Silesian University of Technology
Faculty of Automatic Control,
Electronic and Computer Science
Major: Makro sem.4

Gliwice
Academic year 2018/2019

Numerical Methods
Laboratory Exercise

11. Aproximate Solving of Partial Differential Equations

Group 2, Section 4
Mateusz Warmuz
Szymon Skorupa

Exercise preformed on: 6.06.2019

1. Sourde code

```
#include<iostream>
#include<cmath>
using namespace std;
int main()
{
    double net[100][100] = {{0},{0}};
    double z_step = 1;
    double t_step = 0.001;

    for(int i = 0; i <= 10; i++)
    {
        net[i][0] = 0;
        net[i][1] = 0;
        net[0][i] = sin(M_PI*z_step*i);
    }

    for(int t = 0; t <= 10; t++)
    {
        for(int k = 1; k <= 9; k++)
        {
            net[t+1][k] = net[t][k]+0.1*t_step*pow(z_step,-2)*(net[t]
[k+1]-2*net[t][k]+net[t][k-1]);
        }
    }
    for(int t=0;t<=10;t++){
        for(int k=0;k<=10;k++){
            cout<<net[k][t]<<", ";
        }
        cout<<";"<<endl;
    }
}
```

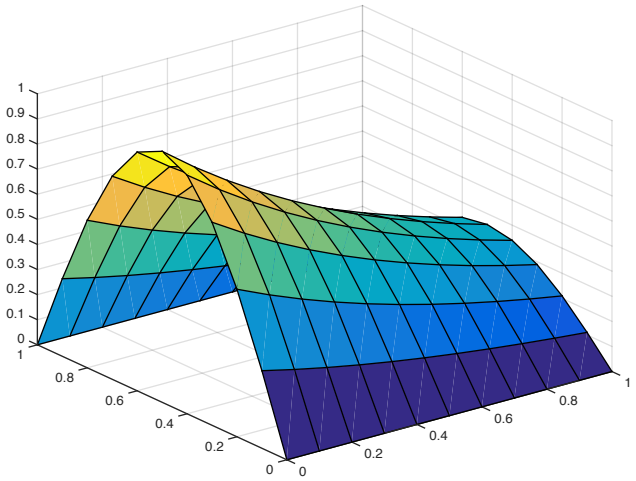
2. Convergence criterion and it's testing:

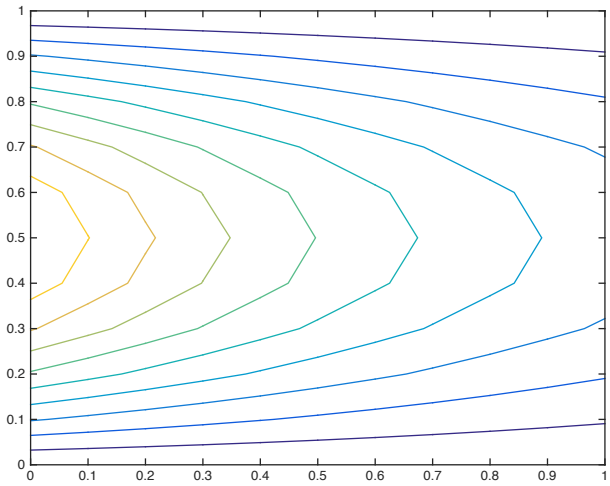
$$\frac{1}{2} \geq \frac{a^2 h}{r^2}$$

Convergence criterion

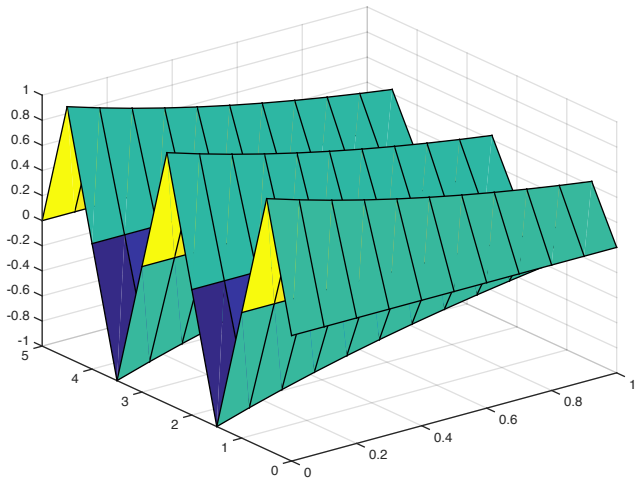
We tested different values of h and r to get values above and below 1/2. The results are presented below on 3D graphs plotted by Matlab

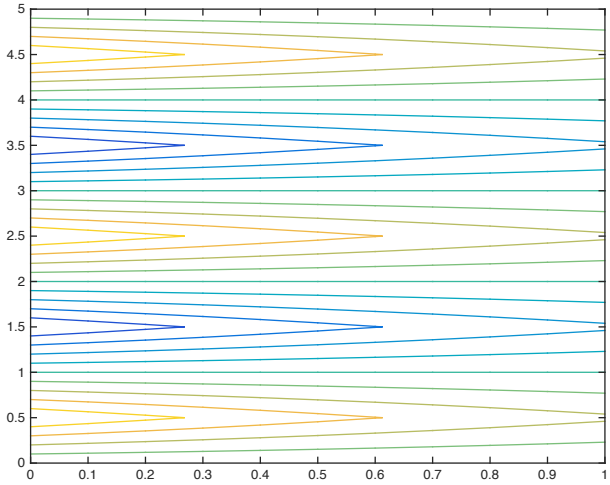
1st test: r = 0.1, h = 0.1 $\frac{1}{2} \ngeq \frac{a^2 h}{r^2}$ Convergence criterion not satisfied.



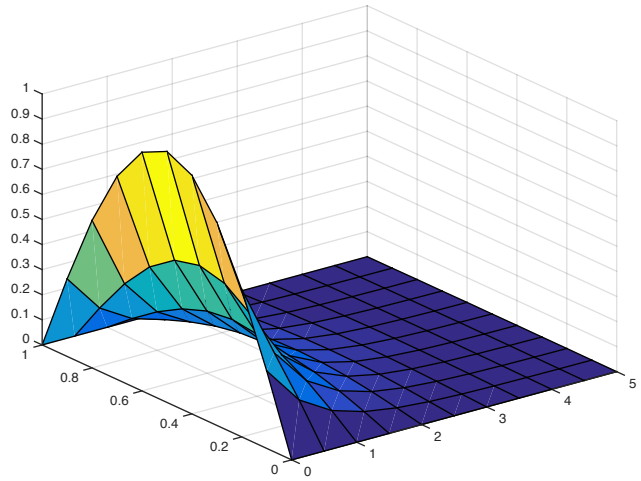


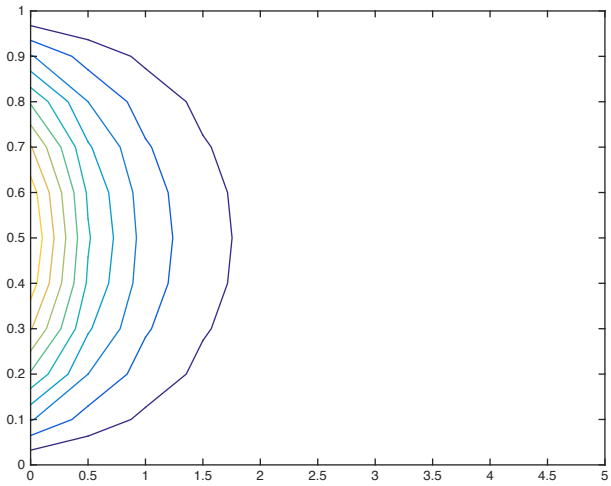
2nd test: r = 0.5, h = 0.1, $\frac{1}{2} \geq \frac{a^2 h}{r^2}$ Convergence criterion satisfied.



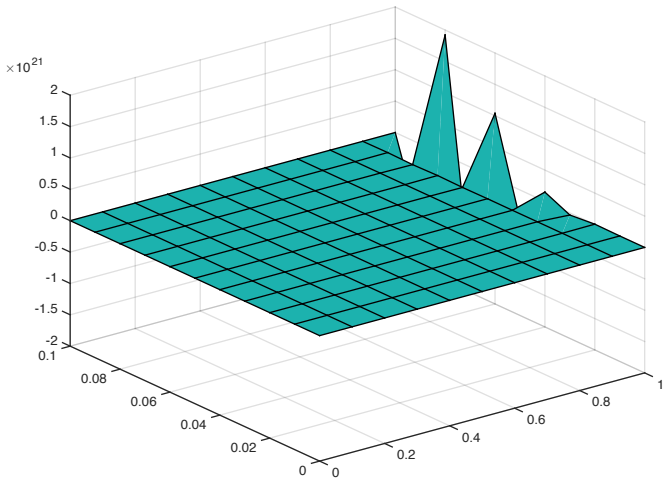


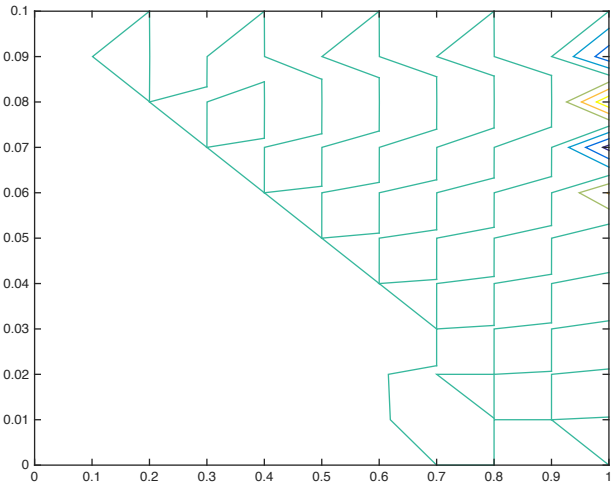
3rd test: r = 0.1 h = 0.5 $\frac{1}{2} \ngeq \frac{a^2 h}{r^2}$ Convergence criterion not satisfied.



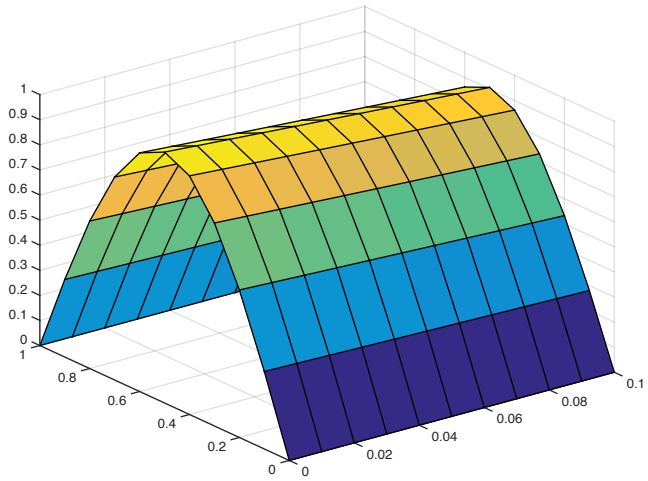


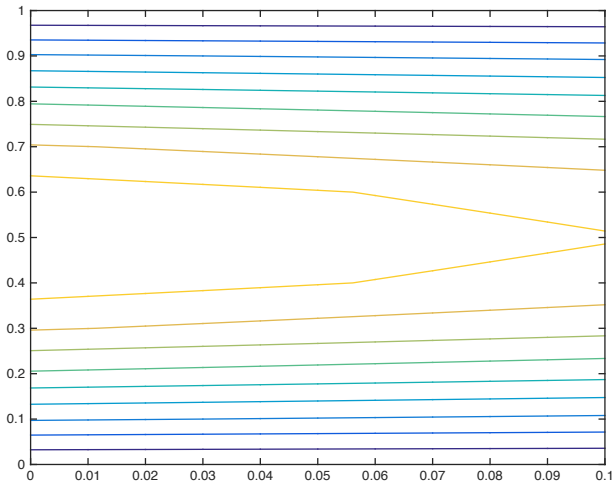
4th test: r = 0.01, h = 0.1 Convergence criterion not satisfied.



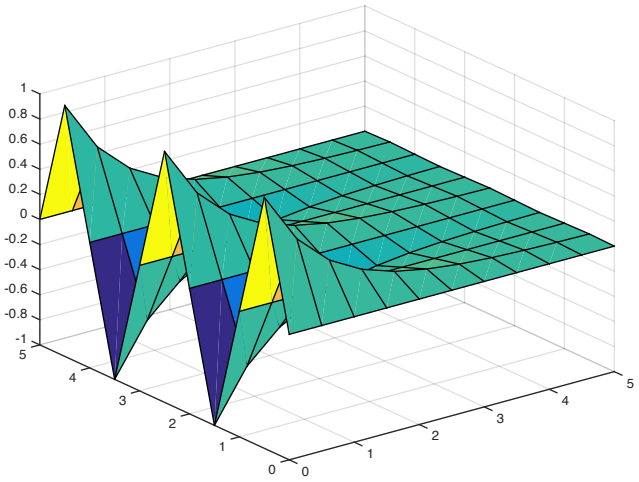


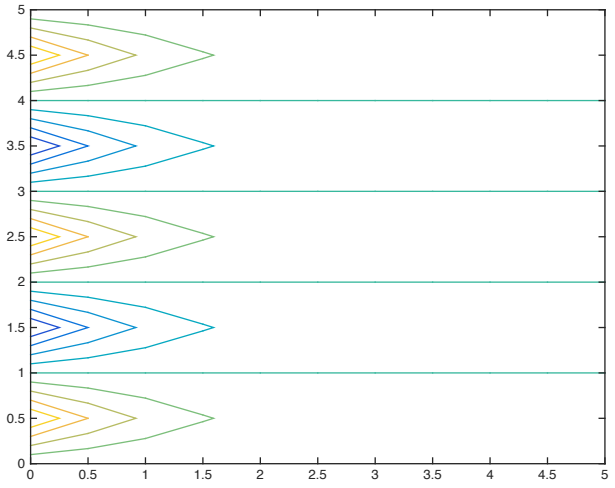
5th test: r = 0.1, h = 0.01 $\frac{1}{2} \geq \frac{a^2 h}{r^2}$ Convergence criterion satisfied.



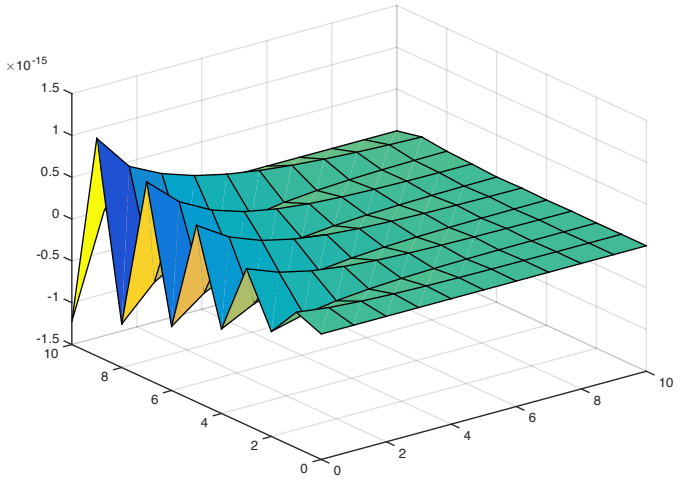


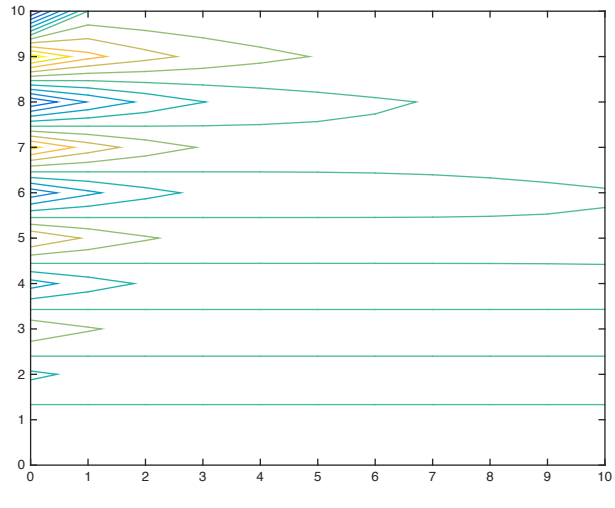
6th test: r = 0.5, h = 0.5 $\frac{1}{2} \ngeq \frac{a^2 h}{r^2}$ Convergence criterion not satisfied.



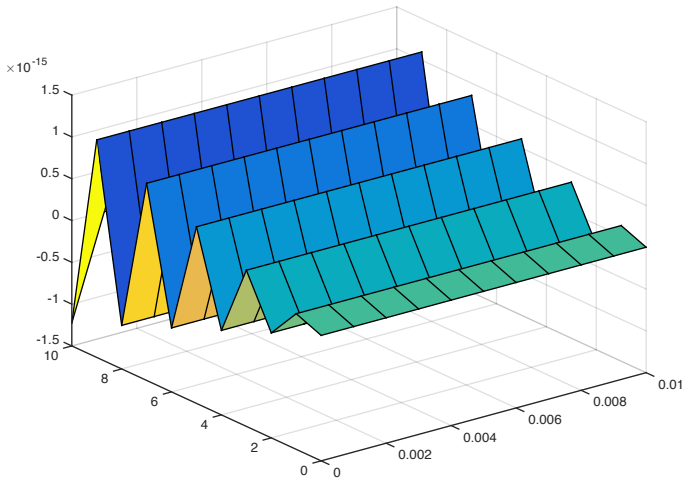


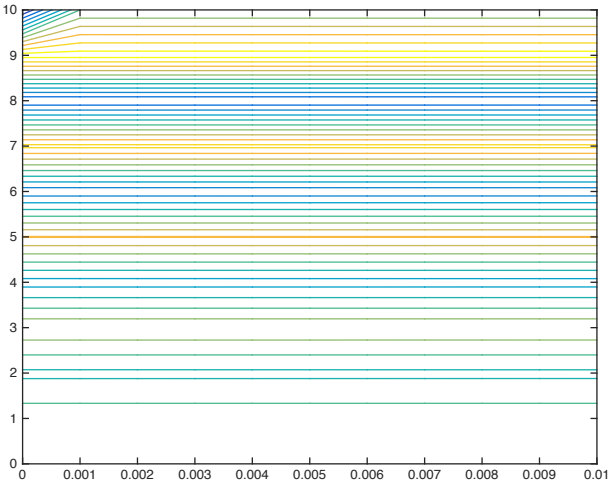
7th test: r = 1, h = $1\frac{1}{2} \geq \frac{a^2 h}{r^2}$ Convergence criterion satisfied.



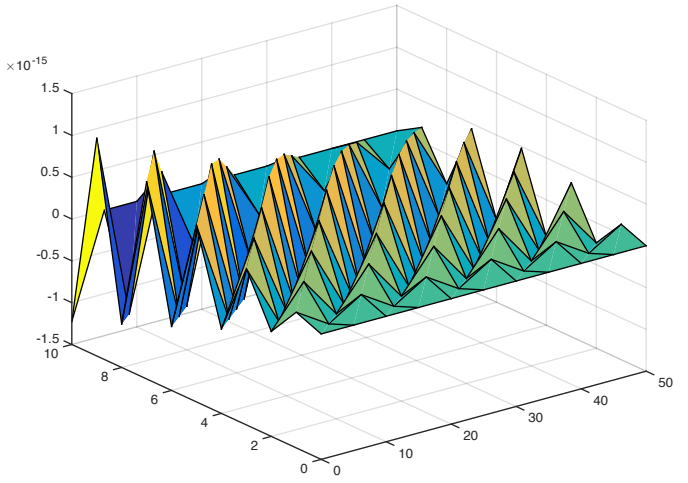


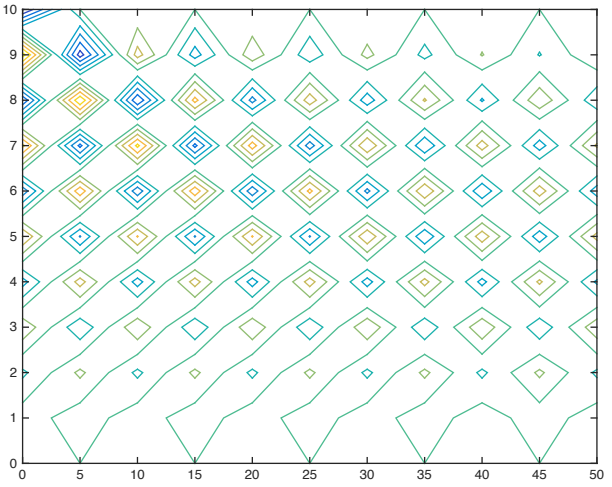
8th test: r = 1, h = 0.001 $\frac{1}{2} \geq \frac{a^2 h}{r^2}$ Convergence criterion satisfied.



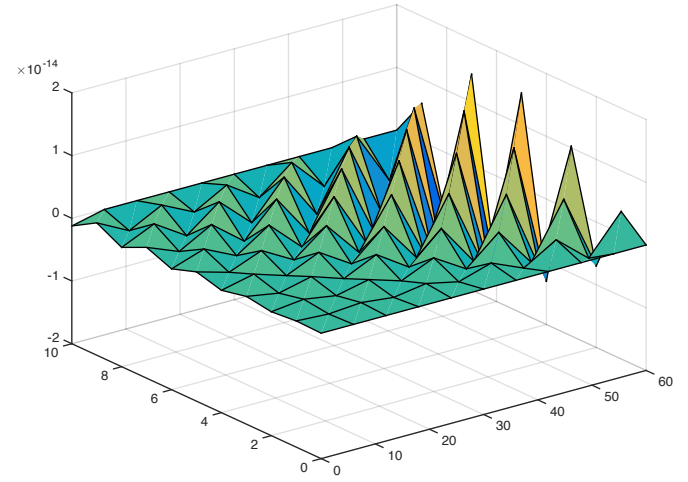


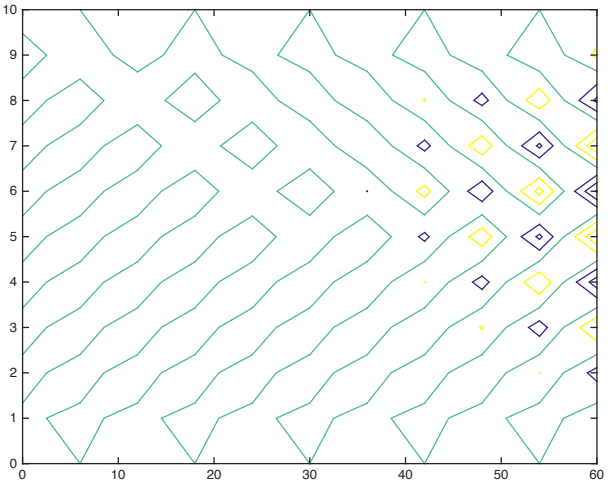
9th test: r = 1, h = 5 $\frac{1}{2} \geq \frac{a^2 h}{r^2}$Convergence criterion satisfied.



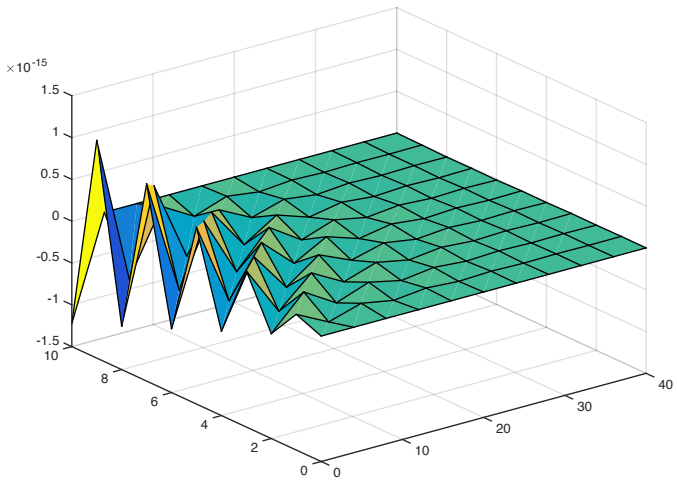


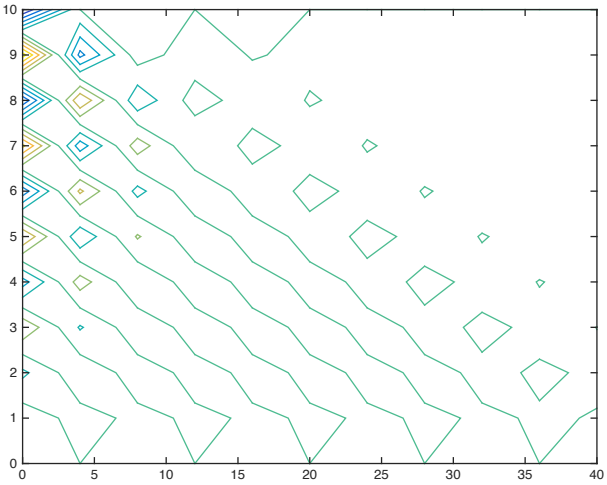
10th test: r = 1, h = 6 $\frac{1}{2} \not\geq \frac{a^2 h}{r^2}$Convergence criterion not satisfied.





11th test: r = 1, h = 4 $\frac{1}{2} \geq \frac{a^2 h}{r^2}$Convergence criterion satisfied.





3. Mean Squared Error

Mean error calculated using Matlab function immse and exact solution $w(t,z) = e^{-\pi^2 a^2 t} sin \pi z$

r	h	MSE
0,5	0,1	0.382904722219846
0,1	0,01	0.225323066150490
1	1	0.204607521083360
1	0,001	0.204607521083360
1	5	0.204607521083360
1	4	0.204607521083360

4. Discuss the results

Net method approach to solving PDE allows for good approximations. From the choice of step r and h depend the accuracy of the approximations, lower step gives better approximation but costs more time in computations.