

Exercises 2-1

Q.1 Four universities - 1, 2, 3 and 4 are participating in a holiday basketball tournament. In the 1st round, 1 will play 2 and 3 will play 4. Then the two winners will play for the championship, and the two losers will also play. One possible outcome can be denoted by 1324 (1 beats 2 and 3 beats 4 in the 1st round games, and then 1 beat 3 and 2 beats 4).

- Ⓐ List all outcomes in S.
- Ⓑ Let A denote the event that 1 wins the tournament. List the outcomes in A.
- Ⓒ Let B denotes the event that 2 gets into championship game. List outcomes in B.
- Ⓓ What are the outcomes in $A \cup B$ and in $A \cap B$? What are the outcomes in A^c .

Ⓐ Ⓑ Ⓒ Ⓓ

So 1)
Q-1 1 3 2 4 → In 1st round 1 wins 2
 2 3 wins 4
2) 1 will be champion if 2 2 wins 4
outcomes are

1 3 2 4
1 3 4 2
1 4 2 3
1 4 3 2

2 will be the champion if the outcome
are

2 3 1 4
2 3 4 1
2 4 1 3
2 4 3 1

3 will be the champion if the outcome
are

3 1 4 2
3 1 2 4
3 2 4 1
3 2 1 4

4 will be champion if the outcome
are

4 1 3 2
4 1 2 3
4 2 1 3
4 2 3 1

(a) Sample space is

$$S = \{ 1324, 1342, 1423, 1432, 2314, 2341, 2413, 2431, 3142, 3124, 3241, 3214, 4132, 4123, 4231, 4213 \}$$

(b) List of outcomes of A denote 1 wins the tournament = {1324, 1342, 1423, 1432}

(c) List of outcomes of B denote 2 gets into the championship game

= B is in either 1st or 2nd position

$$= \{ 2314, 2341, 2413, 2431, 3241, 3214, 4231, 4213 \}$$

(d) Outcomes of $A \cup B$

$$= \{ 1324, 1342, 1423, 1432, 2314, 2341, 2413, 2431, 3241, 3214, 4231, 4213 \}$$

Outcomes of $A \cap B = \emptyset$

Q ②

Suppose that vehicles taking a particular freeway exit can turn right (R), turn left (L) or go straight (S). Consider observing the direction for each of three successive vehicles.

- (a) List all outcomes in the event A that all three vehicles go in the same directions.
- (b) List all outcomes in the event B that all three vehicles take different directions.
- (c) List all outcomes in the event C that exactly two of three vehicles turn right.
- (d) List all outcomes in the event D that exactly two vehicles go in the same directions.
- (e) List outcomes in D' , $C \cap D$ and $C \cup D$.

Sol

Q-2

Directions are

R → right

L → left

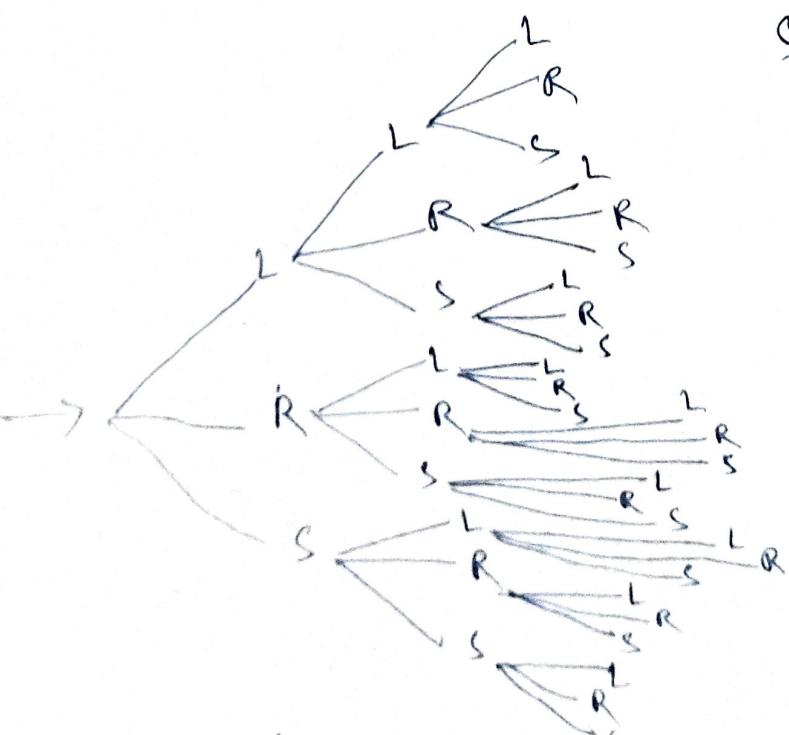
S → straight

Ⓐ A = ~~all three~~ vehicles go in the same direction

B = all three vehicles go in the different direction

C = exactly two vehicles ~~turn right~~
~~go straight~~
~~different directions~~

D = exactly two vehicles go in the same direction



Sample space S

= set of directions
for three vehicles

= {LLL, LLR, LLS,
LRL, LRR, LRS,
RLL, RLQ, RLS,
RRL, RRR, RRS,
SLL, SLR, SLS,
SRL, SRR, SRS,
SSL, SSR, SSS,
RSL, RSR, RSS}

Ⓐ A = {LLL, RRR, SSS}

Ⓑ B = {LRS, LSR, RLS, SLR, SRL, RSL}

④ ~~$C = \{ LRR, RRL, RRS, RSR, SRR, RSS \}$~~
 ~~$RLL, RLR, RRL, RRS, RSR, SRR \}$~~

$$C = \{ LRR, RLR, RRL, RRS, RSR, SRR \}$$

⑤ $D = \{ LLR, LLS, LRL, LRR, LSL, LSS, RLL, RLR, RRL, RRS, RSR, RSS, SLL, SLS, SRR, SRS, SSL, SSR \}$

⑥ $D' = \{ LLL, LRS, LSR, RLS, RRR, RSL, SLR, SRL, SSS \}$

$$C \cup D = D \quad (\because C \subset D)$$

$$C \cap D = C \quad (\because C \subset D)$$

Q3 Verify De Morgan's Laws

~~Ans~~
~~Ans~~

$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

$$\text{Sample space } S = \{1, 2, 3, 4\}$$

$$A = \{1, 2\}, B = \{2, 3\}$$

$$A' = \{3, 4\}, B' = \{1, 4\}$$

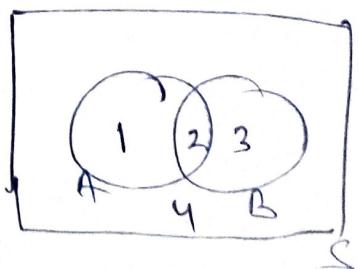
$$A \cup B = \{1, 2, 3\}, (A \cup B)' = \{4\}$$

$$A \cap B = \{2\}, (A \cap B)' = \{1, 3, 4\}$$

$$A' \cup B' = \{1, 3, 4\}, A' \cap B' = \{4\}$$

$$\text{Hence } (A \cup B)' = A' \cap B' = \{4\}$$

$$(A \cap B)' = A' \cup B' = \{4\}$$



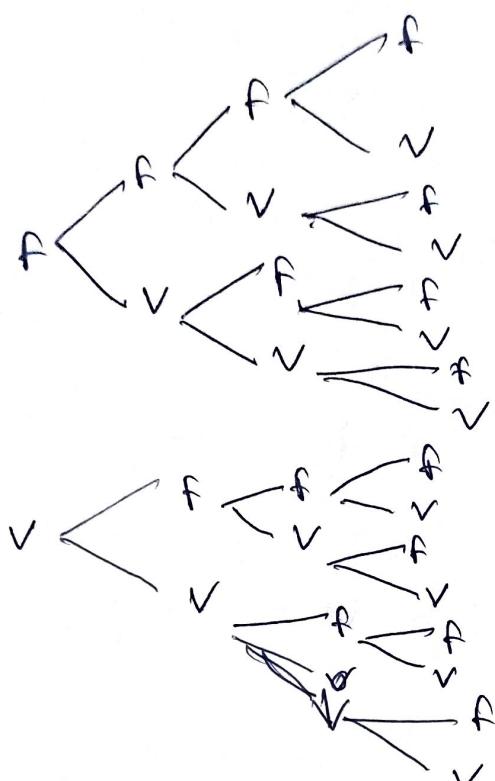
Ans
Ans

Q4) Each of a sample of four mortgages is classified as fixed rate (F) or variable-rate (V).

- (a) What are the 16 ($= 2^4$) outcomes in S?
- (b) Which outcomes are in the event that exactly three of the selected mortgages are fixed rate?
- (c) Which outcomes are in the event that all four mortgages ~~are~~ are of same type?
- (d) Which outcomes are in the event that at most one of four is a variable-rate mortgage?
- (e) What is the union of the events in parts (c) and (d), and what is the intersection of these two events?
- (f) What are the union and intersection of the two events in parts (b) & (c)?

Sol)
① $S = \{ \text{FFFF}, \text{FFFV}, \text{FFVF}, \text{FVFF}, \text{FFVV}, \text{FVFV}, \text{FVVF}, \text{FVVV}, \text{VVVV}, \text{VVVF}, \text{VVVF}, \text{VFVV}, \text{VVFF}, \text{VFVF}, \text{VFFV}, \text{VFFF} \}$

or
soft @



$$S = \{ \text{ffff}, \text{fffv}, \text{ffvf}, \text{ffvv}, \text{fvff}, \text{fvvf}, \text{fvvv}, \text{vfff}, \text{vvff}, \text{vvvf}, \text{vvvv} \}$$

(b)

$A =$ Exactly three of the selected mortgages are fixed rate

$$= \{ \text{ffffv}, \text{ffrvf}, \text{frvff}, \text{vffff} \}$$

(c)

$B =$ All mortgages are of same type

$$= \{ \text{ffff}, \text{vvvv} \}$$

(d)

$C =$ At most one of four is a valuable rate

$$= \{ \text{fffff}, \text{ffffv}, \text{ffvvf}, \text{fvvff}, \text{vvfff} \}$$

(e)

$$B \cup C = \{ \text{ffff}, \text{vvvv}, \text{ffffv}, \text{ffvfv}, \text{fvvff}, \text{vvfff} \}$$

$$B \cap C = \{ \text{ffff} \}$$

(f)

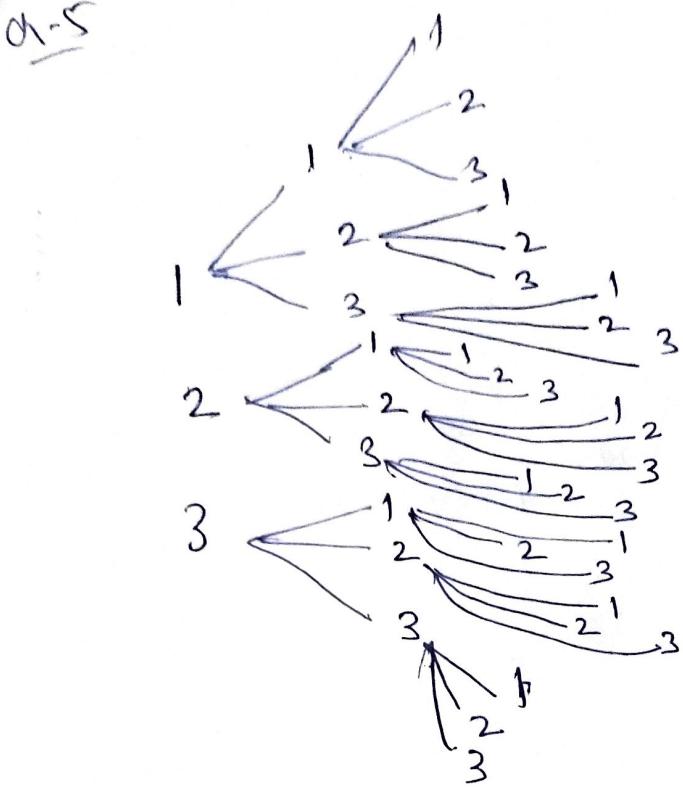
$$A \cup B = \{ \text{fffv}, \text{ffvf}, \text{fvff}, \text{vfff}, \text{ffff}, \text{vvvv} \}$$

$$A \cap B = \emptyset$$

- Q(5) A family consisting of three persons A, B & C - goes to a medical clinic that always has a doctor at each station 1, 2 & 3. During a certain week, each member of the family visits the clinic once and is assigned ~~to~~ at random to a station. ~~The station~~
The experiment consists of recording the station number for each member. One outcome is $(1, 2, 1)$ for A to station 1, B to station 2 and C to station 1.
- a) List the 27 ($= 3^3$) outcomes in the sample space \mathcal{S} .
 - b) List all the outcomes in the event that all three members go to same station.
 - c) List all outcomes in the event that all the members go to different stations
 - d) List all outcomes in the event that no one goes to station B.

Soln
a) Sample space $\mathcal{S} = \{(1, 1, 1), (1, 1, 2), (1, 1, 3), (1, 2, 1), (1, 2, 2), (1, 2, 3), (1, 3, 1), (1, 3, 2), (1, 3, 3), (2, 1, 1), (2, 1, 2), (2, 1, 3), (2, 2, 1), (2, 2, 2), (2, 2, 3), (2, 3, 1), (2, 3, 2), (2, 3, 3), (3, 1, 1), (3, 1, 2), (3, 1, 3), (3, 2, 1), (3, 2, 2), (3, 2, 3), (3, 3, 1), (3, 3, 2), (3, 3, 3)\}$

Sol Sample space $S = \{(x, y, z) \mid A \rightarrow x, B \rightarrow y, C \rightarrow z\}$
 $x, y, z \in \{1, 2, 3\}\}$



$$= \{(1, 1, 1), (1, 1, 2), (1, 1, 3), (1, 2, 1), (1, 2, 2), (1, 2, 3), (1, 3, 1), (1, 3, 2), (1, 3, 3), (2, 1, 1), (2, 1, 2), (2, 1, 3), (2, 2, 1), (2, 2, 2), (2, 2, 3), (2, 3, 1), (2, 3, 2), (2, 3, 3), (3, 1, 1), (3, 1, 2), (3, 1, 3), (3, 2, 1), (3, 2, 2), (3, 2, 3), (3, 3, 1), (3, 3, 2), (3, 3, 3)\}$$

Ⓐ Ⓡ

A = All three members go to the same station

$$= \{(1, 1, 1), (2, 2, 2), (3, 3, 3)\}$$

Ⓒ B = All members go to different stations

$$= \{(1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2), (3, 2, 1)\}$$

Ⓓ C = No one goes to station 2

$$= \{(1, 1, 1), (1, 1, 3), (1, 3, 1), (1, 3, 3), (3, 1, 1), (3, 1, 3), (3, 3, 1), (3, 3, 3)\}$$