

Homework 2

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11/12/2025

1 จงหา Domain และ Range ของ $f(x) = \frac{1}{3-\sqrt{x^2-25}}$

1.1 Domain

1.1.1 Cannot divide by 0

$$3 - \sqrt{x^2 - 25} \neq 0 \quad (1)$$

$$-\sqrt{x^2 - 25} \neq -3 \quad (\text{minus 3 both sides}) \quad (2)$$

$$\sqrt{x^2 - 25} \neq 3 \quad (\text{multiply -1 both sides}) \quad (3)$$

$$x^2 - 25 \neq 9 \quad (\text{square both sides}) \quad (4)$$

$$x^2 - 34 \neq 0 \quad (\text{minus 9 to both sides}) \quad (5)$$

$$(x - \sqrt{34})(x + \sqrt{34}) \neq 0 \quad (6)$$

$$x \neq \pm\sqrt{34} \quad (7)$$

$$x \in (-\infty, -\sqrt{34}) \cup (-\sqrt{34}, \sqrt{34}) \cup (\sqrt{34}, \infty) \quad (8)$$

1.1.2 In square root cannot be less than 0

$$x^2 - 25 \geq 0 \quad (9)$$

$$(x - 5)(x + 5) \geq 0 \quad (10)$$

$$x \in (-\infty, -5] \cup [5, \infty) \quad (11)$$

1.1.3 Combine

$$x \in ((-\infty, -\sqrt{34}) \cup (-\sqrt{34}, \sqrt{34}) \cup (\sqrt{34}, \infty)) \cap ((-\infty, -5] \cup [5, \infty)) \quad (12)$$

$$\therefore x \in (-\infty, -\sqrt{34}) \cup (-\sqrt{34}, -5] \cup [5, \sqrt{34}) \cup (\sqrt{34}, \infty) \quad (13)$$

1.2 Range

$$\text{let } t = \sqrt{x^2 - 25}; \quad t \geq 0, t \neq 3 \quad (14)$$

1.2.1 $t \rightarrow [0, 3)$

$$\text{when } t = 0; \quad y = \frac{1}{3} \quad (15)$$

$$\text{when } t \rightarrow 3^-; \quad y \rightarrow \infty \quad (16)$$

$$y \in [1/3, \infty) \quad (17)$$

1.2.2 $t \rightarrow (3, \infty)$

$$\text{when } t \rightarrow 3^+; \quad y \rightarrow -\infty \quad (18)$$

$$\text{when } t \rightarrow \infty; \quad y \rightarrow 0 \quad (19)$$

$$y \in (-\infty, 0) \quad (20)$$

1.2.3 Combine

$$\therefore y \in (-\infty, 0) \cup [1/3, \infty) \quad (21)$$