Homework 7

68011008 Waroon Ragwongsiri

18/12/2025

1 For all of non-negative integer n, $1 + 2 + 2^2 + ... + 2^n = 2^{n+1} - 1$

1.1 Prove by induction

$$Let\ P(n)\ be\ the\ statement: \tag{1}$$
$$1 + 2 + 2^2 + ... + 2^n = 2^{n+1} - 1 \tag{2}$$

1.1.1 P(base), base = 0

$$2^0 = 2^{0+1} - 1 \tag{3}$$
$$1 = 1 \tag{4}$$
$$\therefore P(base) \equiv True \tag{5}$$

1.1.2 P(n+1)

$$1 + 2 + 2^2 + ... + 2^n + 2^{n+1} = 2^{n+1} - 1 + 2^{n+1} \tag{6}$$
$$1 + 2 + 2^2 + ... + 2^n + 2^{n+1} = 2(2^{n+1}) - 1 \tag{7}$$
$$1 + 2 + 2^2 + ... + 2^n + 2^{n+1} = 2^{n+2} - 1 \tag{8}$$
$$\therefore P(n+1) \equiv True \tag{9}$$

$\therefore$ For all of non-negative integer n, $1 + 2 + 2^2 + ... + 2^n = 2^{n+1} - 1$

2 For all positive integer n, $n \ < \ 2^n$

2.1 Prove by induction

$$Let\ P(n)\ be\ the\ statement: \tag{10}$$
$$n \ < \ 2^n \tag{11}$$

2.1.1 P(base), base = 1

$$1 < 2^1 \tag{12}$$
$$\therefore P(base) \equiv True \tag{13}$$

2.1.2 P(n+1)

$$n + 1 < 2^{n+1} \tag{14}$$
$$n + 1 < 2 \cdot 2^n \tag{15}$$
$$n + 1 < 2^n + 2^n \tag{16}$$
$$from\ n < 2^n\ and\ 1 < 2^n\ from\ P(base)\ for\ n\ \geq 1 \tag{17}$$
$$\therefore P(n+1) \equiv True \tag{18}$$

$\therefore$ For all positive integer n,
$n \ < \ 2^n$