotography, medical imaging, and surveillance necessitates effective restoration methods to mitigate noise and artifacts in digital images. This study explores the implementation of these PDE-based techniques to restore degraded images efficiently.**

Total Variation Inpainting minimizes image intensity variation to preserve edges, although it can produce a "staircase effect" in smooth gradients. Curvature-Driven Diffusion improves upon TV by effectively handling complex geometries using third-order PDEs. Gaussian Filtering, while simple, effectively reduces noise using a Gaussian Kernel but at the cost of blurring fine details. Nonlinear Diffusion, akin to Gaussian filtering, uses diffusion processes to smooth irregularities while preserving edges.

Our results indicate that the choice of method should be context dependent. TV Inpainting is ideal for uniform areas, whereas CDD is better suited for intricate geometrical details. Gaussian Filtering excels at noise reduction for Gaussian noise but compromises image detail. Nonlinear Diffusion balances noise reduction and edge preservation effectively.

This research underscores the importance of selecting appropriate restoration techniques based on specific image degradation characteristics. The findings suggest potential directions for future research, including developing hybrid models combining the strengths of these methods and integrating advanced machine learning techniques for more adaptable and efficient solutions in image restoration.

New methods such as diffusion models for information propagation and AI-driven generative fills are emerging, offering innovative ways to identify inpainting domains and improve restoration quality. These developments build on diffusion techniques and iterative processes, enhancing the efficacy of image restoration methods.

***Index Terms*—partial differential equations, image restoration, portfolio, inpainting, denoising**

I. INTRODUCTION

IMAGE restoration has become a priority in Digital Image Processing due to the need for high quality visuals in increasingly more fields such as astrophotography, medical imaging, surveillance, and others. Effective methods require to be cost effective and fast to make the data usable as soon as possible. With that goal in mind, how can the use of mathematical-based solutions, specifically those utilizing Partial Differential Equations (PDEs), be effectively implemented to restore images degraded by noise and artifacts?

Figure 1: Ecce Home by Elias Garcia (left) and its restoration (right)

This portfolio, 'Portfolio on Image Restoration Using Partial Differential Equations,' aims to answer this question by exploring the implementation and analysis of various PDE-based methods to enhance image quality.

II. A SHORT SUMMARY OF RELEVANT PRIOR WORK

Numerous methods have already been explored and developed as there is a crucial need for high quality visuals. Indeed, images are perpetually damaged either during transmission, capture, or even by various elements in the electronics causing noise, blur, and other artifacts. In this section, we survey many of those powerful methods using Partial Differential Equations (PDEs) to automate the restoration, a task typically left to the artists. They usually tackle inpainting issues (where the image has missing or occluded portions) by propagating the information from around the missing data to fill the insides.

A famous example of what would be human image inpainting would be the painting by Elias Garcia [1, Fig. 1] and its restoration outlining the challenges of the restoration.

Alternatively, well-tuned automated methods can yield better results. To highlight their effects, state-of-the-art methods have been chosen to illustrate the main operations.

A. Total Variation Inpainting

Total Variation (TV) Inpainting is one of the most famous PDE-based methods for inpainting. It operates by minimizing the total variation of the image intensity, effectively removing intruding elements while preserving edges. Conceived by Leonid I. Rudin, Stanley Osher, and Emad Fatemi around 1992 [3] by applying the concept of total variation regularization to image inpainting.
The model is praised for its simplicity and effectiveness, particularly at reconstructing edges and geometric structures. It however struggles to restore smooth gradients, creating a “staircase effect” (see results) where the painted region is pieced in several constant regions [4, Figure 2].

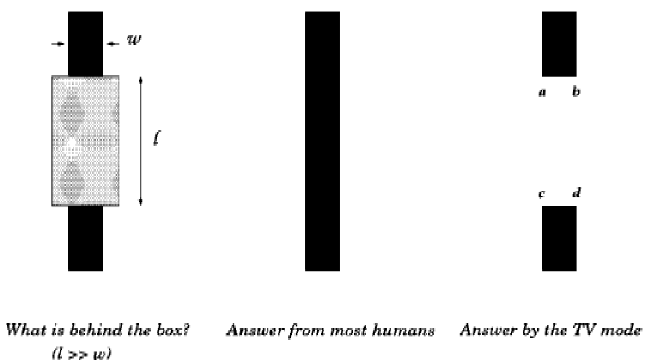

Figure 3: TV mode failures

Its equation reads as:

$$u_t = \nabla \cdot \left(\frac{\nabla u}{|\nabla u|} \right) \tag{1}$$

where u is the image array and ∇u its gradient vector.

B. Curvature Driven Diffusion

Trying to tackle the limitations of the TV model, Tony F. Chan and J. Shen introduced in 2002 [4] the Curvature Driven Scheme as improvement particularly when it comes to handling images with complex geometries. The CDD incorporates a function penalizing large curvatures (where d (∞) = ∞ and d(0) = 0). This new equation reads.

$$u_t = \nabla \cdot \left(\frac{d(k)\, \nabla u}{|\nabla u|} \right) \tag{2}$$

where k is the curvature. This effectively modifies the conductivity coefficient, and by computing the curvatures first of the image, the model effectively uses third-order PDEs to integrate deeply the geometry of the image into the calculations.

C. A type of noise-resistant filter: The Gaussian filter

Unlike previous inpainting methods, the Gaussian filter is a common and simpler method for dealing with noise involving a Gaussian Kernel to smooth out irregularities in the image. This results in a blur in the image, negating noise but also resulting in the loss of delicate details and edges.
In (3), σ denotes the standard deviation.

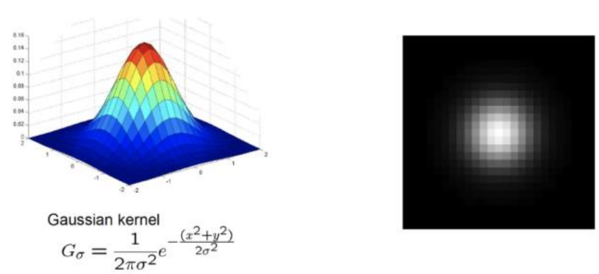

Figure 2: Gaussian distribution filter

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \tag{3}$$

[Figure 3, 5] illustrates the kernel being used for the convolution following a gaussian distribution in 2D. In this function, σ is the standard deviation of the kernel. This convolution relates to the Heat equation:

$$\frac{\partial u}{\partial t} = \alpha \nabla^2 u \tag{4}$$

with α being the diffusivity of the medium.
The heat equation is a second order PDE usually depicting the temperature distribution through a medium over time [7]. It happens that the solution of this equation for a given initial condition with a Gaussian filter can be approximated by the convolution of that initial function with the Gaussian filter.

$$u(x, y, t) = G(x, y, \sigma) * u(x, y, 0)) \tag{5}$$

where σ=$\sqrt{2\sigma t}$
Meaning, by applying convolution, which simulates the smoothing effect of the heat equation over time, we can theoretically recover certain details of the image.

D. Nonlinear diffusion

Just like Gaussian filtering this method by Perona and Malik [6] involves using diffusion in order to smooth out irregularities in the image. It defines the diffusion equation, derived from Fick’s law, as:

$$\frac{\partial u}{\partial t} = \nabla \cdot (g\, \nabla u) \tag{6}$$

where the gradient ∇u acts as an edge detector and the diffusivity constant g is expressed as a function of the image:

$$g(u) = \frac{1}{\sqrt{1 + \frac{|\nabla u|^2}{\lambda^2}}} \tag{7}$$

with λ being the contrast parameter, this relation making the equation nonlinear.

III. Technical Implementation

In this part, we want to discuss the practical aspect of implementing those methods in software. Whether the code must be built up from scratch or if it was borrowed from other

sources.
The programming language chosen to work with is Python not only for simplicity of explanation but also because it offers many scientific libraries to automatically perform some mathematical operation (like "gradient' for example). MATLAB was also an appropriate choice, but it lacks the visualization tools that Python has [8].
Note that many of the methods discussed came with a process to be digitally implemented which means that we mostly relied on the indications given in their respective papers. Those implementations are personal, and their development can be followed in [9]

A. Curvature Driven Diffusion Implementation

The first function to be implemented is the CDD scheme, as it appeared to be the most complicated one, from there the TV scheme could also be easily derived.
To do so the main idea was to follow the Chan and Shen iterative process [4]

$$u^{(n+1)} = u^{(n)} - \Delta t \nabla \cdot j^{(n)} \tag{8}$$

where Δt is the time step, $u^{(n)}$ is the image at a time nΔt, and $\nabla \cdot j_n$ is the divergence of the flux field itself defined as:

$$\boldsymbol{j} = D^{\wedge} \nabla u = -\frac{g(|k|)}{|\nabla u|} \nabla u \tag{9}$$

where k is the curvature at a given pixel and g is our large curvature annihilator function. This outlines a clear list of steps to follow:

- Compute the curvature of the image.
- Compute the flux field.
- Update the image.

1) Computing the curvature

The formula of the curvature is given by:

$$k = \nabla \cdot \left[\frac{\nabla u}{|\nabla u|}\right] \tag{10}$$

meaning that the one thing to compute is the gradient of u along the x and y-axes before computing its magnitude. Since:

$$\nabla u = \begin{matrix} u_x = \dfrac{\partial u}{\partial x} \\ u_y = \dfrac{\partial u}{\partial y} \end{matrix} \tag{11}$$

this can be given using the NumPy library's gradient function.

```
u_x = np.gradient(u, axis=1)
u_y = np.gradient(u, axis=0)
```

Following this the magnitude of the gradient is simply given by

```
magnitude = np.sqrt(u_x**2 + u_y**2) + 1e-8
```

with a small value (1e-8) being added so that when the magnitude reaches too low values, it doesn't cause the conductivity coefficient variable to grow too high and overflow (an issue arising when a digital variable exceeds the maximum that can be represented). Then the curvature formula is applied to output k

```
curvature = np.gradient(u_x/magnitude,
axis=1)+ np.gradient(u_y/magnitude, axis=0)
```

In the initial formula the conductivity coefficient is

$$\hat{D} = \frac{g(k)}{|\nabla u|} \tag{12}$$

which is converted in

```
D = g(np.abs(curvature))/magnitude
```

where g is function is previously defined by the user. Finally, the flux is then given along the x-axis and y-axis by

```
j_x = -D*np.gradient(u, axis=1)
j_y = -D*np.gradient(u, axis=0)
```

Finally, the iteration can be applied to the image by updating the mask region with the divergence variable

```
divergence = np.gradient(j_x, axis=1) +
np.gradient(j_y, axis=0)
u[mask] -= tau * divergence[mask]
```

B. Total Variation inpainting Implementation

As previously stated, the TV model is quite similar to the CDD one. In our case we chose to implement a version close to the original by Leonid I. Rudin, Stanley Osher and Emad Fatemi with a linear fidelity coefficient as proposed by Sobucki Stéphane [10, Fig. 4].
where ∇E is the variation of the image's function energy defined as

Algorithm 1 TV inpainting

```
procedure TV_INPAINTING(f, λ, T, dt)
    t ← 0
    u ← f                                  ▷ f : image endommagée
    while t < T do
        Compute u_xx, u_yy, u_xy
        ∇E ← −(u_xx u_y^2 − 2u_x u_y u_xy + u_yy u_x^2)/(u_x^2 + u_y^2)^(3/2) + 2λ(u − f)
        u^(n+1) ← u^n + dt(−∇E)
        t ← t + dt
    end while
    return u                               ▷ u : image restaurée
end procedure
```

Figure 4: Sobucki's TV implementation

$$\int_{\Omega} |\nabla u|^2 + D\lambda \int_{\Omega} (u-f)^2 \tag{13}$$

computing its variation, we get

$$\begin{aligned} \nabla E &= -\nabla\left(\frac{\nabla u}{|\nabla u|}\right) + 2\lambda(u-f) \\ &= -\left(\frac{u_{xx}u_y^2 - 2u_x u_y u_{xy} + u_{yy}u_x^2}{(u_x^2+u_y^2)^{\frac{3}{2}}}\right) + 2\lambda(u-f) \end{aligned} \tag{14}$$

where u is the updated image and f the original damaged one. In Python this calculation comes as

```
deltaE = -(u_xx * u_y**2 - 2*u_x * u_y * u_xy
+ u_yy * u_x**2) / (0.1 + (u_x**2 +
u_y**2)**(3/2)) + 2 * mask * (lambda_val *
(u - input_img))
```

with 0.1 being added to the denominator to prevent overflow. This can directly be used to update

```
u = dt * (-deltaE ) + u
```

the process being inside of a loop that only stops when *t* reaches T the time limit imposed.

C. Nonlinear diffusion implementation

The process of nonlinear diffusion happened to have been extensively previously outlined [6] allowing to use a Python implementation of Perona and Malik's work, as reminder the original equation is

$$\partial u = \nabla \cdot (g(u)\nabla u), \quad \Omega \in R^2 \tag{15}$$

, expanding the divergence (nabla) operator these output

$$\partial\, \mathrm{u} = \partial_{\mathrm{x}}(\mathrm{g(u)}\ \partial_{\mathrm{x}}\ \mathrm{u}) + \partial_{\mathrm{y}}(\mathrm{g(u)}\ \partial_{\mathrm{y}}\ \mathrm{u}) \tag{16}$$

This is a process we want discretized as we intend to work with image arrays therefore along the x-axis

$$\partial_x(g(u)\,\partial_x u) \approx (g(u)\,\partial_x u)^t_{i+\frac{1}{2},j} - (g(u)\,\partial_x u)^t_{i-\frac{1}{2},j}$$
$$\approx g_{i+\frac{1}{2},j}\left(u^t_{i+1,j} - u^t_{i,j}\right) - g_{i-\frac{1}{2},j}\left(u^t_{i,j} - u^t_{i-1,j}\right) \tag{17}$$

and along the y-axis

$$\partial_y\big(g(u)\partial_y u\big) \approx \big(g(u)\partial_y u\big)^t_{i,j+\frac{1}{2}} - \big(g(u)\partial_y u\big)^t_{i,j-\frac{1}{2}}$$
$$\approx g_{i,j+\frac{1}{2}}\left(u^t_{i,j+1} - u^t_{i,j}\right) - g_{i,j-\frac{1}{2}}\left(u^t_{i,j} - u^t_{i,j-1}\right) \tag{18}$$

with

$$g_{i+\frac{1}{2},j} = \sqrt{g_{i+1,j}\, g_{i,j}} \tag{19}$$

converted in Python as

```
g_pj = math.sqrt(g_padded[j, i+1] *
g_padded[j, i])
                g_nj =
math.sqrt(g_padded[j, i-1] * g_padded[j,
i])
                g_ip =
math.sqrt(g_padded[j+1, i] * g_padded[j,
i])
                g_in =
math.sqrt(g_padded[j-1, i] * g_padded[j,
i])
```

Alongside this a padding is performed for both u and g and g is originally set to 0 in order to maintain the image intensity within boundaries. The temporal derivative is also computed

$$\partial_t u \approx \frac{u^{t+1}_{i,j} - u^t_{i,j}}{\tau} \tag{20}$$

where τ is the time step between iterations.
Finally, by substituting … we get for $u^{t+1}_{i,j}$

$$u^{t+1}_{i,j} = u^t_{i,j} + \tau\Bigg(g_{i+\frac{1}{2},j}\left(u^t_{i+1,j} - u^t_{i,j}\right) - g_{i-\frac{1}{2},j}\left(u^t_{i,j} - u^t_{i-1,j}\right)$$
$$+ g_{i,j+\frac{1}{2}}\left(u^t_{i,j+1} - u^t_{i,j}\right)$$
$$- g_{i,j-\frac{1}{2}}\left(u^t_{i,j} - u^t_{i,j-1}\right)\Bigg) \tag{21}$$

a process mimicked by

```
ux0 =  g_pj * (u_padded[j, i+1] -
u_padded[j, i])
                ux1 = -g_nj * (u_padded[j,
i] - u_padded[j, i-1])
                uy0 =  g_ip *
(u_padded[j+1, i] - u_padded[j, i])
                uy1 = -g_in * (u_padded[j,
i] - u_padded[j-1, i])
update[j-1, i-1] = ux0 + ux1 + uy0 + uy1
```

and used to update the image with respect to the time step

```
u += tau * update
```

D. Gaussian filter implementation

As previously explained the Gaussian filter is essentially convoluting the image with a gaussian kernel with predefined standard deviation. This operation is already embedded in the Skimage library filters collection [11]. They can be imported by

```
from skimage.filters import gaussian
```

Note that since this function applies a convolution filter to the image, the border pixels remain unaffected as expected without padding.
The function can then simply be called

```
gauss_img = gaussian(masked_image, sigma=1)
```

with a sigma being defined by the user and controlling the size of the kernel by taking 3σ on each side to ensure a significant amount of the distribution is considered. For example, for a sigma of 1 we expect a kernel of size

$$(3 * 1) * 2 + 1 = 7$$

with the + 1 term being used to ensure symmetry

IV. Results obtained

It is difficult to properly numerically address what is a "good" image restoration, but there are nonetheless metrics that can provide insight into the efficiency of the model.

A. Metrics of judgment

A common metric when image restoration is discussed is Peak Signal to Noise Ratio (PSNR). It measures the quality of a reconstructed image in reference to the original. It is a representation on a logarithmic scale expressed as

$$PSNR = 10 \cdot log_{10}\left(\frac{MAX^2}{MSE}\right) \tag{22}$$

in dB, where MAX refers to the highest possible value in the image (in a greyscale array, 255) and MSE to the Mean Square Error, the average of the squared differences between corresponding pixels of the original and corrupted image.
This computation can be done using the OpenCV library

```
cv2.PSNR(gray_image, masked_image)
```

and is based on each pixel intensity value (MSE). PSNR is useful as it allows us to define a "good restoration." Typically for a PSNR value below 20 dB the model can be considered to be doing a poor job, while a value above 40 dB supposes an excellent restoration work. However, PSNR fails to always represent the human perception of the image, as a good PSNR restoration can visually appear differently, furthermore this method computes the PSNR over all the image and not just the noisy/inpainted regions meaning that a fair portion

Figure 5:Original Image for tests

of the value can be attributed to the unaffected region. To address this issue the MSE will also be used as metric as it is a straightforward comparison of each equivalent pixel of both images, painting an accurate picture of their proximity both visually and quantitatively. Additionally, the original PSNR and MSE value of the images to improve will be provided to compare.

B. Testing setup

Taking two sets of tests, one where a square mask will be applied to the centre of the image, representing a consistent artifact during capture, that will be called image A and another one where a gaussian noise is applied to simulate transmission noise, named image B. Gaussian noise as the name suggests is a type of noise where pixels are affected following a statistical 2D Gaussian distribution. It is considered "white" noise as modified pixels are uncorrelated with each other. The NumPy function

Figure 6: Image A (top) and B (bottom)

```
noise = np.random.normal(0, noise_level, image.shape)
```

allows to create the noise over random pixels on the image following the Gaussian probability function with the "noise level" parameter controlling the standard deviation. Taking a free-use public image [12, Figure 5] and altering it with those degradations. The models are tested on their best parameters.

N.B: For comparison Image A has a PSNR of 70.8 dB and a MSE of 53.72e-4 and Image B has a PSNR of 65.97 dB and a MSE of 0.01645

C. Tests results

Figure 7: Results of CDD (top) and TV (bottom) on Image A

For the first test the models were tasked with restoring Image A. After running for 40000 iterations (around 8 minutes).
The CDD model achieved a PSNR of 86.06 dB and MSE of 16.098e-6 (a significant improvement from 70.8 dB and a MSE 53.72e-4) displaying an effective restoration both graphically and quantitatively. The TV model restoration resulted in a PSNR of 79.98 dB and a MSE of 65e-6 outlining to a decent inpainting that however did a lesser work than the CDD one.

The two inpainting methods did a great deal at negating the square artifact, on the other hand methods of diffusion cannot affect artifacts like this one.

Conversely when dealing with gaussian noise, CDD inpainting outputs a pitch-black image with low visibility and a really high MSE (0.125 compared to the original MSE of 0.01645). The TV model still gives decent results with a PSNR of 76.46 dB and a MSE of 14.69e-3 (reminder that higher means less similar and that this MSE is really close

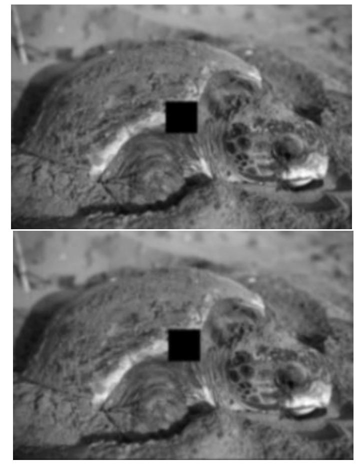

Figure 8: Results of Nonlinear diffusion (top) and Gaussian filtering (bottom) on Image A

to the one of the original noisy image). The Nonlinear Diffusion method makes the noise less prevalent but at the cost of the sharpness of the image (PSNR: 70.87 dB and MSE = 53.18e-4) similarly as the Gaussian filter which, as expected, achieves good results (MSE = 52.46e-4 and PSNR of 70.92 dB) when dealing with noise following a gaussian distribution at the cost of some of the image's integrity.

V. Analysis of the Results

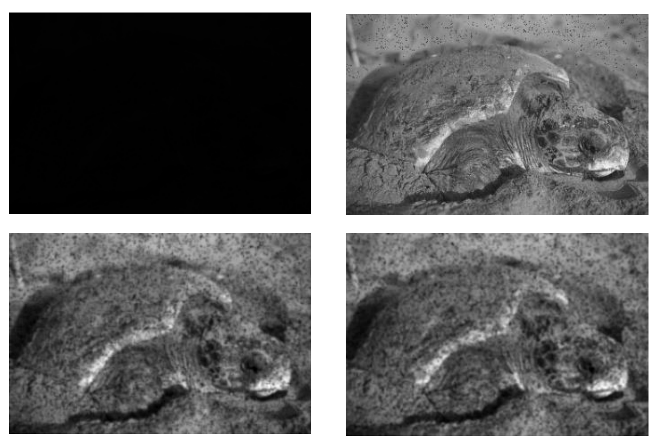

Figure 9: Results of CDD (top left), TV (top right), Nonlinear diff (bottom left) and gaussian (bottom right),

From the results collected and what we know about the methods we can confirm the theory that each method is highly context dependent. When it comes to reconstruct and inpaint over an identified degradation whose domain can be defined, methods like TV inpainting CDD are required when diffusion filters are noneffective. Furthermore, CDD seems to be outdoing TV inpainting at following and reconstructing complex geometries (Figure 7).

On the other hand, when the corruption comes from transmission and that all of the image is affected, CDD completely fails by incorporating the noise into the geometry and blackening the image. This effect is caused by how CDD interprets noise as highly complex geometrical structures, the model integrates those curvatures into the calculations amplifying the noise [4]. This outlines the versatility of TV inpainting that is capable of inpainting over some of the artifacts, improving the image overall unlike CDD inpainting that would require denoising action before to outputs decent results. By blurring the image through diffusion both Nonlinear Diffusion and Gaussian filtering can lower the intensity of the artifacts of Image B (Figure 9) although they do not completely eliminate them. As expected, Gaussian filtering displays better results likely due to the fact that just like the noise in this case, it features a gaussian distribution convolution kernel approximating accurately the pixel underneath the noise.

VI. Conclusion remarks

This study on image restoration using Partial Differential Equations (PDEs) has provided a comprehensive evaluation of four prominent techniques: Total Variation (TV) Inpainting, Curvature-Driven Diffusion (CDD), Gaussian Filtering, and Nonlinear Diffusion. The primary goal was to assess the effectiveness of these methods in restoring images degraded by noise and artifacts, focusing on their ability to balance noise reduction with detail preservation.

The research demonstrated that each method has unique strengths and limitations, emphasizing the importance of selecting the appropriate technique based on the specific characteristics of the image degradation. TV Inpainting proved effective for images with large, uniform areas due to its ability to preserve edges while reducing noise, despite its limitations in handling smooth gradients. In contrast, CDD excelled in handling complex geometries, offering superior restoration for images with intricate details, as indicated by the highest PSNR and lowest MSE scores.

Gaussian Filtering and Nonlinear Diffusion both showed proficiency in noise reduction, particularly for Gaussian noise, though at the cost of losing fine image details. Gaussian Filtering was especially effective due to its convolution with a Gaussian kernel that closely matches the noise distribution, leading to significant improvements in image quality.

A way to upgrade the most effective methods explored would be to integrate denoising into their architecture, a method that has shown promise in most cases[4].

It is important to also acknowledge that many new technologies have been and are being developed to improve the quality of image restoration. New methods to better identify the inpainting area are arising such as information propagation diffusion models [13]. And Artificial Intelligence generative fill keep evolving to provide insightful completion of the image as displayed by Google's Imagen2 software.[14].

As image restoration continues to evolve, the integration of artificial intelligence and machine learning offers promising avenues for future work. Implementing AI-based methods, particularly deep learning models, such as Generative Adversarial Networks (GANs) would be a likely step to significantly improve results. These models can learn intricate details and contextual information from vast datasets, allowing them to mimic human perception, avoiding the issue of PDEs method where metrics don't correspond with image results. Moreover, AI techniques are continually improving in terms of computational efficiency, allowing them to make the best of today's computer power.

While these developments do not completely rely on PDEs it is important to remember that their core are still based on diffusion techniques and iterative processes reminiscent of the many original techniques just discussed. Nonetheless, Machine learning methods are definitely currently ahead and worth investigating.

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APPENDIX A

AN UPDATED LITTERATURE REVIEW OF IMAGE RESTORATION USING PARTIAL DIFFERENTIAL EQUATIONS

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A Comprehensive Review of Image Restoration Using Partial Differential Equations

Literature Survey

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Declaration

I hereby declare that, except where otherwise indicated, this document is entirely my own work and has not been submitted in whole or in part to any other university.

Signed: ELIJAH KI-ZERBO Date: 14/08/2024

PREFACE:

This update of the original paper "A Comprehensive Review of Image Restoration Using Partial Differential Equations" includes an addition of the AI methods discussed in the Portfolio, they were extensively researched to discuss for future work in the Image Inpainting and Restoration domain. An addendum was also made to the CDD scheme's explanation to elaborate on the TV model and its many implementations.

Abstract—**This paper is an investigative review of some of the image restoration methods, focusing on those that use Partial Differential Equations (PDEs). The goal being to provide an overview of various models potentially useful and give some insights on their operations. The work begins by introducing the broader concept of image restoration, addressing issues such as noise, blur, and artifacts as well as emphasizing the importance of image restoration and processing. Then a particular attention is given to the Curvature-Driven Diffusion (CDD) model as a novel technique for digital inpainting that differs from traditional models like Total Variation (TV) by incorporating the geometry of the image into its conductivity coefficient. Further details are given on the mathematical specifications of the model. Following, the paper delves into smoothing methods in image processing discussing the tradeoff between noise cancellation and image preservation. The discussion covers Nonlinear Diffusion, where a PDE incorporating the rate of diffusion is utilized with a focus on edge preservation and gaussian filtering, another prevalent approach for noise reduction, is presented as a convolution operation expressed equivalently as a PDE, specifically the heat equation.**

The paper puts emphasis on models and hints at their implementation for future projects.

I. INTRODUCTION

The domain of image restoration is quite extensive but roughly refers to all methods involved in the process of enhancing the quality of a degraded image. In the digital realm this usually involves using digital signal processing to remove and/or interpret the corrupted data. Typically, the damage to the image is done by noise (during transmission usually), blur (during the capture of the frame) or artifacts (many causes). In engineering the capability to reliably use the information acquired from an image can save projects from many expensive errors which is why researchers have developed many methods of restoration typically adapted to one type of degradation. In this paper the methods that will be discussed will have as a common point that they all use mathematical Partial Differential Equations (PDEs) to operate as such solutions date all the way back to the 1960s [2, 4] and have proven their effectiveness. PDEs in Image Processing problems can usually follow two approaches: Variational design, defining the energy function and the problem as minimization/maximization or Direct Design, writing down the PDEs directly and approximating the closest solution. In this paper several models using PDEs will be discussed as preparation for their future implementation in the project.

First the focus will be on inpainting methods using Variational design PDEs such as the Curvature Driven scheme and then the focus will shift to smoothing methods using diffusion to reduce noise in an image, particularly Nonlinear diffusion and the well-known Gauss filter. Before concluding, a small section discusses the way to solve PDEs.

II. CURVATURE DRIVEN DIFFUSION

A. Introduction to the model

Before introducing Curvature Driven Diffusion precisions need to be added to the definition of inpainting.

Inpainting is a typical practice of restoring an image to its previous state, in simpler terms to restore an image. To do so digitally algorithms have been introduced to fill in the missing data logically. A damaged region is typically selected in the image for the restoration through inpainting. The mathematical model used, such as a partial differential equation, must then simulate computing the expected information meaning there is an interest to find the most appropriate model for the application.

It is a challenging technique due to the need to handle complex textures, preserve fine details, and address subjective preferences while still maintaining decent results in the presence of noise and other corruption sources. This need for evolution is what spurred the topic of the paper: Curvature- Driven Diffusion (CDD) a new technique for digital inpainting [5].

The CDD inpainting scheme is a model whose conductivity coefficient, the coefficient on which depends the strength of diffusion, relies on the curvature of the isophotes. Like other models (like the Total Variation (TV) model) [5] CDD is based on Partial Differential Equations. But unlike the TV model whose conductivity coefficient does not take into account the geometry (figure 1) since its conductivity coefficient is:

$$D^{\wedge} = -\frac{1}{|\nabla u|} \quad (1)$$

where u is the image intensity.

Figure 1: TV model failures

B. TV model implementations

It is worth noting that the Total Variation Inpainting model, being a staple since its introduction in 1992 has seen many implementations.

Stéphane Sobucki's implementation of the Total Variation (TV) Inpainting model offers a practical and efficient approach to image restoration. By closely following the original principles laid out by Rudin, Osher, and Fatemi, this implementation leverages a linear fidelity coefficient to maintain accuracy while reducing computational complexity. Sobucki's approach is convenient for digital applications due to its adaptability and ease of integration with existing image processing frameworks.

The implementation process involves calculating the variation of the image's function energy, which includes terms for both the gradient and the difference between the current and original image. The variation is computed as:

$$\int_{\Omega} |\nabla u|^2 + D\lambda \int_{\Omega} (u - f)^2 \, (11)$$

where u is the updated image, f is the original damaged image, and λ is a regularization parameter. The implementation uses Python's NumPy library to handle gradient calculations and update the image iteratively. This is done by minimizing the energy function and updating the image array in a loop until the desired restoration is achieved.

Sobucki's method is reminiscent of the Curvature-Driven Diffusion (CDD) scheme, as it similarly emphasizes edge preservation and geometric fidelity. This is achieved by incorporating gradient-based calculations that are central to both methods, allowing for effective handling of complex geometries while minimizing artifacts such as the "staircase effect." Sobucki's TV model demonstrates a balance between preserving crucial image details and achieving noise reduction, making it a robust choice for various restoration tasks

C. CDD Implementation

Unlike the TV approach CDD takes shape into account by modifying the conductivity coefficient so that

$$D^{\wedge} = -\frac{g(|k|)}{|\nabla u|} \, (2)$$

Here, *g* is a function that depends on the magnitude of the gradient and the curvature of isophotes, thus making the model geometrically aware. The CDD inpainting model PDE then takes the form:

$$\frac{dt}{du} = \nabla \cdot (\hat{D} \nabla u) (3)$$

A third order PDE describing the evolution of the process over time with u the image function.
For the implementation of the model on an image there is a necessary step of spatial discretization in each pixel so that u (i, j) relates to the full rectangle, then, a time marching scheme is iteratively used until convergence of u [5].

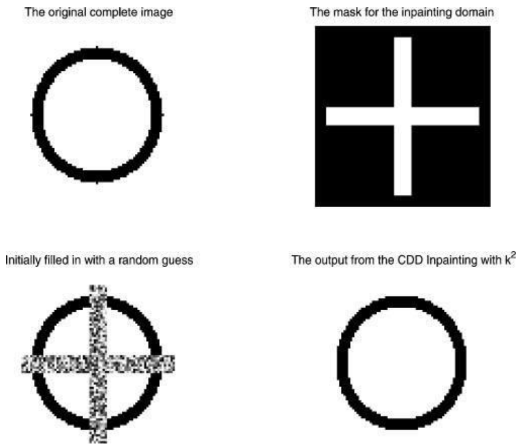

Figure 2 is a good example of how the CDD scheme can deal with more complex geometries.

III. SMOOTHING METHODS

Smoothing methods are techniques used in image processing to reduce noise by applying a frequency filter and simplifying the resolution of the data while preserving the overall structure. There is usually a tradeoff between noise reduction and data preservation when using those.[3]

A. Nonlinear Diffusion

The diffusion equation is a PDE of the form:

$$\frac{du}{dt} = \nabla (g \nabla u) \, (5)$$

In which g is the rate of diffusion.
When g = g (u) the diffusion is nonlinear (inversely when g = 1, the diffusion is **linear**). In the nonlinear case g(u):

$$g(u) = \frac{1}{\sqrt{1 + \frac{|u|^2}{\lambda^2}}} \, (6)$$

Where λ is the contrast parameter. Then similarly to the CDD a discretization step is applied followed by an iterative process where the smoothing is repeated until reaching the desired outcome [3]. The key feature is edge preservation using the gradient of u

B. Gaussian Filtering

Another approach that is much more common to image restoration against noise is Gaussian fitltering. The process involves convolving an image with a gaussian kernel. Since mathematically this is expressed as a convolution integral, the solution is then, equivalently, a PDE.

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} (7)$$

Here, G represents the Gaussian kernel, σ is the standard deviation controlling the spread of the Gaussian and (x, y) are the coordinates.

$$\frac{du}{dt} = \alpha \nabla^{\wedge} 2u (8)$$

The heat equation [3, 4] can be used to describe the smoothing operation over time as it approximates the convolution operation of G when discretized. This makes this PDE a simple tool for denoising.

IV. SOLVING PDES

A small discussion can be held on how digital methods can solve PDEs. Taking an example, the Variational Iteration Method (VIM) [6] aims to find approximate solutions to PDEs by constructing a correction functional. The key idea involves introducing a correction term to an initial approximation, subsequently refining the solution through an iterative process.

$$Xn = \sum n \; ai \; ui \qquad (9)$$

The simplified digital correction functional (8) where a are the coefficients to be determined. Xn is then minimizes with respect to the coefficients until convergence.

$$un+1 = uo + Xn \quad (10)$$

Illustrating this: Consider the nonlinear differential equation

$u'' + u = 0$ subject to initial conditions $u(0) = 0$ and $u'(0) = 1$. Assuming an initial approximation $u0=x(t)$. The correction functional is $Xn = \Sigma n \ aixi(t)$, and the minimization of a function J[Xn] with respect to ai leads to a recursive formula for the coefficients. This is repeated until convergence with u updated at each iteration.

V. RECENT DEVELOPMENTS

A. Hybrid Models

Recent research suggests the development of hybrid models that combine the strengths of different PDE-based methods. These models aim to enhance restoration quality by integrating various techniques, potentially offering more robust solutions. More specifically, models implementing noise diffusions algorithms mixed techniques like CDD usually achieve far higher results than their counterparts [5]. They relieve the inpainting algorithms of their weaknesses to noise and allow them to work at full capacity to restore the image.

B. AI-Driven Techniques

The integration of Artificial Intelligence (AI) and machine learning into image restoration represents a significant advancement in the field. Traditional PDE-based methods, while powerful, often rely on manually tuned parameters and can struggle with complex image degradation scenarios. AI-driven techniques [8], particularly those utilizing deep learning, offer a more adaptive and automated approach to image restoration, learning directly from data to improve performance.

Generative Adversarial Networks (GANs) are a class of AI models that consist of two neural networks [10], the generator and the discriminator, that are trained together in a competitive manner. The generator creates synthetic images that are progressively refined to resemble the original, while the discriminator attempts to distinguish between real and generated images. This adversarial process continues until the generator produces images that are nearly indistinguishable from the originals.

In the context of image restoration, GANs can be used to inpaint missing or corrupted regions of an image by generating plausible content that matches the surrounding areas thanks to their previous training. The ability of GANs to learn complex patterns and textures makes them particularly effective for tasks such as inpainting, where traditional PDE methods might struggle with filling large gaps or restoring intricate details. The paper mentions the potential of GANs to mimic human perception, producing restorations that not only score well on objective metrics like PSNR but also look visually convincing.

With the rise of Text to Image Technologies new tools models become great fillers too.

Google's Imagen2 model, an advanced AI-driven approach that further pushes the boundaries of image restoration. Imagen2 is a diffusion-based model designed to generate high-quality images from textual descriptions, but its underlying technology is also highly effective for inpainting tasks [9]. By leveraging deep learning and extensive training on large datasets, Imagen2 can accurately fill in missing parts of an image with realistic content that seamlessly blends with the surrounding pixels. What sets Imagen2 apart is its ability to understand and replicate complex textures, colors, and patterns, making it a powerful tool for high-quality image restoration. The model's generative capabilities allow it to predict and reconstruct parts of an image that are not just missing, but also require creative synthesis, which is particularly useful in scenarios where traditional methods might fail to produce convincing results. Imagen2 exemplifies the cutting edge of AI in image restoration, demonstrating the potential for AI models to not only restore images but also enhance them in ways previously thought impossible.

VI. CONCLUSION

In summary, this paper explained into the realm of image restoration, emphasizing the efficacity of mathematical Partial Differential Equations (PDEs) in signal degradation issues. The Curvature-Driven Diffusion (CDD) inpainting model, with its unique incorporation of image geometry, showcased its promising results. Additionally, the smoothing methods like nonlinear diffusion and Gaussian filtering demonstrated their effectiveness in basic noise reduction. Finally, a small demonstration of how PDEs could be iteratively solved was done. Further work could have been examined but for the sake of space the models examined were kept to three. A next step would be to discuss the digital implementation of such models as well as their performance so that the most efficient image restoration can discovered.

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APPENDIX B

UPDATED PROJECT DESIGN PLAN

ON IMAGE RESTORATION USING PARTIAL DIFFERENTIAL EQUATIONS

Project Design Plan: Image Restoration Using Partial Differential Equations

Goal of the project

How can partial differential equations (PDEs) methods be effectively utilized to restore images degraded by noise and artifacts?

Project Scope

Included:

- Implementation of image restoration methods:
  - Total Variation (TV) Inpainting
  - Curvature-Driven Diffusion (CDD) Inpainting
  - Gaussian Filtering
  - Nonlinear Diffusion
- Evaluation of restoration quality using metrics such as Peak Signal-to-Noise Ratio (PSNR)
- Analysis and comparison of the effectiveness of these methods in various scenarios of image degradation
- Utilization of Python for implementing the restoration algorithms

Excluded:

- Use of machine learning-based image restoration techniques

- Exploration of restoration methods not based on PDEs

Design Approach

1. Implementation:
   - Implement the identified PDE-based methods.

   - Develop separate modules for TV Inpainting, CDD Inpainting, Gaussian Filtering, and Nonlinear Diffusion.

2. Testing and Validation:
   - Test the implemented methods on various degraded images to assess their performance.
   - Use PSNR and other relevant metrics to quantify the restoration quality.

3. Analysis:
   - Compare the performance of different methods under various conditions.
   - Analyze the strengths and weaknesses of each method.
   - Document findings and suggest potential improvements.

Timeline

Phase	Dates	Tasks	Milestones
Project Design Plan	20 May 2024	Review Literature Plan and the approaches to be taken from now on	Project Design plan completed
Implementation Phase 1	June 2024	- Implement the final forms TV Inpainting and CDD Inpainting - Initial testing	TV Inpainting and CDD Inpainting implemented
Implementation Phase 2	End of June/ Beginning of July 2024	- Implement Gaussian Filtering and Nonlinear Diffusion - Initial testing	Gaussian Filtering and Nonlinear Diffusion implemented
Testing and Validation	July 2024	- Conduct detailed testing of all methods - Validate results using PSNR and other metrics	Testing and validation completed
Analysis and Comparison	Beginning of August 2024	- Compare methods - Strengths and weaknesses - Suggest improvements	Analysis completed
Final Report and Portfolio	August 2024	- Compile results - Write final report - Prepare portfolio	Final report and portfolio completed

Success Criteria

This project will be considered successful if the following criteria are met:

- Completion of a comprehensive literature review on PDE-based image restoration methods.
- Successful implementation of TV Inpainting, CDD Inpainting, Gaussian Filtering, and Nonlinear Diffusion in Python.
- Validation of the implemented methods using PSNR and other relevant metrics.
- Detailed analysis and comparison of the performance of the different methods.
- Submission of a well-documented final report and portfolio by the deadline.

APPENDIX C

ORIGINAL PROJECT DESIGN PLAN

Project Design Plan: Image Restoration Using Partial Differential Equations

Goal of the project

How can partial differential equations (PDEs) methods be effectively utilized to restore images degraded by noise and artifacts?

Project Scope

Included:

- Implementation of image restoration methods:
  - Total Variation (TV) Inpainting
  - Curvature-Driven Diffusion (CDD) Inpainting
  - Gaussian Filtering
  - Nonlinear Diffusion
- Evaluation of restoration quality using metrics such as Peak Signal-to-Noise Ratio (PSNR)
- Analysis and comparison of the effectiveness of these methods in various scenarios of image degradation
- Utilization of Python for implementing the restoration algorithms

Excluded:

- Use of machine learning-based image restoration techniques

- Exploration of restoration methods not based on PDEs

Design Approach

1. Implementation:
   - Implement the identified PDE-based methods.

   - Develop separate modules for TV Inpainting, CDD Inpainting, Gaussian Filtering, and Nonlinear Diffusion.

2. Testing and Validation:
   - Test the implemented methods on various degraded images to assess their performance.
   - Use PSNR and other relevant metrics to quantify the restoration quality.

3. Analysis:
- Compare the performance of different methods under various conditions.
- Analyze the strengths and weaknesses of each method.
- Document findings and suggest potential improvements.

Timeline

Phase	Dates	Tasks	Milestones
Project Design Plan	20 May 2024	Review literature the plan and the approaches to be taken from now on	Project Design plan completed

Implementation Phase 1	June 2024	- Implement the final forms TV Inpainting and CDD Inpainting - Initial testing	TV Inpainting and CDD Inpainting implemented
Implementation Phase 2	Mid-June 2024	- Implement Gaussian Filtering and Nonlinear Diffusion - Initial testing - New techniques in the "Partial Differential Equation Methods for Image Inpainting" by Carola-Bibiane Schonlieb (DOI: ISBN 978-1-107-00100-8) that can be explored.	Gaussian Filtering and Nonlinear Diffusion implemented
Testing and Validation	July 2024	- Conduct detailed testing of all methods - Validate results using PSNR and other metrics	Testing and validation completed
Analysis and Comparison	End of July 2024	- Compare methods - Strengths and weaknesses - Suggest improvements - Work on a visual representation of work	Analysis completed

Final Report and Portfolio	August 2024	- Compile results - Write final report - Prepare portfolio	Final report and portfolio completed

Success Criteria

This project will be considered successful if the following criteria are met:

- Completion of a comprehensive literature review on PDE-based image restoration methods.
- Successful implementation of TV Inpainting, CDD Inpainting, Gaussian Filtering, and Nonlinear Diffusion in Python.
- Validation of the implemented methods using PSNR and other relevant metrics.
- Detailed analysis and comparison of the performance of the different methods.
- Submission of a well-documented final report and portfolio by the deadline.

APPENDIX D

PROJECT RESEARCH LOG ON IMAGE RESTORATION USING PARTIAL DIFFERENTIAL EQUATIONS

Masters Project Research Log

Masters in Electronic and Computer Engineering 2023/2024

Student Name: Elijah Ki-Zerbo
Student ID: 23266931
Project Title: Image Restoration Using Partial Differential Equations

Please read before making entries in this log

The purpose of this Project Research Log is to capture concise, focused summaries of research materials you read, as you progress through your project. The emphasis is to record (i) how the material you have read will determine or influence your project solution approach and (ii) your assessment of the key strengths and weaknesses of the solutions, methods, technologies, etc. proposed in the material you have read.

In the first stage of your project, the literature review, use the Log to capture this information for the key papers you have read (for example, the three most important papers of your 10 literature review references). As your project progresses into the design and implementation phases, you will need to continue to search the literature so you can review, revise and refine your initial thinking and the details of your approach to a project solution. Use this Research Log to capture your continued research reading and its influence on your project design and implementation.

Be selective about what you record in this log. Do not use it as an informal notebook while you are reading a new paper. Only make an entry after you have read a paper that you consider important to the development of your project solution. It is expected that, by the end of the project, you will have made **between 10 and 20 entries (20 maximum)**.

Share your log with your supervisor for viewing throughout the project. You will submit the final version of the log for grading, at the end of the project implementation period. It will be assessed on the basis of how well you have used your analysis of the literature to inform your project design, implementation and the evaluation of your project results. The Research Log contributes **5%** to the overall project mark.

Note: All entries you make in this log must use the prescribed format shown on the next page. You will maintain other notes as you progress through your project but they should not be recorded here. Fill in the details where the *** signs are.

Statement of project problem / research question (maximum 200 words) *This statement should be periodically reviewed and updated, as necessary, as your project progresses and you gain further insight into the detailed project challenges, requirements and objectives as your project work moves from background reading, literature review, initial project design planning and detailed design and implementation. Initially, start by stating your current understanding of the project objectives. After each meeting with your supervisor, review and refine your project problem statement, as required.*
The challenge of this project was developing an effective image restoration model that could handle complex geometrical structures while preserving fine details. The Curvature-Driven Diffusion (CDD) method presented a promising solution due to its advanced handling of image curvature. However, the implementation posed significant challenges, specifically, computational complexity and output instability with high iterations. Additionally, the method's sensitivity to noise was problematic, as CDD sometimes misinterpreted noise as complex geometry. Parameter tuning and iterative testing were required as well as a deep understanding of the scheme.

A complete reference for the paper
Chan, T. F., & Shen, J. (2001). Nontexture Inpainting by Curvature-Driven Diffusions. *Journal of Visual Communication and Image Representation*, 12, 436–449. doi: 10.1006/jvci.2001.0487.
Summary of paper (maximum 100 words)
This paper introduces Curvature-Driven Diffusion (CDD) as an enhancement to Total Variation (TV) Inpainting. The CDD method incorporates image curvature into the diffusion process, allowing it to better restore intricate details while preserving edges. By using third-order partial differential equations, CDD effectively integrates the geometry into computation.
How is this paper relevant to solving your project problem or addressing your research question? (maximum 100 words)
CDD is highly relevant to my project as it offers an advanced technique for inpainting images with complex geometrical features.
What are the strengths and weaknesses of the solutions/methods/technologies proposed in this paper? (maximum 100 words)
The strengths of CDD include its ability to effectively handle complex geometries and preserve fine details, which surpasses the capabilities of traditional TV Inpainting. However, it struggles with random noise, often interpreting it as complex structures, which can degrade the restoration quality.

Statement of project problem / research question (maximum 200 words) *This statement should be periodically reviewed and updated, as necessary, as your project progresses and you gain further insight into the detailed project challenges, requirements and objectives as your project work moves from background reading, literature review, initial project design planning and detailed design and implementation. Initially, start by stating your current understanding of the project objectives. After each meeting with your supervisor, review and refine your project problem statement, as required.*
The issue with this model was primarily the multitude of different implementations that would be confusing. Choosing the appropriate one for Python implementation was a challenge in itself followed then by understanding said implementation. Simply trying to copy the mathematical expression yielded poor results due to my incomprehension of the iterative process .

A complete reference for the paper
Rudin, L. I., Osher, S., & Fatemi, E. (1992). Nonlinear Total Variation-based noise removal algorithms. *Physica D: Nonlinear Phenomena*, 60(1-4), 259–268. doi: 10.1016/0167-2789(92)90242-F.
Summary of paper (maximum 100 words)
Total Variation (TV) Inpainting is a PDE-based method for restoring images by minimizing the total variation of the image intensity. This technique is particularly effective at preserving edges while removing noise, making it ideal for images with large, uniform areas. However, it tends to create a "staircase effect" in smooth gradients, where the inpainted region appears blocky rather than smooth.
How is this paper relevant to solving your project problem or addressing your research question? (maximum 100 words)
TV Inpainting is foundational to my project, as it represents a widely used and effective technique for edge-preserving image restoration. Its simplicity and effectiveness in handling large, uniform areas make it ideal as a reference. Understanding its limitations, such as the staircase effect, provides a basis for seeking improvements or integrating supplementary methods like CDD.
What are the strengths and weaknesses of the solutions/methods/technologies proposed in this paper? (maximum 100 words)
The primary strength of TV Inpainting is its ability to preserve sharp edges while reducing noise, which is crucial for maintaining the integrity of important image features. However, its main weakness lies in its handling of smooth gradients, where it can produce unnatural, blocky artifacts. Its implementation is also quite unclear as it is the initial paper discussing TV inpainting.

A complete reference for the paper
Sobucki, S. (n.d.). *Total Variation Inpainting*. Retrieved from *https://github.com/StephaneSobucki/Total-variation-Inpainting.*
Summary of paper (maximum 100 words)
Stéphane Sobucki's paper on Total Variation (TV) Inpainting provides a detailed implementation of the TV model, focusing on minimizing the energy function to restore damaged images. The method is based on solving the heat equation while applying a fidelity coefficient to maintain the original image's energy, thereby reducing noise and preventing blurring. Sobucki’s approach simplifies the process by using a structured iterative method, making it easier to implement.
How is this paper relevant to solving your project problem or addressing your research question? (maximum 100 words)
Sobucki’s implementation of TV Inpainting was crucial in overcoming the initial challenges faced in the project. The paper provided a clearer, more structured approach to implementing the TV method, addressing the earlier difficulties with the iterative process. By following Sobucki’s method, we benefited from a more reliable and effective TV Inpainting implementation, which significantly improved the quality of image restoration, without excessive blurring or the creation of black regions.
What are the strengths and weaknesses of the solutions/methods/technologies proposed in this paper? (maximum 100 words)
The strengths of Sobucki’s implementation include its clarity and effectiveness in minimizing the energy function while preserving the essential features of the image. The use of a fidelity coefficient ensures that the method does not overly blur the image, which is a common issue with other inpainting techniques. However, it doesn’t address the main weaknesses of the TV mode.

Statement of project problem / research question (maximum 200 words) *This statement should be periodically reviewed and updated, as necessary, as your project progresses and you gain further insight into the detailed project challenges, requirements and objectives as your project work moves from background reading, literature review, initial project design planning and detailed design and implementation. Initially, start by stating your current understanding of the project objectives. After each meeting with your supervisor, review and refine your project problem statement, as required.*
The project aimed to explore advanced image restoration techniques that could effectively reduce noise while preserving important image features such as edges and textures. Implementing Nonlinear Diffusion presented a specific challenge due to the complexity of balancing edge preservation with noise reduction. Early attempts to implement this technique led to inconsistent results, particularly in cases where edge preservation was not as robust as expected.

A complete reference for the paper
Perona, P., & Malik, J. (1990). Scale-space and edge detection using anisotropic diffusion. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 12(7), 629-639. doi: 10.1109/34.56205.
Summary of paper (maximum 100 words)
This paper presents a method of image smoothing using nonlinear diffusion, which preserves edges while reducing noise. The diffusion process is governed by a function that decreases as the gradient increases, allowing the method to smooth regions with low contrast while preserving high-contrast edges.
How is this paper relevant to solving your project problem or addressing your research question? (maximum 100 words)
The paper is relevant to the project as it offers a sophisticated approach to noise reduction that preserves important image features like edges.
What are the strengths and weaknesses of the solutions/methods/technologies proposed in this paper? (maximum 100 words)
The strength of this nonlinear diffusion implementation lies in its ability to effectively smooth noise while preserving crucial image edges, making it superior to linear diffusion methods. However, its implementation can be complex and computationally demanding, requiring careful tuning of parameters such as the diffusion coefficient and the number of iterations. Additionally, it struggles in images where the edges are not clearly defined.

A complete reference for the paper
Olovsson, N. (2022). *Image smoothing by nonlinear diffusion*. Retrieved from https://nils-olovsson.se/articles/image_smoothing_by_nonlinear_diffusion/.
Summary of paper (maximum 100 words)
Nils Olovsson's article on nonlinear diffusion provides an in-depth guide to implementing this image smoothing technique. The method relies on anisotropic (nonlinear) diffusion to selectively reduce noise in images while preserving edges. The article explains the mathematical foundation of the process, including the Perona-Malik model, and offers detailed instructions on discretizing the diffusion equation for practical implementation.
How is this paper relevant to solving your project problem or addressing your research question? (maximum 100 words)
This resource is particularly valuable for understanding how to fine-tune the parameters that control the diffusion process, ensuring that the image is smoothed without losing important structural details. The article was highly relevant to the project as it provided a clear and detailed approach to implementing nonlinear diffusion . The guide helped address the challenges encountered in earlier attempts to implement this method, particularly by offering practical advice on parameter selection and managing the trade-offs between noise reduction and detail preservation. By following Olovsson's methodology, the project was able to achieve more consistent and effective results in image restoration.
What are the strengths and weaknesses of the solutions/methods/technologies proposed in this paper? (maximum 100 words)
The strength of Olovsson's approach lies in its comprehensive explanation of the nonlinear diffusion process, making it accessible for implementation even for those with a basic understanding of image processing. The detailed breakdown of the Perona-Malik model and its practical application is particularly beneficial. However, the method's complexity still exhibits the weakness of the original scheme, and while the ideal lambda parameter can be computed, some parameters still require tuning.

Statement of project problem / research question (maximum 200 words) *This statement should be periodically reviewed and updated, as necessary, as your project progresses and you gain further insight into the detailed project challenges, requirements and objectives as your project work moves from background reading, literature review, initial project design planning and detailed design and implementation. Initially, start by stating your current understanding of the project objectives. After each meeting with your supervisor, review and refine your project problem statement, as required.*
The project sought to explore advanced image restoration techniques and possibly provide insights for new ones to be implemented. The main challenge was to identify these PDE-based inpainting methods and outline relevant ones, particularly for complex cases involving intricate geometrical structures or large damaged regions. This required a deep understanding of various PDE models, their mathematical foundations, and their practical applications in image processing.

A complete reference for the paper
Schönlieb, C.-B. (2015). *Partial Differential Equation Methods for Image Inpainting*. Cambridge University Press. and https://github.com/cb-schonlieb/PDE-Methods-for-Inpainting
Summary of paper (maximum 100 words)
Carola-Bibiane Schönlieb's book, *Partial Differential Equation Methods for Image Inpainting*, serves as a comprehensive guide to the use of PDEs in digital image restoration. It covers a wide range of methods, from second-order diffusion equations to higher-order approaches like Curvature-Driven Diffusion (CDD) and Total Variation (TV) inpainting. The book also discusses applications of these methods in various fields, including art restoration, medical imaging, and satellite data reconstruction. It is an essential resource for understanding both the theoretical underpinnings and practical implementations of PDE-based image inpainting.
How is this paper relevant to solving your project problem or addressing your research question? (maximum 100 words)
Schönlieb's book was highly relevant to the project as it provided a detailed exploration of PDE methods that were directly applicable to the challenges of image restoration. The book's thorough explanations of the mathematical principles behind these methods, combined with practical examples, helped bridge the gap between theory and application.
What are the strengths and weaknesses of the solutions/methods/technologies proposed in this paper? (maximum 100 words)
The book provided an extensive and comprehensive coverage of PDE-based methods, that made complex mathematical concepts accessible. However, the book was not quite intuitive, simply outlining methods without exploring the mathematical origins of the schemes. Furthermore, many schemes appeared to be less efficient than the ones seen at the beginning of the book.