Problem Set - 2.4

Q.45 The population of a particular country consists of three ethnic groups. Each individual belongs to one of the four major blood groups. The accompanying joint probability table gives the proportions of individuals in the various ethnic group-blood combinations.

		Blood Group			
		O	A	B	AB
Ethnic group	1	0.082	0.106	0.008	0.004
	2	0.135	0.141	0.018	0.006
	3	0.215	0.200	0.065	0.020

Suppose that an individual is randomly selected from the population and define events by A = {type A selected}, B = {type B selected} and C = {ethnic group 3 selected}

(a) calculate P(A), P(C) and P(A∩C)

(b) calculate both P(A|C) and P(C|A) and explain in content what each of these probabilities represents.

(c) If the selected individual does not have type B blood, what is the probability that he or she is from ethnic group 1?

Solution:

(a) $P(A) = 0.106 + 0.141 + 0.2 = 0.447$

$P(C) = 0.215 + 0.2 + 0.065 + 0.02$
$= 0.5$

$P(A \cap C) = 0.2$

(b) $P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{0.2}{0.5} = 0.4$

means, given that the individual came from ethnic group 3, the prob. that he or shea has type A blood is 0.4

$P(C|A) = \frac{P(A \cap C)}{P(A)} = \frac{0.2}{0.447} = 0.447427$

means, given that the individual has type A blood, the probability that he or she is from ethnic group 3 is 0.447427

(c) Let $D = \{\text{ethnic group 1 selected}\}$

To find $P(D|B')$: we have

$$P(B) = 0.008 + 0.018 + 0.065$$
$$= 0.091$$
$$\therefore P(B') = 1 - P(B) = 0.909$$
$$P(D \cap B') = 0.082 + 0.106 + 0.004$$
$$= 0.192$$
$$P(D|B') = \frac{P(D \cap B')}{P(B')}$$
$$= \frac{0.192}{0.909} = 0.211221$$

(47) Consider randomly selecting a student at a large university, and let A be the event that the selected student has a Visa-card and B be the analogous event for Master-card. Suppose that $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.25$. Calculate and interprete each of the following probabilities in venn diagram.

(a) $P(B|A)$ (b) $P(B'|A)$

(c) $P(A|B)$ (d) $P(A'|B)$

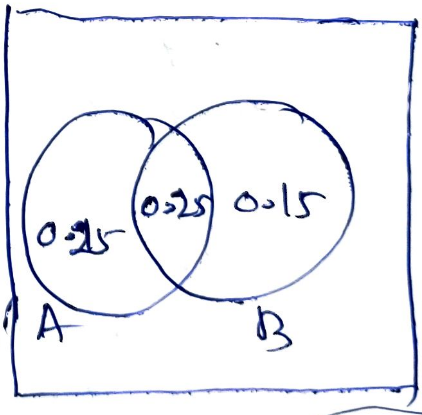
(e) Given that the selected individual has at least one card what is the probability that he or she has a Visa card

Sol)

$P(A) = 0.5$

$P(B) = 0.4$

$P(A \cap B) = 0.25$



(a) $P(B|A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{0.25}{0.5} = 0.5$

(b) $P(B'|A) = 1 - P(B|A) = 1 - 0.5 = 0.5$

(c) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.25}{0.4} = 0.6125$

(d) $P(A'|B) = 1 - P(A|B) = 1 - 0.6125 = 0.3875$

(e) $P(\text{Visa} | \text{visa or Master}) = P(A | A \cup B) = \dfrac{P(A \cap (A \cup B))}{P(A \cup B)} = \dfrac{P(A)}{P(A \cup B)}$

$= \dfrac{0.5}{0.65}$

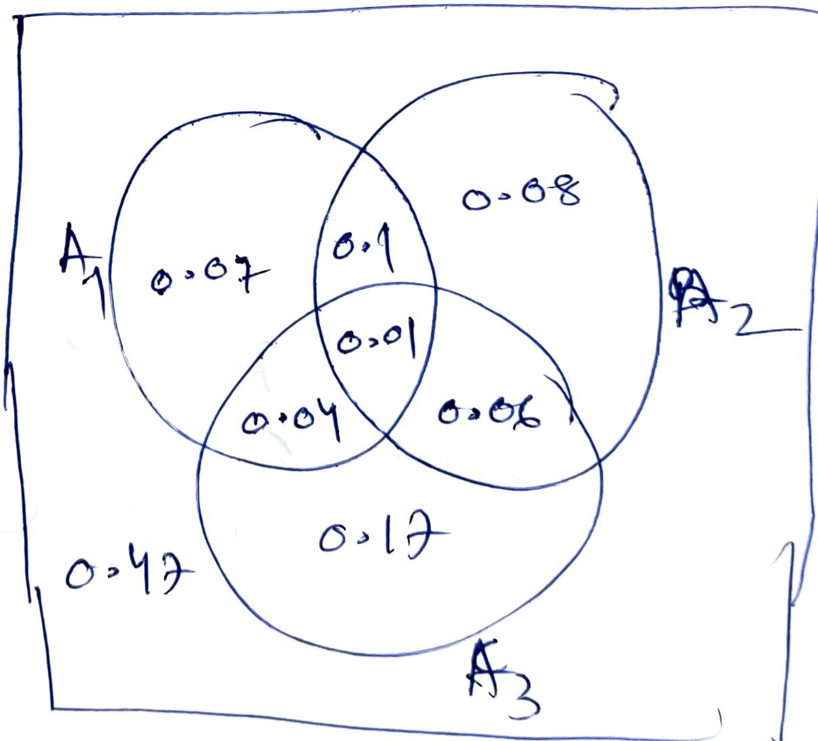
$= 0.7692$

Q(54) A computer consulting firm presently has bids out on three projects.
Let $A_i = \{\text{awarded project } i\}$ for $i = 1,2,3$
and suppose that $P(A_1) = 0.22$, $P(A_2) = 0.25$, $P(A_3) = 0.28$, $P(A_1 \cap A_2) = 0.11$, $P(A_1 \cap A_3) = 0.05$, $P(A_2 \cap A_3) = 0.07$, $P(A_1 \cap A_2 \cap A_3) = 0.01$.
Find the probabilities

(a) $P(A_2 | A_1)$ (b) $P(A_2 \cap A_3 | A_1)$

(c) $P(A_2 \cup A_3 | A_1)$ (d) $P(A_1 \cap A_2 \cap A_3 | A_1 \cup A_2 \cup A_3)$

Sol^n



(a) $P(A_2 | A_1)$

$= \dfrac{P(A_2 \cap A_1)}{P(A_1)}$

$= \dfrac{0.11}{0.22} = 0.5$

~~= 0.3929~~

(b) $P(A_2 \cap A_3 | A_1)$

$= \dfrac{P(A_2 \cap A_3 \cap A_1)}{P(A_1)} = \dfrac{0.01}{0.22} = 0.0455$

(c)

(c) $P(A_2 \cup A_3 | A_1) = \frac{P((A_2 \cup A_3) \cap A_1)}{P(A_1)}$

$= \frac{P[(A_1 \cap A_2) \cup (A_1 \cap A_3)]}{P(A_1)}$

$= \frac{P(A_1 \cap A_2) + P(A_1 \cap A_3) - P(A_1 \cap A_2 \cap A_3)}{P(A_1)}$

$= \frac{0.11 + 0.05 - 0.01}{0.22} = \frac{0.15}{0.22}$

$= 0.6818$

(d) $P[A_1 \cap A_2 \cap A_3 | A_1 \cup A_2 \cup A_3]$

$= \frac{P[(A_1 \cap A_2 \cap A_3) \cap (A_1 \cup A_2 \cup A_3)]}{P[A_1 \cup A_2 \cup A_3]}$

$= \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cup A_2 \cup A_3)} = \frac{0.01}{0.53} = 0.0189$

71. An oil exploration company currently has two active projects, one in Asia and the other in Europe. Let A be the event that the Assian project is successful and B be the event that the Europe project is successful. Suppose that A & B are independent events with $P(A) = 0.4$ & $P(B) = 0.7$

(a) If the Assian project is not successful, what is the probability that the European project is also not successful?

(b) What is the probability that at least one of the two project will be successful?

(c) Given that at least one of the two projects is successful, what is the probability that only Assian project is successful?

Sol^n

Given $A = \{$Assian project successful$\}$

$B = \{$European " " $\}$

$P(A) = 0.4, \; P(B) = 0.7$

Since A & B are independent, $P(A \cap B) = P(A)P(B)$

$= 0.4 \times 0.7$

$= 0.28$

(a) A & B are independent, so A' & B' are also independent,

P(European not successful | Assam not successful)

$= P(B' \mid A')$ $\boxed{A \,\&\, B \text{ are independent} \Rightarrow P(A \mid B) = P(A)}$

$= P(B') = 1 - P(B) = 1 - 0.7 = 0.3$

(b) prob. that at least one least one the ~~pos~~ two projects will be successful

$= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= 0.4 + 0.7 - 0.28$

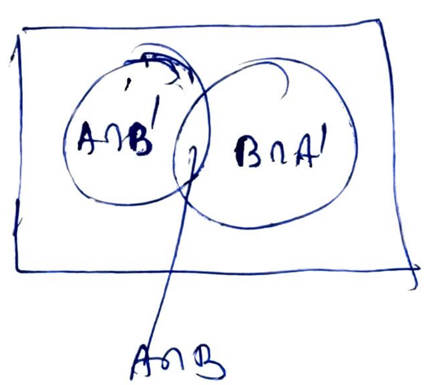
$= 0.82$

(c) P(only Assam | Assam or European)

$= \cancel{P(AB' \mid A \cup B)}$

$= P(A \cap B' \mid A \cup B)$

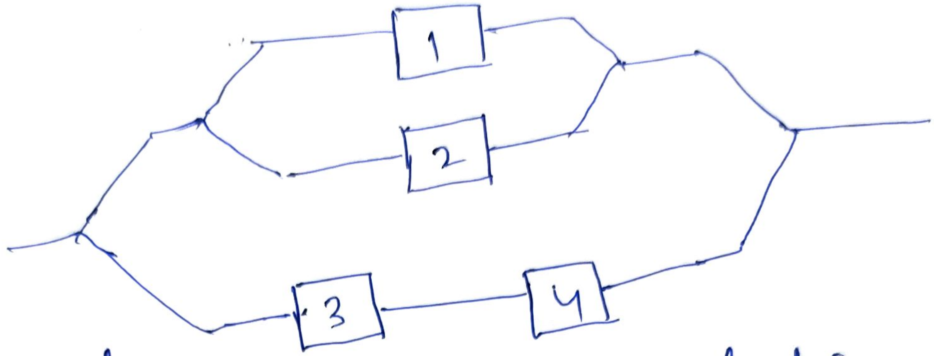
$$= \frac{P[(A \cap B') \cap (A \cup B)]}{P(A \cup B)}$$



$$= \frac{P(A \cap B')}{P(A \cup B)} = \frac{P(A) - P(A \cap B)}{P(A \cup B)} = \frac{0.4 - 0.28}{0.82} = \frac{0.12}{0.82}$$

$= 0.146$

80 Consider system of components connected as in the accompanying picture



Component 1 & 2 are connected in parallel, so that ~~the~~ subsystem works iff either 1 ~~or~~ 2 works; since 3 and 4 are connected in series that subsystem works iff both 3 & 4 work. If component work independently of one another and p(component works) = 0.9, calculate P(system works)

Sol^n Let ~~event~~ event A_i = component i works

then $P(A_i) = 0.9$ for $i = 1, 2, 3, 4$

$P(\text{system works}) = P[(A_1 \cup A_2) \cup (A_3 \cap A_4)]$

$= P[(A_1 \cup A_2 \cup A_3) \cap (A_1 \cup A_2 \cup A_4)]$

$= P(A_1 \cup A_2 \cup A_3) \cdot P(A_1 \cup A_2 \cup A_4)$

$= [P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)]$

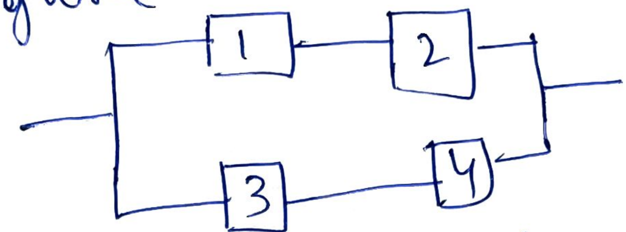
$[P(A_1) + P(A_2) + P(A_4) - P(A_1 \cap A_2) - P(A_1 \cap A_4) - P(A_2 \cap A_4) + P(A_1 \cap A_2 \cap A_4)]$

$\boxed{\begin{array}{l} P(A_i \cap A_j) \\ = P(A_i)P(A_j) \\ P(A_i \cap A_j \cap A_k) \\ = P(A_i)P(A_j)P(A_k) \end{array}}$

$= [3 \times 0.9 - 3 \times 0.9^2 + 0.9^3]^2 = 0.998001$

81) [Article: "Reliability evaluation of Solar Photovoltaic" (Solar enery, 2002, 129–141) presents various configurations of solar photovoltaic arrays consists of crystalline silicon solar cells]

Consider the system illustrated in figure



There are two ~~sy~~ subsystems connected in parallel, each one containing two cells. In order for the system to function, at least one of the two parallel ~~sys~~ subsystem must work. Within each subsystem, the two cells are connected in series, so the ~~sy~~ subsystem will work only if all cells in the subsystem work. Consider a particular life-time value t_0. Let A_i denote the event that the lifetime of cell i exceeds t_0 $(i=1,2,3,4)$. Assume that A_i's are independent events. If $P(A_i)=p$ ~~fo~~ $i=1,2,3,4$ and prob. that system liftime exceed ~~t_0~~ is 0.99. Find p.

Sol^n

Prob. that the system ~~works~~ lifetime exceed t_0

$= P[(A_1 \cap A_2) \cup (A_3 \cap A_4)]$

$= P(A_1 \cap A_2) + P(A_3 \cap A_4) - P(A_1 \cap A_2 \cap A_3 \cap A_4)$

$= P(A_1)P(A_2) + P(A_3)P(A_4) - P(A_1)P(A_2)P(A_3)P(A_4)$

$= p^2 + p^2 - p^4$ ($\because A_j$'s are independent)

$= 2p^2 - p^4$, since $P(A_j) = p$

Given that

~~P~~ P(system lifetime exceeds t_0) = 0.99

$\Rightarrow 2p^2 - p^4 = 0.99$

$\Rightarrow p^4 - 2p^2 + 0.99 = 0$

$\Rightarrow p^2 = \dfrac{2 \pm \sqrt{4 - 4 \times 0.99}}{2} = 0.9$ or 1.1

~~= 0.99 or 1.01~~

~~since~~ $\Rightarrow p = 0.9487$ or 1.049

Since $p \le 1$, we have $p = 0.9487$