(12) Consider randomly selecting a student at a certain university, and let A denote the event that the selected individual has a Visa credit card and B be the analogous event for a Master-card. Suppose that

$P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.25$

(a) compute the prob. that the selected individual has at least one of two types of card.

(b) What is the prob. that the selected individual has neither type of card?

(c) Describe, in terms of A and B, the event that the selected student has a Visa-card but not a Master-card and then calculate the prob. of this event.

Sol^n Event A = Visa-card
B = Master-card

(a) The prob. that the selected individual has at least one of two types of cards A or B

$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.5 + 0.4 - 0.25 = 0.65$

(b) The prob. that the selected individual has neither type of card

$= P[(A \cup B)'] = 1 - P(A \cup B) = 1 - 0.65 = 0.35$

(c) $P(\text{Visacard but not Master card}) = P(A \cap B')$
$= P(A) - P(A \cap B) = 0.5 - 0.25 = 0.25$

(13) A computer consulting firm presently has bids out on three projects.
Let A_i = {awarded project i}, for $i = 1,2,3$ and suppose that $P(A_1) = 0.22$, $P(A_2) = 0.25$, $P(A_3) = 0.28$, $P(A_1 \cap A_2) = 0.11$, $P(A_1 \cap A_3) = 0.05$, $P(A_2 \cap A_3) = 0.07$, $P(A_1 \cap A_2 \cap A_3) = 0.01$.

Express in words each of the following events and compute the probability of each event

(a) $A_1 \cup A_2$ (b) $A_1' \cap A_2'$ (c) $A_1 \cup A_2 \cup A_3$
(d) $A_1' \cap A_2' \cap A_3'$ (e) $A_1' \cap A_2' \cap A_3$
(f) $(A_1' \cap A_2') \cup A_3$

Solⁿ (a) $A_1 \cup A_2$ = awarded project 1 or 2 or both

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$$
$$= 0.22 + 0.25 - 0.11 = 0.36$$

(b) $A_1' \cap A_2'$ = awarded project neither 1 nor 2

$$P(A_1' \cap A_2') = P(A_1 \cup A_2)' = 1 - P(A_1 \cup A_2)$$
$$= 0.64$$

(c) $A_1 \cup A_2 \cup A_3$ = awarded project at least 1 or 2 or 3

$$P(A \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$$
$$= 0.22 + 0.25 + 0.28 - 0.11 - 0.05 - 0.07 + 0.01$$
$$= 0.53$$

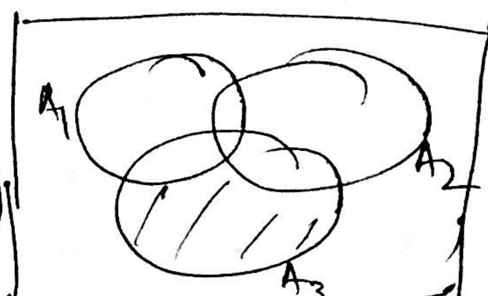
(d) $A_1' \cap A_2' \cap A_3'$ = awarded none of the three projects 1, 2 & 3

$$P(A_1' \cap A_2' \cap A_3') = P(A_1 \cup A_2 \cup A_3)'$$
$$= 1 - P(A_1 \cup A_2 \cup A_3)$$
$$= 1 - 0.53 = 0.47$$

(e) $A_1' \cap A_2' \cap A_3$ = Awarded project 3 but neither 1 nor 2.

We have

$$A_1' \cap A_2' \cap A_3$$
$$= A_3 - [(A_1 \cap A_3) \cup (A_2 \cap A_3)]$$

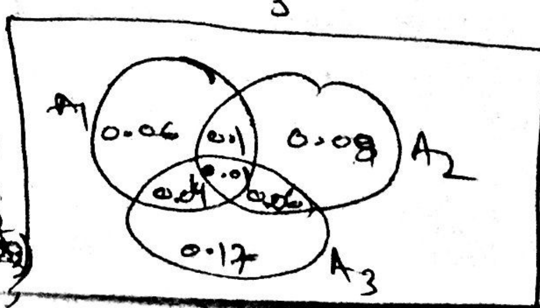


$(A_1 \cap A_3) \cup (A_2 \cap A_3)$ is subset of A_3, we have

$$P(A_1' \cap A_2' \cap A_3) = P(A_3) - P[(A_1 \cap A_3) \cup (A_2 \cap A_3)]$$
$$= P(A_3) - [P(A_1 \cap A_3) + P(A_2 \cap A_3) - P(A_1 \cap A_2 \cap A_3)]$$
$$= P(A_3) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$$
$$= 0.28 - 0.05 - 0.07 + 0.01 = 0.17$$

(f) $(A_1' \cap A_2') \cup A_3$ = Awarded either (neither 1 nor 2) or 3

$$P[(A_1' \cap A_2') \cup A_2]$$
$$= P(A_1' \cap A_2') + P(A_3) - P(A_1' \cap A_2' \cap A_3)$$
$$= 0.64 + 0.28 - 0.17 = 0.75$$



(15) The three most popular options on a certain type of new car ~~are~~ are built-in GPS (A), a sunroof (B) and an ~~auto~~ automatic transmission (C). If 40% of all purchasers request A, 55% request B, 70% request C, 63% request A or B, 77% request A or C, 80% requests B or C, 85% request A or B or C, determine the probabilities of the following events

(a) The next purchaser will request at least one of the three options

(b) The next purchaser will select none of the three options.

(c) The next purchaser will request only an automatic transmission and not either of the other two options

(d) The next purchaser will select exactly one of these three options.

Sol^n Given $A \to$ GPS, $B \to$ Sunroof, $C =$ Automatic

$P(A) = 0.4$, $P(B) = 0.55$, $P(C) = 0.7$

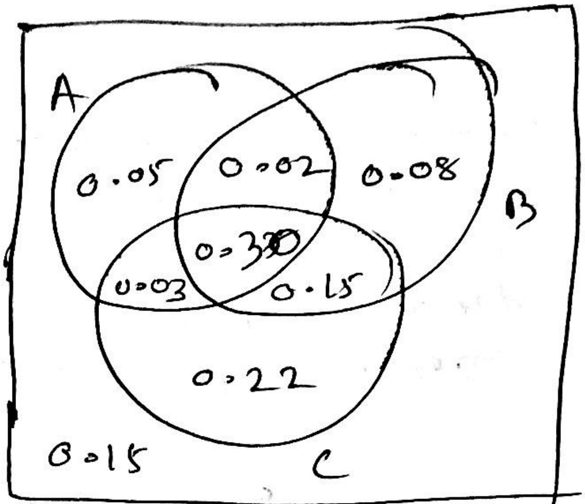
$P(A \cup B) = 0.63$, $P(A \cup C) = 0.77$

$P(B \cup C) = 0.8$, $P(A \cup B \cup C) = 0.85$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.4 + 0.55 - 0.63$

$= 0.32$



$P(A \cap C) = P(A) + P(B) - P(A \cap C)$

$= 0.4 + 0.7 - 0.77$

$= 0.33$

$P(B \cap C) = P(B) + P(C) - P(B \cup C)$

$= 0.55 + 0.70 - 0.8 = 0.45$

(a) P(at least one of three options A, B, C selected)

$= P(A \cup B \cup C) = 0.85$

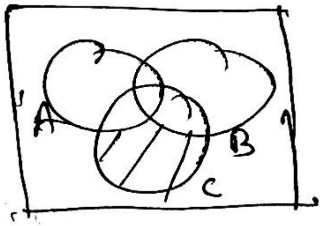
(b) P(none of A, B, C is selected) $= P(A \cup B \cup C)'$

$= 1 - P(A \cup B \cup C) = 0.15$

(c) P(Automatic and neither of GPS & sunroof)

$= P(C \cap A' \cap B')$

$= P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$



Since $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$, we get

$P(A \cap B \cap C) = P(A \cup B \cup C) - P(A) - P(B) - P(C) + P(A \cap B) + P(A \cap C) + P(B \cap C)$

$= 0.85 - 0.4 - 0.55 - 0.7 + 0.32 + 0.33 + 0.45 = 0.30$

Hence $P(C \cap A' \cap B') = 0.70 - 0.33 - 0.45 + 0.30 = 0.22$

(d) P[select exactly one of these three options]

$p = P[(A \cap B' \cap C') \cup (B \cap A' \cap C') \cup (C \cap A' \cap B')]$

where $= P(A \cap B' \cap C') + P(B \cap A' \cap C') + P(C \cap A' \cap B')$

$P(A \cap B' \cap C') = P(A) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$

$= 0.4 - 0.32 - 0.33 + 0.3$

$= 0.05$

$P(B \cap A' \cap C') = P(B) - P(B \cap A) - P(B \cap C) + P(B \cap A \cap C)$

$= 0.55 - 0.32 - 0.45 + 0.3$

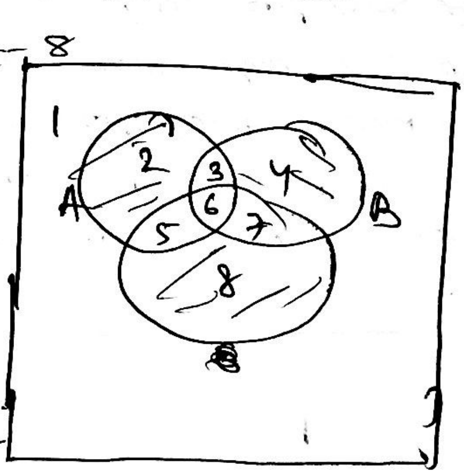
$= 0.08$

$P(C \cap A' \cap B') = 0.22$

Hence the soln is

$p = 0.05 + 0.08 + 0.22 = 0.35$

Alternative ; ~~Region~~ event i, i = 1, 2 ... 8



$P(A \cup B \cup C) = 0.85$,

$P(1) = 0.15$

$P(6) = 0.3$, $P(3) = P(A \cap B) - P(6) = 0.02$

$P(5) = P(A \cap C) - P(6) = 0.03$

$P(2) = P(A) - P(3) - P(6) - P(5) = 0.05$

$P(7) = P(B \cap C) - P(6) = 0.15$, $P(4) = P(B) - P(3) - P(6) - P(7) = 0.08$

$P(8) = P(C) - P(5) - P(6) - P(7) = 0.03$

(16) An individual is presented with three different glasses of cola, labeled C, D & P. He is asked to taste all three and then list them in order of preference. Suppose the same cola has actually been put into the three glasses.

(a) What are the sample events in this ranking experiment, and what probability would you assign to each one?

(b) What is the probability that C is ranked first?

(c) What is the probability that C is ranked first and D is ranked last.

Soln (a) Sample space $= \{CDP, CPD, DCP, DPC, PCD, PDC\}$

prob. of each outcome $= \frac{1}{6}$

(b) P (C ranked first)

$= P(\{CDP, CPD\}) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

(c) P (C ranked first & D last)

$= P(\{CPD\}) = \frac{1}{6}$