ML Homework 3

Due Date: 2020/04/20 (MON) 23:55

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Description:

1. Random Data Generator

- a. Univariate gaussian data generator
  - Input
    - Expectation value or mean: m
    - Variance: s
  - Output: A data point from $N(m, s)$
  - HINT
    - **Generating values from normal distribution**
    - You have to handcraft your geneartor based on one of the approaches given in the hyperlink.
    - You can use uniform distribution function (Numpy)
- b. Polynomial basis linear model data generator
  - $y = W^T \phi(x) + e$
    - W is a $n \times 1$ vector
    - $e \sim N(0, a)$
  - Input: n (basis number), a, w
    - e.g. $n = 2 \rightarrow y = w_0 x^0 + w_1 x^1$,
  - Output: y (a number)
  - Internal constraint
    - $-1.0 < x < 1.0$
    - x is uniformly distributed.

2. Sequential Estimator

- Sequential estimate the mean and variance
  - Data is given from the univariate gaussian data generator (1.a).
- Input: m, s as in (1.a)
- Function:
  - Call (1.a) to get a new data point from $N(m, s)$
  - Use sequential estimation to find the current estimates to m and s

- Repeat steps above until the estimates converge.
- Output: Print the new data point and the current estimiates of m and s in each iteration.
- Notes
    - You should derive the recursive function of mean and variance based on the sequential esitmation.
    - Hint: **Online algorithm**
- Sample input & output (⚠️ *for reference only* ⚠️)

```
1    Data point source function: N(3.0, 5.0)
2
3    Add data point: 3.234685454257290
4    Mean = 3.408993960833291    Variance = 0.030383455464755956
5    Add data point: 0.519242879651157
6    Mean = 2.445743600439247    Variance = 1.875958150575018
7    Add data point: 1.347113997201991
8    Mean = 2.171086199629932    Variance = 1.633278676389248
9    Add data point: 8.979491998496083
10   Mean = 3.532767359403163    Variance = 8.723325264636875
11   Add data point: 3.603448448693051
12   Mean = 3.544547540951477    Variance = 7.270131583917285
13   Add data point: 4.127197937610908
14   Mean = 3.627783311902824    Variance = 6.273110519038578
15   Add data point: 4.992735798186870
16   Mean = 3.798402372688330    Variance = 5.692747751482052
17
18   ...
19
20   Add data point: 4.233592159021013
21   Mean = 2.961576104513964    Variance = 5.045715437349161
22   Add data point: 3.529990930040463
23   Mean = 2.961883688294010    Variance = 5.043159812425648
24   Add data point: 1.125210345431449
25   Mean = 2.960890354955524    Variance = 5.042255747918937
```

3. Baysian Linear regression

- Input
    - The precision (i.e., b) for initial prior $w \sim N(0, b^{-1}I)$
    - All other required inputs for the polynomial basis linear model geneartor (1.b)
- Function
    - Call (1.b) to generate one data point
    - Update the prior, and calculate the parameters of predictive distribution
    - Repeat steps above until the posterior probability converges.
- Output
    - Print the new data point and the current paramters for posterior and predictive distribution.

- After probability converged, do the visualization
  - Ground truth function (from linear model generator)
  - Final predict result
  - At the time that have seen 10 data points
  - At the time that have seen 50 data points
  - Note
    - Except ground truth, you have to draw those data points which you have seen before
    - Draw a black line to represent the mean of function at each point
    - Draw two red lines to represent the variance of function at each point
      - In other words, distance between red line and mean is **ONE** variance
- Hint: Online learning
- Sample input & output (***for reference only***)

1. b = 1, n = 4, a = 1, w = [1, 2, 3, 4]

```
1   Add data point (-0.64152, 0.19039):
2
3   Postirior mean:
4       0.0718294547
5      -0.0460797888
6       0.0295609502
7      -0.0189638408
8
9   Posterior variance:
10      0.6227289276,    0.2420256620,   -0.1552634839,    0.0996041049
11      0.2420256620,    0.8447365161,    0.0996041049,   -0.0638976884
12     -0.1552634839,    0.0996041049,    0.9361023116,    0.0409914289
13      0.0996041049,   -0.0638976884,    0.0409914289,    0.9737033172
14
15  Predictive distribution ~ N(0.00000, 2.65061)
16  --------------------------------------------------
17  Add data point (0.07122, 1.63175):
18
19  Postirior mean:
20      0.6736864869
21      0.2388980107
22     -0.1054659080
23      0.0710615952
24
25  Posterior variance:
26      0.3765992302,    0.1254838660,   -0.1000441911,    0.0627881634
27      0.1254838660,    0.7895542671,    0.1257503020,   -0.0813299447
28     -0.1000441911,    0.1257503020,    0.9237138418,    0.0492510997
29      0.0627881634,   -0.0813299447,    0.0492510997,    0.9681964094
```
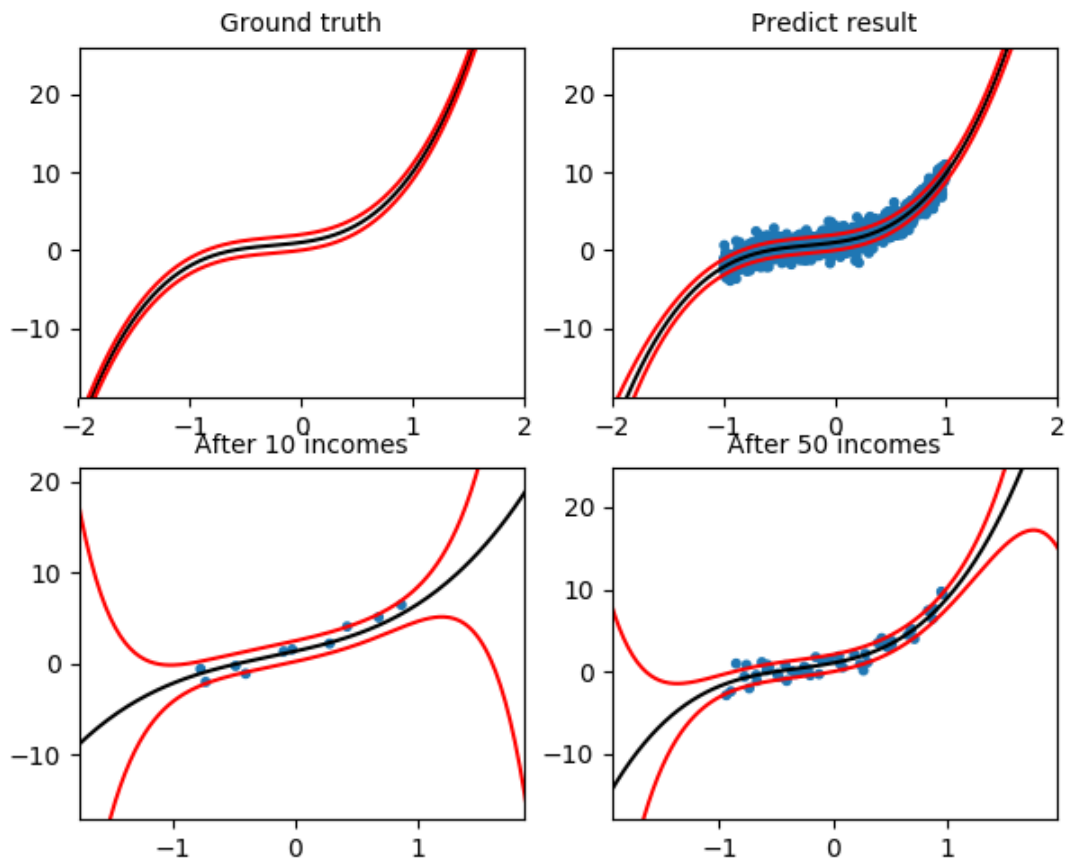
```
Predictive distribution ~ N(0.06869, 1.66008)
--------------------------------------------------
Add data point (-0.19330, 0.24507):

Postirior mean:
    0.5760972313
    0.2450231522
   -0.0801842453
    0.0504992402

Posterior variance:
    0.2867129751,    0.1311255325,   -0.0767580827,    0.0438488542
    0.1311255325,    0.7892001707,    0.1242887609,   -0.0801412282
   -0.0767580827,    0.1242887609,    0.9176812972,    0.0541575540
    0.0438488542,   -0.0801412282,    0.0541575540,    0.9642058389

Predictive distribution ~ N(0.62305, 1.34848)
--------------------------------------------------

...

--------------------------------------------------
Add data point (-0.76990, -0.34768):

Postirior mean:
    0.9107496675
    1.9265499885
    3.1119297129
    4.1312375189

Posterior variance:
    0.0051883836,   -0.0004416700,   -0.0086000319,    0.0008247001
   -0.0004416700,    0.0401966605,    0.0012708906,   -0.0554822477
   -0.0086000319,    0.0012708906,    0.0265353911,   -0.0031205875
    0.0008247001,   -0.0554822477,   -0.0031205875,    0.0937197255

Predictive distribution ~ N(-0.61566, 1.00921)
--------------------------------------------------
Add data point (0.36500, 2.22705):

Postirior mean:
    0.9107404583
    1.9265225090
    3.1119408740
    4.1312734131

Posterior variance:
    0.0051731092,   -0.0004872471,   -0.0085815201,    0.0008842340
```
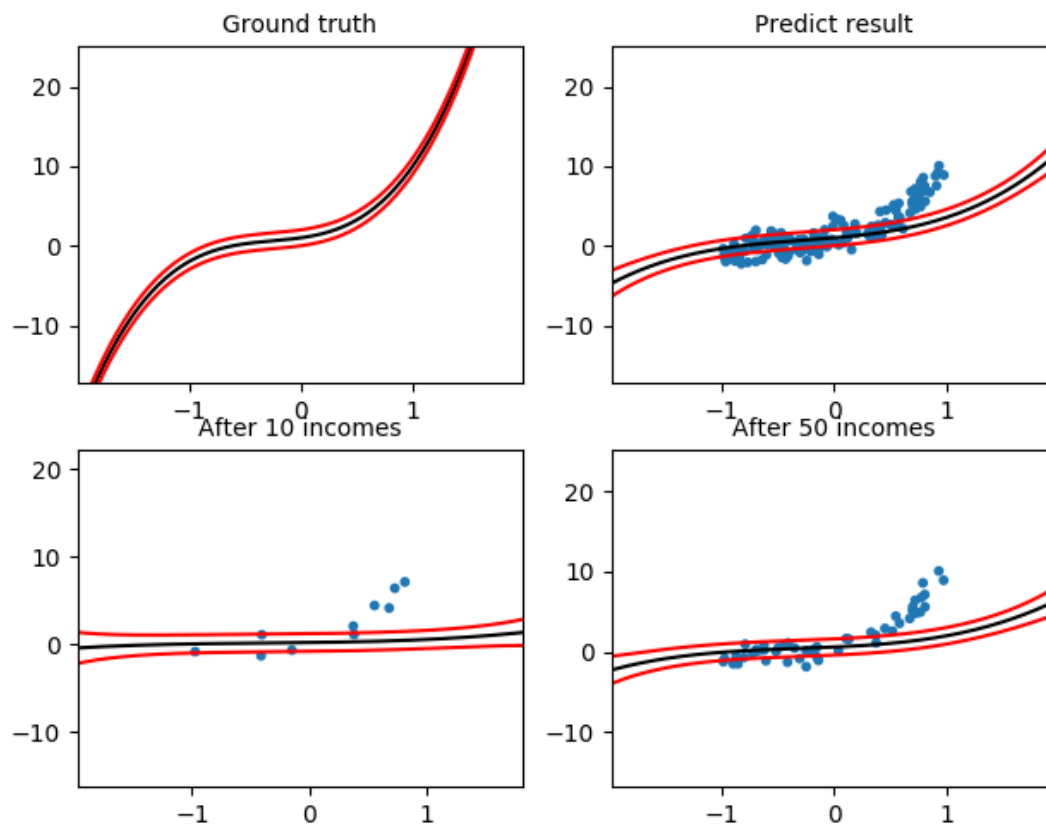
```
79    -0.0004872471,    0.0400606628,    0.0013261280,   -0.0553046044
80    -0.0085815201,    0.0013261280,    0.0265129556,   -0.0031927398
81     0.0008842340,   -0.0553046044,   -0.0031927398,    0.0934876838
82
83  Predictive distribution ~ N(2.22942, 1.00682)
84  --------------------------------------------------
```


Ground truth / Predict result / After 10 incomes / After 50 incomes

2. b = 100, n = 4, a = 1, w = [1, 2, 3, 4]

```
1   (Console output omitted)
```

3. b = 1, n = 3, a = 3, w = [1, 2, 3]

```
(Console output omitted)
```