Abstract—This paper presents an overview of recent advances in the ﬁeld of the vehicle routing problem (VRP), based on papers published in high-quality journals during the period from January 2015 to July 2017. A distinctive feature of the presented survey is its focus on new versions of the VRP, which have recently emerged or gained momentum, and the corresponding new solution methods, with particular emphasis on computational intelligence (CI) approaches. The list of newly proposed or currently popular VRP formulations include last mile and same day delivery, crowdshipping, bike sharing systems, post-disaster response plans, local routing in large production or cargo plants, customer-centric VRP, autonomous delivery, unnamed aerial vehicle delivery, green VRP, waste collection VRP, rich VRP, or VRP with backhauls. Simultaneously, an adequate increase of interest in the application of traditional CI methods (e.g., genetic, memetic, ant colony or particle swarm optimization, simulated annealing, or their various hybrid versions) can be observed in the VRP domain. At the same time, approaches proven efﬁcient in other optimization areas (e.g., hyperheuristics, methods based on Monte Carlo simulations, algorithms rooted in game theory and bi-level optimizationStackelberg games,or cognitively motivated methods) have lately entered the VRP ﬁeld and become a viable alternative to more traditional techniques. Since VRP is one of the fastest growing ﬁelds in the operations research area, we believe that an analysis of the recently published VRP papers from the perspective of their novelty in problem formulation and/or applied solution method can provide a true value for the CI community, especially young researchers entering the ﬁeld and seeking challenges in this interesting and fast developing research area.

摘要本文根据2015年1月至2017年7月在高质量期刊上发表的论文，综述了车辆路径问题（VRP）领域的最新进展。这项调查的一个显著特点是，它侧重于最近出现或取得势头的VRP的新版本，以及相应的新解决方法，特别强调计算智能方法。新提出的或目前流行的VRP配方清单包括最后一英里和当天交付、众包、自行车共享系统、灾后响应计划、大型生产或货运工厂的本地路线、以客户为中心的VRP、自主交付、未命名的飞行器交付、绿色VRP、废物收集VRP、丰富VRP或VRP背拖。同时，可以在VRP领域观察到对传统CI方法（例如，遗传、模因、蚁群或粒子群优化、模拟退火或其各种混合版本）的应用兴趣的充分增加。同时，在其他优化领域（例如，超启发式、基于蒙特卡罗模拟的方法、根植于博弈论和双层优化stackelberg博弈的算法或认知激励方法）中被证明有效的方法最近进入了VRP领域，成为更传统技术的可行替代方法。由于VRP是运筹学领域增长最快的领域之一，我们认为，从问题表述和/或应用解决方法的新颖性的角度分析最近发表的VRP论文，可以为CI社区提供真正的价值，尤其是年轻的研究人员进入这个领域，在这个有趣而快速发展的研究领域寻求挑战。

HE Vehicle Routing Problem (VRP), was introduced in the literature as the truck dispatching problem by Dantzig and Ramser [1] in 1959 and proven to be NP-hard by Lenstra and Kan [2] in 1981. The classical version of the VRP consists in serving a set of customers by a ﬂeet of homogeneous vehicles with routes beginning and ending at a speciﬁed depot. The optimization goal is to minimize the cost of delivery, i.e., the total length of routes of all vehicles. In the majority of cases, a capacity limit is imposed on the vehicles, leading to the Capacitated VRP (CVRP) formulation.//容量约束

The VRP (CVRP) belongs to the most widely researched problems in the domain of Operations Research, mainly due to its practical relevance and combinatorial complexity. The importance of the VRP stems from its direct application to everyday business routines of distribution/serviceproviding companies. Due to a huge variety of practical implementations of the problem, the VRP literature covers a broad range of possible extensions to the classical problem formulation.

车辆路径问题（VRP）是Dantzig和Ramser[1]于1959年提出的卡车调度问题，Lenstra和Kan[2]于1981年证明是NP-hard问题。VRP的经典版本包括通过一系列同质车辆为一组客户提供服务，这些车辆的路线起点和终点都在指定的车辆段。优化的目标是最小化交付成本，即所有车辆的路线总长度。在大多数情况下，对车辆施加了容量限制，从而形成了电容化VRP（CVRP）公式。

VRP（CVRP）是运筹学领域研究最为广泛的问题之一，其主要原因是它的实用相关性和组合复杂性。VRP的重要性源于它直接应用于分销/服务提供公司的日常业务。由于问题的各种实际实现，VRP文献涵盖了对经典问题公式的广泛可能扩展。

On a general note, there are two main factors which determine both the intrinsic difﬁculty of a given VRP formulation and the potential applicability of certain solution methods. These two dimensions along which the majority of VRP variants can be grouped are static vs. dynamic and deterministic vs. stochastic formulations.

Following [3], in static problems partial vehicle routes constructed during a solution process do not change afterwards, while in dynamic formulations they may, and usually do, change (sometimes signiﬁcantly) due to dynamic factors which are unknown beforehand and are only revealed later, when partial vehicle routes are already designed and frozen.

In deterministic variants of the VRP all variables are deterministic, although in the dynamic case some of them are unknown a priori. In contrast, in stochastic versions some problem parameters are stochastic (*e.g. customer demands, travel times, service times or service probabilities*). The easiest variants are those which are static and deterministic.

Dynamic and stochastic formulations are generally located on the other end of the difﬁculty spectrum.

一般而言，有两个主要因素决定了给定VRP公式的内在困难和某些解决方法的潜在适用性。在这两个维度上，大多数VRP变体可以被分组为静态与动态、确定性与随机性公式。在[3]之后，在静态问题中，在求解过程中构建的部分车辆路线不会在之后发生变化，而在动态公式中，它们可能（通常是）由于动态因素而发生变化（有时是显著的），这些动态因素在之前是未知的，只有在之后才被揭示，当部分车辆路线已经设计并冻结时。在VRP的确定性变量中，所有变量都是确定性的，尽管在动态情况下，其中一些变量是先验未知的。相反，在随机版本中，一些问题参数是随机的（例如，客户需求、旅行时间、服务时间或服务概率）。最简单的变体是静态的和确定性的变体。动态和随机公式通常位于困难谱的另一端。

From the application point of view, besides dynamism and stochasticity, there are several other features which differentiate the VRP formulations. The list of most popular problem aspects includes:

time windows (hard or soft),

pickups and deliveries or backhauls,

a mixed (heterogeneous) vehicle ﬂeet,

split deliveries,

loading/capacity constraints,

multiple depots,

open routes (with no requirement to return to the depot at the end of a route [4]).

Clearly the above list refers to the most popular aspects of the VRP only, and by no means should be treated as exhaustive. A recently published book [5] provides a comprehensive description of the most popular problem variants. For a detailed VRP taxonomy the reader can refer to [6].

从应用的角度来看，除了动态性和随机性之外，还有其他几个特征可以区分VRP公式。最受欢迎的问题方面包括：

时间窗口（硬或软）、

取货和交货或回程、

混合（异构）车辆、

分批交货、

装载/容量限制、

多个仓库、

开放路线（不要求在路线结束时返回仓库[4]）。

显然，上述清单仅提及VRP最受欢迎的方面，绝不应视为详尽无遗。最近出版的一本书[5]对最流行的问题变体进行了全面的描述。关于详细的VRP分类，读者可以参考[6]。

The majority of the recently published VRP books or review papers can be assigned to one of the following two categories:

surveys describing major types of the VRP from a general perspective of the entire VRP ﬁeld, e.g. [5]–[7];

surveys referring to particular types of the VRP, e.g. Dynamic VRP [3], Green VRP [8], Multi-attribute VRP [9], Waste Collection VRP [10], City VRP [11] or Periodic VRP [12], or considering particular aspects of the problem, e.g. the VRP taxonomy [6], [13].

最近出版的大多数VRP书籍或评论论文可分为以下两类：

从整个VRP领域的总体角度描述VRP主要类型的调查，例如[5]–[7]；

参考特定类型VRP的调查，如动态VRP[3]、绿色VRP[8]、多属性VRP[9]、废物收集VRP[10]、城市VRP[11]或周期性VRP[12]，或考虑问题的特定方面，如VRP分类法[6]、[13]。

A distinctive feature of this survey is its interest in emerging problem formulations and new CI-related solution methods. In terms of problem types the paper surveys:

new VRP formulations which have emerged only recently, e.g. last mile and same day delivery, crowdshipping, bike sharing systems, post-disaster response plans, local routing in large production or cargo plants, customer-centric VRP, autonomous delivery, or UAV delivery,

problem formulations which have been around for some time, but gained momentum over the last several years, e.g. Green VRP, Waste Collection VRP, Rich VRP or VRP with Backhauls.

这项调查的一个显著特点是它对新出现的问题公式和新的CI相关解决方法感兴趣。根据问题类型，论文调查：

最近才出现的新VRP方法，例如最后一英里和同一天交付、众包、自行车共享系统、灾后响应计划、大型生产或货运工厂的本地路由、以客户为中心的VRP、自主交付或无人机交付

已经存在了一段时间，但在过去几年中势头强劲的问题公式，例如绿色VRP、废物收集VRP、丰富VRP或带回程的VRP。

With respect to solution methods the focus is on:

recently developed approaches, previously not utilized in the VRP domain, that belong to the broadly understood CI ﬁeld, e.g. methods based on Monte Carlo simulations, algorithms rooted in game theory and bi-level optimization (Stackelberg games), hyperheuristics, or cognitively motivated methods, as well as innovative applications of hybrid CI methods.

The remainder of the paper is organized as follows: the next section describes the applied methodology of paper selection and presents some statistics related to journals and papers chosen in this review. Section III discusses new, interesting problem formulations and is divided into subsections grouping particular types of the VRP. Section IV presents innovative and promising Computational Intelligence (CI) methods that have recently emerged in the ﬁeld. The last section contains conclusions.

关于解决方法，重点是：

最近开发的方法，以前未在VRP领域使用，属于广泛理解的领域，例如基于蒙特卡罗模拟的方法、根植于博弈论和双层优化（Stackelberg博弈）的算法、超启发式或认知激励方法，以及混合CI方法的创新应用。

论文的其余部分安排如下：  
下一节介绍了论文选择的应用方法，并给出了与本文所选期刊和论文相关的一些统计数据。第三节讨论了新的，有趣的问题公式，并被分为分组特定类型的VRP子节。

1. 介绍了最近在该领域出现的创新和有前途的计算智能（CI）方法。
2. 最后一节包含结论。

**part3部分阅读笔记**

The main criterion of paper selection in this survey was the presence of an innovative problem formulation or solution method. While each of the baseline VRP aspects mentioned in the introductory section (time frame, loading constraints, open routes, etc.) single-handedly adds a new layer of complexity to the plain VRP/CVRP formulation, such one-dimensional VRP extensions are, nevertheless, well-researched in the literature and gradually give ground to more complex problem variants. On a general note, the mainstream VRP research is developing in the three following directions:

在这次调查中，论文选择的主要标准是有一个创新的问题公式或解决方法。虽然导论部分中提到的每个基线VRP方面（时间框架、加载约束、开放路径等）单枪匹马地为普通VRP/CVRP公式增加了一个新的复杂层，但是这种一维VRP扩展是，在文献中得到了很好的研究，并逐渐为更复杂的问题变体奠定了基础。总的来说，主流的VRP研究正朝着以下三个方向发展：

new challenging problem extensions reﬂecting contemporary technical and societal challenges, demands and opportunities, e.g. Green VRP, ad-hoc delivery, or last-mile delivery,

新的挑战性问题扩展反映了当代技术和社会挑战、需求和机遇，例如绿色VRP、点对点交付或最后一英里交付，

synergetic combinations of several VRP aspects which lead to essentially new problem formulations, usually practically-motivated, e.g. VRP with backhauls or various formulations of Rich VRP,

多个VRP方面的协同组合，导致基本上新的问题公式，通常是出于实际动机，例如，VRP与回程或rich VRP的各种公式，

new problem formulations addressing speciﬁc business or industry settings, e.g. VRP for transportation of hazardous materials, humanitarian transportation in post-disaster settings, or local VRP in large industry or cargo plants.

针对特定商业或行业环境的新问题配方，例如用于危险材料运输的VRP、灾后环境中的人道主义运输，或大型工业或货运厂中的本地VRP。

## 带回程的：

One of the fast developing variants of the VRP in recent years is VRP with Backhauls (VRPB), initially proposed in [14], which includes, as one of its main subcases, VRP with Simultaneous Pick-up and Delivery (VRPSPD). Simultaneous pick-up and delivery assumes that both linehauls (delivering goods) and backhauls (pick-up tasks) are requested simultaneously at a client’s site [15]–[17]. While this situation is quite frequent in real-life scenarios, there are also two other VRPB formulations which gained momentum in recent years [18]. In the ﬁrst one, known as VRP with Delivery Before Pick-up (VRPDBP) [19], [20],linehauls must be served before backhauls. In the other one, referred to as VRP with Backhauls and Mixed-load (VRPBM) or VRP with Mixed Pickup and Delivery (VRPMPD) [21], linehauls and backhauls can be served in any order.

近年来快速发展的VRP变体之一是回程VRP（VRPB），最初在文献[14]中提出，它包括作为其主要子类之一的同时提货的VRP（VRPSPD）【同时派送/收集的车辆路线规划问题】。同时提货和交货假设在客户现场同时请求联运（交货）和回运（提货任务）[15]-[17]。虽然这种情况在现实生活中相当常见，但还有另外两种VRPB公式近年来获得了发展势头[18]。在第一种情况下，即所谓的“先交货后提货”（VRPDBP）[19]，[20]，必须在回程运输之前提供货物运输服务。在另一种情况下，称为具有回程和混合装载（VRPBM）的VRP或具有混合取货和交货（VRPMPD）[21]的VRP，可以按任何顺序提供直线运输和回程运输。

The three above-mentioned variants of VRPB were initially introduced as stand-alone extensions of the VRP. Lately, they started to serve as a basis for new VRPB versions that address speciﬁc business requirements, most notably, loading constraints described below.

Two-dimensional loading VRP with clustered backhauls (2LVRPB), which assumes that both types of demands (pickups and deliveries) are composed of non-stackable items, was proposed in [22]. Quite surprisingly, despite relatively high commonness in everyday transportation logistics, the problem has not been formally considered in the literature until recently [22]. The problem is approached with the Large Neighborhood Search (LNS) metaheuristic, with dedicated routing and packing local search heuristics.

上述三种VRPB变体最初作为VRP的独立扩展引入。最近，它们开始成为解决特定业务需求的新VRPB版本的基础，尤其是下面描述的加载约束。

文献[22]提出了具有集群回程的二维装载VRP（2LVRPB），假设两种需求（提货和交货）都是由不可堆叠的物品组成。令人惊讶的是，尽管在日常运输物流中具有较高的共性，但直到最近才在文献中正式考虑到这个问题[22]。采用大邻域搜索（LNS）元启发式算法、专用路由和打包局部搜索启发式算法对该问题进行求解。

VRPSPD is extended by adding constraints related to the transportation of non-stackable rectangular items was also proposed in [23] under the name VRP with Simultaneous Pick-ups and Deliveries and Two-Dimensional Loading Constraints (2LSPD). A combination of 2L-CVRP [24] and VRPSPD makes the resulting problem truly demanding as 2D loading feasibility must be veriﬁed for each arc of the traveled routes. The problem can be further extended by imposing LIFO (Last In, First Out) constraints which prohibit the rearrangement of items on the route. Both versions (with and without LIFO) are solved with a two step approach: ﬁrst a heuristic initial solution is constructed, which is then optimized with the help of three local exchange operators, previously used and evaluated by the authors in [25]. Also a new set of benchmarks [26] corresponding to 2L-SPD formulation is proposed in [23].

VRPSPD的扩展是通过增加与不可堆叠矩形物品的运输相关的约束，在文献[23]中也以VRP的名义提出了同时提货和交货以及二维装载约束（2LSPD）。2L-CVRP[24]和VRPSPD的结合使得所产生的问题真正需要解决，因为必须验证每个行驶路线弧的二维加载可行性。这个问题可以通过强制后进先出（Last-In，First-Out）约束进一步扩展，这些约束禁止重新排列路线上的项目。两个版本（有和没有后进先出）都采用两步方法求解：首先构造一个启发式初始解，然后在三个局部交换算子的帮助下进行优化，作者以前在[25]中使用和评估过。文[23]还提出了一组新的与2L-SPD公式相对应的基准[26]。

An extension along the lines of the classical VRPSPD formulation consisting in adding three-dimensional loading constraints, denoted as 3L-VRP, was proposed in [27]. The speciﬁcation of the problem assumes that each demand is in the form of a set of 3D rectangular items (boxes) which must be carried by vehicles with a given 3D rectangular loading space. Furthermore, it may additionally be assumed that boxes, once loaded, must not be moved before the ﬁnal unloading, i.e., no reloading effort is required while serving the customers (which is a variant of the LIFO constraint). This problem version is abbreviated as 3L-PDP. The solution method combines a routing procedure and a packing heuristic. The former is an extension of the LNS method used for the 1D-PDP (i.e. VRPSPD). The latter relies on a speciﬁc tree search problem representation. The authors propose a new set of 54 benchmark problems which are available.

文献[27]提出了一种沿经典VRPSPD公式的扩展，该公式包括添加三维载荷约束，即3L-VRP。问题的具体说明假设每个需求都是一组三维矩形物品（盒子），这些物品必须由具有给定三维矩形装载空间的车辆携带。此外，还可以假设，箱子一旦装载，在最终卸载之前不得移动，即在为客户服务时不需要重新装载（这是后进先出约束的一种变体）。此问题版本缩写为3L-PDP。该求解方法结合了路由过程和包装启发式。前者是用于1D-PDP（即VRPSPD）的LNS方法的扩展【A quick large neighborhood search ( LNS) algorithm快速大规模领域搜索】。后者依赖于特定的树搜索问题表示。

Another interesting extension of VRPB was proposed in [29], where the problem is combined with Multiple-Trip VRP (MTVRP) [30] (which assumes that a vehicle may perform several trips within a given time period), leading to the MT-VRPB variant. A MILP formulation is proposed and solved with the IBM ILOG CPLEX 12.5 optimizer for small and mid-size MTVRPB instances. For large-size problems the Two-level VNS algorithm is applied which alternately switches between the two stages: outer and inner. The former is responsible for the construction of the transitory solution and the later aims at its local search-based improvement. The algorithm is tested on a new set of 168 MT-VRPB benchmark problems [31] proposed by the authors.

文献[29]提出了VRPB的另一个有趣的扩展，其中该问题与多行程VRP（MTVRP）[30]相结合（假设车辆在给定的时间段内可以执行多个行程），从而导致MT-VRPB变体。针对中小型MTVRPB实例，提出了MILP公式，并用ibmilogcplex12.5优化器求解。对于大尺寸问题，采用两级VNS算法，在外部和内部两个阶段之间交替切换。前者负责构建临时解决方案，后者则以局部搜索为基础进行改进。该算法在作者提出的168mt-VRPB基准问题[31]上进行了测试。