Google开源框架or-tools解决路径规划问题

问题列表：

- 旅行商TSP问题
- 路径规划VRP问题
- 容量限制CVRP问题
- 取货送货pickups and deliveries问题
- VRP时间窗约束问题（time window）

TSP问题

TSP问题是旅行商问题的简写，问题非常简单：从原点出发经过所有需求点并回到原点，使得途经的距离最短。

```
"""Simple travelling salesman problem between cities."""

from __future__ import print_function
from ortools.constraint_solver import routing_enums_pb2
from ortools.constraint_solver import pywrapcp



def create_data_model():
    """Stores the data for the problem."""
    data = {}
    #节点之间的距离矩阵，对称矩阵
    data['distance_matrix'] = [
        [0, 2451, 713, 1018, 1631, 1374, 2408, 213, 2571, 875, 1420, 2145, 1972],
        [2451, 0, 1745, 1524, 831, 1240, 959, 2596, 403, 1589, 1374, 357, 579],
        [713, 1745, 0, 355, 920, 803, 1737, 851, 1858, 262, 940, 1453, 1260],
        [1018, 1524, 355, 0, 700, 862, 1395, 1123, 1584, 466, 1056, 1280, 987],
        [1631, 831, 920, 700, 0, 663, 1021, 1769, 949, 796, 879, 586, 371],
        [1374, 1240, 803, 862, 663, 0, 1681, 1551, 1765, 547, 225, 887, 999],
        [2408, 959, 1737, 1395, 1021, 1681, 0, 2493, 678, 1724, 1891, 1114, 701],
        [213, 2596, 851, 1123, 1769, 1551, 2493, 0, 2699, 1038, 1605, 2300, 2099],
        [2571, 403, 1858, 1584, 949, 1765, 678, 2699, 0, 1744, 1645, 653, 600],
        [875, 1589, 262, 466, 796, 547, 1724, 1038, 1744, 0, 679, 1272, 1162],
        [1420, 1374, 940, 1056, 879, 225, 1891, 1605, 1645, 679, 0, 1017, 1200],
        [2145, 357, 1453, 1280, 586, 887, 1114, 2300, 653, 1272, 1017, 0, 504],
        [1972, 579, 1260, 987, 371, 999, 701, 2099, 600, 1162, 1200, 504, 0],
```

```
    ]  # yapf: disable
    #货车数量
    data['num_vehicles'] = 1
    data['depot'] = 0
    return data

#打印结果
def print_solution(manager, routing, solution):
    """Prints solution on console."""
    print('Objective: {} miles'.format(solution.ObjectiveValue()))
    index = routing.Start(0)
    plan_output = 'Route for vehicle 0:\n'
    route_distance = 0
    while not routing.IsEnd(index):
        plan_output += ' {} ->'.format(manager.IndexToNode(index))
        previous_index = index
        index = solution.Value(routing.NextVar(index))
        route_distance += routing.GetArcCostForVehicle(previous_index,
index, 0)
    plan_output += ' {}\n'.format(manager.IndexToNode(index))
    print(plan_output)
    plan_output += 'Route distance: {}miles\n'.format(route_distance)


def main():
    """Entry point of the program."""
    # Instantiate the data problem.
    data = create_data_model()

    # Create the routing index manager.
    manager = pywrapcp.RoutingIndexManager(len(data['distance_matrix']),
                                           data['num_vehicles'],
data['depot'])

    # Create Routing Model.
    routing = pywrapcp.RoutingModel(manager)

    #距离回调函数，将两节点序号转化为两节点距离
    def distance_callback(from_index, to_index):
        """Returns the distance between the two nodes."""
        # Convert from routing variable Index to distance matrix NodeIndex.
        from_node = manager.IndexToNode(from_index)
        to_node = manager.IndexToNode(to_index)
        return data['distance_matrix'][from_node][to_node]

    transit_callback_index =
routing.RegisterTransitCallback(distance_callback)

    #定义每条弧的权重
    # Define cost of each arc.
    routing.SetArcCostEvaluatorOfAllVehicles(transit_callback_index)

    # Setting first solution heuristic.
    search_parameters = pywrapcp.DefaultRoutingSearchParameters()
    search_parameters.first_solution_strategy = (
        routing_enums_pb2.FirstSolutionStrategy.PATH_CHEAPEST_ARC)

    # Solve the problem.
```

```
    solution = routing.SolveWithParameters(search_parameters)

    # Print solution on console.
    if solution:
        print_solution(manager, routing, solution)


if __name__ == '__main__':
    main()
```

VRP问题

VRP问题是车辆路径问题的缩写。问题是：有N辆车，都从原点出发，每辆车访问一些点后回到原点，要求所有的点都要被访问到，求最短的车辆行驶距离或最少需要的车辆数或最小化最长行驶距离。常见的限制要求包括：车辆容量限制、时间窗限制、点访问顺序要求等。

```
"""Vehicles Routing Problem (VRP)."""

from __future__ import print_function
from ortools.constraint_solver import routing_enums_pb2
from ortools.constraint_solver import pywrapcp


def create_data_model():
    """Stores the data for the problem."""
    data = {}
    #对称矩阵
    data['distance_matrix'] = [
        [
            0, 548, 776, 696, 582, 274, 502, 194, 308, 194, 536, 502, 388,
354,
            468, 776, 662
        ],
        [
            548, 0, 684, 308, 194, 502, 730, 354, 696, 742, 1084, 594,
480, 674,
            1016, 868, 1210
        ],
        [
            776, 684, 0, 992, 878, 502, 274, 810, 468, 742, 400, 1278,
1164,
            1130, 788, 1552, 754
        ],
        [
            696, 308, 992, 0, 114, 650, 878, 502, 844, 890, 1232, 514,
628, 822,
            1164, 560, 1358
        ],
        [
            582, 194, 878, 114, 0, 536, 764, 388, 730, 776, 1118, 400,
514, 708,
            1050, 674, 1244
        ],
        [
            274, 502, 502, 650, 536, 0, 228, 308, 194, 240, 582, 776, 662,
628,
```

```
            514, 1050, 708
        ],
        [
            502, 730, 274, 878, 764, 228, 0, 536, 194, 468, 354, 1004,
890, 856,
            514, 1278, 480
        ],
        [
            194, 354, 810, 502, 388, 308, 536, 0, 342, 388, 730, 468, 354,
320,
            662, 742, 856
        ],
        [
            308, 696, 468, 844, 730, 194, 194, 342, 0, 274, 388, 810, 696,
662,
            320, 1084, 514
        ],
        [
            194, 742, 742, 890, 776, 240, 468, 388, 274, 0, 342, 536, 422,
388,
            274, 810, 468
        ],
        [
            536, 1084, 400, 1232, 1118, 582, 354, 730, 388, 342, 0, 878,
764,
            730, 388, 1152, 354
        ],
        [
            502, 594, 1278, 514, 400, 776, 1004, 468, 810, 536, 878, 0,
114,
            308, 650, 274, 844
        ],
        [
            388, 480, 1164, 628, 514, 662, 890, 354, 696, 422, 764, 114,
0, 194,
            536, 388, 730
        ],
        [
            354, 674, 1130, 822, 708, 628, 856, 320, 662, 388, 730, 308,
194, 0,
            342, 422, 536
        ],
        [
            468, 1016, 788, 1164, 1050, 514, 514, 662, 320, 274, 388, 650,
536,
            342, 0, 764, 194
        ],
        [
            776, 868, 1552, 560, 674, 1050, 1278, 742, 1084, 810, 1152,
274,
            388, 422, 764, 0, 798
        ],
        [
            662, 1210, 754, 1358, 1244, 708, 480, 856, 514, 468, 354, 844,
730,
            536, 194, 798, 0
        ],
    ]
```

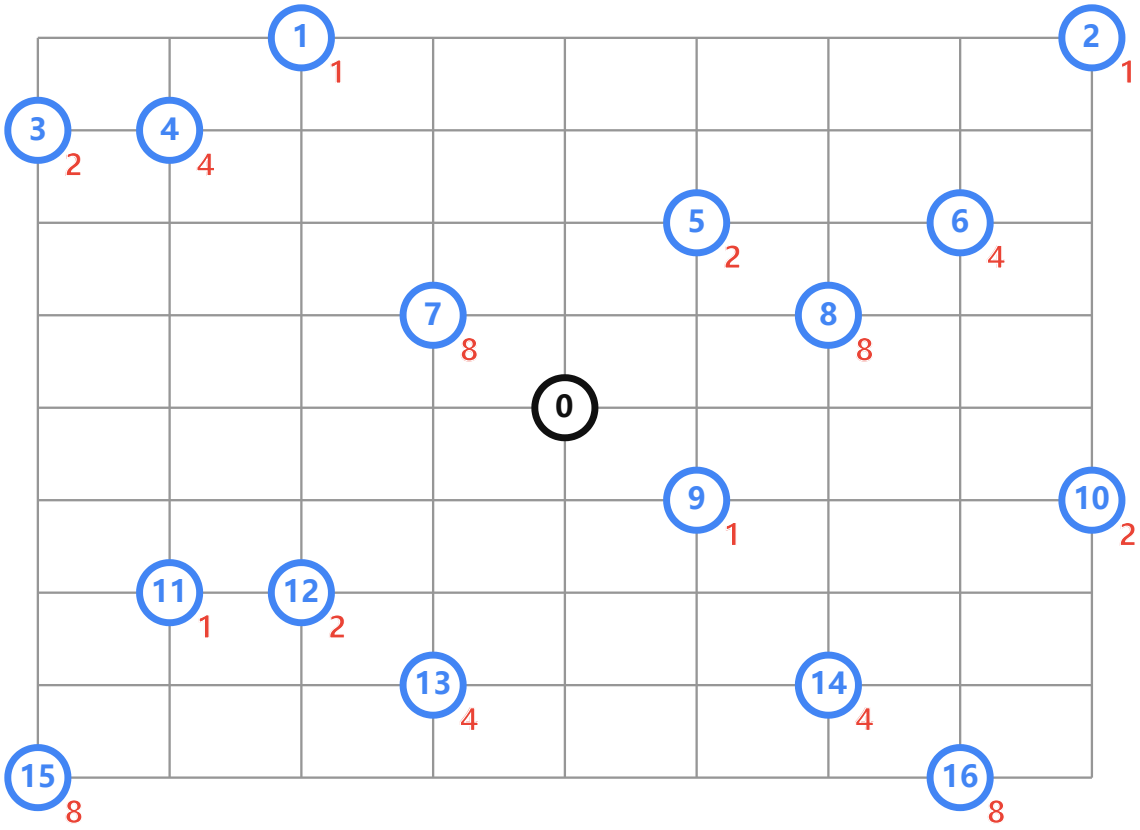
```
    #货车数量
    data['num_vehicles'] = 4
    data['depot'] = 0
    return data


def print_solution(data, manager, routing, solution):
    """Prints solution on console."""
    max_route_distance = 0
    for vehicle_id in range(data['num_vehicles']):
        index = routing.Start(vehicle_id)
        plan_output = 'Route for vehicle {}:\n'.format(vehicle_id)
        route_distance = 0
        while not routing.IsEnd(index):
            plan_output += ' {} -> '.format(manager.IndexToNode(index))
            previous_index = index
            index = solution.Value(routing.NextVar(index))
            route_distance += routing.GetArcCostForVehicle(
                previous_index, index, vehicle_id)
        plan_output += '{}\n'.format(manager.IndexToNode(index))
        plan_output += 'Distance of the route:
{}m\n'.format(route_distance)
        print(plan_output)
        max_route_distance = max(route_distance, max_route_distance)
    print('Maximum of the route distances:
{}m'.format(max_route_distance))



def main():
    """Solve the CVRP problem."""
    # Instantiate the data problem.
    data = create_data_model()

    # Create the routing index manager.
    manager = pywrapcp.RoutingIndexManager(len(data['distance_matrix']),
                                           data['num_vehicles'],
data['depot'])

    # Create Routing Model.
    routing = pywrapcp.RoutingModel(manager)


    # Create and register a transit callback.
    def distance_callback(from_index, to_index):
        """Returns the distance between the two nodes."""
        # Convert from routing variable Index to distance matrix
NodeIndex.
        from_node = manager.IndexToNode(from_index)
        to_node = manager.IndexToNode(to_index)
        return data['distance_matrix'][from_node][to_node]

    transit_callback_index =
routing.RegisterTransitCallback(distance_callback)

    # Define cost of each arc.
    routing.SetArcCostEvaluatorOfAllVehicles(transit_callback_index)
```

```
135
136     # Add Distance constraint.
137     dimension_name = 'Distance'
138     routing.AddDimension(
139         transit_callback_index,
140         0,  # no slack
141         #货车最大送货距离
142         3000,  # vehicle maximum travel distance
143         True,  # start cumul to zero
144         dimension_name)
145     distance_dimension = routing.GetDimensionOrDie(dimension_name)
146     distance_dimension.SetGlobalSpanCostCoefficient(100)
147
148     # Setting first solution heuristic.
149     search_parameters = pywrapcp.DefaultRoutingSearchParameters()
150     search_parameters.first_solution_strategy = (
151         routing_enums_pb2.FirstSolutionStrategy.PATH_CHEAPEST_ARC)
152
153     # Solve the problem.
154     solution = routing.SolveWithParameters(search_parameters)
155
156     # Print solution on console.
157     if solution:
158         print_solution(data, manager, routing, solution)
159
160
161 if __name__ == '__main__':
162     main()
```

CVRP问题

CVRP指的是有容量（capacity）限制的VRP模型，是最常见的VRP模型。

每个点多了一个送货需求。每辆车的最大容量是15，最小化总运输距离。
dimension可以使用AddDimensionWithVehicleCapacity方法，和AddDimension唯一的区别就是，第三个参数从一个固定值变成了一个列表，表示每一辆车有自己单独的最大容量限制。


1
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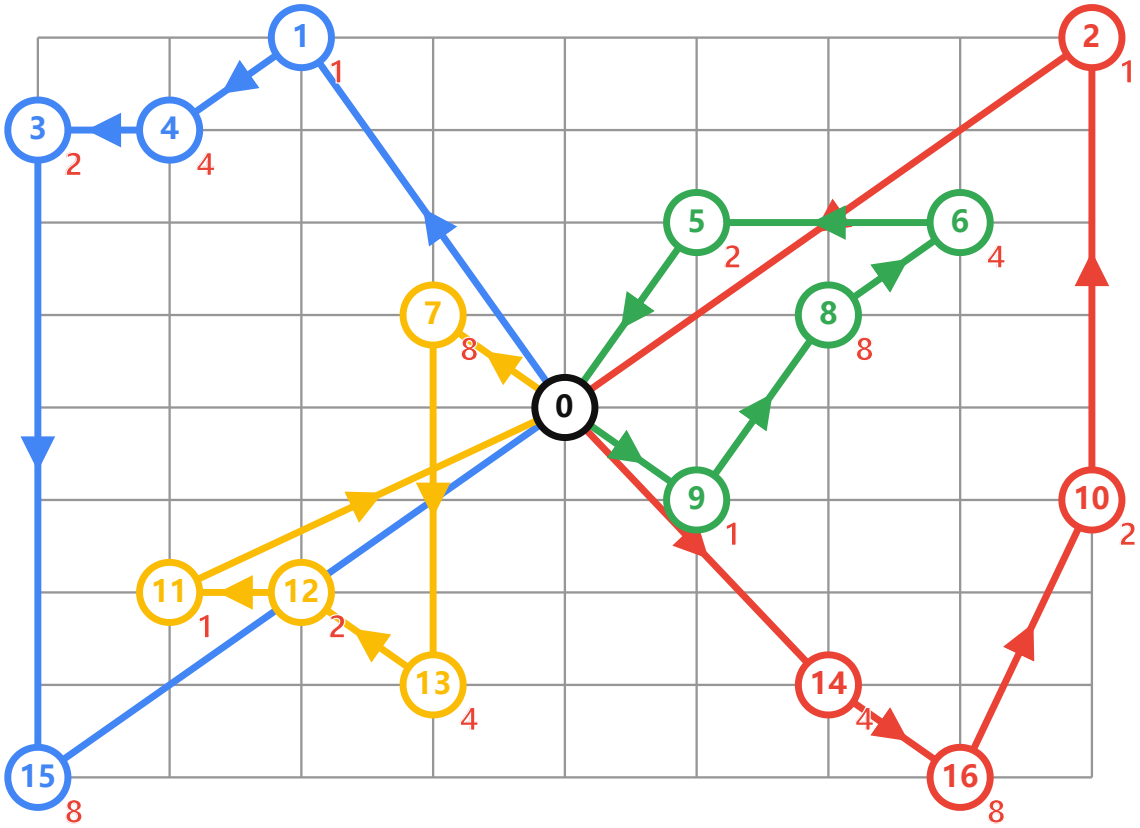
1
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1
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2
13
4
14
4
15
8
16
8

```
Route for vehicle 0:
 0 Load(0) ->  1 Load(1) ->  4 Load(5) ->  3 Load(7) ->  15 Load(15) ->  0 Load(15)
Distance of the route: 2192m
Load of the route: 15

Route for vehicle 1:
 0 Load(0) ->  14 Load(4) ->  16 Load(12) ->  10 Load(14) ->  2 Load(15) ->  0 Load(15)
Distance of the route: 2192m
Load of the route: 15

Route for vehicle 2:
 0 Load(0) ->  7 Load(8) ->  13 Load(12) ->  12 Load(14) ->  11 Load(15) ->  0 Load(15)
Distance of the route: 1324m
Load of the route: 15

Route for vehicle 3:
 0 Load(0) ->  9 Load(1) ->  8 Load(9) ->  6 Load(13) ->  5 Load(15) ->  0 Load(15)
Distance of the route: 1164m
Load of the route: 15

Total Distance of all routes: 6872m
```

```python
"""Capacited Vehicles Routing Problem (CVRP)."""

from __future__ import print_function
from ortools.constraint_solver import routing_enums_pb2
from ortools.constraint_solver import pywrapcp


def create_data_model():
    """Stores the data for the problem."""
    data = {}
    #对称矩阵
    data['distance_matrix'] = [
        [
            0, 548, 776, 696, 582, 274, 502, 194, 308, 194, 536, 502, 388, 354,
            468, 776, 662
        ],
        [
            548, 0, 684, 308, 194, 502, 730, 354, 696, 742, 1084, 594, 480, 674,
            1016, 868, 1210
        ],
        [
            776, 684, 0, 992, 878, 502, 274, 810, 468, 742, 400, 1278, 1164,
            1130, 788, 1552, 754
        ],
        [
            696, 308, 992, 0, 114, 650, 878, 502, 844, 890, 1232, 514, 628, 822,
            1164, 560, 1358
        ],
        [
            582, 194, 878, 114, 0, 536, 764, 388, 730, 776, 1118, 400, 514, 708,
            1050, 674, 1244
        ],
```

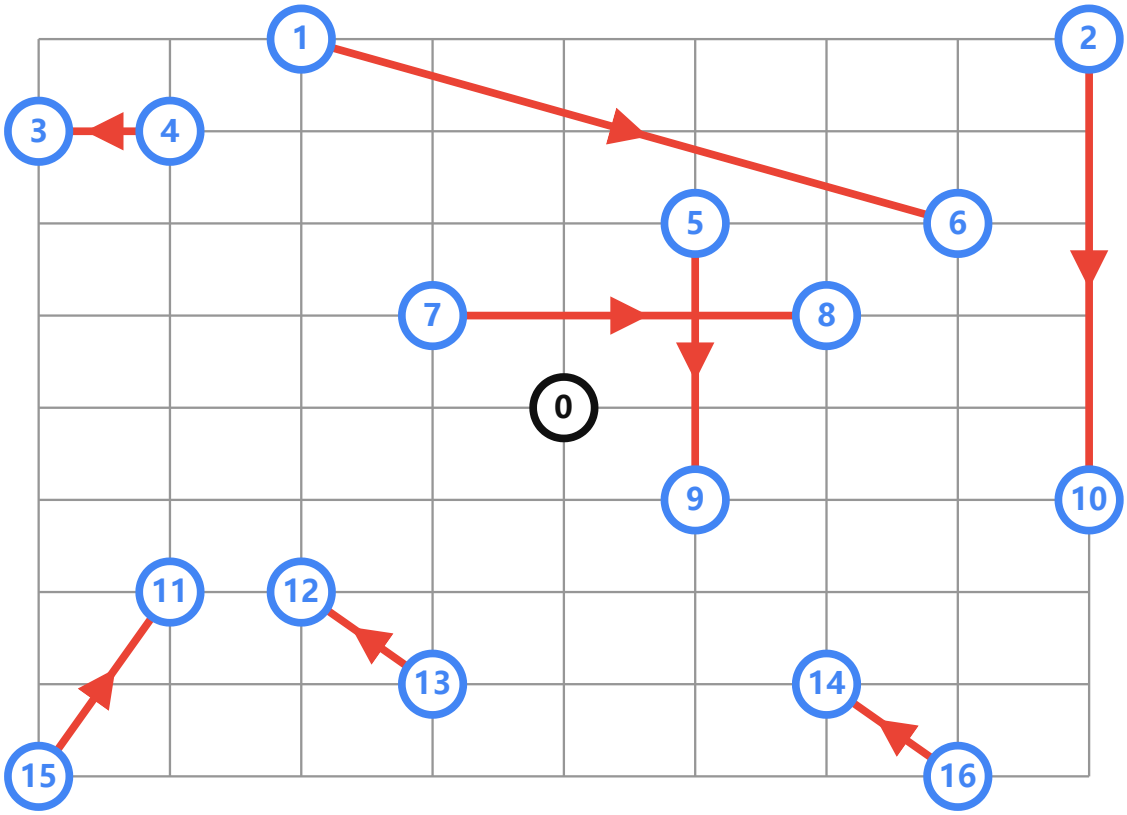
```
        [
            274, 502, 502, 650, 536, 0, 228, 308, 194, 240, 582, 776, 662,
628,
            514, 1050, 708
        ],
        [
            502, 730, 274, 878, 764, 228, 0, 536, 194, 468, 354, 1004,
890, 856,
            514, 1278, 480
        ],
        [
            194, 354, 810, 502, 388, 308, 536, 0, 342, 388, 730, 468, 354,
320,
            662, 742, 856
        ],
        [
            308, 696, 468, 844, 730, 194, 194, 342, 0, 274, 388, 810, 696,
662,
            320, 1084, 514
        ],
        [
            194, 742, 742, 890, 776, 240, 468, 388, 274, 0, 342, 536, 422,
388,
            274, 810, 468
        ],
        [
            536, 1084, 400, 1232, 1118, 582, 354, 730, 388, 342, 0, 878,
764,
            730, 388, 1152, 354
        ],
        [
            502, 594, 1278, 514, 400, 776, 1004, 468, 810, 536, 878, 0,
114,
            308, 650, 274, 844
        ],
        [
            388, 480, 1164, 628, 514, 662, 890, 354, 696, 422, 764, 114,
0, 194,
            536, 388, 730
        ],
        [
            354, 674, 1130, 822, 708, 628, 856, 320, 662, 388, 730, 308,
194, 0,
            342, 422, 536
        ],
        [
            468, 1016, 788, 1164, 1050, 514, 514, 662, 320, 274, 388, 650,
536,
            342, 0, 764, 194
        ],
        [
            776, 868, 1552, 560, 674, 1050, 1278, 742, 1084, 810, 1152,
274,
            388, 422, 764, 0, 798
        ],
        [
            662, 1210, 754, 1358, 1244, 708, 480, 856, 514, 468, 354, 844,
730,
```

```
            536, 194, 798, 0
        ],
    ]
    #每个客户的需求
    data['demands'] = [0, 1, 1, 2, 4, 2, 4, 8, 8, 1, 2, 1, 2, 4, 4, 8, 8]
    #每个货车的货物装载容量
    data['vehicle_capacities'] = [15, 15, 15, 15]
    data['num_vehicles'] = 4
    data['depot'] = 0
    return data


def print_solution(data, manager, routing, solution):
    """Prints solution on console."""
    total_distance = 0
    total_load = 0
    for vehicle_id in range(data['num_vehicles']):
        index = routing.Start(vehicle_id)
        plan_output = 'Route for vehicle {}:\n'.format(vehicle_id)
        route_distance = 0
        route_load = 0
        while not routing.IsEnd(index):
            node_index = manager.IndexToNode(index)
            route_load += data['demands'][node_index]
            plan_output += ' {0} Load({1}) -> '.format(node_index,
route_load)
            previous_index = index
            index = solution.Value(routing.NextVar(index))
            route_distance += routing.GetArcCostForVehicle(
                previous_index, index, vehicle_id)
        plan_output += ' {0}
Load({1})\n'.format(manager.IndexToNode(index),
                                                 route_load)
        plan_output += 'Distance of the route:
{}m\n'.format(route_distance)
        plan_output += 'Load of the route: {}\n'.format(route_load)
        print(plan_output)
        total_distance += route_distance
        total_load += route_load
    print('Total distance of all routes: {}m'.format(total_distance))
    print('Total load of all routes: {}'.format(total_load))


def main():
    """Solve the CVRP problem."""
    # Instantiate the data problem.
    data = create_data_model()

    # Create the routing index manager.
    manager = pywrapcp.RoutingIndexManager(len(data['distance_matrix']),
                                           data['num_vehicles'],
data['depot'])

    # Create Routing Model.
    routing = pywrapcp.RoutingModel(manager)


    # Create and register a transit callback.
```

```
    def distance_callback(from_index, to_index):
        """Returns the distance between the two nodes."""
        # Convert from routing variable Index to distance matrix
NodeIndex.
        from_node = manager.IndexToNode(from_index)
        to_node = manager.IndexToNode(to_index)
        return data['distance_matrix'][from_node][to_node]

    transit_callback_index =
routing.RegisterTransitCallback(distance_callback)

    # Define cost of each arc.
    routing.SetArcCostEvaluatorOfAllVehicles(transit_callback_index)


    # Add Capacity constraint.
    def demand_callback(from_index):
        """Returns the demand of the node."""
        # Convert from routing variable Index to demands NodeIndex.
        from_node = manager.IndexToNode(from_index)
        return data['demands'][from_node]

    demand_callback_index = routing.RegisterUnaryTransitCallback(
        demand_callback)
    routing.AddDimensionWithVehicleCapacity(
        demand_callback_index,
        0,  # null capacity slack
        data['vehicle_capacities'],  # vehicle maximum capacities
        True,  # start cumul to zero
        'Capacity')

    # Setting first solution heuristic.
    search_parameters = pywrapcp.DefaultRoutingSearchParameters()
    search_parameters.first_solution_strategy = (
        routing_enums_pb2.FirstSolutionStrategy.PATH_CHEAPEST_ARC)

    # Solve the problem.
    solution = routing.SolveWithParameters(search_parameters)

    # Print solution on console.
    if solution:
        print_solution(data, manager, routing, solution)


if __name__ == '__main__':
    main()
```

pickups and deliveries问题

每一个需要运送的货物都有自己对应的pickup起点和delivery终点。我们需要从depot出发，依次将货物从起点送到终点。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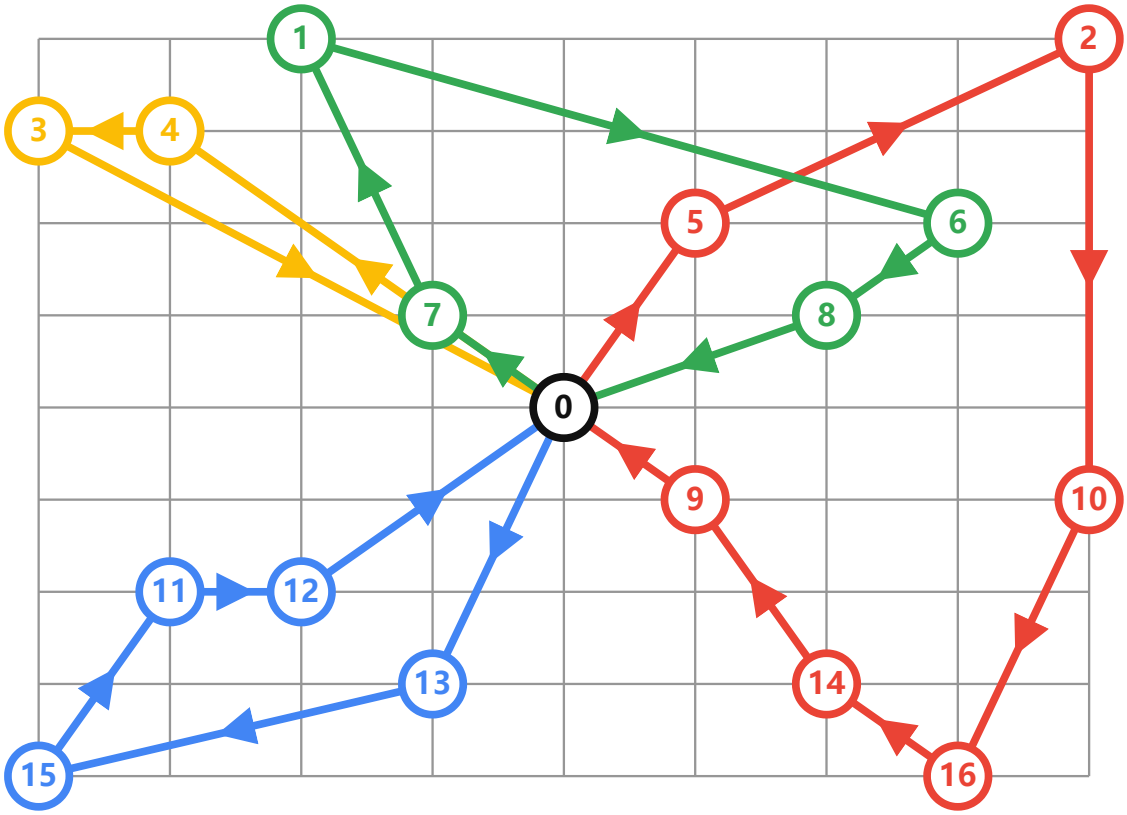
1
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4
5
6
7
8
0
9
10
11
12
13
14
15
16

```
Objective: 226116
Route for vehicle 0:
 0 ->  13 ->  15 ->  11 ->  12 -> 0
Distance of the route: 1552m



Route for vehicle 1:
 0 ->  5 ->  2 ->  10 ->  16 ->  14 ->  9 -> 0
Distance of the route: 2192m



Route for vehicle 2:
 0 ->  4 ->  3 -> 0
Distance of the route: 1392m



Route for vehicle 3:
 0 ->  7 ->  1 ->  6 ->  8 -> 0
Distance of the route: 1780m



Total Distance of all routes: 6916m
```

```python
"""Simple Pickup Delivery Problem (PDP)."""

from __future__ import print_function
from ortools.constraint_solver import routing_enums_pb2
from ortools.constraint_solver import pywrapcp


def create_data_model():
    """Stores the data for the problem."""
    data = {}
    data['distance_matrix'] = [
        [
            0, 548, 776, 696, 582, 274, 502, 194, 308, 194, 536, 502, 388, 354,
            468, 776, 662
        ],
        [
            548, 0, 684, 308, 194, 502, 730, 354, 696, 742, 1084, 594, 480, 674,
            1016, 868, 1210
        ],
        [
            776, 684, 0, 992, 878, 502, 274, 810, 468, 742, 400, 1278, 1164,
            1130, 788, 1552, 754
        ],
        [
            696, 308, 992, 0, 114, 650, 878, 502, 844, 890, 1232, 514, 628, 822,
            1164, 560, 1358
        ],
        [
```

```
            582, 194, 878, 114, 0, 536, 764, 388, 730, 776, 1118, 400,
514, 708,
            1050, 674, 1244
        ],
        [
            274, 502, 502, 650, 536, 0, 228, 308, 194, 240, 582, 776, 662,
628,
            514, 1050, 708
        ],
        [
            502, 730, 274, 878, 764, 228, 0, 536, 194, 468, 354, 1004,
890, 856,
            514, 1278, 480
        ],
        [
            194, 354, 810, 502, 388, 308, 536, 0, 342, 388, 730, 468, 354,
320,
            662, 742, 856
        ],
        [
            308, 696, 468, 844, 730, 194, 194, 342, 0, 274, 388, 810, 696,
662,
            320, 1084, 514
        ],
        [
            194, 742, 742, 890, 776, 240, 468, 388, 274, 0, 342, 536, 422,
388,
            274, 810, 468
        ],
        [
            536, 1084, 400, 1232, 1118, 582, 354, 730, 388, 342, 0, 878,
764,
            730, 388, 1152, 354
        ],
        [
            502, 594, 1278, 514, 400, 776, 1004, 468, 810, 536, 878, 0,
114,
            308, 650, 274, 844
        ],
        [
            388, 480, 1164, 628, 514, 662, 890, 354, 696, 422, 764, 114,
0, 194,
            536, 388, 730
        ],
        [
            354, 674, 1130, 822, 708, 628, 856, 320, 662, 388, 730, 308,
194, 0,
            342, 422, 536
        ],
        [
            468, 1016, 788, 1164, 1050, 514, 514, 662, 320, 274, 388, 650,
536,
            342, 0, 764, 194
        ],
        [
            776, 868, 1552, 560, 674, 1050, 1278, 742, 1084, 810, 1152,
274,
            388, 422, 764, 0, 798
```

```
        ],
        [
            662, 1210, 754, 1358, 1244, 708, 480, 856, 514, 468, 354, 844,
730,
            536, 194, 798, 0
        ],
    ]
    #需要取货送货的节点对
    data['pickups_deliveries'] = [
        [1, 6],
        [2, 10],
        [4, 3],
        [5, 9],
        [7, 8],
        [15, 11],
        [13, 12],
        [16, 14],
    ]
    data['num_vehicles'] = 4
    data['depot'] = 0
    return data


def print_solution(data, manager, routing, solution):
    """Prints solution on console."""
    total_distance = 0
    for vehicle_id in range(data['num_vehicles']):
        index = routing.Start(vehicle_id)
        plan_output = 'Route for vehicle {}:\n'.format(vehicle_id)
        route_distance = 0
        while not routing.IsEnd(index):
            plan_output += ' {} -> '.format(manager.IndexToNode(index))
            previous_index = index
            index = solution.Value(routing.NextVar(index))
            route_distance += routing.GetArcCostForVehicle(
                previous_index, index, vehicle_id)
        plan_output += '{}\n'.format(manager.IndexToNode(index))
        plan_output += 'Distance of the route:
{}m\n'.format(route_distance)
        print(plan_output)
        total_distance += route_distance
    print('Total Distance of all routes: {}m'.format(total_distance))


def main():
    """Entry point of the program."""
    # Instantiate the data problem.
    data = create_data_model()

    # Create the routing index manager.
    manager = pywrapcp.RoutingIndexManager(len(data['distance_matrix']),
                                           data['num_vehicles'],
data['depot'])

    # Create Routing Model.
    routing = pywrapcp.RoutingModel(manager)


```

```
    # Define cost of each arc.
    def distance_callback(from_index, to_index):
        """Returns the manhattan distance between the two nodes."""
        # Convert from routing variable Index to distance matrix
NodeIndex.
        from_node = manager.IndexToNode(from_index)
        to_node = manager.IndexToNode(to_index)
        return data['distance_matrix'][from_node][to_node]

    transit_callback_index =
routing.RegisterTransitCallback(distance_callback)
    routing.SetArcCostEvaluatorOfAllVehicles(transit_callback_index)

    # Add Distance constraint.
    dimension_name = 'Distance'
    routing.AddDimension(
        transit_callback_index,
        0,  # no slack
        3000,  # vehicle maximum travel distance
        True,  # start cumul to zero
        dimension_name)
    distance_dimension = routing.GetDimensionOrDie(dimension_name)
    distance_dimension.SetGlobalSpanCostCoefficient(100)

    # Define Transportation Requests.
    for request in data['pickups_deliveries']:
        pickup_index = manager.NodeToIndex(request[0])
        delivery_index = manager.NodeToIndex(request[1])
        routing.AddPickupAndDelivery(pickup_index, delivery_index)
        #单个货车实现取货送货
        routing.solver().Add(
            routing.VehicleVar(pickup_index) == routing.VehicleVar(
                delivery_index))
        #取货必须先于送货
        routing.solver().Add(
            distance_dimension.CumulVar(pickup_index) <=
            distance_dimension.CumulVar(delivery_index))

    # Setting first solution heuristic.
    search_parameters = pywrapcp.DefaultRoutingSearchParameters()
    search_parameters.first_solution_strategy = (

 routing_enums_pb2.FirstSolutionStrategy.PARALLEL_CHEAPEST_INSERTION)

    # Solve the problem.
    solution = routing.SolveWithParameters(search_parameters)

    # Print solution on console.
    if solution:
        print_solution(data, manager, routing, solution)


if __name__ == '__main__':
    main()
```

VRPTW（VRP with Time Windows）问题

当VRP问题有到达时间的约束条件时，问题变为VRPTW（VRP with Time Windows）

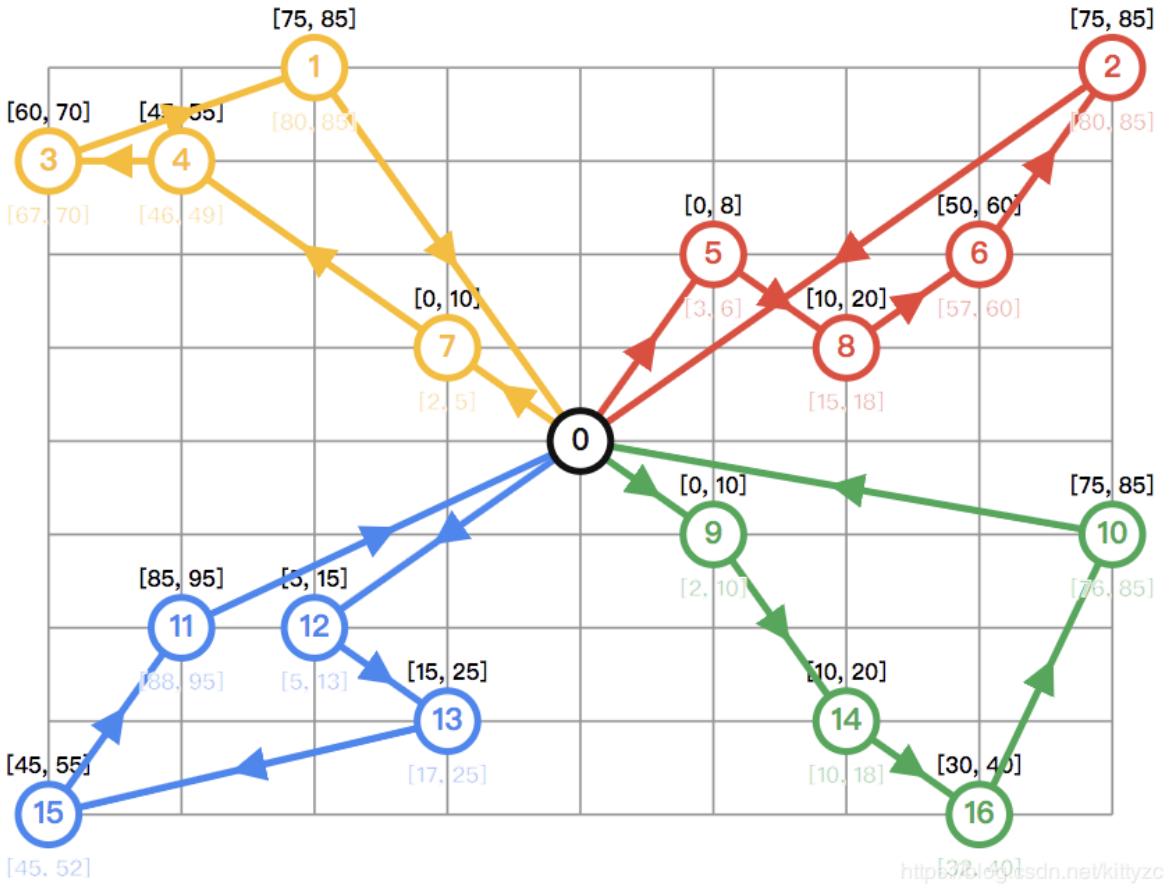
```
Route for vehicle 0:
 0 Time(0,0) ->  9 Time(2,3) ->  14 Time(7,8) -> 16 Time(11,11) ->  0 Time(18,18)
Time of the route: 18min

Route for vehicle 1:
 0 Time(0,0) -> 7 Time(2,4) -> 1 Time(7,11) -> 4 Time(10,13) -> 3 Time(16,16) -> 0 Time(24,
Time of the route: 24min

Route for vehicle 2:
 0 Time(0,0) -> 12 Time(4,4) -> 13 Time(6,6) -> 15 Time(11,11) -> 11 Time(14,14) -> 0 Time(
Time of the route: 20min

Route for vehicle 3:
 0 Time(0,0) -> 5 Time(3,3) -> 8 Time(5,5) -> 6 Time(7,7) -> 2 Time(10,10) -> 10 Time(14,14
 0 Time(20,20)
Time of the route: 20min

Total time of all routes: 82min
```




```python
"""Vehicles Routing Problem (VRP) with Time Windows."""

from __future__ import print_function
from ortools.constraint_solver import routing_enums_pb2
from ortools.constraint_solver import pywrapcp


def create_data_model():
    """Stores the data for the problem."""
    data = {}
```

```
    #时间矩阵
    data['time_matrix'] = [
        [0, 6, 9, 8, 7, 3, 6, 2, 3, 2, 6, 6, 4, 4, 5, 9, 7],
        [6, 0, 8, 3, 2, 6, 8, 4, 8, 8, 13, 7, 5, 8, 12, 10, 14],
        [9, 8, 0, 11, 10, 6, 3, 9, 5, 8, 4, 15, 14, 13, 9, 18, 9],
        [8, 3, 11, 0, 1, 7, 10, 6, 10, 10, 14, 6, 7, 9, 14, 6, 16],
        [7, 2, 10, 1, 0, 6, 9, 4, 8, 9, 13, 4, 6, 8, 12, 8, 14],
        [3, 6, 6, 7, 6, 0, 2, 3, 2, 2, 7, 9, 7, 7, 6, 12, 8],
        [6, 8, 3, 10, 9, 2, 0, 6, 2, 5, 4, 12, 10, 10, 6, 15, 5],
        [2, 4, 9, 6, 4, 3, 6, 0, 4, 4, 8, 5, 4, 3, 7, 8, 10],
        [3, 8, 5, 10, 8, 2, 2, 4, 0, 3, 4, 9, 8, 7, 3, 13, 6],
        [2, 8, 8, 10, 9, 2, 5, 4, 3, 0, 4, 6, 5, 4, 3, 9, 5],
        [6, 13, 4, 14, 13, 7, 4, 8, 4, 4, 0, 10, 9, 8, 4, 13, 4],
        [6, 7, 15, 6, 4, 9, 12, 5, 9, 6, 10, 0, 1, 3, 7, 3, 10],
        [4, 5, 14, 7, 6, 7, 10, 4, 8, 5, 9, 1, 0, 2, 6, 4, 8],
        [4, 8, 13, 9, 8, 7, 10, 3, 7, 4, 8, 3, 2, 0, 4, 5, 6],
        [5, 12, 9, 14, 12, 6, 6, 7, 3, 3, 4, 7, 6, 4, 0, 9, 2],
        [9, 10, 18, 6, 8, 12, 15, 8, 13, 9, 13, 3, 4, 5, 9, 0, 9],
        [7, 14, 9, 16, 14, 8, 5, 10, 6, 5, 4, 10, 8, 6, 2, 9, 0],
    ]
    #每个客户要求的时间窗
    data['time_windows'] = [
        (0, 5),  # depot
        (7, 12),  # 1
        (10, 15),  # 2
        (16, 18),  # 3
        (10, 13),  # 4
        (0, 5),  # 5
        (5, 10),  # 6
        (0, 4),  # 7
        (5, 10),  # 8
        (0, 3),  # 9
        (10, 16),  # 10
        (10, 15),  # 11
        (0, 5),  # 12
        (5, 10),  # 13
        (7, 8),  # 14
        (10, 15),  # 15
        (11, 15),  # 16
    ]
    data['num_vehicles'] = 4
    data['depot'] = 0
    return data


def print_solution(data, manager, routing, solution):
    """Prints solution on console."""
    time_dimension = routing.GetDimensionOrDie('Time')
    total_time = 0
    for vehicle_id in range(data['num_vehicles']):
        index = routing.Start(vehicle_id)
        plan_output = 'Route for vehicle {}:\n'.format(vehicle_id)
        while not routing.IsEnd(index):
            time_var = time_dimension.CumulVar(index)
            plan_output += '{0} Time({1},{2}) -> '.format(
                manager.IndexToNode(index), solution.Min(time_var),
                solution.Max(time_var))
            index = solution.Value(routing.NextVar(index))
```

```
        time_var = time_dimension.CumulVar(index)
        plan_output += '{0} Time({1},
{2})\n'.format(manager.IndexToNode(index),

 solution.Min(time_var),

 solution.Max(time_var))
        plan_output += 'Time of the route: {}min\n'.format(
            solution.Min(time_var))
        print(plan_output)
        total_time += solution.Min(time_var)
    print('Total time of all routes: {}min'.format(total_time))


def main():
    """Solve the VRP with time windows."""
    # Instantiate the data problem.
    data = create_data_model()

    # Create the routing index manager.
    manager = pywrapcp.RoutingIndexManager(len(data['time_matrix']),
                                           data['num_vehicles'],
data['depot'])

    # Create Routing Model.
    routing = pywrapcp.RoutingModel(manager)


    # Create and register a transit callback.
    def time_callback(from_index, to_index):
        """Returns the travel time between the two nodes."""
        # Convert from routing variable Index to time matrix NodeIndex.
        from_node = manager.IndexToNode(from_index)
        to_node = manager.IndexToNode(to_index)
        return data['time_matrix'][from_node][to_node]

    transit_callback_index =
routing.RegisterTransitCallback(time_callback)

    # Define cost of each arc.
    routing.SetArcCostEvaluatorOfAllVehicles(transit_callback_index)

    # Add Time Windows constraint.
    time = 'Time'
    routing.AddDimension(
        transit_callback_index,
        #单个节点最大等待时间
        30,  # allow waiting time
        #每个货车最大运送时间
        30,  # maximum time per vehicle
        False,  # Don't force start cumul to zero.
        time)
    time_dimension = routing.GetDimensionOrDie(time)
    # Add time window constraints for each location except depot.
    for location_idx, time_window in enumerate(data['time_windows']):
        if location_idx == 0:
            continue
        index = manager.NodeToIndex(location_idx)
```

```
    #硬时间窗口
    time_dimension.CumulVar(index).SetRange(time_window[0], time_window[1])
     # Add time window constraints for each vehicle start node.
     for vehicle_id in range(data['num_vehicles']):
         index = routing.Start(vehicle_id)
         time_dimension.CumulVar(index).SetRange(data['time_windows'][0]
[0],
                                                 data['time_windows'][0]
[1])

     # Instantiate route start and end times to produce feasible times.
     for i in range(data['num_vehicles']):
         routing.AddVariableMinimizedByFinalizer(
             time_dimension.CumulVar(routing.Start(i)))
         routing.AddVariableMinimizedByFinalizer(
             time_dimension.CumulVar(routing.End(i)))

     # Setting first solution heuristic.
     search_parameters = pywrapcp.DefaultRoutingSearchParameters()
     search_parameters.first_solution_strategy = (
         routing_enums_pb2.FirstSolutionStrategy.PATH_CHEAPEST_ARC)

     # Solve the problem.
     solution = routing.SolveWithParameters(search_parameters)

     # Print solution on console.
     if solution:
         print_solution(data, manager, routing, solution)


if __name__ == '__main__':
     main()
```