信息基础 **2021 春**

Homework 1: Fundamentals of Information Science

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This lecture's notes illustrate some uses of various LaTeX macros. Take a look at this and imitate.

1.1 Summarize what you learned from the videos about Claude Shannon and Alan Turing:

- 香农被称为是“信息论之父”，香农是数字电路和信息论的创始人，人们尊崇香农为信息论及数字通信时代的奠基之父，其提出的“香农”信息论从概率论的角度解释了信息的不确定性定义，他证明了‘与’、‘或’和‘非’等简单的连接器。香农创造的信息论广泛应用于各种学科，如编码学、集成电路、人工智能、密码学与密码分析学、数据传输、压缩甚至政治学。小到电话、调制解调器，大到人造卫星都离不开香农创造的信息论。并且香农的信息论还激发了后来发明设计移动电话和计算机的人，可见香农的信息论是多么的伟大。
- 而图灵被称作“人工智能之父”，图灵在科学、特别在数理逻辑和计算机科学方面，构成了现代计算机技术的基础。令我印象最深刻的是图灵进行的图灵测试，指测试者与被测试者（一个人和一台机器）隔开的情况下，通过一些装置（如键盘）向被测试者随意提问。进行多次测试后，如果机器让平均每个参与者做出超过百分之三十的误判，那么这台机器就通过了测试，并被认为具有人类智能。图灵能够在那个时代提出人工智能、“机器人能思考吗”这样的问题，是一个划时代之作。

1.2 Coin flips. A fair coin is flipped until the first head occurs. Let X denote the number of flips required. Find the entropy H(x) in bits.:

$$\because H(X) = \sum_{i=1}^{n} \log_2 \frac{1}{p(x)}$$

$$\begin{aligned}
\therefore H(X) \quad & = 0.5 \times \log_2 2 + 0.5^2 \times \log_2 4 + 0.5^3 \times \log_2 8 + ... \\
& = 0.5 + 2 \times 0.5^2 + 2 \times 0.5^3 + ... \\
& = \frac{0.5}{0.5^2} \\
& = 2(bits)
\end{aligned}$$

1.3 Entropy of a sum. Let X and Y be random variables that take on values $x_1, x_2..., x_r$ and $y_1, y_2, ...y_s$, respectively. Let$Z = X + Y$.

- Show that $H(Z|X) = H(Y|X)$.Argue that if X,Y are independent,then $H(Y) \leq H(Z)$ and $H(X) \leq H(Z)$.Thus,the addition of independent random variables adds uncertainty.

$$\begin{aligned} H(Z|X) \quad &= E[log_2 \frac{1}{p(Z|X)}] \\ &= E[log_2 \frac{1}{p(Y|X)}] \end{aligned}$$

$$\because H(Y|X) = E[log_2 \frac{1}{p(Y|X)}]$$
$$\therefore H(Z|X) = H(Y|X)$$
$$I(X;Z) = H(Z) - H(Z|X)$$
$$\because H(Z|X) = H(Y|X)$$
$$X, Y \quad are \quad independent$$
$$\therefore H(Y|X) = H(Y)$$
$$\therefore I(X;Z) = H(Z) - H(Y)$$
$$\because I(X;Z) \geq 0$$
$$\therefore H(Z) \geq H(Y)$$
$$\therefore H(Z) \geq H(X)$$

- Give an example of(necessarily dependent)random variables in which $H(X) > H(Z)$ and $H(Y) > H(Z)$.

$$if \quad X = -Y = \begin{cases} 1 & , \quad \text{p=0.5} \\ 0 & , \quad \text{p=0.5} \end{cases}$$
$$then \quad H(X) = 0.5 \times \log_2 2 + 0.5 \times \log_2 2 = 1(bits)$$
$$H(Y) = 1(bits)$$
$$H(Z) = 1 \times \log_2 1 = 0$$
$$\therefore H(X) > H(Z)$$
$$H(Y) > H(Z)$$

- Under what conditions does $H(Z) = H(X) + H(Y)$?

$$\begin{aligned} H(X) \quad &= H(X,Z) - H(Z|X) \\ &= H(X,Z) - H(Y|X) \end{aligned}$$

$$\begin{aligned} H(Z) &= H(X,Z) - H(X|Z) \\ &= H(Y,Z) - H(Y|Z) \end{aligned}$$

$$H(Y) = H(Y,Z) - H(Z|Y)$$

if

$$H(Z) = H(X) + H(Y)$$
$$\because H(Z|X) = H(Y|X)$$
$$H(Z|Y) = H(X|Y)$$

$$\begin{aligned} H(X) + H(Y) &= H(X,Z) - H(Y|X) + H(Y,Z) - H(X|Y) \\ &= H(X) + H(Z|X) + H(Y) + H(Z|Y) - H(Y|X) - H(X|Y) \end{aligned}$$

$$\because H(X) - H(X|Y) = H(Y) - H(Y|X)$$

$$\begin{aligned} \therefore H(X) + H(Y) &= H(Z|X) + H(Z|Y) \\ &= H(Y|X) + H(X|Y) \\ &= H(X,Y) - H(X) + H(X,Y) - H(Y) \end{aligned}$$

$$\therefore H(X) + H(Y) = H(X,Y)$$

then if

$$H(X,Z) = H(Z) \quad and \quad X,Y \quad are \quad independent$$
$$H(Z) = H(X) + H(Y)$$