Silesian University of Technology
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Numerical Methods
Laboratory Exercise

Ex.8 Determining eigenvalues and eigenvectors of matrix

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1. Calculate eigenvalues and eigenvectors for a given square matrix A, using Krylov method:

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Taking as an initial vector $y^{(0)} = \left[1,0,0\right]^T$

To calculate components $y_j^{(i)}$ of vectors $y^{(i)}$ for i, j =1, 2, …, n the following relation is applied

$$y^{(n)} = Ay^{(n-1)}$$

$$y^{(1)} = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$y^{(2)} = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 6 \end{bmatrix}$$

$$y^{(3)} = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ 6 \\ 6 \end{bmatrix} = \begin{bmatrix} 39 \\ 36 \\ 27 \end{bmatrix}$$

Using a system of linear equations for determination of characteristic polynomial

$$\begin{bmatrix} y_1^{(n-1)} & y_1^{(n-2)} & \cdots & y_1^{(0)} \\ y_2^{(n-1)} & y_2^{(n-2)} & \cdots & y_2^{(0)} \\ \vdots & \vdots & \ddots & \vdots \\ y_k^{(n-1)} & y_k^{(n-2)} & \cdots & y_k^{(0)} \\ \vdots & \vdots & \ddots & \vdots \\ y_n^{(n-1)} & y_n^{(n-2)} & \cdots & y_n^{(0)} \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix} = - \begin{bmatrix} y_1^{(n)} \\ y_2^{(n)} \\ \vdots \\ y_n^{(n)} \end{bmatrix}$$

Coefficients takes the form:

$$\begin{aligned} 9p_1 + p_2 + p_3 &= -39 \\ 6p_1 + 2p_2 &= -36 \\ 6p_1 + p_2 &= -27 \end{aligned} \quad \Rightarrow \quad \begin{aligned} p_1 &= -3 \\ p_2 &= -9 \\ p_3 &= -3 \end{aligned}$$

Characteristic equation of matrix A is of the form:

$$\lambda^3 - 3\lambda^2 - 9\lambda - 3 = 0$$

Characteristic equation roots of matrix A are follows:

$\lambda_1 = -1.5525$ $\lambda_2 = -0.39091$ $\lambda_3 = 4.9434$

Setting g_3=1 in system for eigenvalues, coefficients g_1 and g_2 can be calculated from equations:

$$g_2 = \lambda_{1,2,3} + p_1$$
$$g_1 = \lambda_{1,2,3} g_2 + p_2$$

For λ_1:

$g_2 = -4.5525$
$g_1 = -1.93224$

For λ_2

$g_2 = -3.39091$
$g_1 = -7.67446$

For λ_3

$g_2 = 1.9431$
$g_1 = 0.604938$

To calculate eigenvector we can x^i expressed as a linear combination of vectors $y^0, y^1, \ldots, y^{n-1}$

$$x^i = g_1 y^0 + g_2 y^1 + g_3 y^2 + \ldots + g_n y^{n-1}$$

$$x^1 = -1.93224 \cdot \begin{bmatrix}1\\0\\0\end{bmatrix} - 4.5525 \cdot \begin{bmatrix}1\\2\\1\end{bmatrix} + 1 \cdot \begin{bmatrix}9\\6\\6\end{bmatrix} = \begin{bmatrix}2.51526\\-3.105\\1.4475\end{bmatrix}$$

$$x^2 = -7.67446 \cdot \begin{bmatrix}1\\0\\0\end{bmatrix} - 3.39091 \cdot \begin{bmatrix}1\\2\\1\end{bmatrix} + 1 \cdot \begin{bmatrix}9\\6\\6\end{bmatrix} = \begin{bmatrix}-2.06537\\-0.78182\\2.60909\end{bmatrix}$$

$$x^3 = 0.604938 \cdot \begin{bmatrix}1\\0\\0\end{bmatrix} + 1.9431 \cdot \begin{bmatrix}1\\2\\1\end{bmatrix} + 1 \cdot \begin{bmatrix}9\\6\\6\end{bmatrix} = \begin{bmatrix}11.548038\\9.8862\\7.9431\end{bmatrix}$$

Eigen vectors of matrix A:

$$x^1 = \begin{bmatrix}2.51526\\-3.105\\1.4475\end{bmatrix} \qquad x^2 = \begin{bmatrix}-2.06537\\-0.78182\\2.60909\end{bmatrix} \qquad x^3 = \begin{bmatrix}11.548038\\9.8862\\7.9431\end{bmatrix}$$

To compare our result we use Wolfram, after normalisation we se that our result are quite accurate

Wolfram:	Krylov method:
Input: eigenvalues $\begin{pmatrix} 1 & 3 & 2 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$	
Results:	
$\lambda_1 \approx 4.94338$	$\lambda_1 = 4.9434$
$\lambda_2 \approx -1.55247$	$\lambda_2 = -1.5525$
$\lambda_3 \approx -0.390906$	$\lambda_3 = -0.39091$
Corresponding eigenvectors:	
$v_1 \approx (1.45407, 1.24465, 1)$	$x^1 = [1,45384522, 1.24462741, 1]$
$v_2 \approx (1.73754, -2.14501, 1)$	$x^2 = [1.73766, -2.14508, 1]$
$v_3 \approx (-0.791608, -0.299649, 1)$	$x^3 = [-0.7916055, -0.29965237, 1]$

2. Using Power method determine the highest modulus of eigen value of matrix A

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Source code:

```
#include <iostream>
using namespace std;
int main() {
    const int n=3;      //Size of matrix
    const int k=8;      //amount of iterratons

    double A[n][n]={{1, 3, 2},{2, 1, 2},{1, 2, 1}};     //Matrix
in which we determine highest eigenvalue
    double y[n][k];
    double h[n];
    double result=0;

    y[0][0]=1;
    for(int i=1 ; i<n ; i++){
        y[i][0]=0;
    }

    for(int i=1 ; i<=k ; i++){
        for(int j=0 ; j<n ; j++){
            y[j][i]=0;

            for(int z=0 ; z<n; z++){
                y[j][i] += A[j][z] * y[z][i-1];
            }
        }
    }

    for(int i=0 ; i<n ; i++){
        h[i]=y[i][k]/y[i][k-1];
        result += h[i];
    }
    result /= n;
    cout<<"The highest-modulus eigenvalue: "<<result<<endl;
}
```

The program screen value 4.94364, which is the highest modulus eigenvalue of matrix A, for eight iteration. As we can see the value is quite accurate of Wolfram’s result.