Probability

Introduction

The term probability refers to the study of randomness and uncertainty. Theory of probability is the theory of uncertainty. In probability theory, the mathematical models of processes are set up which are effected by chance.

In mathematical statistics or statistics, these models are checked by observable reality. This is called ~~mathemat~~ statistical inference.

Samples: The statistical inference is done by sampling, that is, by drawing random samples or samples

Population: The set of ~~samples~~ values drawn from a much larger set of values that could be studied, called the population.

Example

From a lot of (population) 1000 screws, select 20 screws (samples). In facts samples are selected from population.

Independent sample values:

These samples will be obtained in experents with an infinite sample space. Sampling with replacement and without replacement has no difference in an infinite sample space. but in a finite sample space, it effects the dependence.

Ex- 5 screws from 1000 screws.

Relation between probability theory and statistical theory

Probability and statistics are related each other in many ways:

① The theory of probability forms the basis for the statistical inferences

② Probability is used to evaluate the stability of inferences made about the population when we have ~~evaluate~~ only sample ~~information~~ information.

③ If the population is known, then probability is used to describe the likelihood of observing a particular sample outcome.

Experiments, outcomes, Events

An experiment is a process of measurement or observation, in a laboratory, or in a factory, or on the street.

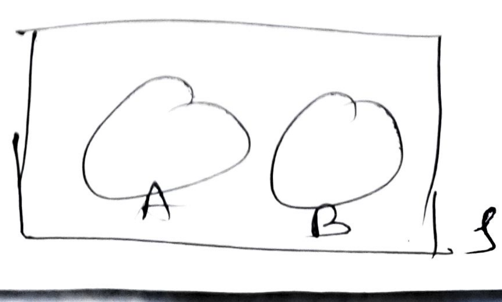
A trial is a single performance of an experiment.

The result of the trial is called outcome or a sample point.

The set of all possible outcomes is called sample space. The sample point is an element of the sample space.

Collection of some sample points obtained with a specific rule is event. Event is a subset of sample space.

Mutually exclusive event (disjoint): The events A & B of a sample space S are mutually exclusive if they have no common points, i.e., $A \cap B = \phi$



Equally likely events: for each event $A \subset S$, $P(A) = p$ (For a fair coin, $p = \frac{1}{2}$)

Complement Event: The event B is complement of A in the sample space S, if $A \cup B = S$ and $A \cap B = \phi$.

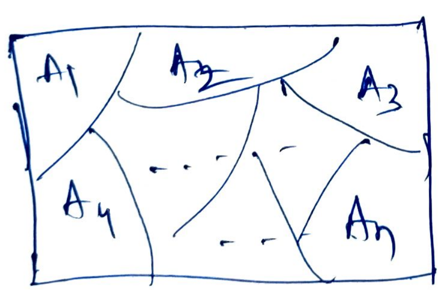
In particular, the complement of A is A' or $\bar{A}$ or A^c.

In this case, A & A' are called mutually exclusive & exhaustive.

In general the events $A_1, A_2 \cdots A_n$ of sample space S are mutually exclusive & exhaustive if

$$\bigcup_{j=1}^{n} A_j = S \quad \text{and} \quad A_i \cap A_j = \phi$$

for each i & j.



Properties of the events

① $(A \cup B)' = A' \cap B'$
② $(A \cap B)' = A' \cup B'$ } De Morgan's laws

③ $(A')' = A$ ④ $S' = \phi$ ⑤ $\phi' = S$

⑥ $A \subseteq B \Leftrightarrow A \cup B = B$ & $A \cap B = A$

Exhaustive events

The total no. of all possible elementary outcomes in a random experiment is called exhaustive event.

The event $A \subset S$ is exhaustive, when there is no other possible events ~~exists~~ at the same time.

Ex - A fair coin tossed once, the event of getting a head is exhaustive event.

Independent events:

Two event A & B are independent in a trial if the outcome of one event A does not effect the outcome of the event B & vice versa.

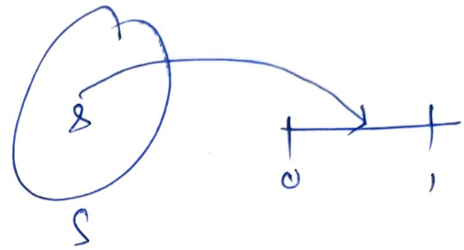
Ex - Two switch in a electric board are independent.

In general, $A_1, A_2 --- A_n$ are independent if ~~either~~ out of any event A_i does not effect the outcomes of the other events $A_1, A_2 -- A_{i-1}, A_{i+1}, -- A_n$ in the trial.

In mathematical language, probability (P) is considered as a mapping from the sample space to the set $[0,1]$.

i.e., $P : S \to [0,1]$

defined by



$$P(s) \in [0,1]$$

for $s \in S$

1) For any event $A \subseteq S$,

$$P(A) = \sum_{s \in A} P(s)$$

2) $P(S) = 1$

3) $P(\phi) = 0$

<u>Types of probability :</u>

→ classical Approach

→ frequency approach
or
relative frequency approach

→ Axiomatic approach

<u>classical approach</u>

for any event $A \subseteq S$,

$$P(A) = \frac{|A|}{|S|} \quad \text{where } P(s) = \frac{1}{|S|}$$

for each $s \in S$

$|A| \to$ no of elements

Proof

since for each $s \in S$,

$$P(s) = \frac{1}{|S|}$$

So

$$P(A) = \sum_{s \in A} P(s) = \sum_{s \in A} \frac{1}{|S|}$$

$$= \frac{|A|}{|S|}$$ proved

Properties

① $0 \le P(A) \le 1$, $A \subseteq S$

② $P(A') = 1 - P(A)$

③ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

④ If $A_1, A_2 \dots A_n$ are mutually exclusive, then

$$P\left(\bigcup_{j=1}^{n} A_j\right) = P(A_1 \cup A_2 \cup \dots \cup A_n)$$

$$= P(A_1) + P(A_2) + \dots + P(A_n)$$

$$= \sum_{j=1}^{n} P(A_j)$$

Proof ④

$$P(A_1 \cup A_2 \cup \dots \cup A_n) = P(A_1) + \dots + P(A_n) - \sum_{1 \le i < j \le n}\sum P(A_i \cap A_j)$$

$$+ \sum_{1 \le i < j < k \le n}\sum\sum P(A_i \cap A_j \cap A_k)$$

$$- + \dots + (-1)^{n-1} P(A_1 \cap \dots A_n)$$

Since $A_1, A_2, \ldots A_n$ are mutually exclusive & exhaustive

$P(A_i \cap A_j) = 0$ for each $i \neq j$

and so on.

Hence

$$P\left(\bigcup_{j=1}^{n} A_j\right) = P(A_1) + P(A_2) + \cdots + P(A_n)$$

$$= \sum_{j=1}^{n} P(A_j) \quad \text{proved}$$

Proof ①

Since $|\phi| = 0$, we have

$$P(\phi) = \frac{|\phi|}{|S|} = 0$$

$\phi \subseteq A \Rightarrow |\phi| \leq |A| \Rightarrow 0 \leq |A|$

$\Rightarrow \frac{0}{|S|} \leq \frac{|A|}{|S|}$

$\Rightarrow 0 \leq P(A)$

$A \subset S \Rightarrow |A| \leq |S|$

$\Rightarrow \frac{|A|}{|S|} \leq 1 \Rightarrow P(A) \leq 1$

Hence $0 \leq P(A) \leq 1$

Proof ② A & A' are mutually exclusive & exhaustive so $P(A') = 1 - P(A)$

as $P(A \cup A') = P(S) = 1 \Rightarrow 1 = P(A) + P(A') - P(A \cap A')$

$= P(A) + P(A')$

Statistical (or Empirical) approach

This approach is based on frequency and relative frequency. From n trials, the no. of times of an event A is the frequency of A or $f(A)$. Then the relative frequency for the event A is

$$f_{rel}(A) = \frac{f(A)}{n}$$

$$= \frac{\text{No. of times A occurs}}{\text{No. of trials}}$$

Properties

① If A did not occur, then

$$f(A) = 0 \Rightarrow f_{rel}(A) = 0$$

② If $A = S$, then $f_{rel}(A) = 1$

③ If A & B are mutually exclusive & exhaustive then $f_{rel}(A \cup B) = f_{rel}(A) + f_{rel}(B)$

④ For any mutually exclusive & exhaustive events $A_1, A_2 -- A_m$)

$$f_{rel}(A_1 \cup A_2 \cup -- A_m)$$

$$= \sum_{j=1}^{m} f_{rel}(A_j)$$

Axiomatic approach

This approach is based on axioms means the statements are reasonably true and are ~~~~ accepted withoutseeking any proof.

Ex According a farmer, the possibility of getting rain in a rainy day is 80%, i.e, 0.8
For a environment scientist it is 0.9 or 90%.

In these cases, we don't need any proof.